Princeton University Press, 2017

slides

chapter 9

nominal rigidity

exchange rates, and

unemployment

Introduction

• Chapter 9 develops a theoretical framework in which nominal rigidities result in inefficient adjustment to aggregate disturbances

• The framework can be used in an intuitive graphical manner to demonstrate how nominal rigidities amplify the business cycle in open economies

• The framework can also be used to derive quantitative predictions useful for policy evaluation

Some Motivation: Peripheral Europe and the Global Crisis of 2008

Take a look at the next slide.

• The inception of the Euro in 1999 was followed by massive capital inflows into the region, possibly driven by expectations of quick convergence of peripheral and core Europe.

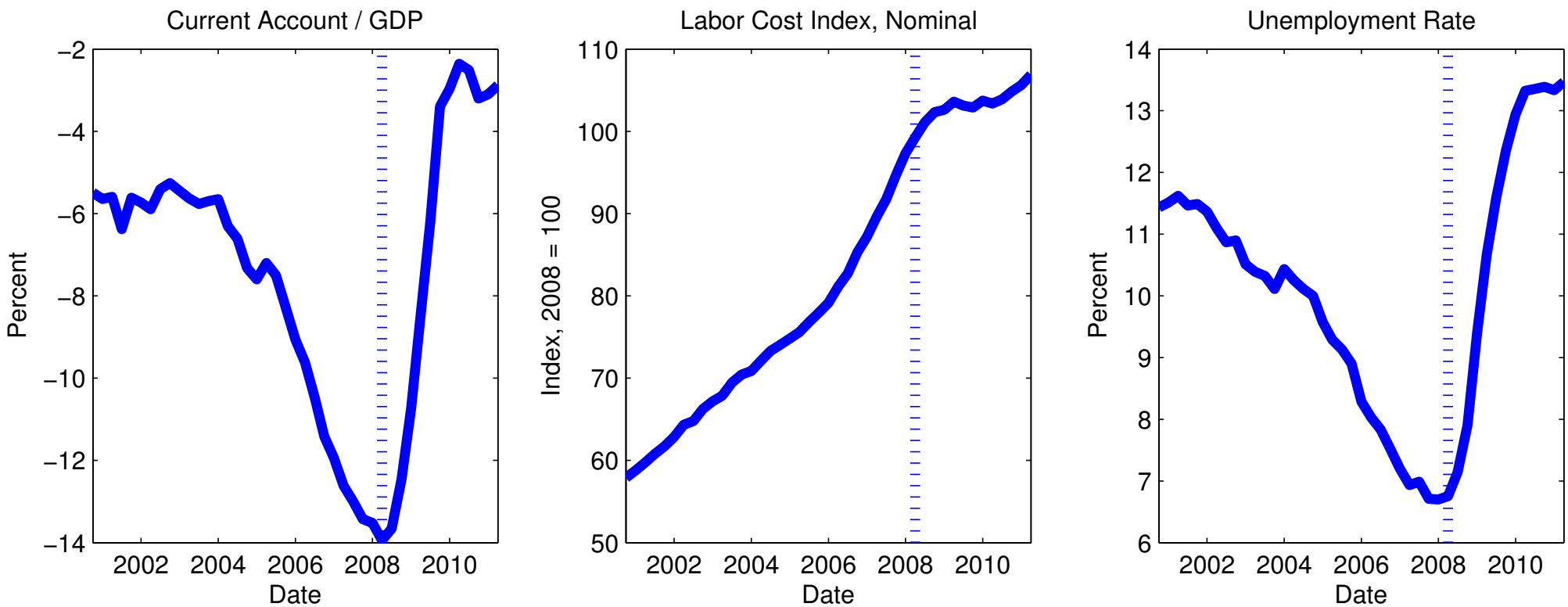
• Large current account deficits and large increases in nominal hourly wages, with declining rates of unemployment between 2000 and 2008.

• When the global financial crisis of 2008 starts, capital inflows dry up abruptly. Peripheral Europe suffers a severe **sudden stop** (sharp reductions in current account deficits).

• In spite of the collapse in aggregate demand and the lack of a devaluation, nominal hourly wages remain as high as at the peak of the boom.

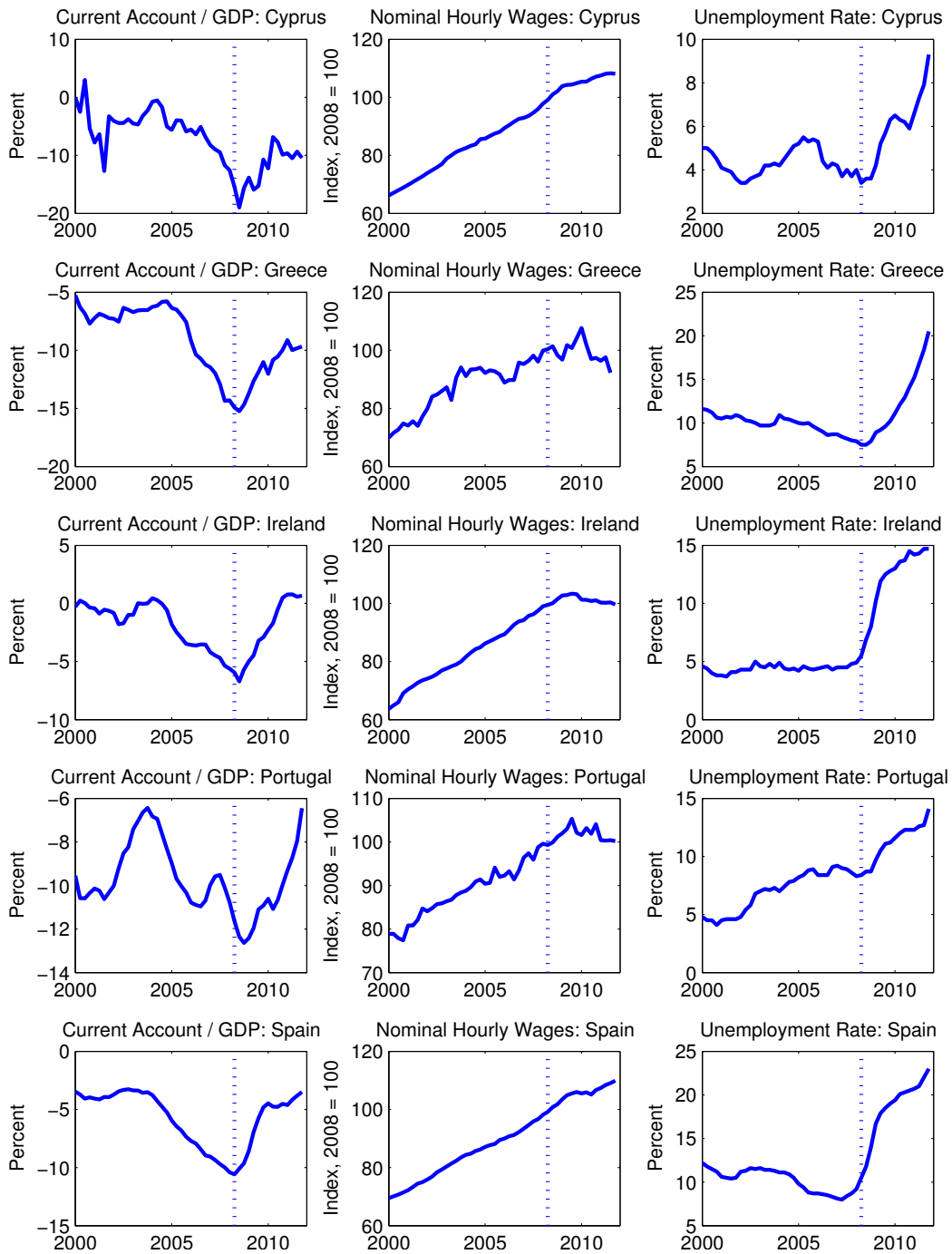
• Massive unemployment affects all countries in the region.

Figure 9.1 Boom-Bust Cycle in Peripheral Europe: 2000-2011



Data Source: Eurostat. Labor Cost Index, Nominal, is the nominal hourly wage rate in manufacturing, construction and services (including the public sector, but for Spain.)

Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.



