Economics G6712 Part 2

Monetary Theory and Policy

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Homework 2

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The Transmission of Business Cycles From the U.S. to Canada: The Role of Asset Markets

In this problem set, we explore the transmission of productivity shocks from the United States to Canada via complete asset markets.

Consider a two-country model composed of a large economy (The U.S.) and a small economy (Canada). Assume a limit case in which the U.S. is so large relative to Canada that it can be modeled as a closed economy. At the same time, assume that Canada is so small that it takes the U.S. business cycle as exogenously determined. Assume the existence of complete asset markets.

U.S. consumer have preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta_u^t \frac{\left[c_t^u (1-h_t^u)^{\theta_u}\right]^{1-\sigma_u} - 1}{1-\sigma_u}$$

where c_t^u denotes consumption and h_t^u denotes hours worked, and $\beta_u \in (0,1)$, $\theta_u > 0$, and $\sigma_u > 0$, are parameters. U.S. output is produced with the technology

$$y_t^u = a_t^u (k_t^u)^{\alpha_u} (h_t^u)^{1-\alpha_u},$$

where y_t^u denotes output, a_t^u denotes an exogenous and stochastic productivity shock, and k_t^u denotes physical capital. The evolution of capital is given by

$$k_{t+1}^u = (1-\delta_u) k_t^u + i_t^u,$$

where i_t^u denotes gross investment in the U.S. in period t. The productivity shock follows an autoregressive process of order one of the form

$$\ln(a_t^u/a_u) = \rho_u \ln(a_{t-1}^u/a_u) + \epsilon_t^u,$$

where ϵ_t^u is a white noise with mean zero and variance $\sigma_{\epsilon^u}^2$, and $a_u > 0$ and $\rho_u \in (0,1)$ are parameters.

Calibrate the U.S. economy as follows: $a_u = 2.1$, $\rho_u = 0.8$, $\sigma_{\epsilon^u} = 0.02$, $\alpha_u = 0.42$, $\delta_u = 0.1$, $\sigma_u = 1$, $\beta_u = 1/1.04$, and $h^u = 0.2$. Here, h^u denotes the steady-state level of hours worked.

1. Find the implied value of θ_u. Also, report the implied steady-state value of consumption per capita in the U.S.

In modeling the Canadian economy, mimic the small open RBC economy model with complete asset markets studied in class and analyzed in detail in chapter 4 of the lecture notes, with three modifications. First, impose that the productivity factor A_t is constant and normalized to 1. Second, because we are now considering a two-country model, you must appropriately replace the assumption of a constant parameter ψ_4. Third, assume that in the deterministic steady state Canada and the United States enjoy the same level of consumption per capita. Use the functional forms and parameter values (except for the steady-state level of consumption per capita) as in the complete-asset-market example of chapter 4 of the lecture notes. Report the steady-state value of the trade balance to output ratio.

2. Using a first-order approximation to the equilibrium conditions of this two-country model, produce a table displaying the Canadian unconditional standard deviation, serial correlation, and correlation with output of output, consumption,investment, hours, and the trade-balance-to-output ratio.

3. Produce a figure with 6 plots depicting the impulse responses to a one-percent innovation in U.S. productivity of Canadian output, consumption, investment, hours, the trade-balance-to-output ratio, and the marginal utility of wealth (λ_t).
4. Now replace the values of ρ_u and σ_{ϵ^u} given above with values chosen to match the serial correlation and standard deviation of detrended output in the U.S. Compute these two statistics by removing a linear trend from the log of annual U.S. output per capita. Attached to this problem set, you will find a .mat file containing a time series of U.S. real GDP per capita at annual rate over the period 1955-2006. Use this data to compute the desired statistics. Answer questions 2 and 3 using the resulting parameterization.
5. Based on your answer to the previous question, evaluate the ability of the two-country RBC model of the small open economy with complete asset markets to explain business cycles in Canada. Compare the performance of this model with that of the model driven by domestic (i.e., Canadian) productivity shocks analyzed in chapter 4 of the lecture notes.