Slides for Chapter 3

An Intertemporal Theory of the Current Account

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February 10, 2020
Motivation

- Build a model of an open economy to study the determinants of the trade balance and the current account.

- Study the response of the trade balance and the current account to a variety of economic shocks
  - Changes in current income
  - Changes in future income

- Pay special attention to how those responses depend on whether the shocks are perceived to be temporary or permanent.
A Small Open Economy

What does ‘small’ and ‘open’ mean in this context?

- An economy is small when world prices and interest rates are independent of domestic economic conditions.
- An economy is open when it trades in goods and financial assets with the rest of the world.
- Most countries in the world are small open economies:
  - Examples of developed small open economies: the Netherlands, Switzerland, Austria, New Zealand, Australia, Canada, Norway.
  - Examples of emerging small open economies: Chile, Peru, Bolivia, Greece, Portugal, Estonia, Latvia, Thailand.
  - Examples of large open economies: United States, Japan, Germany, and the United Kingdom.
  - Examples of large emerging economies: China, India
  - Examples of closed economies: Perhaps the most notable cases are North Korea, Venezuela, and to a lesser extent Cuba and Iran.
- Economic and geographic size not necessarily related: Australia and Canada vs. UK and Japan.
The Model Economy

• A two-period small open economy: periods 1 and 2.

• Households receive endowments $Q_1$ and $Q_2$ in periods 1 and 2, respectively.

• Initial asset holdings $B_0^*$ inherited from the past, paying the interest rate $r_0$ in period 1.

• In period 1, households choose consumption, $C_1$, and bond holdings, $B_1^*$, which pay the interest rate $r_1$ in period 2.
Sequential Budget Constraints

The period-1 budget constraint

\[ C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1. \]  

The period-2 budget constraint

\[ C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2, \]  

Because the world ends after period 2, no one is going to be around to pay or collect debts. So bond holdings must be nil at the end of period 2, that is,

\[ B_2^* = 0. \]  

This expression is known as the transversality condition.
3.1 The Intertemporal Budget Constraint

Combine (1), (2), and (3) to eliminate $B_1^*$ and $B_2^*$. This yields

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}. \quad (4)$$

This is the intertemporal budget constraint. It says that the present discounted value of the endowment plus the initial financial wealth (the right-hand side) must be enough to pay for the present discounted value of consumption (the left-hand side).

The following figure provides a graphical representation of the intertemporal budget constraint.
Note. The figure is drawn under the assumption that the initial net foreign asset position is zero, $B_0^* = 0$. 
Properties of the Intertemporal Budget Constraint

• It’s downward sloping. Its slope is \(-(1 + r_1)\), because if you sacrifice one unit of consumption today and put it in the bank for one period, you get \(1 + r_1\) units next period.

• The set of feasible consumption paths \((C_1, C_2)\) are those inside or at the borders of the triangle formed by the vertical axis, the horizontal axis, and the intertemporal budget constraint. Points A, B, C, and D are all feasible consumption paths.

• Point D violates the trasversality condition, because households leave money on the table at the end of period 2 \((B_2 > 0)\).

• Points outside that triangle, such as point E, are infeasible. They violate the transversality condition because households leave unpaid debts at the end of period 2 \((B_2^* < 0)\).

• What feasible point the household will choose depends on its preferences. We turn to this issue next.
3.2 The Lifetime Utility Function
We assume that the household’s happiness increases with the consumption of goods in periods 1 and 2. Preferences for consumption in periods 1 and 2 are described by the lifetime utility function, which is assumed to be of the form

\[ U(C_1) + \beta U(C_2), \]

where \( U(\cdot) \) denotes the period utility function and is assumed to be increasing and concave. The parameter \( \beta > 0 \) denotes the subjective discount factor. Typically it is assumed that \( \beta \leq 1 \), which means that in period 1 households care less about period-2 consumption than about period-1 consumption.
**Indifference Curves**

An indifference curve is the set of consumption baskets \((C_1, C_2)\) that delivers the same level of welfare. The following figure displays examples of indifference curves.
Properties of Indifference Curves

- If $C_1$ and $C_2$ are goods (i.e., objects for which more is preferred to less), indifference curves are downward sloping.
- An indifference curve located northeast of another one yields higher utility.
- One (and only one) indifference curve is associated with each point in the positive quadrant crosses; they densely populate it.
- Indifference curves do not cross one another.
- Indifference curves that we focus on are convex. If you are consuming a lot in period 1 and almost nothing in period 2, you are not willing to give up lots of period-2 consumption for an additional unit of period-1 consumption. But if you are consuming very little in period 1 and a lot in period 2, you are willing to give up a lot of period-2 consumption for an additional unit of period-1 consumption. This property of preferences is known as diminishing marginal rate of substitution of $C_2$ for $C_1$. 
3.3 The Optimal Intertemporal Allocation of Consumption

