Slides for Chapter 9: The Real Exchange Rate and Purchasing Power Parity

Columbia University

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Introduction

You might have noticed that sometimes Europe seems much cheaper than the United States and sometimes it is the other way around. The real exchange rate measures how expensive a foreign country is relative to the home country. The real exchange rate tracks the evolution over time of the price of a basket of goods abroad in terms of baskets of goods at home. When prices expressed in the same currency are equalized at home and abroad, we say that purchasing power parity holds.

Two important empirical questions:
• how large and persistent are deviations from purchasing power parity?
• what factors determine deviations from purchasing power parity?

This chapter is devoted to studying these and other related questions.
The Law of One Price
When a good costs the same abroad and at home, we say that the \textit{law of one price} (LOOP) holds.

Formally, the LOOP holds for a given good if

\[ P = \varepsilon P^*, \]

where

\( P \) = domestic-currency price of a particular good in the domestic country,

\( P^* \) = foreign-currency price of the same good in the foreign country, and

\( \varepsilon \) = the nominal exchange rate (domestic-currency price of one unit of foreign currency).
Should the LOOP hold?

In a frictionless world, yes. If a can of coke costs 2 dollars in the US and 5 dollars in Guatemala, you could become infinitely rich buying cans of coke in the US and selling them in Guatemala.

Reasons why the LOOP may not hold: International transportation costs, distribution costs (loading and unloading, domestic transportation, storage, advertising, and retail services).

Type of goods for which the LOOP holds pretty well: commodities (e.g., gold, oil, soy beans, wheat), luxury consumer goods (e.g., Rolex watches, Hermes neckties, and Montblanc fountain pens).

Type of goods for which the LOOP doesn’t hold well: personal services (e.g., health care, education, restaurant meals, domestic services, and personal care, such as haircuts), housing, transportation, and utilities.
The Big Mac and the LOOP

Does the LOOP hold for Big Macs? This is an interesting case because: (1) Big Macs are produced more or less the same way all over the world; (2) prices are readily available. (3) it’s a popular good.

The Big Mac real exchange rate: measures how many U.S. Big Macs it takes to buy one Big Mac abroad. Formally,

$$e_{\text{BigMac}} = \frac{\varepsilon P_{\text{BigMac}*}}{P_{\text{BigMac}}}.$$

where

- $e_{\text{BigMac}}$ = Big Mac real exchange rate.
- $P_{\text{BigMac}}$ = dollar price of a Big Mac in the United States.
- $P_{\text{BigMac}*}$ = foreign-currency price of a Big Mac in a foreign country.

LOOP holds for the Big Mac when $e_{\text{BigMac}} = 1$. 
## The Big-Mac Real Exchange Rate, January 2019

<table>
<thead>
<tr>
<th>Country</th>
<th>$P_{\text{BigMac}}$</th>
<th>$\epsilon$</th>
<th>$\epsilon P_{\text{BigMac}}$</th>
<th>$e_{\text{BigMac}}$</th>
<th>$\epsilon_{\text{BigMac}}$</th>
<th>$PPP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>6.50</td>
<td>1.02</td>
<td>6.62</td>
<td>1.19</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>5.58</td>
<td>1</td>
<td>5.58</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>6.77</td>
<td>0.75</td>
<td>5.08</td>
<td>0.91</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>Euro area</td>
<td>4.05</td>
<td>1.15</td>
<td>4.64</td>
<td>0.83</td>
<td>1.38</td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>20.90</td>
<td>0.15</td>
<td>3.05</td>
<td>0.55</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>178</td>
<td>0.01</td>
<td>2.55</td>
<td>0.46</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td>110.17</td>
<td>0.01</td>
<td>1.65</td>
<td>0.30</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>
Takeaways from the table

• The table shows that the law of one price does not hold well for the Big Mac:
  • Example 1: in Russia a Big Mac sells for the equivalent of $1.65, whereas in the United States it sells for $5.58 ⇒ 1 Big Mac in Russia buys you only 0.3 Big Macs in the US. That is, the Big Mac real exchange rate is 0.3.
  • Example 2: In Switzerland a Big Mac sells for the equivalent of $6.62. So one Big Mac in Switzerland buys you 1.19 Big Macs in the US. The Big Mac real exchange rate is 1.19.

Why is the Big Mac so expensive in some countries and so cheap in others? Look at the international tradability of its components:

(1) Highly Tradable: grain (wheat and sesame seeds), meat, and dairy (cheese). These components represent a small fraction of the total cost of a Big Mac.
(2) Hardly Tradable: labor (compare the wage per hour of a short-order cook in the US vs India), rent, electricity, and water. Large share of the cost of a Big Mac.
Changes in Big Mac Real Exchange Rates from 2006 to 2019

- **What's plotted**: the change in the Big Mac real exchange rate, $e_{\text{BigMac}} = \frac{E_{\text{BigMac}^*}}{P_{\text{BigMac}}}$, between 2006 and 2019 against the Big Mac real exchange rate in 2006 for 40 countries.

- **How to read it**: Because, by definition, the Big Mac real exchange rate is always equal to one for the United States, this country is located at coordinate (1, 0). Countries left of the vertical line were relatively cheaper than the US in 2006. Countries below the horizontal line became relatively cheaper than the US between 2006 and 2019.