The Disaggregated Story: Boom-Bust Cycles in Cyprus, Greece, Ireland, Portugal, and Spain.

The previous two figures suggest the following narrative:

Countries in the periphery of the European Union, such as Ireland, Portugal, Greece, and a number of small eastern European countries adopted a fixed exchange rate regime by joining the Euroarea. Most of these countries experienced an initial transition into the Euro characterized by low inflation, low interest rates, and economic expansion.

However, history has shown time and again that fixed exchange rate arrangements are easy to adopt but difficult to maintain. (Example: Argentina's 1991 convertibility plan.)

The Achilles' heel of currency pegs is that they hinder the efficient adjustment of the economy to negative external shocks, such as drops in the terms of trade or hikes in the interest-rate. Such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency, in order to bring about an expenditure switch away from tradables and toward nontradables. In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both.

The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables. However, when nominal wages are downwardly rigid, producers of nontradables are reluctant to lower prices, for doing so might render their enterprises no longer profitable. As a result, the necessary real depreciation takes place too slowly, causing recession and unemployment along the way.

This narrative goes back at least to Keynes (1925) who argued that Britain's 1925 decision to return to the gold standard at the 1913 parity despite the significant increase in the aggregate price level that took place during World War I would force deflation in nominal wages with deleterious consequences for unemployment and economic activity. Similarly, Friedman's (1953) seminal essay points at downward nominal wage rigidity as the central argument against fixed exchange rates.

To formalize this narrative let's build an open economy model with

- downward nominal wage rigidity
- a traded and a nontraded sector
- involuntary unemployment

To produce quantitative predictions

- Estimate the key parameters of the model (with particular attention to the parameter governing downward wage rigidity) and estimate the driving forces.
- Characterize response to large negative external shocks under a peg and show that the model can explain the observed effects of sudden stops.
- Characterize optimal exchange rate policy.
- Quantify the costs of currency pegs in terms of unemployment and welfare.

The material is based on Schmitt-Grohé and Uribe (JPE, 2016).

9.1 An Open Economy with Downward Nominal Wage Rigidity

(The DNWR Model)

Overview of the Model

• Downward Nominal Wage Rigidity: $W_t \geq \gamma\, W_{t-1}$, $W_t =$ nominal wage rate in period t, $\gamma =$ degree of downward wage rigidity.
• Stochastic endowment of tradable goods:* y_t^T.
• Stochastic country interest rate: r_t.
• Nontraded goods, y_t^N, produced with labor, h_t: $y_t^N = F(h_t)$.
• Law of one price holds for tradables: $P_t^T = \mathcal{E}_t P_t^*$.
• P_t^T, nominal price of tradable goods.
• $\mathcal{E}_t$, nominal exchange rate, domestic-currency price of one unit of foreign currency ($\mathcal{E}_t \uparrow$ depreciation of domestic currency).
• P_t^*, foreign currency price of tradable goods.
• Assume that $P_t^* = 1$, so that $P_t^T = \mathcal{E}_t$

*Section 9.14 considers a production economy.

9.1.1 Households

$$\max_{\{c_t^T, c_t^N, d_{t+1}\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{9.1}$$

subject to

$$c_t = A(c_t^T, c_t^N) \tag{9.2}$$

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t d_t = P_t^T y_t^T + W_t h_t + \mathcal{E}_t \frac{d_{t+1}}{1 + r_t} + \Phi_t \tag{9.3}$$

$$h_t \leq \bar{h} \tag{9.7}$$

• First constraint: Consumption is a composite of traded and non-traded goods. $A(.,.)$ increasing, concave, and HD1.
• Second constraint: d_t = one-period debt chosen in t, due in $t+1$. Debt is denominated in units of foreign currency $\rightarrow$ full liability dollarization. $\rightarrow$*Original Sin:* In emerging countries almost 100% of external debt issued in foreign currency (Eichengreen, Hausmann, and Panizza, 2005). Country interest rate, r_t, is stochastic.
• Third constraint: Workers supply $\bar{h}$ hours inelastically,* but may not be able to sell them all. They take $h_t \leq \bar{h}$ as given.

*Section 9.13 relaxes this assumption.

Optimality Conditions Associated with the Household Problem

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t \tag{9.5}$$

$$\lambda_t = U'(A(c_t^T, c_t^N))A_1(c_t^T, c_t^N)$$

$$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1}$$

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t d_t = P_t^T y_t^T + W_t h_t + \mathcal{E}_t \frac{d_{t+1}}{1 + r_t} + \Phi_t$$

$$h_t \leq \bar{h}$$

The Demand For Nontradables

Look again at the optimality condition (9.5)

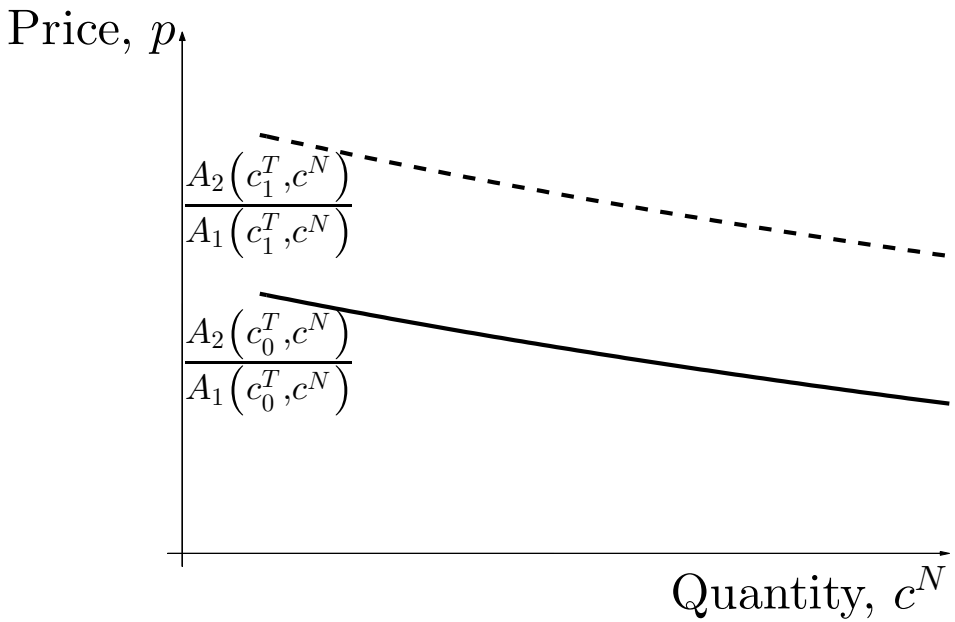
$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t. \tag{9.5}$$

If $A(c^T, c^N)$ is concave and HD1, then given c_t^T, the left-hand side is decreasing in c_t^N. This means that, all other things equal, an increase in p_t reduces the desired demand for nontradables, giving rise to the downward sloping demand schedule shown in the next slide.

Note that c_t^T acts as a shifter of the demand schedule for nontradables: given p_t, an increase in c_t^T is associated with an equiproportional desired increase in c_t^N. Of course, this shifter is endogenously determined.

Figure 9.2 The Demand For Nontradables

$$p_t = \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} \qquad (9.5)$$



- here we treat c_t^T as a shifter of the demand schedule.
- An increase in c_t^T from c_0^T to $c_1^T > c_0^T$, shifts the demand schedule up and to the right.

9.1.2 Firms

Nontraded output, y_t^N, is produced by perfectly competitive firms. Profits, Φ_t:

$$\Phi_t = P_t^N F(h_t) - W_t h_t$$

P_t^N, nominal price of nontradables.

Firms maximize profits taking as given P_t^N and W_t. Optimality Condition:

$$P_t^N F'(h_t) = W_t$$

Divide by $P_t^T = \mathcal{E}_t$ and rearrange

$$p_t = \frac{W_t/\mathcal{E}_t}{F'(h_t)} \tag{9.16}$$

$p_t \equiv \frac{P_t^N}{P_t^T}$, relative price of nontradables in terms of tradables. Interpret this optimality conditions as a supply schedule for nontradables, see next slide.

The Supply Schedule of Nontradables

Let's derive the supply schedule for nontradables in the space (y^N, p) given the real wage, $W_t/\mathcal{E}_t$.

