Slides for Chapter 9:

Determinants of the Real Exchange Rate

*International Macroeconomics*

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The LOOP

• LOOP stands for the Law of One Price.
• The law of one price says that a good should cost the same abroad and at home when its price is measured in the same currency. Formally, if the LOOP holds for good $i$, then

$$P_i = P_i^* S,$$

where

- $P_i$ = domestic currency price of good $i$.
- $P_i^*$ = foreign currency price of good $i$.
- $S$ = nominal exchange rate, defined as the domestic currency price of 1 unit of foreign currency.

Examples of goods for which the LOOP holds:

**Commodities:** Gold, Oil, Wheat,

**Luxury Consumer Goods:** Hermes neckties, MontBlanc pens, Rolex watches, etc.

Examples of goods for which the LOOP fails:

**Personal Services:** health, restaurant meals, haircuts, etc.

**Local Goods:** housing, transportation.
Reasons Why the LOOP May Fail

- a good has non-traded inputs such as:
  - labor
  - rent
  - electricity

- government policies/regulations (taxes)

- barriers to trade (tariffs, quotas)

- International price discrimination, such as pricing to market (pharmaceuticals, clothing)
PPP

• PPP stands for Purchasing Power Parity.
• PPP generalizes the LOOP to baskets of goods instead of individual goods. Let
  \( P = \) domestic currency price of a domestic basket of goods.
  \( P^* = \) foreign currency price of a foreign basket of goods
  \( S = \) nominal exchange rate (domestic currency per unit of foreign currency).
• The real exchange rate (RER), denoted
  \[ e = \frac{SP^*}{P} \] (*)

In words, the real exchange rate is the relative price of a foreign basket of goods in terms of comparable domestic baskets of goods.
• When \( e \) increases the foreign country becomes relatively more expensive and we say that the real exchange rate depreciated. Similarly, when \( e \) decreases the foreign country becomes relatively cheaper and we say that the real exchange rate appreciated.

• if \( e = 1 \), we say that PPP holds.
From LOOP to PPP
Suppose the LOOP holds for all goods. Does PPP (i.e., $e = 1$) have to hold?

Not necessarily, because the foreign and domestic baskets could:
– contain different items
– have different weights for the same items.
Absolute and Relative PPP

Absolute PPP
We say that absolute PPP holds, when

$$e = 1$$

Relative PPP
We say that relative PPP holds, when

$$\Delta e = 0$$

The distinction between absolute and relative PPP is relevant, because sometimes $P$ and $P^*$ are proxied by price indices, such as the domestic and foreign consumer price indices. In these cases, the level of $e$ is meaningless, but variations thereof are informative.
**Testing the Long-Run Validity of Relative PPP**

Test 1: Begin from the definition of the real exchange rate, $e_t = \frac{S_tP^*_t}{P_t}$, and take logs on both sides to get

$$\ln e_t = \ln (S_tP^*_t) - \ln (P_t)$$

If relative PPP holds, then $\Delta \ln e_t = 0$ and hence $\ln (S_tP^*_t)$ should be moving over time in tandem with $\ln (P_t)$.

The next graph tests the long-run validity of relative PPP by plotting $\ln (S_tP^*_t)$ and $\ln (P_t)$ for the dollar pound real exchange rate over the period 1820 to 2001.

The broken line is $\ln (S_tP^*_t)$, and the solid line is $\ln (P_t^{US})$.

The vertical difference between the broken and the solid lines is $\ln e_t$, the log of the dollar-pound real exchange rate.
Observations on the figure

1. Overall the comovement between U.S. and U.K. prices over the past 180 years has been very high. This suggests that relative PPP holds in the long run.