- **Takeaway**: most countries were cheaper than the United States in 2006 and many of them became even cheaper by 2019 (diverged from LOOP).
Purchasing Power Parity
Purchasing power parity (PPP) is the generalization of the idea of the law of one price for broad baskets of goods representative of households’ actual consumption, as opposed to a single good.
The Real Exchange Rate

The real exchange rate, denoted $e$, is defined as

$$e = \frac{\mathcal{E}P^*}{P}.$$  

where

- $P$ = domestic-currency price of a basket of goods in the domestic country.
- $P^*$ the foreign-currency price of a basket of goods in the foreign country.

- The real exchange rate measures how expensive the foreign country is relative to the home country: it indicates the relative price of a consumption basket in the foreign country in terms of consumption baskets in the home country.
- When $e = 1$, we say that absolute purchasing power parity holds.
Absolute PPP

• Testing whether absolute PPP holds requires data on the prices of the domestic basket, \( P \), and the foreign basket, \( P^* \).

• Such data is difficult, time consuming, and expensive to collect.

• The only available data source is the World Bank’s International Comparison Program (ICP). About every six years ICP collects price level data of more than 1,000 individual goods for 199 countries.

• From this raw data, the ICP produces, for each country, an estimate of \( P \) and \( P^* \). Data for the nominal exchange rate, \( \varepsilon \), is readily available and thus one can compute \( e \), which allows testing for absolute PPP.
Empirical Evidence on Absolute PPP

Look at the next table.

• **What’s Shown:**
  — the dollar real exchange rate, $e = \varepsilon P^*/P^U$, 
  — the nominal exchange rate, $\varepsilon$ (dollar price of one unit of foreign currency), and 
  — the PPP exchange rate, $\varepsilon^{PPP} = P^U/P^*$. (Ignore this one for now, we’ll discuss it later.)

• **Notation:** The variable $P^*$ denotes the foreign-currency price of a basket in the foreign country, and $P^U$ denotes the dollar price of a basket in the United States.

• **Where, When?:** in selected countries in 2011.

• **How To Interpret It?:** If absolute PPP held, then a basket of goods that costs 100 dollars in the United States should also cost 100 dollars in every country.

• **Takeaway:** The table suggests that there are large deviations from absolute PPP. For example, a basket that in 2011 cost 100 dollars in the United States cost 163 dollars in Switzerland and only 27 dollars in Egypt.
### Deviations From Absolute PPP in Selected Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>$\epsilon$</th>
<th>$\mathcal{E}$</th>
<th>$\mathcal{E}^{PPP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>1.63</td>
<td>1.13</td>
<td>0.69</td>
</tr>
<tr>
<td>Australia</td>
<td>1.56</td>
<td>1.03</td>
<td>0.66</td>
</tr>
<tr>
<td>Japan</td>
<td>1.35</td>
<td>0.0125</td>
<td>0.00931</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.12</td>
<td>1.60</td>
<td>1.43</td>
</tr>
<tr>
<td>Germany</td>
<td>1.08</td>
<td>1.39</td>
<td>1.28</td>
</tr>
<tr>
<td>United States</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.7711</td>
<td>0.00009023</td>
<td>0.00117</td>
</tr>
<tr>
<td>China</td>
<td>0.54</td>
<td>0.15</td>
<td>0.29</td>
</tr>
<tr>
<td>Vietnam</td>
<td>0.33</td>
<td>4.88e-05</td>
<td>0.000149</td>
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<tr>
<td>India</td>
<td>0.32</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>0.31</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Pakistan</td>
<td>0.28</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Egypt</td>
<td>0.27</td>
<td>0.17</td>
<td>0.62</td>
</tr>
</tbody>
</table>

How does the ICP real exchange rate compare with the Big Mac real exchange rate?

This is shown in the graph on the next slide with plots $e$ against $e_{\text{BigMac}}$. 
Comparing the ICP and Big-Mac Real Exchange Rates in 2011

- **What’s Shown:** the World Bank’s International Comparison Program (ICP) real exchange rate (RER) against The Economist’s Big-Mac real exchange rate for 57 countries in 2011.
- **Interpretation:** the Big-Mac real exchange rate is highly correlated (0.81) with its ICP counterpart.
- **Takeaway:** the Big Mac real exchange rate is a good proxy measure of how expensive different countries are relative to one another.
- **Why is this relevant:** The ICP RER is difficult and costly to construct and is produced only every 6 years. The Big Mac RER is easy to construct and is produced at least once a year by The Economist.
PPP Exchange Rates
Definition

The PPP exchange rate is the nominal exchange rate that would make the consumption basket in two countries equally expensive. Formally, letting $\mathcal{E}^{PPP}$ denote the PPP exchange rate, we have that

$$\mathcal{E}^{PPP} P^* = P,$$

where $P$ = price level in the domestic country

$P^*$ = price level in the foreign country.

If $\mathcal{E}^{PPP} > \mathcal{E}$ ⇒ the domestic country is more expensive than the foreign country ($P > \mathcal{E} P^*$) and we say that the foreign currency is undervalued.

If $\mathcal{E}^{PPP} < \mathcal{E}$ ⇒ the domestic country is less expensive than the foreign country ($P < \mathcal{E} P^*$) and we say that the foreign currency is overvalued.
Big Mac PPP Exchange Rates

Following the above definition, is it is given by

$$\varepsilon^\text{BigMac PPP} = \frac{P^\text{BigMac US}}{P^\text{BigMac *}}.$$  

Take another look at the table on slide 7, which shows the Big Mac PPP exchange rate and is reproduced on the next slide.