Note that real marginal cost of one unit of nontraded good

$$\text{marginal cost} = \frac{W_t/\mathcal{E}_t}{F'(h_t)}$$

Use $h = F^{-1}(y^N)$ to obtain

$$\text{marginal cost} = \frac{W_t/\mathcal{E}_t}{F'(F^{-1}(y^N))}$$

By the profit maximization condition marginal cost equals price or

$$p_t = \frac{W_t/\mathcal{E}_t}{F'(F^{-1}(y_t^N))}$$

Interpret this relation as a supply schedule of nontradables given the real wage.

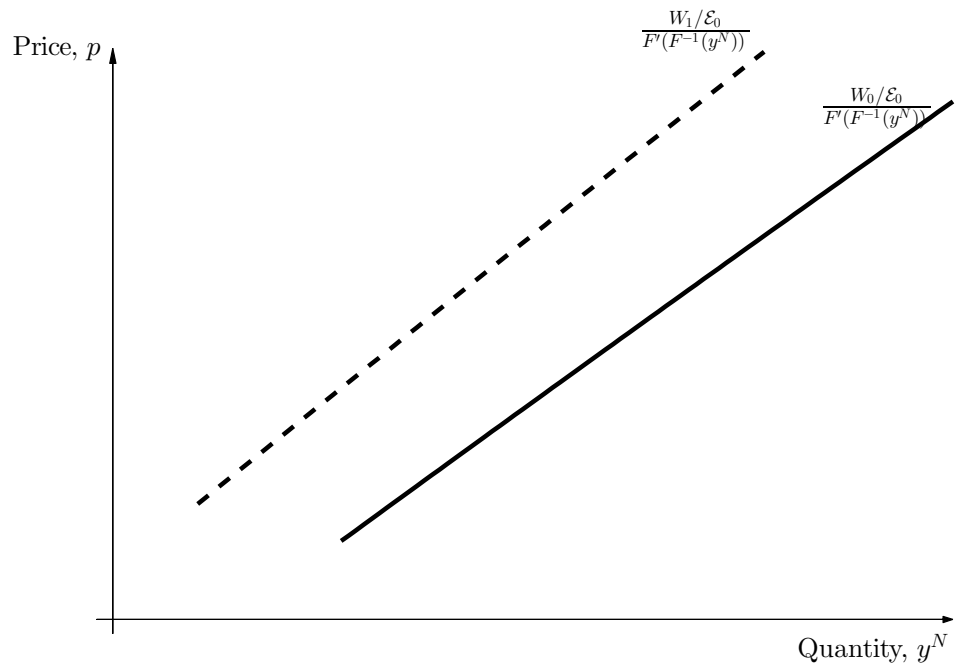
Figure 9.3 The Supply Of Nontradables

Supply schedule: $p_t = \frac{W_t/\mathcal{E}_t}{F'(F^{-1}(y_t^N))}$

Properties:
– upward sloping, the higher the price, the more a firm wishes to produce, given factor prices.

– A decrease in nominal wage from W_1 to $W_0 < W_1$ shifts the supply schedule down and to the right.

– A devaluation $\mathcal{E}_t \uparrow$ (not shown) shifts the supply schedule in the same manner as a nominal wage cut.



9.1.3 Downward Nominal Wage Rigidity and the Labor Market

Nominal wages are downwardly rigid

$$W_t \geq \gamma W_{t-1} \tag{9.6}$$

Labor demand may not exceed supply

$$h_t \leq \bar{h} \tag{9.7}$$

Impose the following slackness condition:

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0 \tag{9.8}$$

This slackness condition says that, if there is involuntary unemployment ($h_t < \bar{h}$), then the lower bound on nominal wages must be binding. It also says that if the lower bound on nominal wages is not binding ($W_t > \gamma W_{t-1}$), then the labor market must feature full employment.

9.1.4 Equilibrium

Market clearing in the nontraded sector:

$$c_t^N = y_t^N = F(h_t)$$

The (gross) devaluation rate

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$$

If $\epsilon_t > 1$, then the domestic currency depreciates. And if $\epsilon_t < 1$, then the domestic currency appreciates.

A competitive equilibrium is a set of stochastic processes $\{c_t^T,\ h_t,\ w_t,\ d_{t+1},\ p_t,\ \lambda_t\}_{t=0}^{\infty}$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t} \tag{9.9}$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)) \tag{9.13}$$

$$\frac{\lambda_t}{1+r_t} = \beta E_t \lambda_{t+1} \tag{9.14}$$

$$p_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} \tag{9.15}$$

$$p_t = \frac{w_t}{F'(h_t)} \tag{9.16}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t} \tag{9.17}$$

$$h_t \leq \bar{h} \tag{9.18}$$

$$(\bar{h} - h_t)\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) = 0 \tag{9.19}$$

given an exchange rate policy $\{\epsilon_t\}_{t=0}^{\infty}$, initial conditions w_{-1} and d_0, and exogenous stochastic processes $\{r_t,\ y_t^T\}_{t=0}^{\infty}$.

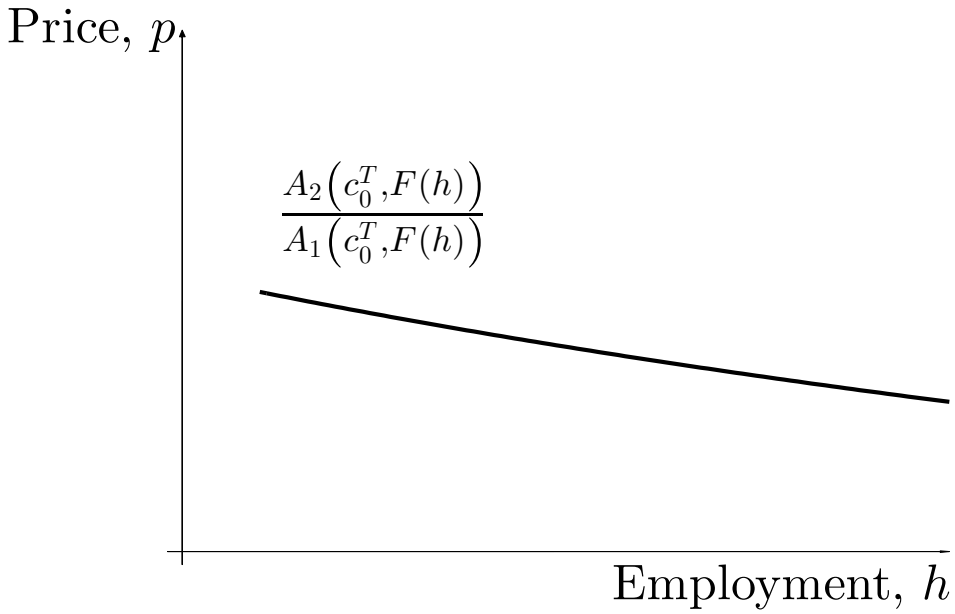
To characterize the eqm we must specify the exchange-rate regime. We will turn to this next.

A Graphical Representation of (Partial) Equilibrium

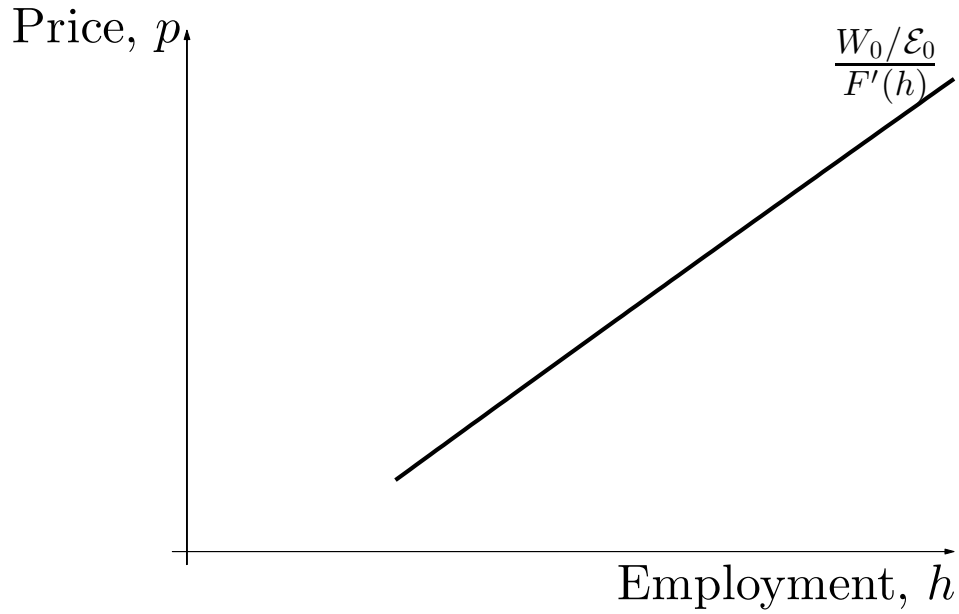
Why partial, because for the moment we take c^T as given.