- The household chooses consumption in periods 1 and 2 to maximize its utility function, subject to its intertemporal budget constraint (4).
- The next slide provides a graphical representation of how the optimal consumption path is determined. For simplicity, the graph is drawn assuming zero initial assets, $B_0^* = 0$.
- The endowment point $(Q_1, Q_2)$ is point A.
- The optimal consumption path is point $B$. This point is on the intertemporal budget constraint and belongs to an indifference curve that is tangent to the intertemporal budget constraint.
- The graph is drawn so that at point $B$, the household consumes more than its endowment. This means that it must borrow in period 1. In period 2, the household consumes less than his endowment, and uses the difference to pay back its debt including interest.
- In general, the optimal level of period-1 consumption does not need to be higher than the period-1 endowment. Whether period-1 consumption is higher, equal, or lower than period-1 endowment depends on preferences, present and future endowments, initial wealth, and the interest rate.
The Optimal Consumption Path, $(C_1, C_2)$
Deriving the Optimal Consumption Path

Formally, the household problem is

$$\max_{\{C_1, C_2\}} U(C_1) + \beta U(C_2)$$

subject to

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}.$$  \hspace{1cm} (4)

The household takes as given all objects on the right-hand side of the intertemporal budget constraint. Therefore, to save notation, let’s call the right-hand side $\bar{Y}$:

$$\bar{Y} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}.$$ 

Solve the intertemporal budget constraint for $C_2$ to get

$$C_2 = (1 + r_1)(\bar{Y} - C_1).$$  \hspace{1cm} (5)

Use this expression to eliminate $C_2$ from the lifetime utility function.
Deriving the Optimal Consumption Path (Continued)

The household maximization problem then becomes

$$\max_{\{C_1\}} U(C_1) + \beta U((1 + r_1)(\bar{Y} - C_1))$$

To maximize this expression, take the derivative with respect to $C_1$, equate it to zero, and rearrange:

$$U'(C_1) = \beta(1 + r_1)U'(C_2)$$

This optimality condition is known as the consumption Euler equation.

Rearrange terms to write the Euler equation as

$$-\frac{U'(C_1)}{\beta U'(C_2)} = -(1 + r_1)$$

which says that at the optimal consumption path, the (negative of the) marginal rate of substitution is equal to the (negative of the) gross interest rate, or graphically the slope of the indifference curve is equal to the slope of the intertemporal budget constraint.
3.4 The Interest Rate Parity Condition

We assume that there is free international capital mobility. That is, households can borrow and lend in the international financial market. Let $r^*$ be the world interest rate. Then, free capital mobility guarantees that the domestic interest rate be equal to the world interest rate. That is,

$$r_1 = r^*.$$

We will refer to this condition as the interest rate parity condition. Any difference between $r_1$ and $r^*$ would give rise to an arbitrage opportunity that would allow someone to make infinite profits. For instance, if $r_1 > r^*$, then one could make infinite amounts of profits by borrowing in the international market and lending in the domestic market. Similarly, if $r_1 < r^*$, unbounded profits could be obtained by borrowing domestically and lending abroad. These arbitrage opportunities disappear when $r_1 = r^*$. 
3.5 Equilibrium in the small open economy
An equilibrium then is a consumption path \((C_1, C_2)\) and an interest rate \(r_1\) that satisfy the country’s intertemporal resource constraint, the consumption Euler equation, and the interest rate parity condition, that is,

\[
C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1},
\]

(6)

\[
U'(C_1) = (1 + r_1)\beta U'(C_2),
\]

(7)

and

\[
r_1 = r^*,
\]

(8)

given the exogenous variables \(r_0, B_0^*, Q_1, Q_2,\) and \(r^*.\)
Graphical Representation of Equilibrium

Slide 19 is a graphical representation of the three equilibrium conditions shown in slide 17 that determine $C_1$, $C_2$, and $r_1$.

• The Equilibrium is at point B.

• Point B is on the Intertemporal resource constraint, as required by equilibrium condition (6).

• The indifference curve that crosses point B is tangent to the Intertemporal budget constraint, whose slope is $-(1 + r_1)$. This means that equilibrium condition (7) holds.