• for Switzerland $\varepsilon^\text{BigMac PPP} < \varepsilon \Rightarrow$ Switzerland is more expensive than the US, and the Swiss franc is overvalued relative to the dollar.

• for India $\varepsilon^\text{BigMac PPP} > \varepsilon \Rightarrow$ India is cheaper than the US, and the Indian rupee is undervalued relative to the dollar.
## The Big-Mac Real Exchange Rate, January 2019

<table>
<thead>
<tr>
<th>Country</th>
<th>$P^{\text{BigMac}^*}$</th>
<th>$\varepsilon$</th>
<th>$\varepsilon P^{\text{BigMac}^*}$</th>
<th>$e^{\text{BigMac}}$</th>
<th>$\varepsilon^{\text{BigMac}}$</th>
<th>$PPP$</th>
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</thead>
<tbody>
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<td>Switzerland</td>
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<td>6.62</td>
<td>1.19</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>5.58</td>
<td>1</td>
<td>5.58</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Canada</td>
<td>6.77</td>
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<td>0.03</td>
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</tr>
<tr>
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<td>110.17</td>
<td>0.01</td>
<td>1.65</td>
<td>0.30</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>
Does China Manipulate Its Currency to Gain Competitiveness?

• A number of policymakers and observers have suggested that China has put in place policies conducive to an undervaluation of its currency with the intention to boost China’s competitiveness in international trade.

• The table shows that $E_{BigMac}^{PPP}$ for China is 0.27 dollar per yuan, while the market exchange rate, $E$, is 0.15 dollar per yuan. This means that, according to the Big Mac PPP exchange rate, the yuan is undervalued relative to the dollar. This gives credence to the critics.

• But later we will argue that if we take into account the different levels of economic development in the US and China, this conclusion is far less clear.
PPP Exchange Rates for Baskets of Goods

<table>
<thead>
<tr>
<th>Country</th>
<th>$e$</th>
<th>$\mathcal{E}$</th>
<th>$\mathcal{E}^{PPP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>1.63</td>
<td>1.13</td>
<td>0.69</td>
</tr>
<tr>
<td>United States</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>China</td>
<td>0.54</td>
<td>0.15</td>
<td>0.29</td>
</tr>
<tr>
<td>India</td>
<td>0.32</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>Egypt</td>
<td>0.27</td>
<td>0.17</td>
<td>0.62</td>
</tr>
</tbody>
</table>

This table shows an excerpt from the table presented on slide 15 earlier. The last column displays PPP exchange rates. PPP exchange rates are constructed using the price level data ($P^*$) from the 2011 ICP program for baskets of goods, which contain hundreds of goods.

- **Takeaways:** Similar pattern of under- and over-valuation of currencies as suggested by the Big Mac PPP exchange rates: In particular
  - the Swiss franc is overvalued and
  - the Indian rupee and Chinese yuan are significantly undervalued.
PPP Exchange Rates and Standard of Living Comparisons

Standard of Living Comparison are Tricky

• The problem is that prices of similar goods vary widely across borders.

• Example: in 2011 GDP per capita was 49,782 dollars in the United States but only 1,533 dollars in India. Can we conclude that the average American is 32 times richer than the average Indian? What if a given amount of dollars buys more goods and services in India than in the United States?

• Let’s calculate how many burgers one can buy with each per capita GDP: A Big Mac costs 5.58 dollars in the United States but only 2.55 dollars in India. So one U.S. per capita GDP buys 8,922 Big Macs and one Indian per capita GDP buys 601 Big Macs. Thus, in terms of Big Macs, Americans are 15 times richer than Indians.

• Still a big income gap, but not as large as the one suggested by the simple ratio of dollar GDPs.
Per Capita GDP at PPP Exchange Rate  Now compare GDPs per capita measured in units of baskets of goods. Let \( GDP^I \) = GDP per capita in India in Indian rupees. 
\( P^I \) = the rupee price of one basket of goods in India.
\( \Rightarrow GDP^I / P^I \) = per capita GDP in India in units of baskets of goods.
Similarly, let \( GDP^{US} \) and \( P^{US} \) be GDP in the United States and the price of one basket of goods in the United States, both measured in dollars.
\( \Rightarrow GDP^{US} / P^{US} \) = per capita GDP in the United States measured in units of baskets of goods.

\[ \Rightarrow \text{Ratio of Incomes in Baskets of Goods} = \frac{GDP^{US} / P^{US}}{GDP^I / P^I} \]

\[ = \frac{1}{P^{US} / P^I} \cdot \frac{GDP^{US}}{GDP^I} \]

\[ = \frac{GDP^{US}}{\varepsilon^{PPP,IGDP^I}}. \]

\( \varepsilon^{PPP,IGDP^I} \) is called per capita GDP at PPP exchange rate; it’s denoted \( GDP^{PPP,IGDP^I} \) and represents per capita GDP in India when baskets of goods are priced in dollar prices of the United States.
Look at the Next Table

It shows GDP per capita in dollars at market exchange rates and at PPP exchange rates.
### GDP Per Capita at Market and PPP Exchange Rates in 2011