2. Over the past 180 years the dollar has appreciated vis-a-vis the Pound in real terms (i.e., the United States became relatively more expensive). This can be see from the fact that at the beginning of the sample the solid line is below the broken line, whereas at the end of the sample the latter is below the former.
Relative PPP in the Short Run

The plot shows that the vertical distance between the solid and broken lines changes quite a bit from year to year. This means that the real exchange rate moves significantly at an annual frequency, suggesting that PPP does not hold in the short run.

Also recall from Chapter 8 that in monthly data we observed the following average changes in real exchange rates: over the short period September 1982 to January 1988

<table>
<thead>
<tr>
<th>Country</th>
<th>( \ln e_t - \ln e_{t-1} ) (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-6.35 %</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-8.35%</td>
</tr>
<tr>
<td>France</td>
<td>-6.25%</td>
</tr>
<tr>
<td>Mexico</td>
<td>-3.32%</td>
</tr>
</tbody>
</table>

We conclude that in the short-run relative PPP does not hold. In fact \( e_t \) is VERY volatile in the short run.
Test 2 of Relative PPP in the long run:

\( P_t \) — U.S. Price level in dollars;
\( P_t^* \) — foreign price level in foreign currency

Take the k-period log difference of (\(*\))

\[
\ln e_t - \ln e_{t-k} = \ln \left( \frac{P_t^*}{P_t^*_{t-k}} \right) - \ln \left( \frac{P_t}{P_{t-k}} \right) + \ln \left( \frac{S_t}{S_{t-k}} \right)
\]
If relative PPP holds in the long run, then

$$\ln e_t - \ln e_{t-k} = 0$$

This implies that

$$\ln \left( \frac{P_t^*}{P_{t-k}^*} \right) - \ln \left( \frac{P_t}{P_{t-k}} \right) = -\ln \left( \frac{S_t}{S_{t-k}} \right)$$

This equation says that the difference between foreign long run inflation and U.S. long run inflation should be equal to the rate of depreciation of the foreign currency against the dollar.

This is intuitive, the currency of a country with a higher rate of inflation than the United States should depreciate against the dollar.
Taylor and Taylor test whether relative PPP holds in the long run by considering average inflation differentials and average depreciation rates against the U.S. dollar over the 29 year period 1970 to 1998 for 20 industrialized countries and 26 developing countries. Each country is one observation. If relative PPP holds in the long run, then a plot of long-run inflation differentials against long-run depreciation rates against the dollar should lie on the 45 degree line.

The next graph shows that this is indeed the case, providing more support to the claim that in the long-run relative PPP holds.
Inflation Differential with the United States and Exchange Rate Depreciation vis-à-vis the U.S. Dollar: Averages from 1970 to 1998

Note: The figure shows countries’ cumulative inflation rate differentials against the United States in percent (vertical axis) plotted against their cumulative depreciation rates against the U.S. dollar in percent (horizontal axis). The sample includes data from 20 industrialized countries and 26 developing countries. Source: Alan M. Taylor and Mark P. Taylor, “The Purchasing Power Parity Debate,” Journal of Economic Perspectives 18, Fall 2004, 135-158.
Summary:

- Relative PPP holds over the long run.
- Relative PPP fails to hold over the short run.
Absolute PPP

Absolute PPP holds if $e_t = 1$, that is if the purchasing power of $1$ is the same in the domestic country and abroad.

To test relative PPP all we needed were observations of the nominal exchange rate and price indices. By contrast, to test absolute PPP we need to observe the actual price of the basket of goods, not just a price index. This type of data is much harder to come by.

We will proceed in 2 steps. First, we will analyze whether absolute PPP holds for a single but highly familiar good, namely, MacDonald’s Big Mac sandwich. Then, we will look at price data for a larger set of goods from the International Comparison Program.
Q: Does absolute PPP hold for the Big Mac

Compute the real exchange rate for a Big Mac

\[ e_{\text{BigMac}} = \frac{S \times P_{\text{BigMac}^*}}{P_{\text{BigMac}}} \]

\( e_{\text{BigMac}} = 1 \) tells you how many U.S. Big Macs it takes to buy one foreign Big Mac.