In equilibrium, $c^N = y^N = F(h)$. This means that we can draw Figures 9.2 and 9.3 in the employment-relative price of nontradables space, that is, in the space (h, p)

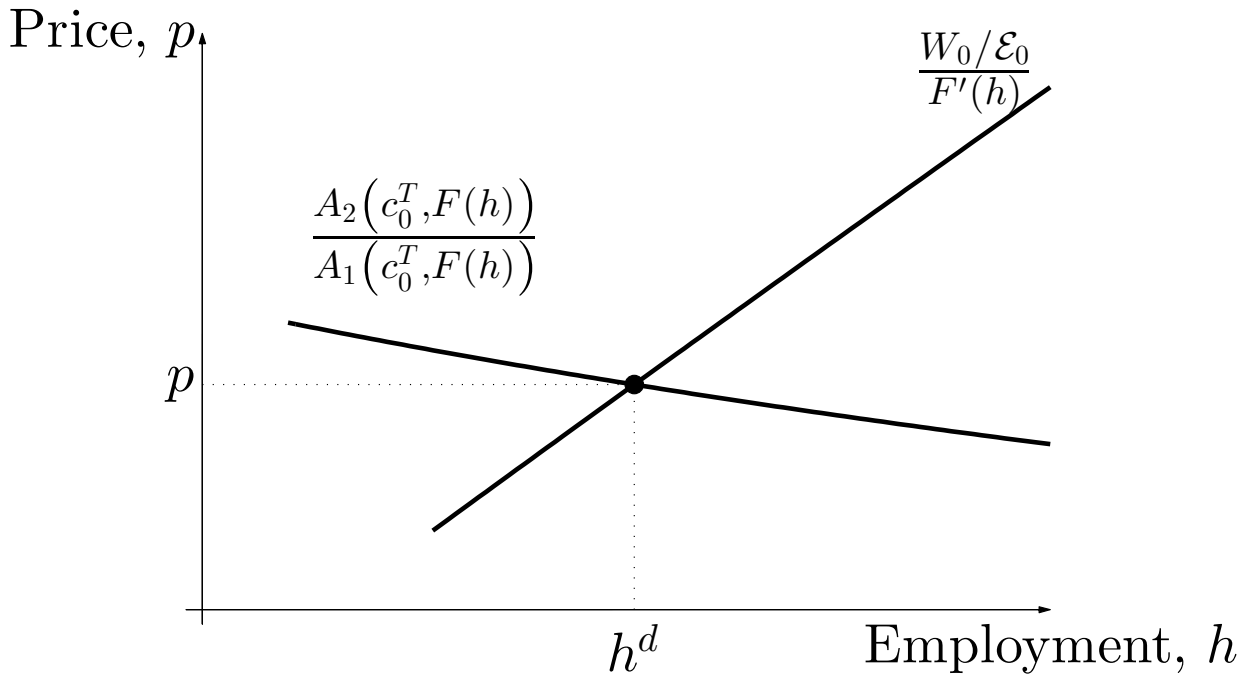
The Demand for Nontradables, (9.15)



The Supply of Nontradables, (9.16)

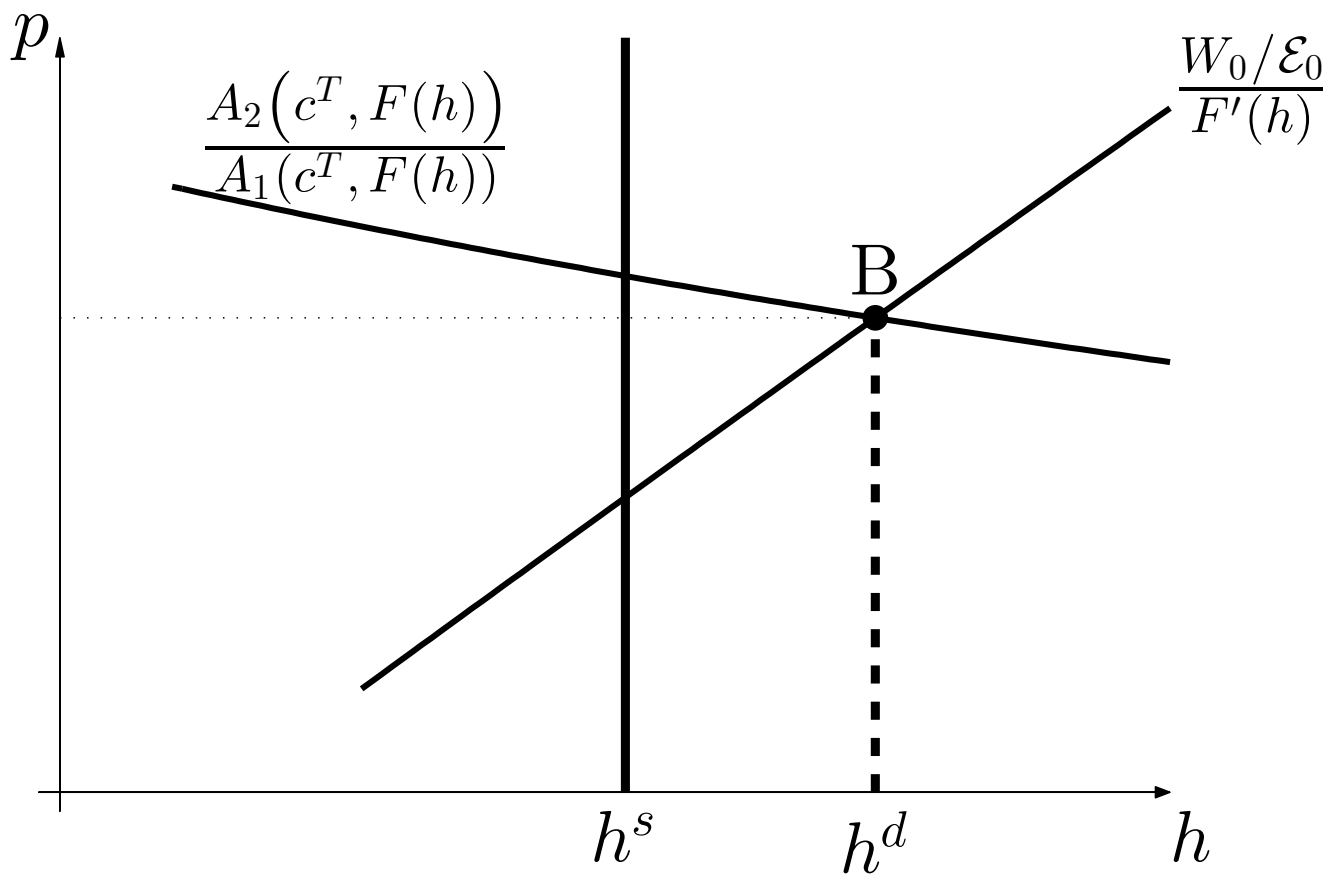


In eqm both (9.15) and (9.16) must hold. We refer to the value of h at which these schedules intersect for given $W_0/\mathcal{E}_0$ and c_0^T as labor demand, and denote it h^d