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP</th>
<th>GDP(_{PPP})</th>
<th>(\frac{GDP^{US}}{GDP})</th>
<th>(\frac{GDP^{US}}{GDP^{PPP}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway</td>
<td>99035</td>
<td>61879</td>
<td>0.50</td>
<td>0.80</td>
</tr>
<tr>
<td>Switzerland</td>
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<td>0.97</td>
</tr>
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<td>Australia</td>
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<tr>
<td>United States</td>
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<td>1</td>
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<tr>
<td>Japan</td>
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<td>South Korea</td>
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<td>10057</td>
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<td>Egypt</td>
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<td>India</td>
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<td>Pakistan</td>
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<td>11.19</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>874</td>
<td>2800</td>
<td>56.95</td>
<td>17.78</td>
</tr>
</tbody>
</table>
In India, GDP per capita was 1,533 dollars when measured at market exchange rates but 4,735 dollars when measured at PPP exchange rates.

per capita GDP in the United States in 2011 was 49,782 dollars.

⇒ the average American is 32 times as rich as the average Indian when GDP is converted into dollars at market exchange rates, but 11 times as rich when GDP is converted at PPP exchange rates.

• This shows that on average prices in India are lower than in the United States.

Is this a more general pattern? Are poor countries cheaper than rich countries? Look at the next graph.
Rich Countries are More Expensive than Poor Countries

Higher Prices in Rich Countries

- **What’s shown?** the dollar real exchange rate, $e = \varepsilon P^*/P^{US}$, against per capita GDP at market exchange rates in 2011 for 177 countries.
- **Takeaway:** countries with higher per capita incomes tend to be more expensive.
- **Note:** China is not an outlier casting doubt on the claim that the yuan is manipulated.
Relative Purchasing Power Parity
Relative Purchasing Power Parity

• Most studies of purchasing power parity focus on changes in the real exchange rate, rather than on its level.
• Why? To calculate the change in the real exchange rate, one can use Consumer Price Indices, which are readily available for many countries at a relatively high frequency (typically monthly).
• The consumer price index does not provide information about the price of a basket of goods, but about its change.

\[ \Delta e_t \equiv \Delta \frac{E_t P^*_t}{P_t} = 0, \]

where \(e_t\) = real exchange rate at time \(t\), \(E_t\) = nominal exchange rate at time \(t\), \(P_t\), and \(P^*_t\) = domestic and foreign consumer price indices at time \(t\), and \(\Delta = \text{change over time}\).

• When \(\Delta e_t < 0\) we say that the real exchange appreciates. The domestic country becomes more expensive.
• When \(\Delta e_t > 0\) we say that the real exchange depreciates. The domestic country becomes less expensive.
Does Relative PPP Hold in the Long Run?

- The next figure plots the logs of the price index in the United States, $P_t^{US}$, and the price index in the United Kingdom expressed in U.S. dollars, $\varepsilon_t P_t^{UK}$, over the period 1870-2018.

- Recall that the level of the price index in a particular year is meaningless; only its change provides information. So, without loss of information, the figure normalizes $P_t^{US}$ and $\varepsilon_t P_t^{UK}$ to 1 (or their logs to 0) in 1870.

- The figure shows that over the past 148 years the United States did not become systematically cheaper or more expensive than the United Kingdom.

- This suggests that relative PPP holds in the long run between these two countries.
Does Relative PPP Hold in the Long Run for Other Countries?

Let $e_t$ be the real exchange rate of a given country with the United States in period $t$. Then,

$$e_t = \frac{\mathcal{E}_t P_{tUS}^U}{P_t}.$$ 

where $P_t$ and $P_{tUS}$ = the consumer price indices in the country considered and the US at time $t$. $\mathcal{E}_t$ = exchange rate at time $t$, defined as the price of one dollar in terms of the country's currency. The real depreciation rate of the country's currency against the U.S. dollar, denoted $\epsilon_r^t$, is the growth rate of $e_t$,

$$1 + \epsilon_r^t = \frac{e_t}{e_{t-1}} = \frac{(\mathcal{E}_t/\mathcal{E}_{t-1})(P_{tUS}^U/P_{tUS}^U)}{P_t/P_{t-1}}.$$ 

Let $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1} - 1$ denote the nominal depreciation rate of the country's currency against the U.S. dollar, and $\pi_t = P_t/P_{t-1} - 1$ and $\pi_{tUS}^U = P_{tUS}^U/P_{tUS}^U - 1$ inflation rates in the country considered and in the United States. This yields

$$1 + \epsilon_r^t = \frac{(1 + \epsilon_t)(1 + \pi_{tUS}^U)}{(1 + \pi_t)}.$$
Taking the natural logarithm and using the approximation \( \ln(1+x) \approx x \)

\[ \epsilon^r_t = \epsilon_t + \pi_t^{US} - \pi_t. \]

Relative \( PPP \) holds if the real exchange rate does not change over time, \( \epsilon^r_t = 0 \):

Relative \( PPP \) holds if \( \epsilon_t = \pi_t - \pi_t^{US} \).

In words, relative \( PPP \) holds if the rate of depreciation of the country’s currency against the dollar, \( \epsilon_t \), is equal to the inflation differential between the country considered and the United States, \( \pi_t - \pi_t^{US} \).

Let \( \epsilon, \pi, \) and \( \pi^{US} \) be the averages of \( \epsilon_t, \pi_t, \) and \( \pi_t^{US} \) over a long period of time. Then,

Relative \( PPP \) holds in the long run if \( \epsilon = \pi - \pi^{US} \).

