Department of Economics
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Economics G6220

Advanced Macroeconomic Analysis

Problem Set 3

Due February 20

1. **Anticipated Endowment Shocks**

   Consider a small open endowment economy enjoying free capital mobility. Preferences are described by the utility function

   $$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t(c_t-\bar{c})^2,$$

   with $\beta\in(0,1)$. Agents have access to an internationally traded bond paying the constant interest rate r^*, satisfying $\beta(1+r^*)=1$. The representative household starts period zero with an asset position b_{-1}. Each period $t\geq 0$, the household receives an endowment y_t, which obeys the law of motion, $y_t=\rho y_{t-1}+\epsilon_{t-1}$, where ϵ_t is an i.i.d shock with mean zero and standard deviation σ_ϵ. Notice that households know already in period $t-1$ the level of y_t with certainty.

   (a) Derive the equilibrium process of consumption and the current account.

   (b) Compute the correlation between the current account and output. Compare your result with the standard case in which y_t is known only in period t.

2. **An Economy with Labor** Consider a small open economy populated by a large number of households with preferences described by the utility function

   $$E_0\sum_{t=0}^{\infty}\beta^t U(c_t,h_t),$$

   where U is a period utility function given by

   $$U(c,h)=-\frac{1}{2}\left[(c-\bar{c})^2+h^2\right],$$

   where $\bar{c}>0$ is a satiation point. The household's budget constraint is given by

   $$d_t=(1+r)d_{t-1}+c_t-y_t,$$

   where d_t denotes real debt acquired in period t and due in period $t+1$, $r>0$ denotes the world interest rate. To avoid inessential dynamics, we impose

   $$\beta(1+r)=1.$$

   The variable y_t denotes output, which is assumed to be produced by the linear technology

   $$y_t=Ah_t.$$

   Households are also subject to the no-Ponzi-Game constraint $\lim_{j\to\infty}E_t d_{t+j}/(1+r)^j\leq 0$.

(a) Compute the equilibrium laws of motion of consumption, debt, the trade balance, and the current account.

(b) Assume that in period 0, unexpectedly, the productivity parameter A increases permanently to $A' > A$. Establish the effect of this shock on output, consumption, the trade balance, the current account, and the stock of debt.

3. **Anticipated Productivity Shocks**

Consider a perfect-foresight economy populated by a large number of identical households with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t),$$

where c_t denotes consumption, U is a period utility function assumed to be strictly increasing, strictly concave, and twice continuously differentiable, and $\beta \in (0, 1)$ is a parameter denoting the subjective rate of discount. Households are subject to the following four constraints

$$b_t = (1 + r)b_{t-1} + y_t - c_t - i_t,$$

$$y_t = \theta_t F(k_t)$$

$$k_{t+1} = k_t + i_t,$$

and

$$\lim_{j \to \infty} \frac{b_{t+j}}{(1 + r)^j} \geq 0,$$

given b_{-1}, k_0, and $\{\theta_t\}_{t=0}^{\infty}$. The variable b_t denotes period-t purchases of an internationally traded bond that matures in period $t + 1$ and pays the constant interest rate r, y_t denotes output, k_t denotes the (predetermined) stock of physical capital in period t, and i_t denotes gross investment., F is a production function assumed to be strictly increasing, strictly concave, and to satisfy the Inada conditions, and $\theta_t > 0$ is an exogenous productivity factor. Suppose that $\beta(1 + r) = 1$. Assume further that up until period -1 inclusive, the productivity factor was constant and equal to $\bar{\theta} > 0$. and that the economy was in a steady state with a constant level of capital and a constant net asset position. Suppose further that in period 0 the productivity factor also equals $\bar{\theta}$, but that agents learn that in period 1 it will jump permanently to $\theta' > \bar{\theta}$. That is, in period 0, households know that the path of the productivity factor is given by

$$\theta_t = \begin{cases} \bar{\theta} & t \leq -1 \\ \bar{\theta} & t = 0 \\ \theta' > \bar{\theta} & t \geq 1 \end{cases}$$

(a) Characterize the equilibrium paths of output, consumption, investment, capital, the net foreign asset position, the trade balance, and the current account.

(b) Compare your answer to the case of an unanticipated permanent increase in productivity studied in class.

(c) Now assume that the anticipated productivity shock is transitory. Specifically, assume that the information available to households at $t = 0$ is

$$\theta_t = \begin{cases} \bar{\theta} & t \leq -1 \\ \bar{\theta} & t = 0 \\ \theta' > \bar{\theta} & t = 1 \\ \bar{\theta} & t \geq 2 \end{cases}$$

i. Compare your answer to the case of an anticipated permanent increase in productivity studied in class.

ii. Compare your answer to the case of an anticipated endowment shocks in the endowment economy studied in question 1.