If absolute PPP holds, then \( e_{\text{BigMac}} = 1 \).

The next table presents data on the BigMac real exchange rate between the dollar and other currencies.
The Big-Mac Real Exchange Rate January 2015 Between the U.S. Dollar and other Currencies

<table>
<thead>
<tr>
<th>Country</th>
<th>$P_{\text{BigMac}}^*$</th>
<th>$S$</th>
<th>$SP_{\text{BigMac}}^*$</th>
<th>$e_{\text{BigMac}} \equiv \frac{SP_{\text{BigMac}}^*}{P_{\text{BigMacUS}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>6.50</td>
<td>1.16</td>
<td>7.56</td>
<td>1.58</td>
</tr>
<tr>
<td>Norway</td>
<td>48.00</td>
<td>0.13</td>
<td>6.30</td>
<td>1.32</td>
</tr>
<tr>
<td>Brazil</td>
<td>13.50</td>
<td>0.39</td>
<td>5.21</td>
<td>1.09</td>
</tr>
<tr>
<td>Sweden</td>
<td>40.70</td>
<td>0.12</td>
<td>4.97</td>
<td>1.04</td>
</tr>
<tr>
<td>United States</td>
<td>4.79</td>
<td>1.00</td>
<td>4.79</td>
<td>1.00</td>
</tr>
<tr>
<td>France</td>
<td>3.90</td>
<td>1.16</td>
<td>4.53</td>
<td>0.95</td>
</tr>
<tr>
<td>Italy</td>
<td>3.85</td>
<td>1.16</td>
<td>4.48</td>
<td>0.93</td>
</tr>
<tr>
<td>Britain</td>
<td>2.89</td>
<td>1.52</td>
<td>4.38</td>
<td>0.91</td>
</tr>
<tr>
<td>Australia</td>
<td>5.30</td>
<td>0.81</td>
<td>4.31</td>
<td>0.90</td>
</tr>
<tr>
<td>Germany</td>
<td>3.67</td>
<td>1.16</td>
<td>4.27</td>
<td>0.89</td>
</tr>
<tr>
<td>Turkey</td>
<td>9.25</td>
<td>0.43</td>
<td>3.97</td>
<td>0.83</td>
</tr>
<tr>
<td>South Korea</td>
<td>4100.00</td>
<td>0.00</td>
<td>3.78</td>
<td>0.79</td>
</tr>
<tr>
<td>Mexico</td>
<td>49.00</td>
<td>0.07</td>
<td>3.35</td>
<td>0.70</td>
</tr>
<tr>
<td>Argentina</td>
<td>28.00</td>
<td>0.12</td>
<td>3.25</td>
<td>0.68</td>
</tr>
<tr>
<td>Japan</td>
<td>370.00</td>
<td>0.01</td>
<td>3.14</td>
<td>0.66</td>
</tr>
<tr>
<td>China</td>
<td>17.20</td>
<td>0.16</td>
<td>2.77</td>
<td>0.58</td>
</tr>
<tr>
<td>India</td>
<td>116.25</td>
<td>0.02</td>
<td>1.89</td>
<td>0.39</td>
</tr>
<tr>
<td>Russia</td>
<td>89.00</td>
<td>0.02</td>
<td>1.36</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Source: The Economist Magazine.
Observations on the table

- Absolute PPP fails. Big Mac real exchange rates are not equal to unity. They are as high as 1.58 (for Switzerland) and as low as 0.28 (for Russia).
- In 2015 BigMac prices in Switzerland, Norway, Brazil (yes), and Sweden were higher than in the United States.
- Big Mac real exchange rates tend to be high for rich countries and low for poor countries. For example, look at India, the Big Mac real exchange rate is 0.39, which means that for 1 Big Mac in India one can only buy 39 percent of a Big Mac in the United States. Later in this chapter, we will explore in more detail whether absolute prices are lower in poor and emerging countries than in rich countries.
Absolute PPP: Evidence from the International Comparison Program (ICP)

How representative is the failure of absolute PPP suggested by the BigMac real exchange rate? Here we analyze evidence on PPP from a large sample of prices across countries.