The next slide suggests that this expression holds for many countries \ldots
**What’s Shown?** Each marker represents a country. There are 45 countries: 13 rich, 17 emerging, and 15 poor.

**Takeaway:** Most countries line up close to the 45-degree line, indicating that relative PPP holds well in the long run.
Does Relative PPP Hold in the Short Run?
Take another look at the figure on slide 33 displaying the price levels of the UK and the US expressed in the same currency.

Notice that sometimes the two prices get closer together and sometimes they move apart. This means that sometimes the US becomes relatively cheaper and sometimes relatively more expensive than the UK.

The next figure displays the dollar/pound real depreciation rate, $\epsilon_t^r$, between 1870 and 2018. The object plotted is the year-to-year change in the distance between the solid and the broken lines in the figure on slide 33.

The takeaway of the figure is that $\epsilon_t^r$ moves around quite a bit. The standard deviation of $\epsilon_t^r$ is 9.3 percent. This means that typically, from one year to the next, the United States becomes almost 10 percent more expensive or cheaper than the United Kingdom.

This suggests that relative PPP does not hold in the short run.
The figure shows that the dollar-pound real exchange rate changes significantly from one year to the next, suggesting that relative PPP does not hold in the short run.
How Wide Is the Border?
Are short-run deviations from relative PPP due to the existence of a country border or do they hold more broadly across different geographic locations?

Two factors that can explain failures in relative PPP across cities: 

**(1) Transportation Costs:** It might pay for NYC households to shop in nearby Newark to exploit relatively small differences in prices. But it would take much larger price differences to induce NYC households to shop in Philadelphia. Therefore, differences in prices are likely to be larger between NYC and Philly than between NYC and Newark.

**(2) An International Border:** It can introduce impediments for prices to equalize across locations, including: 

(a) movements in nominal exchange rates in combination with rigidities in local currency prices; 
(b) movements in trade frictions that contribute to market segmentation across countries, such as tariffs, quotas, and government regulation.

So we could observe that even though New York is much farther away from Los Angeles than from Toronto, relative PPPs fail more significantly for the NY-Toronto pair than for the NY-LA pair.
The Effect of Distance and a Border on Deviations from Relative PPP

The real exchange rate between cities $c_1$ and $c_2$ for a basket of goods $g$ is given by

$$e_{g,c_1,c_2,t}^g = \frac{\mathcal{E}_{c_1,c_2,t}P_{g,c_2,t}^g}{P_{g,c_1,t}^g},$$

where $P_{g,c_1,t}^g$ and $P_{g,c_2,t}^g$ are the price indices of basket $g$ in cities $c_1$ and $c_2$ at time $t$, and

$\mathcal{E}_{c_1,c_2,t} =$ nominal exchange rate between cities $c_1$ and $c_2$ in period $t$. (Clearly, if $c_1$ and $c_2$ are in the same country, then $\mathcal{E}_{c_1,c_2,t} = 1$.)

Let $\sigma_{c_1,c_2}^g$ be the standard deviation of $\Delta \ln e_{c_1,c_2,t}^g$ taken across time periods.

A large value of $\sigma_{c_1,c_2}^g$ indicates large changes in violations of relative PPP in the short run.
How Distance and Borders Affect Relative PPP

Engel and Rogers* estimate the following regression:

\[
\sigma_{c1,c2}^g = \text{constant} + 0.00106 \ln d_{c1,c2} + 0.0119 B_{c1,c2} + \mu_{c1,c2}^g,
\]

where \( d_{c1,c2} \) denotes the distance in miles between cities \( c1 \) and \( c2 \), \( B_{c1,c2} \) is a variable that takes the value 1 if cities \( c1 \) and \( c2 \) are separated by an international border and zero otherwise, and \( \mu_{c1,c2}^g \) is a regression residual.

They used bi-monthly data on prices of 14 baskets of goods, 14 US cities, and 9 Canadian cities observed over the period September 1978 to December 1994. So they have 3542 values of \( \sigma_{c1,c2}^g \).

**Takeaway:** Short-run deviations from relative PPP are increasing in distance and in the existence of a border.

The Border Effect

Counterfactual Experiment: Suppose cities $c_1$ and $c_2$ are separated by a border. If we were to remove the border, by how much should we increase the distance between $c_1$ and $c_2$ so that $\sigma_{c_1,c_2}^g$ stays the same? To answer this question, note that:

- The border increases $\sigma_{c_1,c_2}^g$ by 0.0119.
- Increasing the distance by one mile raises $\sigma_{c_1,c_2}^g$ by

$$\frac{\partial \sigma_{c_1,c_2}^g}{\partial d_{c_1,c_2}} = 0.00106 \frac{\partial \ln d_{c_1,c_2}}{\partial d_{c_1,c_2}} = 0.00106 \frac{1}{d_{c_1,c_2}}.$$  

The average distance between two cities in the Engel-Rogers dataset is 1,100 miles.

Then on average each mile increases $\sigma_{c_1,c_2}^g$ by

$$\frac{\partial \sigma_{c_1,c_2}^g}{\partial d_{c_1,c_2}} = 0.00106 \frac{1}{1100} = 0.00000096364.$$  

Therefore, the border is equivalent to increasing the distance by $0.0119/0.00000096364 \approx 12,000$ miles. This is quite a wide border!
Takeaway

The evidence suggests that

(a) the amplitude of short-run deviations from relative PPP is increasing in the distance separating two locations; and

(b) the mere existence of an international border separating two locations adds significantly to this amplitude.

