The Small-Open-Economy Real Business Cycle Model

Some Empirical Regularities

Variable	Canadian Data		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1
c	2.5	0.7	0.59
i	9.8	0.31	0.64
h	2	0.54	0.8
$\frac{tb}{y}$	1.9	0.66	-0.13

Source: Mendoza (AER, 1991)

Comments

- Volatility ranking: $\sigma_{tb/y} < \sigma_c < \sigma_y < \sigma_i$.
- Consumption, investment, and hours are procyclical.
- The trade-balance-to-output ratios is countercyclical.
- All variables considered are positively serially correlated.
- Similar stylized facts emerge from other small developed countries (see, e.g., Aguiar and Gopinath, JPE, 2006).

An RBC Model with Uzawa Preferences

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t),$$

$$\theta_0 = 1,$$

$$\theta_{t+1} = \beta(c_t, h_t)\theta_t \qquad t \geq 0,$$

The Sequential Budget Constraint

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t),$$

with $\Phi(0) = \Phi'(0) = 0$.

Technology

$$y_t = A_t F(k_t, h_t),$$

Evolution of the Capital Stock

$$k_{t+1} = i_t + (1 - \delta)k_t,$$

No-Ponzi-Game Constraint

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\prod_{s=1}^{j}(1 + r_s)} \leq 0.$$

Optimality Conditions

Define $\tilde{U}(c_t, h_t, \eta_t) = U(c_t, h_t) - \eta_t \beta(c_t, h_t)$.

$$\tilde{U}_c(c_t, h_t, \eta_t) = \lambda_t$$

$$-\tilde{U}_h(c_t, h_t, \eta_t) = \lambda_t A_t F_h(k_t, h_t)$$

$$\lambda_t = \beta(c_t, h_t)(1 + r_t) E_t \lambda_{t+1}$$

$$\begin{aligned} \lambda_t[1 + \Phi'_t] &= \beta(c_t, h_t) E_t \lambda_{t+1} [A_{t+1} F_k(k_{t+1}, h_{t+1}) \\ &+ 1 - \delta + \Phi'_{t+1}] \end{aligned}$$

$$\eta_t = -E_t U(c_{t+1}, h_{t+1}) + E_t \eta_{t+1} \beta(c_{t+1}, h_{t+1})$$

Interpreting the multiplier η_t

$$\eta_t = -E_t \sum_{j=1}^{\infty} \left(\frac{\theta_{t+j}}{\theta_{t+1}} \right) U(c_{t+j}, h_{t+j})$$

$\Rightarrow \eta_t$ is next period's lifetime utility.

Evolution of Total Factor Productivity

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1};$$

$$\epsilon_{t+1} \sim NIID(0, \sigma_\epsilon^2); \quad t \geq 0.$$

Free Capital Mobility

$$r_t = r,$$

where r is the world interest rate, assumed to be constant.

Functional Forms

Period Utility Function

$$U(c,h) = \frac{\left[c - \omega^{-1} h^{\omega}\right]^{1-\gamma} - 1}{1-\gamma}$$

Subjective Discount Factor

$$\beta(c,h) = \left[1 + c - \omega^{-1} h^{\omega}\right]^{-\psi_1}$$

Production Function

$$F(k,h) = k^{\alpha} h^{1-\alpha}$$

Adjustment Cost Function

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0.$$

Calibration

γ	ω	ψ_1	α	ϕ	r	δ	ρ	σ_ϵ
2	1.455	.11	.32	0.028	0.04	0.1	0.42	0.0129

Calibration Strategy

ψ_1: Match Canadian trade balance-to-output ratio

ϕ: Match Canadian investment volatility

ρ: Match Canadian Output serial correlation

σ_ϵ: Match Canadian output volatility

Empirical and Theoretical Second Moments

Variable	Canadian Data			Model		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1	3.1	0.61	1
c	2.5	0.7	0.59	2.3	0.7	0.94
i	9.8	0.31	0.64	9.1	0.07	0.66
h	2	0.54	0.8	2.1	0.61	1
$\frac{tb}{y}$	1.9	0.66	-0.13	1.5	0.33	-0.012
$\frac{ca}{y}$				1.5	0.3	0.026