The ICP represents the most extensive and thorough effort to measure absolute PPP rates across countries. The ICP was established in the late 1960s on the recommendation of the United Nations Statistical Commission (UNSC). It began as a research project carried out jointly by the United Nations Statistical Office and the University of Pennsylvania. The first comparison, conducted in 1970, covered 10 economies. Now, 40 years later, the ICP is a worldwide statistical operation whose latest comparison—ICP 2011—involved 199 economies. The program is led and coordinated by the ICP Global Office hosted by the World Bank.

The 2011 ICP round collected over 7 million prices from 199 economies in eight regions, with the help of 15 regional and international partners. It is the most extensive effort to measure PPPs ever undertaken.
The ICP reports the real exchange rate, which is referred to as the ‘Price Level Index’

\[ e = \text{PLI} \equiv \frac{SP^*}{P} \]

where now \( P^* \) and \( P \) are actual price levels (and not indices!). Here is what they find for the year 2011 (most recent available):

<table>
<thead>
<tr>
<th>Foreign Country</th>
<th>( e \times 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>100</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>29</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>31</td>
</tr>
<tr>
<td>India</td>
<td>32</td>
</tr>
<tr>
<td>Pakistan</td>
<td>28</td>
</tr>
<tr>
<td>China</td>
<td>54</td>
</tr>
<tr>
<td>Germany</td>
<td>108</td>
</tr>
<tr>
<td>Sweden</td>
<td>136</td>
</tr>
<tr>
<td>Switzerland</td>
<td>162</td>
</tr>
<tr>
<td>Japan</td>
<td>135</td>
</tr>
</tbody>
</table>

What do these numbers mean? Take India, a PLI of 32 means that you can buy a basket that costs $100 dollars in the United States for only 32 dollars in India.

\( \Rightarrow \textbf{Absolute PPP fails.} \)

Over and Undervaluation of a Currency

Let $S_{t}^{ppp}$ denote the PPP exchange rate, defined as the nominal exchange rate that would make PPP hold. That is,

$$S_{t}^{ppp} P_{t}^{i} = P_{t}^{US}$$

This is the nominal exchange rate that would make the real exchange rate equal to one.

If

$$S_{t}^{PPP} > S_{t},$$

then

$$P_{t}^{US} > S_{t} P_{t}^{i}$$

then we say that the dollar is overvalued (and the foreign currency undervalued). In this case, the United States is relatively expensive.

and if $S_{t}^{PPP} < S_{t}$, then

$$P_{t}^{US} < S_{t} P_{t}^{i}$$
and we say the dollar is undervalued (and the foreign currency overvalued). In this case the United States is relatively cheap.

If one believes that in the long run PPP should hold for Big Macs, then one would regard the currencies of Switzerland, Norway, Brazil, and Sweden as overvalued and would expect them to depreciate against the U.S. dollar over time. Similarly, one would expect all the countries with a Big Mac real exchange rate less than one to appreciate against the U.S. dollar over time.

This is an argument sometimes suggested in The Economist Magazine. This week’s homework asks you to evaluate this hypothesis empirically.
Application: PPP Exchange Rates and World Shares in GDP

What is the world share of GDP of high income, middle-income, and low-income economies? Comparisons of the size of economies at market exchange rates tend to overstate the size of rich countries and understate the size of poor countries.