In turn, (b) implies that factors such as exchange rate volatility, local price rigidities, tariffs, quotas, and cross-border regulations play an important role in determining the size of changes in real exchange rates in the short run.
Nontradable Goods and Deviations from Purchasing Power Parity
** Tradable and Nontradable Goods**
- We have documented the existence of large and persistent deviations from purchasing power parity. — Example: in 2011 a basket of goods that cost 100 dollar in the United States cost only 32 dollars in India.
- One reason why price differences tend to persist is that not all goods are internationally tradable.
- For these goods, transportation costs are too large for international trade to be profitable.
  — Example: no one would fly to India just because a haircut there is 10 times cheaper.
- This type of goods and services are called nontradable goods.
  — Examples of nontradable goods: services (haircuts, restaurant meals, housing, primary health, education).
  — Examples of tradable goods: agricultural commodities (wheat, corn, and soybeans, metals, minerals, oil) and many manufactured goods.
- Nontradables make up a significant share output, typically above 50 percent.
Nontradables and Deviations from PPP

Let $P_T$ = domestic price of tradables, and $P_N$ = domestic price of nontradables.
$P_T^*$ and $P_N^*$ the corresponding foreign prices.
Suppose the LOOP holds for tradable goods,

$$P_T = \varepsilon P_T^*.$$ 

But the LOOP does not hold for nontradable goods $P_N \neq \varepsilon P_N^*$.

The price level, $P$, is is an average of $P_T$ and $P_N$,

$$P = \phi(P_T, P_N),$$

the function $\phi(\cdot, \cdot)$ is increasing in $P_T$ and $P_N$ and homogeneous of degree one (HD1). HD1 means that if $P_T$ and $P_N$ increase by $x$ percent, then $P$ also increases by $x$ percent.
Examples of HD1 functions: $\phi(P_T, P_N) = (P_T + P_N)/2$ and $\phi(P_T, P_N) = (P_T)^\gamma (P_N)^{1-\gamma}$, with $\gamma \in (0, 1)$. 

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The price level in the foreign country is also an average of the prices of tradables and nontradables

\[ P^* = \phi(P_T^*, P_N^*). \]

The real exchange rate is

\[ e = \frac{\mathcal{E}P^*}{P} = \frac{\mathcal{E}\phi(P_T^*, P_N^*)}{\phi(P_T, P_N)} = \frac{\mathcal{E}P_T^*\phi(1, P_N^*/P_T^*)}{P_T\phi(1, P_N/P_T)} = \frac{\phi(1, P_N^*/P_T^*)}{\phi(1, P_N/P_T)}. \]

**Interpretation:** the real exchange rate depends on the relative price of nontradables in terms of tradables across countries.

\[ e < 1 \text{ if } \frac{P_N^*}{P_T^*} < \frac{P_N}{P_T}. \]

Back to the India-US example: This expression says that India is cheaper than the US because the relative price of nontradables in terms of tradables is lower in India than in the US.
Trade Barriers and Real Exchange Rates
We now analyze deviations from PPP that may arise even when all goods are traded.

Specifically, we study deviations from PPP that arise because governments impose trade barriers, such as import tariffs, export subsidies, and quotas, that artificially distort relative prices across countries.

Consider, for simplicity, an economy in which all goods are internationally tradable.

Suppose further that there are two types of tradable goods, importables and exportables.

Importable goods are goods that are either imported or produced domestically but coexist in the domestic market with identical or highly substitutable imported goods.

Exportable goods are goods that are produced domestically and sold in foreign and possibly domestic markets.
The Real Exchange Rate Without Trade Barriers

Let $P^*_M$ and $P^*_X$ denote the world prices of importables and exportables and $P_M$ and $P_X$ the domestic prices of importables and exportables.

The LOOP holds for each type of good

$$P_X = \mathcal{E}P^*_X \quad \text{and} \quad P_M = \mathcal{E}P^*_M,$$

The domestic and foreign price levels are

$$P = \phi(P_X, P_M) \quad \text{and} \quad P^* = \phi(P^*_X, P^*_M).$$

The real exchange rate, $e = \mathcal{E}P^*/P$, can then be written as

$$e = \frac{\mathcal{E}P^*}{P} = \frac{\mathcal{E}\phi(P^*_X, P^*_M)}{\phi(P_X, P_M)} = \frac{\phi(\mathcal{E}P^*_X, \mathcal{E}P^*_M)}{\phi(P_X, P_M)} = \frac{\phi(P_X, P_M)}{\phi(P_X, P_M)} = 1.$$ 

**Interpretation:** If all goods are tradable and no trade barriers are in place, both countries are equally expensive.
The Real Exchange Rate with Import Tariffs

Suppose the domestic government imposes a tariff $\tau$ on imports.

The importer pays $E P^*_M$ to the foreign producer and $\tau E P^*_M$ to the local government.

The domestic price of the importable good increases by a factor of $1 + \tau$

$$P_M = (1 + \tau)E P^*_M.$$  

And the real exchange rate becomes

$$e = \frac{E \phi(P^*_X, P^*_M)}{\phi(P_X, P_M)} = \frac{\phi(E P^*_X, E P^*_M)}{\phi(E P^*_X, (1 + \tau)E P^*_M)} < 1,$$

**Interpretation:** Import tariffs make the domestic country more expensive relative to the foreign country.
Home Bias and the Real Exchange Rate
• Thus far we have seen that PPP may fail because not all goods are tradable or because of the presence of trade barriers, such as import tariffs.