Comments

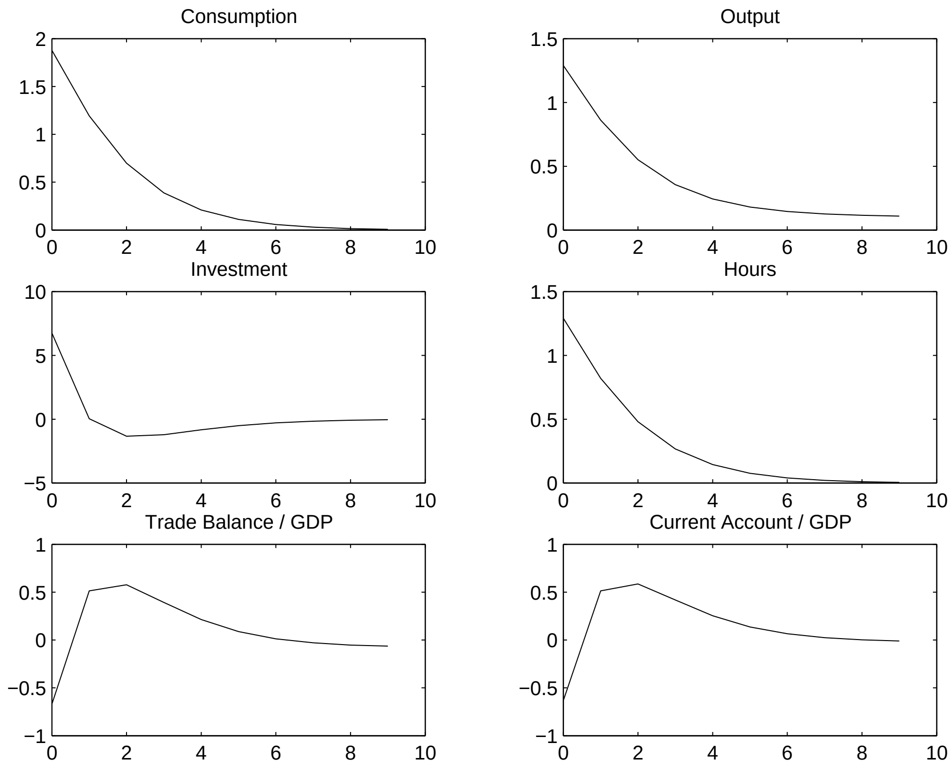
- Parameters ϕ, σ_ϵ, and ρ picked to match σ_i, σ_y, and ρ_{yy}. So no real test here.

- The model matches the volatility ranking $\sigma_c < \sigma_y < \sigma_i$.

- Empirical and theoretical trade-balance-to-output ratios are countercyclical.

- The model overestimates the correlations of hours and consumption with output.

Response to a Positive Technology Shock



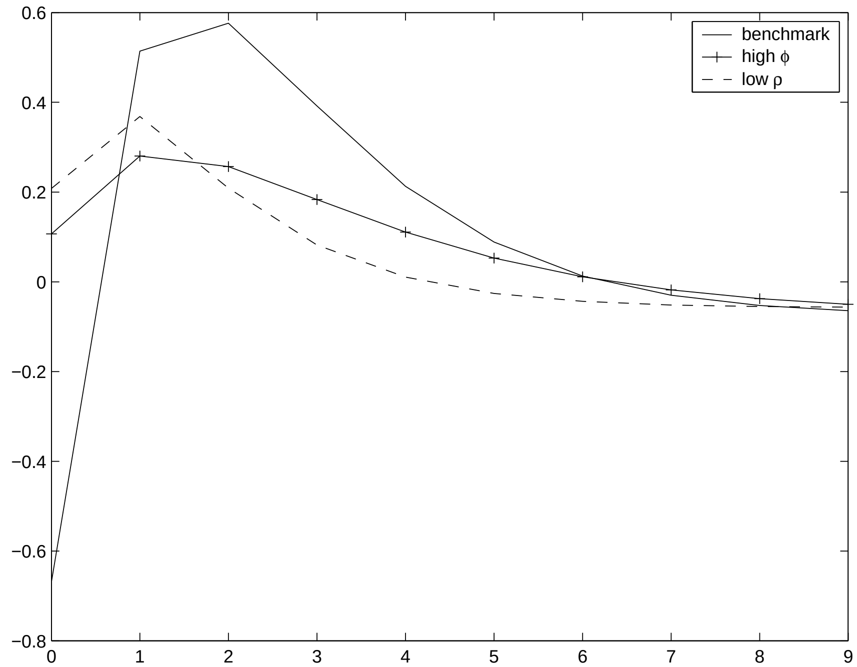
Source: Schmitt-Grohé and Uribe (JIE, 2003)

Comments:

- Output, consumption, investment, and hours expand.