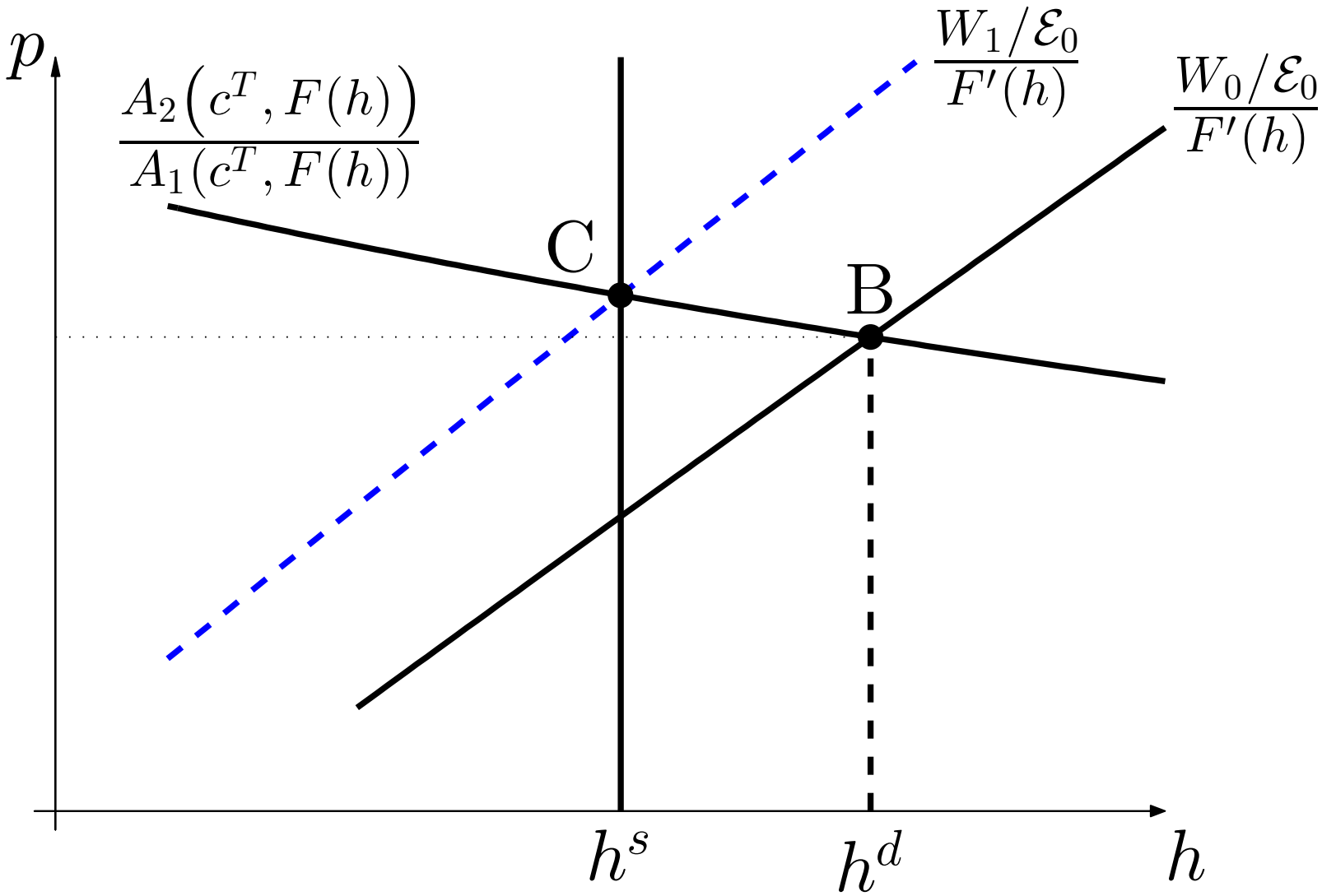


Next add the labor supply schedule to the graph. Two generic cases emerge: at given real wages, $W_0/\mathcal{E}_0$, labor demand exceeds labor supply, or labor supply exceeds labor demand. Let's consider the first case first.

Case 1: Suppose given $W_0/\mathcal{E}_0$, labor demand exceeds labor supply: $h^d > h^s$

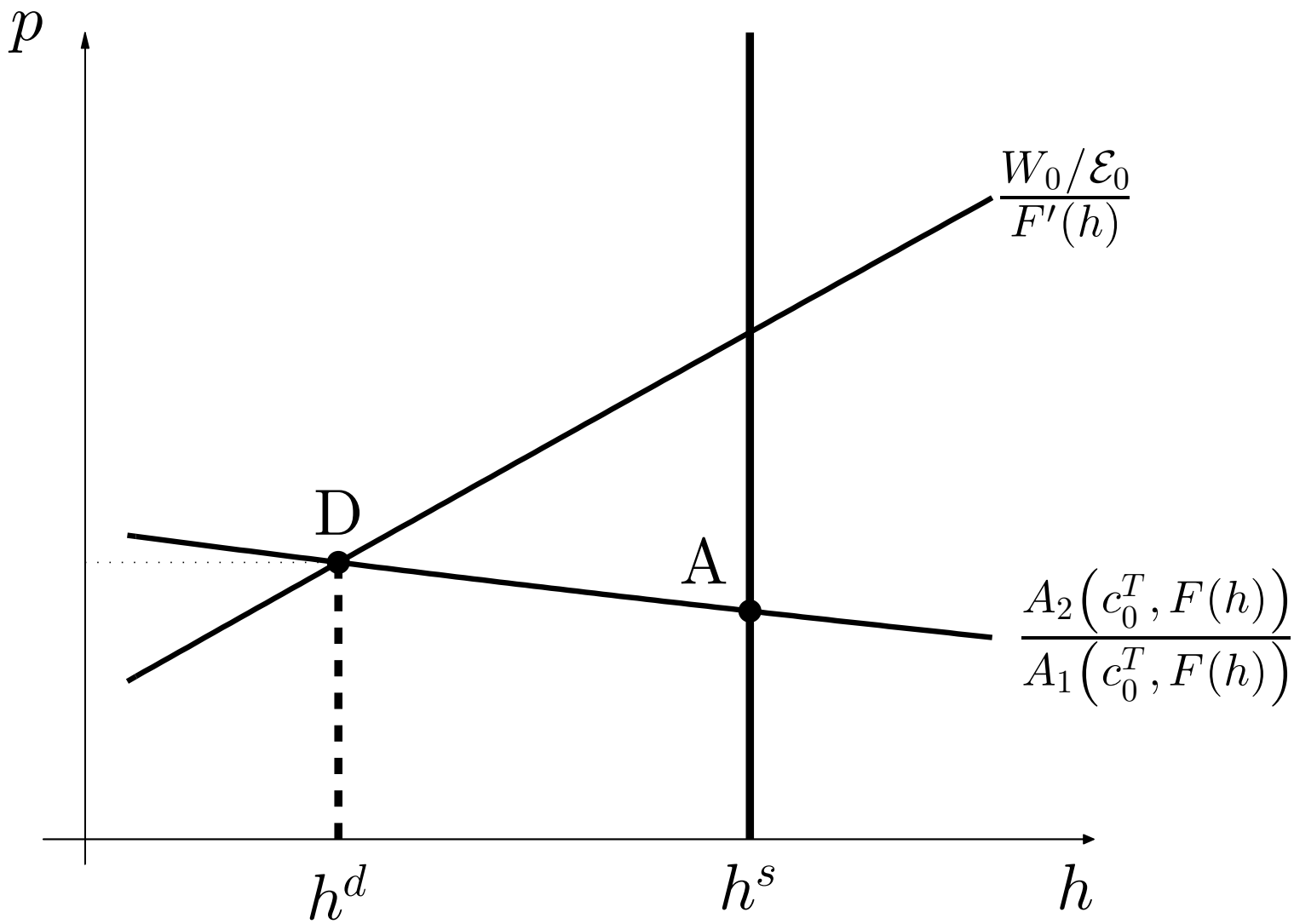


$\Rightarrow$ real wage rises to clear the labor market, $W_0 \uparrow$, until $h^d = h^s$, as shown in the next slide.



In eqm (point C), the nominal wage rises from W_0 to $W_1 > W_0$ until there is full employment, $h^s = h^d$.

Case 2: Now suppose given $W_0/\mathcal{E}_0$, labor supply exceeds labor demand, $h^s > h^d$ (point D); Full employment woud occurr at A.