• We now study how PPP can fail when the weights with which a particular good enters in the consumption basket is different across countries.

• Such differences in weights reflect primarily differences in tastes across countries. In turn, national tastes could be acquired via resource abundance or production specialization. Example: Argentines might spend a larger fraction of their budgets on beef than Germans do, and Germans might spend relatively more on cars than Argentines.

• Such a preference for domestically produced goods is called home bias.

• Intuition: If the price of beef relative to the price of cars goes up, the price index goes up proportionally more in Argentina than in Germany, simply because Argentines spend more on beef.
Let’s Formalize the Idea of Home Bias

Suppose that the consumer price indices in Argentina and Germany, $P$ and $P^*$, are geometric averages of the prices of beef and cars in each country, $P_b$ and $P_c$ in Argentina and $P_b^*$ and $P_c^*$ in Germany:

\[ P = (P_b)\gamma(P_c)^{1-\gamma} \]

and

\[ P^* = (P_b^*)\gamma^*(P_c^*)^{1-\gamma^*}. \]

where $\gamma \in (0, 1)$ and $\gamma^* \in (0, 1)$ are parameters, capturing the importance of beef in the Argentine and German consumption baskets.

Now, if Argentines have a strong preference for beef and Germans for cars, we have

\[ \text{home bias } \Rightarrow \gamma > \gamma^*. \]
The Real Exchange Rate with Home Bias

Suppose beef and cars are freely traded so that the LOOP holds:

\[ P_b = \varepsilon P_b^* \quad \text{and} \quad P_c = \varepsilon P_c^*, \]

where \( \varepsilon \) is the nominal exchange rate (pesos per euro). Then, the real exchange rate is

\[ e = \frac{\varepsilon P^*}{P} = \frac{\varepsilon (P_b^*)^\gamma (P_c^*)^{1-\gamma}}{(P_b)^\gamma (P_c)^{1-\gamma}} = \frac{\varepsilon (P_b/\varepsilon)^\gamma (P_c/\varepsilon)^{1-\gamma}}{(P_b)^\gamma (P_c)^{1-\gamma}} = \left( \frac{P_c}{P_b} \right)^{\gamma-\gamma^*}. \]

**Result:** Because \( \gamma > \gamma^* \), an increase in the price of beef relative to cars causes a real appreciation of the peso (a fall in \( e \)).

**Intuitively**, if the relative price of beef increases, then the price of the Argentine consumption basket, \( P \), increases by more than the price of the German consumption basket, \( P^* \), since beef has a larger weight in the Argentine basket than in the German basket.

- **Note:** If \( \gamma = \gamma^* \), then \( e = 1 \) and PPP holds. So any departure from PPP here is due to home bias.
Price Indices and Standards of Living
We have seen that the price level \( \phi(P_T, P_N) \), should be increasing and homogeneous of degree one in the prices of the goods that enter the consumption basket (in this example, tradable and nontradable goods).

We stated that the function \( \phi(P_T, P_N) \) is an average of \( P_T \) and \( P_N \).

But what type of average? What weights should the average place on \( P_T \) and \( P_N \)? Is the price index useful to measure standards of living?

Example, suppose your money income increases by 10 percent and the price level increases by 11 percent. Are you better off or worse off? Would you be worse off if all of the increase in the price level is accounted for by an increase in the price of beef and you are a vegetarian?

In this section, we analyzes these and other related questions.
Microfoundations of the Price Level

Suppose the utility function is

$$U(C),$$

where $C = \text{current consumption}$ and $U(\cdot)$ is an increasing function.

Suppose that $C$ is a composite of tradable and nontradable consumption given by the aggregator function

$$C = C_T^\gamma C_N^{1-\gamma}, \quad (1)$$

where $C_T = \text{consumption of tradable goods}$ and $C_N = \text{consumption of nontradable goods}$, and $\gamma \in (0, 1)$ is a parameter.

Example: A Big Mac ($C$) is a composite of tradable goods ($C_T$, e.g., beef, wheat, cheese) and nontradable goods ($C_N$, e.g., the labor provided by cooks, store rent, utilities).
**Definition:** The consumer price level, $P$, is the minimum amount of money necessary to purchase one unit of the composite consumption good $C$. Formally,

$$P = \min_{\{C_T, C_N\}} \{ P_TC_T + P_N C_N \}$$

subject to

$$C_T^\gamma C_N^{1-\gamma} = 1,$$

To solve this problem, begin by solving the constraint for $C_N$ to get

$$C_N = C_T^{-\gamma}. \quad (2)$$

Use this to eliminate $C_N$ from the objective function to get

$$P = \min_{\{C_T\}} \left\{ P_TC_T + P_N C_T^{-\gamma} \right\}. \quad (3)$$
Take the derivative and set it to 0:

\[ P_T - \frac{\gamma}{1-\gamma} P_N C_T^{1-\gamma} = 0. \]

Solving for \( C_T \), we obtain

\[ C_T = \left[ \frac{\gamma P_N}{1 - \gamma P_T} \right]^{1-\gamma}. \] (4)

Use this to eliminate \( C_T \) from (2) to get

\[ C_N = \left[ \frac{\gamma P_N}{1 - \gamma P_T} \right]^{-\gamma}. \] (5)

**Intuition:** the last two expressions say that as the nontradable good becomes relatively more expensive \((P_N/P_T \text{ goes up})\), there is a substitution away from nontradables and toward tradables in the production of the unit of composite consumption.
Finally, use (4) and (5) to eliminate $C_T$ and $C_N$ from (3) to obtain

$$P = P_T^\gamma P_N^{1-\gamma} A,$$

where $A \equiv \gamma^{-\gamma}(1 - \gamma)^{-(1-\gamma)}$ is a constant.