• And the slope of the intertemporal resource constraint is $-(1 + r^*)$, which means that $r^* = r_1$, as stated in equilibrium condition (8).
Equilibrium in the Small Open Economy

\[ \text{slope} = -(1 + r^*) \]
3.6 The Trade Balance and the Current Account

We can now answer the question posed at the beginning: What determines the trade balance and the current account? The trade balance is the difference between output and consumption,

\[ TB_1 = Q_1 - C_1 \]

\[ TB_2 = Q_2 - C_2. \]

The current account equals the trade balance plus investment income

\[ CA_1 = TB_1 + r_0 B_0^* \]

\[ CA_2 = TB_2 + r^* B_1^* \]
3.7 Adjustment to Temporary and Permanent Output Shocks
3.7.1 Adjustment to temporary output shocks
Assume that output in period 1 falls from $Q_1$ to $Q_1 - \Delta < Q_1$ and output in period 2, $Q_2$, is unchanged.

A is the endowment point before the shock $(Q_1, Q_2)$
A' the endowment point after the shock $(Q_1 - \Delta, Q_2)$

The temporary decline in output shifts the intertemporal budget constraint down and to the left.
How will the household adjust to the shock? Assume that both $C_1$ and $C_2$ are normal goods (i.e., goods whose consumption increases with income)

Adjustment to a temporary decline in output

\[
Q_1 - \Delta \quad Q_1 \quad C_1' \quad C_1
\]

B is the optimal pre-shock consumption path
B' is the optimal post-shock consumption path
In smoothing consumption over time, the country runs a larger trade deficit in period 1 (recall that it was running a trade deficit even in the absence of the shock) and finances it by acquiring additional foreign debt. Thus, the current account deteriorates. In period 2, the country must generate a larger trade surplus than the one it would have produced in the absence of the shock in order to pay back the additional debt acquired in period 1.

The important principle to take away from this example is that temporary negative income shocks are smoothed out by borrowing from the rest of the world rather than by fully adjusting current consumption by the size of the shock.
3.7.2 Adjustment to Permanent Output Shocks
Suppose $Q_1$ and $Q_2$ both fall by $\Delta$.

$A'$ is the new endowment point $(Q_1 - \Delta, Q_2 - \Delta)$.

In general the decline in consumption should be expected to be close to $\Delta$, implying that a permanent output shock has little consequences for the trade balance and the current account.
Comparing the effects of temporary and permanent output shocks on the current account, the following general principle emerges:

Economies tend to finance temporary shocks (by borrowing or lending on international capital markets) and adjust to permanent ones (by varying consumption in both periods up or down).

Thus, temporary shocks tend to produce large movements in the current account while permanent shocks tend to leave the current account largely unchanged.
3.8 Anticipated Income Shocks

The figure depicts the adjustment to an anticipated increase in $Q_2$ equal to $\Delta > 0$. The intertemporal budget constraint shifts up by $\Delta$. The increase in the period-2 endowment causes an increase in period-1 consumption from $C_1$ to $C'_1$. Because the endowment in period 1 is unchanged, the period-1 trade balance and current account deteriorate.
An economy with logarithmic preferences

Assume that preferences are logarithmic

\[ U(C_1) + \beta U(C_2) = \ln C_1 + \ln C_2 \]

Intertemporal budget constraint

\[ C_1 + \frac{C_2}{1 + r_1} = \bar{Y}, \]

Solving the intertemporal budget constraint for \( C_2 \) and using the result to eliminate \( C_2 \) from the lifetime utility function,

household’s optimization problem reduces to choosing \( C_1 \) to maximize

\[ \ln(C_1) + \ln((1 + r_1)(\bar{Y} - C_1)). \]

The first-order condition associated with this problem is

\[ \frac{1}{C_1} - \frac{1}{\bar{Y} - C_1} = 0. \]
Solving for $C_1$ yields

$$C_1 = \frac{1}{2} \bar{Y}.$$  \hspace{1cm} (9)

This result says that households find it optimal to consume half of their lifetime wealth in the first half of their lives. Combining (5) and (9) yields

$$C_2 = \frac{1}{2} \bar{Y} (1 + r_1).$$  \hspace{1cm} (10)

This is also intuitive. The household consumes half of $\bar{Y}$ in period 1 and puts the other half in the bank, receiving $\frac{1}{2} \bar{Y} (1 + r_1)$ for consumption in period 2.
Deriving the Optimal Consumption Path (Continued)

Now recall that \( \bar{Y} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1} \) to write (9), as

\[
C_1 = \frac{1}{2} \left[ (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1} \right]
\]

According to this expression, consumption is increasing in \( Q_1 \), \( Q_2 \), and \( (1 + r_0)B_0^* \), and decreasing in the interest rate \( r_1 \).