$\Rightarrow$ to clear the labor market the real wage must fall, but nominal wages cannot fall due to DNWR (say, $\gamma = 1$). Thus, unless the monetary authority devalues, $\mathcal{E} \uparrow$, the labor market will fail to clear. Hence, in eqm, absent a devaluation there is unemployment. How much? $h^s - h^d$.

In the following sections, we will use this graphical framework (as well as quantitative methods) to analyze adjustments to shocks under alternative monetary arrangements.

9.2 Currency Pegs

The exchange rate policy is a peg

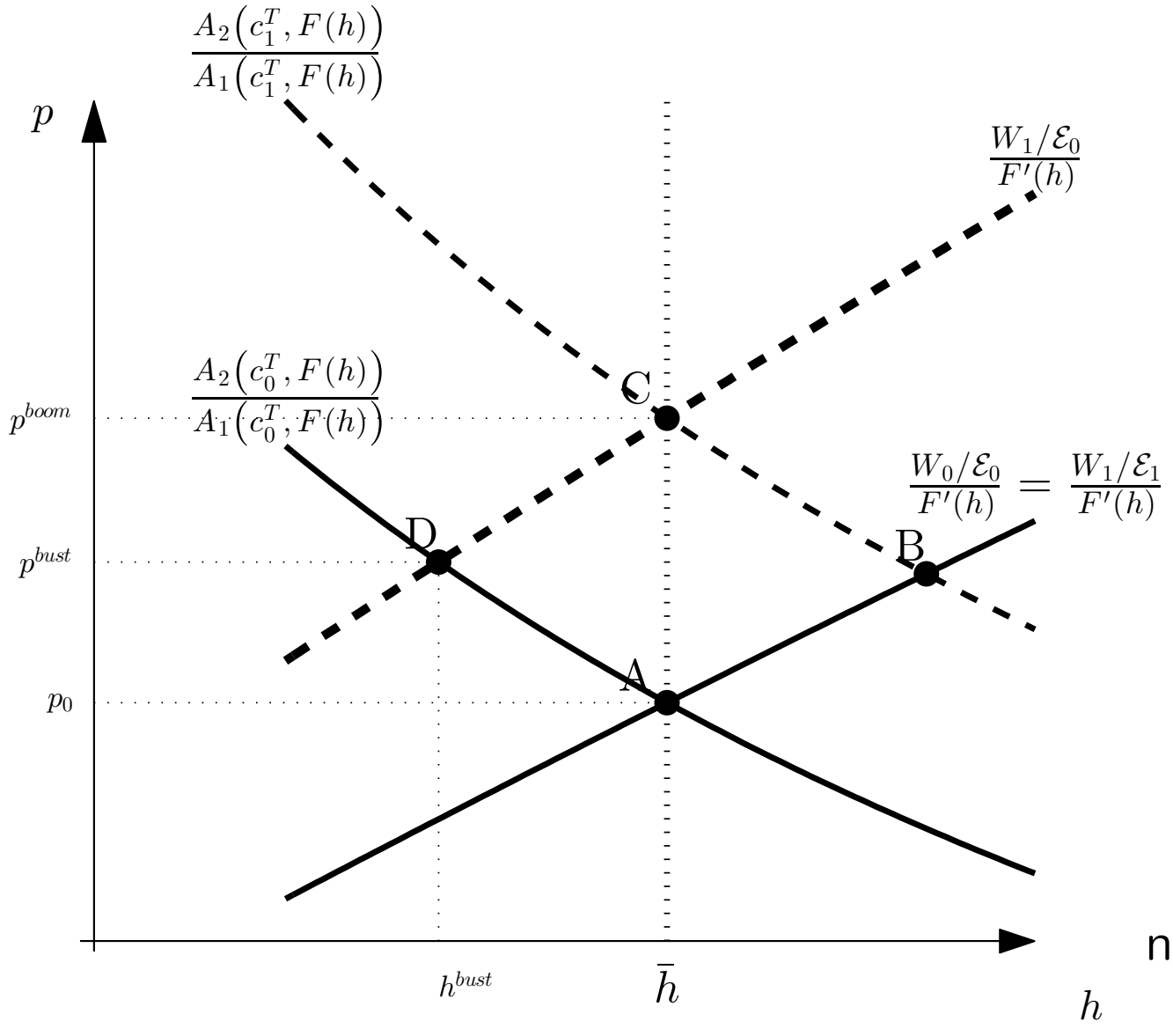
$$\mathcal{E}_t = \mathcal{E}_0; \quad \forall t \geq 0.$$

Use the graphical apparatus just developed to show that a **boom-bust cycle** leads—as documented in Figure 9.1—to

– nominal wage growth and real appreciation during the boom phase

– involuntary unemployment and insufficient real depreciation during the bust phase

Figure 9.4 Adjustment to a Boom-Bust Cycle under a Currency Peg



$c_1^T < c_0^T$

negative external shock

possibly caused by $r_t \uparrow$

Observations on Figure 9.4: Adjustment in a Boom-Bust Episode

The initial situation is point A. At point A there is full employment, $h = \bar{h}$.

Now a boom starts. We capture this by an increase in c^T (perhaps because r falls). Given nominal wages the economy moves to point B. But at point B, there is excess demand for labor. Thus nominal wages will rise. By how much? Until the excess demand for labor has disappeared. That will be at point C. Thus the boom leads to an increase in nominal wages ($W \uparrow$) and a real appreciation ($p \uparrow$). The economy continues to operate at full employment.

Next the boom is over and the bust comes. We capture this by assuming that c^T falls back to its original level, c_0^T.

This shifts the demand for nontradables back to its original position. The new intersection between supply and demand is at point D. At D, labor supply exceeds labor demand. However, because nominal wages are downwardly rigid and the nominal exchange rate is fixed, the supply schedule does not shift, (for simplicity, in the figure, we assume $\gamma = 1$). Thus the economy is stuck at point D. At point D, there is involuntary unemployment ($\bar{h} - h^{bust}$) and there is insufficient real depreciation, i.e., p_t does not fall enough (that is, does not fall to p_0) to restore full employment.

9.2.1 The Peg-Induced Externality

Under a currency peg and downward nominal wage rigidity, a good shock, in the example that follows a fall in the country interest rate r_t, can be the prelude to bad things to happen later. The reason is that individual agents do not internalize that during the boom nominal wages increase too much, putting the economy in a vulnerable position once the good shock fades away.

In Chapter 10, we show that if agents internalize the effects of their current choices on wages and hence on future employment, they will choose to respond less to an expansionary shock (for example, consume less traded goods in response to an interest rate decline) in order to avoid future unemployment. In that chapter, we present the constrained efficient allocation of a social planner who internalizes the wage rigidity and compare it to the competitive equilibrium. We show that a trade-off between an inefficient intertemporal allocation and unemployment emerges.

9.2.3 Adjustment to a Temporary Fall in the Interest Rate

To illustrate the source of the peg-induced externality, consider the following analytical example.

$$U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$$

$$F(h_t) = h_t^\alpha; \quad 0 < \alpha < 1$$

$$\bar{h} = 1; \quad y_t^T = y^T > 0; \quad \gamma = 1; \quad \beta(1+r) = 1; \quad d_0 = 0; \quad w_{-1} = \alpha y^T$$

$$r_t = \begin{cases} r & t > 0 \\ \underline{r} < r & t = 0 \end{cases}$$

Example of Peg-Induced Externality (Continued)

In a couple of slides, you'll find the solution of the equilibrium in algebraic and graphical form. To help the interpretation of those slides, it is of use to discuss intuitively what is going on in this economy:

— The fall in the interest rate in period 0 induces an expansion in the desired demand for consumption goods, of all types, tradables and nontradable.

— The increased demand for tradables causes a trade balance deficit, a deficit in the current account, and an increase in external debt in period 0.

— The increased demand for nontradables causes a rise in wages and a rise in the relative price of nontradables (i.e., an appreciation of the real exchange rate).

Example of the Peg-Induced Externality Continued

— In period 1, the interest rate goes back up to its permanent value r, causing a contraction in the demand for consumption goods (both tradables and nontradables), and a reversal in the trade balance and the current account.

— The contraction in the demand for nontradables causes a derived contraction in the demand for labor. However, because nominal wages are downwardly rigid and the nominal exchange rate is fixed, the real wage fails to fall, causing involuntary unemployment.