**Important to Note:** the weights assigned to the prices of tradable and nontradable goods are related to the weights assigned to the corresponding goods in the aggregator function. Now we know where the price level comes from.
Prices, Income, and Welfare

The amount of consumption one can buy with a given income $Y$ is

$$C = \frac{Y}{P}.$$ 

Since utility depends on $C$, we have that, if the price level is correctly measured, changes in real income, $Y/P$, reflect changes in welfare.

**Example:** suppose that $\gamma = 0.25$, and that in the course of one year nominal income increased by 10 percent ($%\Delta Y = 0.1$), the price of tradables by 12 percent ($%\Delta P_T = 0.12$) and the price of nontradables by 8 percent ($%\Delta P_N = 0.08$). Is the household better off or worse off relative to the previous year?
The percent increase in the amount of consumption the consumer can enjoy this year is given by

\[
\%\Delta C = \%\Delta \frac{Y}{P} = \%\Delta Y - \%\Delta P
\]

\[
= \%\Delta Y - \gamma \%\Delta P_T - (1 - \gamma) \%\Delta P_N
\]

\[
= 0.1 - 0.25 \times 0.12 - 0.75 \times 0.08
\]

\[
= 1\%.
\]

This means that the consumer is better off. **Intuition:** The intuition behind this increase in welfare is that the price that increases proportionally more than income corresponds to a good (the tradable good) that is not too important in the generation of composite consumption (\(\gamma\) small).
What if the Weight $\gamma$ is Badly Measured?

Let’s redo the exercise using a weight $\tilde{\gamma} = 0.75$. Under this incorrect weight, the change in real income is

$$%\Delta \frac{Y}{P} = 0.1 - 0.75 \times 0.12 - 0.25 \times 0.08$$

$$= -1\%,$$

which leads to the misleading conclusion that consumers are worse off and can afford 1% less consumption than in the previous year.

Intuition: The problem here is that the statistical office is assigning too much weight to the price that increased more.

What can the statistical office do to measure $\gamma$ correctly?

Obviously, asking people what they think $\gamma$ is in their utility function wouldn’t go too long.
How to Estimate $\gamma$

Fortunately, there is a way to calculate $\gamma$.

Dividing (4) by (5) and solving for $\gamma$ yields

$$\gamma = \frac{P_T C_T}{P_T C_T + P_N C_N}.$$ 

This expression says that $\gamma$ equals the ratio of expenditure on tradable in total expenditure.

Statistical agencies periodically conduct surveys asking individuals about their expenditure behavior and use this information as an input in the construction of price indices.
**Summing Up** This chapter is concerned with differences in costs of living across countries.

Key concepts: the law of one price (LOOP), purchasing power parity (PPP), the real exchange rate, tradable and nontradable goods, home bias, and price indices.

- The LOOP states that the same good must have the same price across countries or regions when expressed in a common currency.
- There exist large and systematic deviations from the LOOP.
- Absolute PPP extends the concept of the LOOP to baskets of goods. It says that consumption baskets must have the same price across countries or regions when expressed in a common currency.
- Observed deviations from absolute PPP are large and persistent.
- The real exchange rate is the relative price of a basket of goods in a foreign country in terms of a basket of goods in the domestic country. By definition, when absolute PPP holds, the real exchange rate is one.
- Relative PPP holds when the real exchange rate does not change over time.
- In the data, relative PPP holds over the long run, but fails over the short run.
- Rich countries are systematically more expensive than poor countries.
$\textbf{Summing Up (cont.)}$

- The PPP exchange rate is the nominal exchange rate that makes PPP hold. Evaluating GDP per capita at PPP exchange rates allows for meaningful comparisons of standards of living.

- Deviations from PPP across locations are accounted for by a number of factors:
  - Distance between the two locations, which can be a reflection of transportation costs.
  - The existence of an international border separating the two locations. This can reflect the presence of nominal exchange-rate volatility, local-currency price rigidity, cross-border regulations, import and export tariffs, and trade quotas.
  - The existence of nontradable goods. Nontradable goods are goods that are domestically produced, but are neither importable nor exportable.

- If the weights on prices that enter the consumer price index differ across countries, then variations in relative prices can lead to variations in real exchange rates. If domestically produced goods make up a bigger share of the domestic basket than of the foreign basket and hence receive a larger weight in the home country’s price index than in the foreign country’s price index, then we say there is home bias in consumption.

- The imposition of import tariffs or export subsidies makes the country more expensive relative to the rest of the world, that is, it causes a real exchange rate appreciation.
Summing Up (concluded)

- The optimal price index assigns weights to individual prices that correspond to the weight of the associated good in the consumer's utility function. When nominal income is deflated by the optimal price index, variations in real income represent variations in welfare in the same direction.