Department of Economics
Columbia University

Professor Martín Uribe
Spring 2013

Economics G6220
**Advanced Macroeconomic Analysis**
**Problem Set 4**
Due March 6

1. **A Small Open Economy with an AR(2) TFP Process**

   In this question you are asked to show that the SOE RBC model can predict consumption to be more volatile than output when the productivity shock follows a second-order autoregressive process displaying a hump-shaped impulse response. The theoretical model to be used is the External Debt-Elastic Interest Rate (EDEIR) model presented in chapter 4 of Uribe's *Open Economy Macroeconomics* textbook. Replace the AR(1) process with the following AR(2) specification:

   $$\ln A_{t+1} = 1.42 \ln A_t - 0.43 \ln A_{t-1} + \sigma_\epsilon \epsilon_{t+1}.$$

   Otherwise use the same calibration and functional forms as presented in the textbook. Download the matlab files for the EDEIR model from Uribe's webpage located at `http://www.columbia.edu/~mu2166/closing.htm` . Then modify them to accommodate the present specification.

   (a) Produce a table displaying the unconditional standard deviation, contemporaneous correlation with output, and serial correlation of output, consumption, investment, hours, the trade-balance-to-output ratio, and the current-account-to-output ratio.

   (b) Produce a $3 \times 2$ figure displaying the impulse responses of output, consumption, investment, hours, the trade-balance-to-output ratio, and TFP to a unit innovation in TFP.

   (c) Compare and contrast the predictions of the model under the AR(1) and the AR(2) TFP processes. Provide intuition.

2. **A Small Open Economy With Durable Consumption**

   Consider an economy populated by a large number of identical households with preferences described by the lifetime utility function

   $$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(c_t^n - \frac{h_t^\omega}{\omega}\right) s_t^\gamma\right]^{1-\sigma} - 1}{1-\sigma},$$

   where $c_t^n$ denotes consumption of nondurable goods, $h_t$ denotes hours worked, and $s_t$ denotes the stock of durable consumption goods. The parameter $\beta \in (0,1)$ denotes the subjective discount factor, $\gamma, (\omega - 1), (\sigma - 1) > 0$ are preference parameters, and $E_t$ denotes the expectations operator conditional on information available in period $t$.

The law of motion of the stock of durables is assumed to be of the form

$$s_t = (1-\delta)s_{t-1} + c_t^d,$$

where $c_t^d$ denotes durable consumption in period $t$, and $\delta \in (0,1)$ denotes the depreciation rate. The sequential budget constraint of the household is given by

$$d_t = (1+r_{t-1})d_{t-1} + c_t^n + c_t^d + \frac{\phi^d}{2}(s_t - s_{t-1})^2 + i_t + \frac{\phi^k}{2}(k_{t+1} - k_t)^2 - A_t k_t^\alpha h_t^{1-\alpha},$$

where $d_t$ denotes debt acquired in period $t$ and maturing in period $t+1$, $r_t$ denotes the interest rate on assets held between periods $t$ and $t+1$, $i_t$ denotes gross investment, $k_t$ denotes the stock of physical capital, and $A_t$ represents a technology factor assumed to be exogenous and stochastic. The parameters $\phi^d, \phi^k > 0$ govern the degree of adjustment costs in the accumulation of durable consumption goods and physical capital, respectively. The parameter $\alpha$ resides in the interval $(0,1)$. The capital stock evolves over time according to the law of motion

$$k_{t+1} = (1-\delta)k_t + i_t.$$

Note that we assume that physical capital, $k_t$, is predetermined in period $t$ and that investment, $i_t$, takes one period to become productive capital. By contrast, the stock of consumer durables, $s_t$ is non-predetermined in period $t$, and expenditures in consumer durables in period $t$, $c_t^d$, become productive immediately. Finally, assume that the interest rate is debt elastic,

$$r_t = r^* + \psi\left[e^{\tilde{d}_t - \bar{d}} - 1\right],$$

where $\tilde{d}_t$ denotes the cross-sectional average level of debt per capita, and $r^*$, $\bar{d}$, and $\psi$ are parameters. The productivity factor $A_t$ evolves according to the expression

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1},$$

where $\epsilon_t$ is a white noise with mean zero and variance $\sigma_\epsilon^2$, and $\rho \in (0,1)$ is a parameter. Assume that $\beta(1+r^*) = 1$.