The response \( C_1 \) to an increase in \( Q_1 \) depends crucially on what households expect to happen with future consumption. If the increase in \( Q_1 \) is temporary, so that \( Q_2 \) is not expected to change, then consumption increase by \( 1/2 \) the change in output. Households leave half for future consumption. But if the increase in \( Q_1 \) is expected to be associated with an equal increase in \( Q_2 \), households consume most of the output increase (a fraction \( (1+r_1/2)/(1+r_1) \)). In this case there is no need to leave much for tomorrow, because output will also be high next period.

We do not observe \( Q_2 \) in period 1, so the reaction of \( C_1 \), which we do observe, allows us to infer in period 1 whether the change in \( Q_1 \) is expected to be temporary or permanent.
Effect of a Temporary Output Shock on the Current Account
Suppose that output increases in period 1, but is expected not to change in period 2. That is, assume that

$$\Delta Q_1 > 0 \text{ and } \Delta Q_2 = 0$$

Recall that

$$CA_1 = TB_1 + r_0B_0^* = Q_1 - C_1 + r_0B^*0$$

Differentiating this expression, we have

$$\Delta CA_1 = \Delta Q_1 - \Delta C_1 = \left(1 - \frac{1}{2}\right) \Delta Q_1 = \frac{1}{2} \Delta Q_1$$

The current account improves by half the increase in output. Households know the output increase is temporary. So, because they like to smooth consumption over time, they save half of it for consumption next period.
Effect of a Permanent Output Shock on the Current Account

Suppose that output increases by the same amount in both periods 1 and 2. That is, assume that

$$\Delta Q_1 = \Delta Q_2 > 0$$

Then, differentiating the expression for the current account obtained in the previous slide, we have

$$\Delta CA_1 = \frac{1}{2} \left[ \Delta Q_1 - \frac{\Delta Q_2}{1 + r^*} \right]$$

Since $\Delta Q_1 = \Delta Q_2$, we can write

$$\Delta CA_1 = \frac{1}{2} \frac{r^*}{1 + r^*} \Delta Q_1$$

The increase in the current account is now only a fraction $\frac{1}{2} \frac{r^*}{1 + r^*}$ of the change in output, much smaller than in the case of a temporary shock. This makes sense: Why save a large part of the output increase if output is also expected to increase next period?
Intuition

If you lose your lunch money one day, it’s not a big problem. You simply borrow from a friend. Next time, you pay his lunch. However, if you father cuts your monthly allowance, you will have to make plans to reduce your spending accordingly. We have seen that a similar principle is at work with the current account. We summarize this principle as follows:

A General Principle

*Finance temporary output shocks (by running current account deficits or surpluses without much change in spending) and adjust to permanent output shocks (by changing spending, without much change in the current account).*
An Anticipated Increase in Future Output

Suppose $Q_2$ increases by 1, while $Q_1$ is unchanged.

By equations (??)-(??), $Q_1$ increases by $\frac{1}{2(1+r^*)}$ and $TB_1$ and $CA_1$ both deteriorate by $\frac{1}{2(1+r^*)}$.

**Intuition:** The increase in $Q_2$ makes households richer, inducing them to increase $C_1$ and $C_2$. With $Q_1$ unchanged, the increase in $C_1$ causes a fall in $TB_1$, which must be financed by external borrowing, that is, by a fall in $CA_1$. Thus, good news about the future causes a deterioration of the current account. This shows that current account deficits are not necessarily an indication of a weak economy.
Summing Up

This chapter presents an intertemporal model of the current account with 3 building blocks:

- Households face an intertemporal budget constraint.
- Households have preferences over present and future consumption. They choose a consumption path that maximizes lifetime utility subject to the intertemporal budget constraint.
- Free capital mobility equalizes the domestic and world interest rates.
- The model delivers the following key insight: In response to temporary income shocks, countries use the current account to smooth consumption over time. Positive temporary shocks cause an improvement in the current account and negative temporary shocks cause a deterioration. In response to permanent income shocks, countries adjust consumption without much movement in the current account.
- In response to an anticipated increase in future income, the trade balance and the current account deteriorate.