- The trade balance deteriorates.

Adjustment Costs, Persistence of Shocks, and the Trade Balance-To-Output Ratio



Comment

- The more persistent the shock, the more countercyclical the response of the trade balance.

- The weaker the cost of adjusting capital, the more countercyclical the response of the trade-balance-to-output ratio.

Endogenous Discount Factor Without Internalization

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t \qquad t \geq 0,$$

$$\theta_0 = 1,$$

where $\tilde{c}_t$ and $\tilde{h}_t$ denote per capita consumption and hours worked.

$$\lambda_t = \beta(\tilde{c}_t, \tilde{h}_t)(1 + r_t)E_t\lambda_{t+1}$$

$$\lambda_t = U_c(c_t, h_t)$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t)$$

$$\begin{aligned} \lambda_t[1 + \Phi'_t] &= \beta(\tilde{c}_t, \tilde{h}_t)E_t\lambda_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) \\ &+ 1 - \delta + \Phi'_{t+1}] \end{aligned}$$

In Equilibrium

$$c_t = \tilde{c}_t \text{ and } h_t = \tilde{h}_t$$

Debt-Elastic Interest Rate (external)

$$r_t = r + p(\tilde{d}_t),$$

$$\theta_t = \beta^t,$$

$$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1}$$

$$U_c(c_t, h_t) = \lambda_t,$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t).$$

$$\begin{aligned} \lambda_t[1 + \Phi'_t] &= \beta E_t \lambda_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) \\ &+ 1 - \delta + \Phi'_{t+1}] \end{aligned}$$

$$\tilde{d}_t = d_t.$$

Functional Form for Country Spread

$$p(d) = \psi_2 \left(e^{d - \bar{d}} - 1 \right),$$

Calibration

β	$\bar{d}$	ψ_2	r
0.96	0.7442	0.000742	$\beta^{-1} - 1$

Internal Debt-Elastic Interest Rate

$$r_t = r + p(d_t),$$

The Euler equation becomes

$$\lambda_t = \beta[1 + r + p(d_t) + p'(d_t)d_t]E_t\lambda_{t+1}$$

$$p(d) = \psi_2 \left(e^{d-\bar{d}} - 1 \right),$$

Calibration: Same as in the external case. Note that the steady-state value of debt is no longer equal to $\bar{d}$. Instead, d solves

$$(1 + d)e^{d-\bar{d}} = 1 \Rightarrow d = 0.4045212.$$

Portfolio Adjustment Costs

$$d_t = (1+r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t) + \frac{\psi_3}{2}(d_t - \bar{d})^2$$

$$\lambda_t[1 - \psi_3(d_t - \bar{d})] = \beta(1 + r_t)E_t\lambda_{t+1}$$

Calibration

β	$\bar{d}$	ψ_3	r
0.96	0.7442	0.00074	$\beta^{-1} - 1$

Complete Asset Markets

$$E_t r_{t+1} b_{t+1} = b_t + y_t - c_t - i_t - \Phi(k_{t+1} - k_t),$$

$$\lim_{j \to \infty} E_t q_{t+j} b_{t+j} \geq 0,$$

$$q_t = r_1 r_2 \ldots r_t,$$

$$\lambda_t r_{t+1} = \beta \lambda_{t+1}.$$

$$\lambda_t^* r_{t+1} = \beta \lambda_{t+1}^*.$$

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*}.$$

$$\lambda_t = \xi \lambda_t^*,$$

$$\lambda_t = \psi_4,$$

Calibration: Set ψ_4 so that steady-state consumption equals steady-state consumption in the model with Uzawa preferences.