(a) Derive the complete set of equilibrium conditions.

(b) Derive the deterministic steady state. Specifically, find analytical expressions for the steady state values of $c_t^n$, $h_t$, $s_t$, $k_{t+1}$, $d_t$, $r_t$, $i_t$, $tb_t$, and $ca_t$ in terms of the structural parameters of the model $\sigma$, $\beta$, $\delta$, $\omega$, $\alpha$, $\gamma$, $r^*$, and $\bar{d}$. Here, $tb_t$ and $ca_t$ denote, respectively, the trade balance and the current account.

(c) Assume the following parameter values: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\alpha = 0.3$, and $\omega = 1.455$. Calibrate $\bar{d}$ and $\gamma$ so that in the steady state the debt to output ratio is 25 percent and the nondurable consumption to output ratio is 68 percent. Report the implied numerical values of $\gamma$, $\bar{d}$. Also, display the the steady states of $r_t$, $d_t$, $h_t$, $k_t$, $c_t^n$, $s_t$, $c_t^d$, $i_t$, $tb_t$, $ca_t$, and $y_t \equiv A_t k_t^\alpha h_t^{1-\alpha}$.

(d) Approximate the equilibrium dynamics using a first-order perturbation technique. In performing this approximation, express all variables in logs, except for the stock of debt, the interest rate, the trade balance, the current account, the trade-balance-to-output ratio, and the current-account-to-output ratio. You are asked to complete the calibration of the model by setting values for $\psi$, $\phi^d$, $\phi^k$, $\rho$, and $\sigma_\epsilon$ to target key empirical regularities of medium-size emerging countries documented in chapter 1 of Uribe's *Open Economy Macroeconomics* textbook. Specifically, the targets are a standard deviation of output, $\sigma_y$, of 8.99 percent, a relative standard deviation of consumption, $\sigma_c/\sigma_y$, of 0.93, a relative standard deviation of gross investment, $\sigma_i/\sigma_y$, of 2.86, a serial correlation of output of 0.84, and a correlation between the trade-balance-to-output ratio and output of -0.24. In general, you will not be able to hit these targets exactly. Instead, you are required to define a distance between the targets and their corresponding theoretical counterparts and devise a numerical algorithm to minimize it. Define the distance as follows. Let $z(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon) \equiv x(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon) - x^*$, where $x^*$ is the 5x1 vector of empirical targets (the 5 numbers given above) and $x(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon)$ is the 5x1 vector of theoretical counterparts as a function of the parameters. Let $D(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon) \equiv \sqrt{z(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon)' z(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon)}$ be the distance between the target and its theoretical counterpart. Report (a) the values of $\psi$, $\phi^d$, $\phi^k$, $\rho$, and $\sigma_\epsilon$ that you find and (b) complete the following table:

| | Data | Prediction of the Model |
|---|---|---|
| $\sigma_y$ | 8.99 | |
| $\sigma_c/\sigma_y$ | 0.93 | |
| $\sigma_i/\sigma_y$ | 2.86 | |
| $\text{corr}(y_t, y_{t-1})$ | 0.84 | |
| $\text{corr}(tb_t/y_t, y_t)$ | -0.24 | |

(e) Produce a table displaying the model predictions. The table should contain the unconditional standard deviation, serial correlation, and correlation with output of output, consumption, investment, consumption of durables, consumption of nondurables, the trade-balance-to-output ratio, and the current-account-to-output ratio. For consumption, consumption of durables, consumption of nondurables, and investment report the standard deviation relative to output. Discuss how well the model is able to explain actual observed second moments that were not targeted in the calibration. Again use table 1.2 of Uribe's textbook as the source of the observed second moments of medium-size emerging countries.