Look at the next chart. It shows that in 2011, middle-income countries produced 32 percent of world GDP at market exchange rates but 48.2 of world GDP at PPP exchange rates. The flip-side of this is that GDP of high-income economies becomes significantly smaller when PPP-based GDPs are used, their share in world GDP falls from 67.3 percent to 50.3 percent. The largest relative difference obtains for low-income countries whose share in world GDP doubles from 0.7 percent when measured at market exchange rates to 1.5 percent when measured at PPP exchange rates.
The income categories are as follows: low income—per capita gross national income (GNI) less than $1,025 (32 countries); middle income—per capita GNI from $1,026 to $12,475 (84 countries); and high income—per capita GNI greater than $12,475 (56 countries).
Is China the largest economy in the world?

Not yet, in 2011, ICP reports that it is the second largest both at PPP exchange rates and at market exchange rates. U.S. GDP at market prices still twice as large as China’s. And to assess economic power, it probably makes more sense to look at GDP at market exchange rates.

<table>
<thead>
<tr>
<th>Rank</th>
<th>GDP (PPP-based) World Share</th>
<th>GDP (market based) World Share</th>
<th>Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.1</td>
<td>22.1</td>
<td>United States</td>
</tr>
<tr>
<td>2</td>
<td>14.9</td>
<td>10.4</td>
<td>China</td>
</tr>
<tr>
<td>3</td>
<td>6.4</td>
<td>2.7</td>
<td>India</td>
</tr>
<tr>
<td>4</td>
<td>4.8</td>
<td>8.4</td>
<td>Japan</td>
</tr>
<tr>
<td>5</td>
<td>3.7</td>
<td>5.2</td>
<td>Germany</td>
</tr>
</tbody>
</table>

Data Source: Table 7.1 of "Purchasing Power Parities and Real Expenditures of World Economies, Summary of Results and Findings of the 2011 International Comparison Program."
Application: PPP Exchange Rates and Standard of Living Comparisons

• standard of living comparisons are tricky because of differences in relative prices.

• comparisons of per capita income (in $) tend to overstate the differences in real purchasing power between rich and poor countries because rich countries are systematically more expensive than poor countries.
Higher Prices in Rich Countries

Data Source: ICP, 2011.
GDP per capita comparisons at such huge differences in relative prices cause important differences in the basic measurements of real incomes and real standards of living.

<table>
<thead>
<tr>
<th>Country</th>
<th>Per Capita GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US$</td>
</tr>
<tr>
<td>United States</td>
<td>49,782</td>
</tr>
<tr>
<td>India</td>
<td>1,533</td>
</tr>
<tr>
<td>US/India</td>
<td>32</td>
</tr>
</tbody>
</table>

At market exchange rates GDP per capita in the United States in 2011 was 32 times as large as that of India. However, at PPP exchange rate U.S. per capita GDP was only 11 times as large as that of India.

⇒ $1,533 can buy 3 times as many goods in India (at Indian prices) than it can in the U.S. at U.S. prices.
Why does Absolute PPP fail?

One reason is that many goods are not traded internationally, and hence price discrepancies will not be arbitraged away via trade.

The price index is an average of all prices in the economy, traded goods prices, $P_T$, and nontraded goods prices, $P_N$

$$P = \phi(P_T, P_N)$$

Example:

$$P = (P_T)^\alpha(P_N)^{1-\alpha}$$

Suppose the LOOP holds for traded goods:

$$P_T = SP_T^*$$
but not for nontraded goods

\[ P_N \neq SP_N^* \]

Suppose the foreign price level, \( P^* \), is constructed as

\[ P^* = \phi(P_T^*, P_N^*) \]

The real exchange rate then is:

\[
e = \frac{SP^*}{P} = \frac{S\phi(P_T^*, P_N^*)}{\phi(P_T, P_N)} = \frac{SP_T^* \phi(1, P_N^*/P_T^*)}{P_T \phi(1, P_N/P_T)} = \frac{\phi(1, P_N^*/P_T^*)}{\phi(1, P_N/P_T)}.
\]

(1)
If the relative price of nontradables is higher in the foreign country, then the real exchange rate is greater than 1,

\[
\text{if } \frac{P_N^*}{P_T^*} > \frac{P_N}{P_T}, \text{ then } e > 1.
\]
Determinants of the Real Exchange Rate in the Medium Term:

The Balassa-Samuelson Model
Recall that:

\[ e = \frac{\phi \left( 1, \frac{P_N^*}{P_T^*} \right)}{\phi \left( 1, \frac{P_N}{P_T} \right)} \]

From here clear that the real exchange rate depreciates if \( \frac{P_N^*}{P_T^*} \) increases relative to \( \frac{P_N}{P_T} \).