— Involuntary unemployment is highly persistent. (In fact, in this example, because $\gamma = 1$, it never disappears.)

Example with Peg-Induced Externality Continued

The equilibrium has the following closed-form solution:

$$c_0^T = y^T \left[\frac{1}{1+\underline{r}} + \frac{r}{1+r} \right] > y^T$$

$$c_t^T = y^T \left[\frac{1}{1+r} + \frac{r}{1+r} \frac{1+\underline{r}}{1+r} \right] < y^T,$$

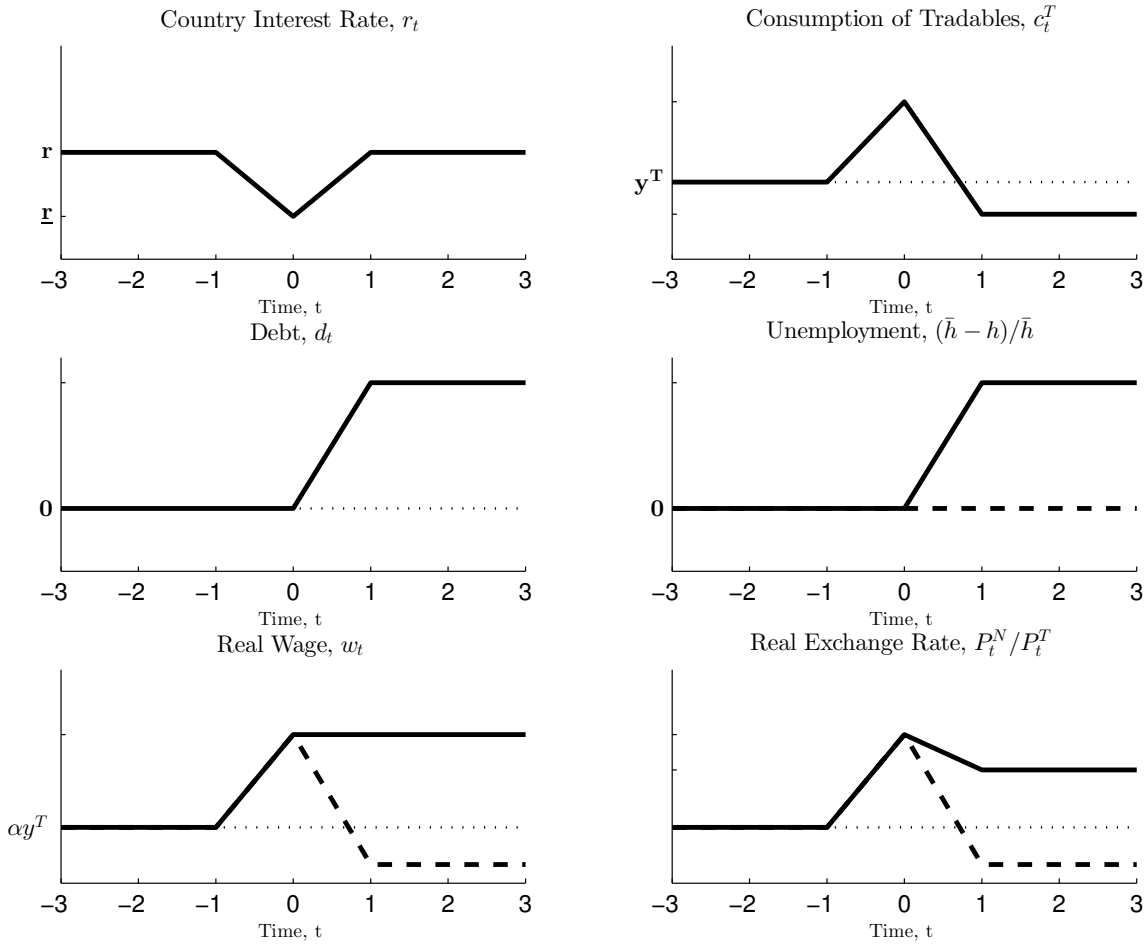
$$d_t = y^T \left[1 - \frac{1+\underline{r}}{1+r} \right] > 0,$$

$$h_0 = 1;$$

$$h_1 = h_2 \cdots = \frac{1+\underline{r}}{1+r} < 1$$

The following slide displays the same information graphically.

Figure 9.5 A Temporary Decline in the Country Interest Rate



____ currency peg _ _ _ _ flexible wage economy or optimal exchange rate economy

9.2.2 The Link between Volatility and Average Unemployment

The present model predicts that aggregate volatility increases the mean level of unemployment.

This prediction gives rise to large welfare benefits of stabilization policy.

— This prediction is not due to the assumption of downward nominal wage rigidity, but due to the assumption that employment is determined by the minimum of labor demand and labor supply. (Note: key difference with Calvo-style sticky wage models in which employment is always demand determined.)

— Downward nominal wage rigidity amplifies the connection between aggregate volatility and mean unemployment.

To see this consider the following example:

$U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$

$d_t = 0$ (no access to international financial markets)

$\Rightarrow c_t^T = y_t^T$.

$$y_t^T = \begin{cases} 1+\sigma & \text{prob } \frac{1}{2} \\ 1-\sigma & \text{prob } \frac{1}{2} \end{cases}$$

$E(y_t^T) = 1$ and $\text{var}(y_t^T) = \sigma^2$.

$F(h_t) = h_t^\alpha$

$\bar{h} = 1$

$\mathcal{E}_t = \mathcal{E}$ (currency peg)

$W_{-1} = \alpha\mathcal{E}$

The equilibrium conditions associated with this economy are (we list all except wage adjustment, for which we will consider two cases):

$$\frac{c_t^T}{c_t^N} = p_t$$

$$\alpha p_t (h_t)^{\alpha-1} = W_t/\mathcal{E}$$

$$c_t^T = y_t^T$$

$$c_t^N = h_t^\alpha$$

Step 1: Find labor demand: $h_t^d = \frac{\alpha y_t^T}{W_t/\mathcal{E}}$

Step 2: Find equilibrium labor as $h_t = \min\{\bar{h}, h_t^d\}$

Case 1: Assume bi-directional nominal wage rigidity.

$$W_t = \alpha \mathcal{E}$$

Then, $h_t^d = y_t^T$, and the equilibrium level of employment is

$$h_t = \begin{cases} 1-\sigma & \text{if } y_t^T = 1-\sigma \\ 1 & \text{if } y_t^T = 1+\sigma \end{cases}$$

Let $u_t \equiv \bar{h} - h_t$ denote the unemployment rate. It follows that the equilibrium distribution of u_t is given by

$$u_t = \begin{cases} \sigma & \text{with probability } \frac{1}{2} \\ 0 & \text{with probability } \frac{1}{2} \end{cases}.$$

The unconditional mean of the unemployment rate is then given by

$$E(u_t) = \frac{\sigma}{2}.$$

Average level of unemployment increases linearly with the volatility of tradable endowment, in spite of the fact that wage rigidity is symmetric!

Case 2: assume 'only' downward nominal wage rigidity, $W_t \geq W_{t-1}$

Then, $W_t/\mathcal{E} = \alpha(1+\sigma) > \alpha$ and

$$h_t = \begin{cases} \frac{1-\sigma}{1+\sigma} & \text{if } y_t^T = 1-\sigma \\ 1 & \text{if } y_t^T = 1+\sigma \end{cases}$$

$E(u_t) = \sigma/(1+\sigma) > \sigma/2$ (recall that σ must be less than 1).

Thus uni-directional wage rigidity exacerbates the link between mean unemployment and volatility.

9.3

Optimal Exchange Rate Policy

Motivation

• We have just seen that under an exchange rate peg a negative external shock may lead to involuntary unemployment. How would optimal exchange rate policy look like?

• In this section we show that under optimal policy there is (1) full employment and (2) negative external shocks call for devaluations.

• We begin with a graphical explanation.

The graph on the next slide illustrates how the optimal exchange rate policy works

The initial situation is point A. Suppose a negative shock (possibly an increase in the country interest rate), shifts the demand schedule down and to the left. Without government intervention, the supply schedule does not move, because W is downwardly rigid. The equilibrium would then be point B, with involuntary unemployment equal to $\bar{h} - h^{PEG}$.

