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Economics G6220
Advanced Macroeconomic Analysis
Problem Set 5
Due March 27

A Model of the U.S.-Canada Business Cycle

Consider a world with two economies, Canada and the United States, index by $i = Can, US$, respectively. Suppose that both economies are populated by a large number of identical households with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^i - \frac{(h_t^i)^\omega}{\omega}\right]^{1-\sigma} - 1}{1-\sigma},$$

where c_t^i and h_t^i denote, respectively, consumption and hours worked in country i in period t. In both countries, households operate a technology that produces output, denoted y_t^i, using labor and capital, denoted k_t^i. The production technology is Cobb-Douglas and given by

$$y_t^i = A_t^i (k_t^i)^\alpha (h_t^i)^{1-\alpha},$$

where A_t^i denotes a productivity shock in country i, which evolves according to the following AR(1) process:

$$\ln A_{t+1}^i = \rho^i \ln A_t^i + \sigma^i \epsilon_{t+1}^i,$$

where ϵ_t^i is an i.i.d. innovation with mean zero and variance equal to one, and ρ^i and σ^i are country-specific parameters. Both countries produce the same good. The evolution of capital obeys the following law of motion:

$$k_{t+1}^i = k_t^i + \frac{1}{\phi^i}\left[\left(\frac{i_t^i}{\delta k_t^i}\right)^{\phi^i} - 1\right]\delta k_t^i,$$

where i_t^i denotes investment in country i, and ϕ^i is a country-specific parameter.

Assume that asset markets are complete and that there exists free mobility of goods and financial assets between the United States and Canada, but that labor and installed capital are immobile across countries. Finally, assume that Canada has measure zero relative to the United States, so that the latter can be modeled as a closed economy.

The following two tables display observed standard deviations, serial correlations, and correlations with output for Canada and the United States over the period 1960-2011. The source is World Development Indicators. The data are annual and in per capita terms. The series y, c, and i are in logs, and the series tb/y is in levels. All series were quadratically detrended. Standard deviations are measured in percentage points.

Variable	Canadian Data 1960-2011		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	3.71	0.86	1.00
c	2.19	0.70	0.62
i	10.31	0.69	0.80
tb/y	1.72	0.76	0.12

Variable	U.S. Data 1960-2011		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.94	0.75	1.00
c	3.00	0.82	0.90
i	10.36	0.67	0.80
tb/y	0.94	0.79	-0.51

1. Calibrate the model as follows: Assume that the deterministic steady-state levels of consumption per capita are the same in Canada and the United States. Set $\beta = 1/1.04$, $\sigma = 2$, $\omega = 1.455$, $\alpha = 0.32$, and $\delta = 0.10$. Set the remaining six parameters, ρ^i, σ^i, and ϕ^i, for $i = Can, US$, to match the observed standard deviations and serial correlations of output and the standard deviations of investment in Canada and the United States. Use a distance minimization procedure as in exercise 2 of Problem Set 4.

2. Approximate the equilibrium dynamics up to first order. Produce the theoretical counterparts of the two tables shown above.

3. Comment on the ability of the model to explain observed business cycles in Canada and the United States.

4. Plot the response of Canadian output, consumption, investment, hours, and the trade-balance-to-output ratio to a unit innovation in the Canadian productivity shock. On the same plot, show the response of the Canadian variables to a unit innovation to the U.S. productivity shock. Discuss the differences in the responses to a domestic and foreign technology shock and provide intuition.

5. Compare, by means of a graph and a discussion, the predicted responses of Canada and the United States to a unit innovation in the U.S. productivity shock. The graph should include the same variables as the one for the previous item.

6. Compute the fraction of the volatilities of Canadian output and trade-balance-to-output ratio explained by the U.S. productivity shock according to the present model. To this end, set $\sigma^{Can} = 0$ and compute the two standard deviations of interest. Then, take the ratio of these standard deviations to their respective counterparts when both shocks are active. Relate your results to those obtained by Lan and Wataru in their EF colloquium presentation.

7. This question aims to quantify the importance of common shocks as drivers of the U.S.-Canada business cycle. Replace the process for the Canadian productivity shock with the folloiwng one

$$\ln A_{t+1}^{Can} = \rho^{Can} \ln A_t^{Can} + \sigma^{Can} \epsilon_{t+1}^{Can} + \nu \epsilon_{t+1}^{US}.$$

All other aspects of the model are as before. Recalibrate the model using an augmented version of the strategy described above that includes an additional parameter, ν, and an additional target, the cross-country correlation of output, which in the sample used here is 0.64. Report the new set of calibrated parameters. Compute the variance of Canadian output. Now set $\nu = 0$ and recalculate the variance of Canadian output. Explain.