Q: What could make \( \frac{P_N^*}{P_T^*} \) go up relative to \( \frac{P_N}{P_T} \)?

A: Productivity growth in the traded sector relative to the non-traded sector in the foreign country being faster than in the domestic country.
The Balassa-Samuelson effect is the tendency for countries with higher productivity growth in tradables compared to nontradables to have higher prices (and hence appreciated real exchange rates.

The Model

2 goods: $Q_T$ and $Q_N$

$Q_T$ = traded output
$Q_N$ = nontraded output
Production

Production of Tradables:

\[ Q_T = a_T L_T \]

Production of Nontradables:

\[ Q_N = a_N L_N \]

\( L_T \) = labor input in the traded sector
\( L_N \) = labor input in the nontraded sector
\( a_T \) = exogenous labor productivity in the traded sector
\( a_N \) = exogenous labor productivity in the nontraded sector
\( W \) = wage rate
Traded Goods Sector
Firms choose $Q_T$ and $L_T$ to maximize profits

$$\text{profits} = P_T Q_T - W L_T.$$ 

subject to

$$Q_T = a_T L_T.$$ 

Eliminate $Q_T$

$$\text{profits} = P_T a_T L_T - W L_T.$$ 

Choose $L_T$ to maximize profits

$$\frac{\partial \text{profits}}{\partial L_T} = 0 \Rightarrow P_T a_T = W \quad (*)$$
Nontraded Goods Sector
Firms choose $Q_N$ and $L_N$ to maximize profits

\[ \text{profits} = P_N Q_N - W L_N. \]

subject to

\[ Q_N = a_N L_N. \]

Eliminate $Q_N$

\[ \text{profits} = P_N a_N L_N - W L_N \]

Choose $L_N$ to maximize profits

\[ \frac{\partial \text{profits}}{\partial L_N} = 0 \Rightarrow P_N a_N = W \]  \hfill (**)
Combining (*) with (**) yields

\[
\frac{P_N}{P_T} = \frac{a_T}{a_N}
\]

That is, the Balassa-Samuelson model predicts that in equilibrium the relative price of nontradables in terms of tradables is inversely related to the ratio of labor productivity in the traded sector to that in the nontraded sector.

Is this prediction of the Balassa-Samuelson model borne out in the data?
Take natural logarithms of (2) and consider changes over time:

$$\%\Delta \left( \frac{P_N}{P_T} \right) = \%\Delta a_T - \%\Delta a_N$$

This expression says that the percent change in the relative price of nontradables is equal to the growth rate differential between factor productivity in the traded sector and the nontraded sector.

We wish to test whether this relationship holds over the long run in actual data.
De Gregorio, Giovannini, and Wolf (EER, 1994) collect data on \(\%\Delta \left( \frac{P_N}{P_T} \right)\) and on \(\%\Delta a_T - \%\Delta a_N\) for 14 OECD countries over the period 1970-1985.

That is they have 14 observations.

The next slide shows what they find.
Differential Factor Productivity Growth and Changes in the Relative Price of Nontradables

Note: The figure plots the average annual percentage change in the relative price of nontradables in terms of tradables, \( \% \Delta \left( \frac{P_N}{P_T} \right) \), (vertical axis) against the average annual growth in total factor productivity differential between the traded sector and the nontraded sector, \( \% \Delta a_T - \% \Delta a_N \), (horizontal axis) over the period 1970-1985 for 14 OECD countries. Source: José De Gregorio, Alberto Giovannini, and Holger C. Wolf, “International Evidence on Tradable and Nontradable Inflation,” European Economic Review 38, June 1994, 1225-1244.
Comments on the figure

- If Balassa-Samuelson model is true, then all 14 observations should line up on the 45 degree line. This is not quite the case.