Calibrations

Debt-Elastic Interest Rate (internal and external)

β	$\bar{d}$	ψ_2	r
0.96	0.7442	0.000742	$\beta^{-1}-1$

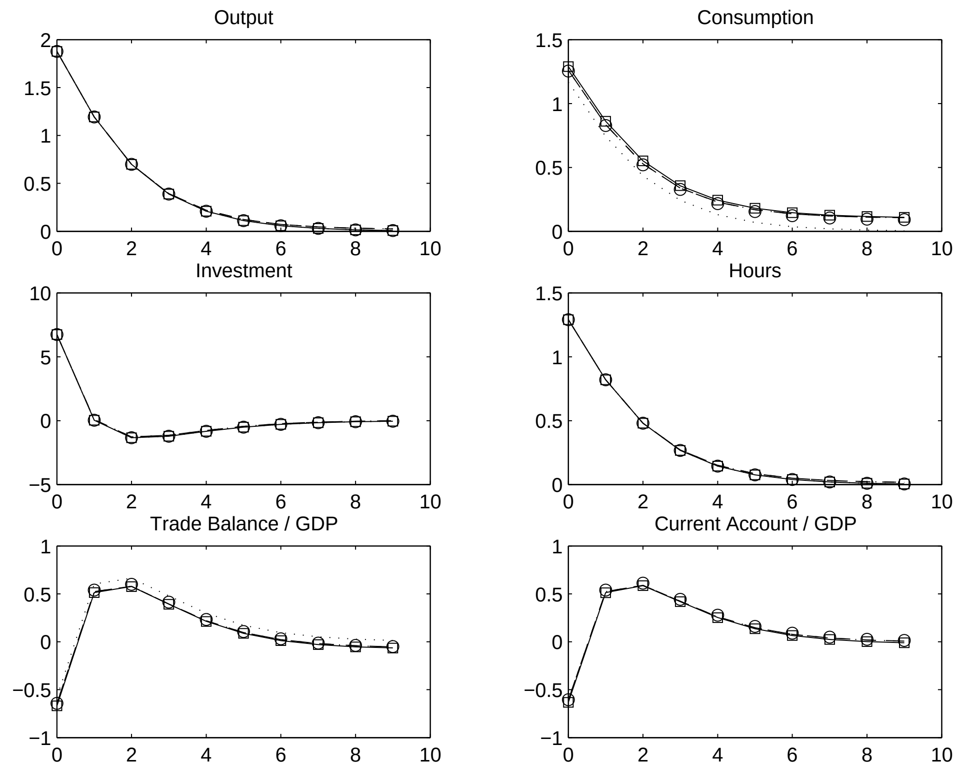
Portfolio Adjustment Costs

β	$\bar{d}$	ψ_3	r
0.96	0.7442	0.00074	$\beta^{-1}-1$

Complete Asset Markets

Set ψ_4 so that steady-state consumption equals steady-state consumption in the model with Uzawa preferences.

Impulse Response to a Unit Technology Shock in Models 1 Through 5



Source: Schmitt-Grohé and Uribe (JIE, 2003)

Note. Solid line, endogenous discount factor. Squares, endogenous discount factor without internalization. Dashed line, Debt-elastic interest rate. Dash-dotted line, Portfolio adjustment cost. Dotted line, complete asset markets. Circles, No stationarity inducing elements.

Observed and Implied Second Moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
Standard Deviations						
y	2.8	3.1	3.1	3.1	3.1	3.1
c	2.5	2.3	2.3	2.7	2.7	1.9
i	9.8	9.1	9.1	9	9	9.1
h	2	2.1	2.1	2.1	2.1	2.1
tb/y	1.9	1.5	1.5	1.8	1.8	1.6
ca/y		1.5	1.5	1.5	1.5	
Serial Correlations						
y	0.61	0.61	0.61	0.62	0.62	0.61
c	0.7	0.7	0.7	0.78	0.78	0.61
i	0.31	0.07	0.07	0.069	0.069	0.07
h	0.54	0.61	0.61	0.62	0.62	0.61
tb/y	0.66	0.33	0.32	0.51	0.5	0.39
ca/y		0.3	0.3	0.32	0.32	
Correlations with Output						
c	0.59	0.94	0.94	0.84	0.85	1
i	0.64	0.66	0.66	0.67	0.67	0.66
h	0.8	1	1	1	1	1
tb/y	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
ca/y		0.026	0.025	0.05	0.051	

Source: Schmitt-Grohé and Uribe (JIE, 2003)

Note. Standard deviations are measured in percent per year.