- Still the figure demonstrates that 15-year averages for OECD countries display a positive relationship between $\%\Delta \left(\frac{P_N}{P_T}\right)$ and $\%\Delta \left(\frac{a_T}{a_N}\right)$, as predicted by the Balassa Samuelson model.
What are the predictions of the Balassa-Samuelson Model for the Behavior of the Real Exchange Rate?

A relationship like equation (2) must also hold in the foreign country

\[
\frac{P_N^*}{P_T^*} = \frac{a_T^*}{a_N^*}
\]

where

\(P_T^*\) = foreign currency price of traded goods abroad

\(P_N^*\) = foreign currency priced of nontraded goods abroad

\(a_T^*\) = exogenous labor productivity in the traded sector abroad

\(a_N^*\) = exogenous labor productivity in the nontraded sector abroad
Thus the Balassa Samuelson model predicts that,

\[
e = \frac{\phi \left( 1, \frac{P_N^*}{P_T^*} \right)}{\phi \left( 1, \frac{P_N}{P_T} \right)}
\]

\[
e = \frac{\phi \left( 1, \frac{a_T^*}{a_N^*} \right)}{\phi \left( 1, \frac{a_T}{a_N} \right)}
\]

Hence \( e \uparrow \), if \( \frac{a_T^*}{a_N^*} \) increases relative to \( \frac{a_T}{a_N} \), that is, the real exchange rate of the domestic country depreciates if relative productivity growth in the traded sector relative to productivity growth in the nontraded sector is faster in the foreign country than in the domestic country.
Suppose the price index is given by

$$\phi(P_T, P_N) = P_T^{1-\alpha} P_N^\alpha, \quad \alpha \in (0, 1)$$

then the Balassa Samuelson model predicts that

$$\% \Delta e = \alpha \left[ \% \Delta \left( \frac{a^*_T}{a^*_N} \right) - \% \Delta \left( \frac{a_T}{a_N} \right) \right]$$

This prediction can be confronted with data. What is needed? Observations of growth rates of technology in the traded and nontraded sector and observations on real exchange rate changes.
Can the Balassa Samuelson model explain the observed real depreciation of the German mark against the Japanese Yen and against the Italian Lira in the 1970s and 1980s?

Canzoneri, Cumby, and Diba (JIE, 1999) collect data on productivity differential for the United States, Germany, Italy and Japan over the period 1970 to 1993.

Consider the bilateral real exchange rate between the German Mark (DM) and the Italian Lira (£): According to the Balassa Samuelson model we have

\[
\%\Delta e^{DM/£} = \alpha \left[ \%\Delta \left( \frac{a_I}{a_N} \right) - \%\Delta \left( \frac{a_N}{a_I} \right) \right]
\]
Over the long run, here 1970 to 1993, Balassa Samuelson explains well the observed real depreciation of the German mark against the Italian Lira. For it shows that the relative growth rate of labor productivity in the traded sector, relative to the nontraded sector, was higher in Italy than in Germany.

These authors, however, also present evidence that the Balassa Samuelson model fails to explain the observed real appreciation of the Japanese yen against the U.S. dollar (not shown).

What to make of this? In some episodes long-run changes in real exchange rates can be explained well by differences in relative productivity growth rates but not always. This is not necessarily evidence against Balassa Samuelson because clearly there can be other explanations for real exchange rate movements.

We will turn to such alternative explanations next.
Add discussion on effects of trade barriers on real exchange rates
Add discussion on micro foundation of price indices.