Suppose now that the government devalues the domestic currency from $\mathcal{E}_0$ to $\mathcal{E}_1 > \mathcal{E}_0$. The devaluation lowers the real wage from $W_0/\mathcal{E}_0$ to $W_0/\mathcal{E}_1$, causing the supply schedule to shift down and to the right.

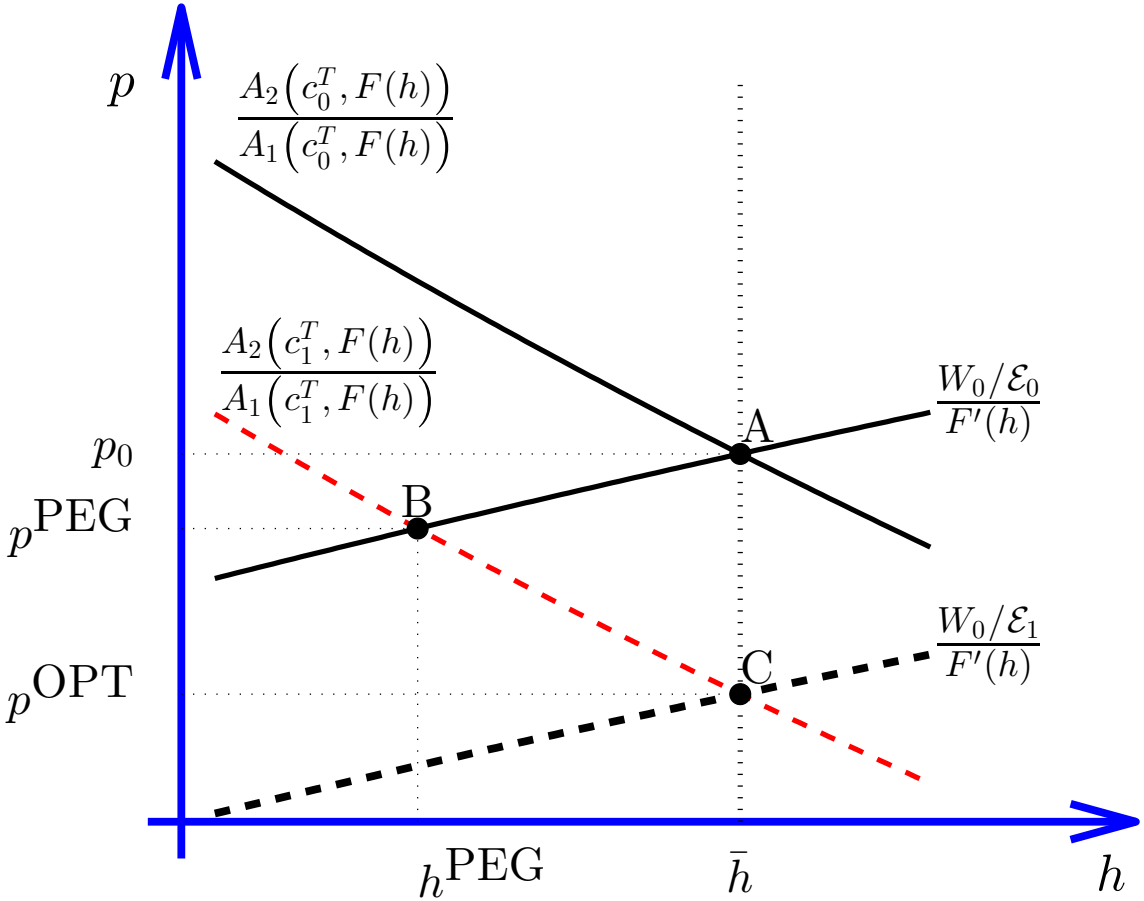
If the devaluation is just right, the new supply schedule will cross the new demand schedule at point C, preserving full employment (we say preserving and not restoring full employment because the economy jumps from A to C, without visiting B).

Note: the fall in labor cost caused by the drop in the real wage allows firms to cut prices from p_0 to p^{OPT} and induces households to switch expenditure away from tradables and toward nontradables.

Take another look at the graph for the analytical example in of a temporary fall in r_0. The broken lines display the equilibrium under optimal exchange-rate policy.

Optimal Exchange-Rate Policy

(again, assume $\gamma = 1$)



$c_1^T < c_0^T$ (negative shock, possibly $r_t \uparrow$)

$\mathcal{E}_1 > \mathcal{E}_0$ (optimal devaluation)

The Ramsey Optimal Exchange-Rate Policy

The Ramsey government solves the problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t)))$$

subject to (9.9) and (9.13)-(9.19).

Strategy: solve a less constrained problem and then show that its solution satisfies (9.9) and (9.13)-(9.19).

Consider the less restricted problem of choosing $\{c_t^T, h_t, d_{t+1}\}_{t=0}^{\infty}$ to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t)))$$

subject to the following subset of the equilibrium conditions:

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \tag{9.9}$$

$$h_t \leq \bar{h} \tag{9.18}$$

Clearly, the solution for labor is $h_t = \bar{h}$ for all t or full employment at all times. Intuition: One nominal friction and one instrument that can fully offset it. Hence possible to obtain the flexible wage allocation (which here coincides with the first best). But to show that this is indeed the allocation under the optimal exchange-rate policy, we must show that the solution to the above social planner's problem satisfies all of the competitive equilibrium conditions, that is, conditions (9.9) and (9.13)-(9.19). To see this, proceed by construction: set λ_t to satisfy (9.13), p_t to satisfy (9.15), w_t to satisfy (9.16), ϵ_t to satisfy (9.17), (9.18) is a constraint of the social planner's problem, and so is (9.9). Because $h_t = \bar{h}$, (9.19) holds. That (9.14) holds follows from the definition of λ_t and the first-order condition of the social planner.

An important reference point: The **full-employment real wage**, denoted $\omega(c_t^T)$, defined as the real wage that clears the labor market,

$$\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}); \quad \omega'(c_t^T) > 0$$

Set the (gross) devaluation rate, $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1}$, to eliminate unemployment:

$$\epsilon_t \geq \frac{\gamma W_{t-1}/\mathcal{E}_{t-1}}{\omega(c_t^T)}$$

Note: There is a whole family of optimal exchange-rate policies. Under any member of this policy, $h_t = \bar{h}$ and $w_t = \omega(c_t^T)$ for all t.

9.3.3 When is it inevitable to devalue?

Optimal exchange rate policy is:

$$\epsilon_t \geq \frac{\gamma W_{t-1}/\mathcal{E}_{t-1}}{\omega(c_t^T)}$$

Because $\omega'(c_t^T) > 0$, optimal devaluations occur in periods of contraction of aggregate demand. It follows that contractions are devaluatory as opposed to devaluations being contractionary.

Under optimal exchange rate policy external debt and tradable consumption are determined by the solution to

$$v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \left\{ U(A(c_t^T, F(\bar{h})) + \beta E_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1}) \right\}$$

subject to

$$y_t^T + \frac{d_{t+1}}{1 + r_t} = d_t + c_t^T$$

9.4 Empirical Evidence On Downward Nominal Wage Rigidity

• Downward nominal wage rigidity is the central friction in the present model $\Rightarrow$ natural to ask if it is empirically relevant.

• Downward nominal wage rigidity has been studied empirically from a number of perspectives:

— Evidence from micro and macro data.

— Studies focusing on rich, emerging, and poor countries.

— Studies focusing on formal and informal labor markets.

• **By product:** Will obtain an estimate of the parameter γ governing wage stickiness in the model (useful for quantitative analysis).

Downward Nominal Wage Rigidity

A.) Evidence From Micro Data from Developed Countries

1. United States, 1986-1993, SIPP panel data (Gottschalk, 2005)

2. United States, 1996-1999, SIPP panel data (Barattieri, Basu, Gottschalk, 2012)

3. United States, 1997-2017, CPS panel data (Jo, Schmitt-Grohe, Uribe, 2017)

4. United States, 1979-2017, CPS and SIPP panel data (Jo, 2019)

5. Micro Evidence On Downward Nominal Wage Rigidity From Other Developed Countries

1.) United States, 1986-1993, SIPP panel data

Probability of Decline, Increase, or No Change in Wages

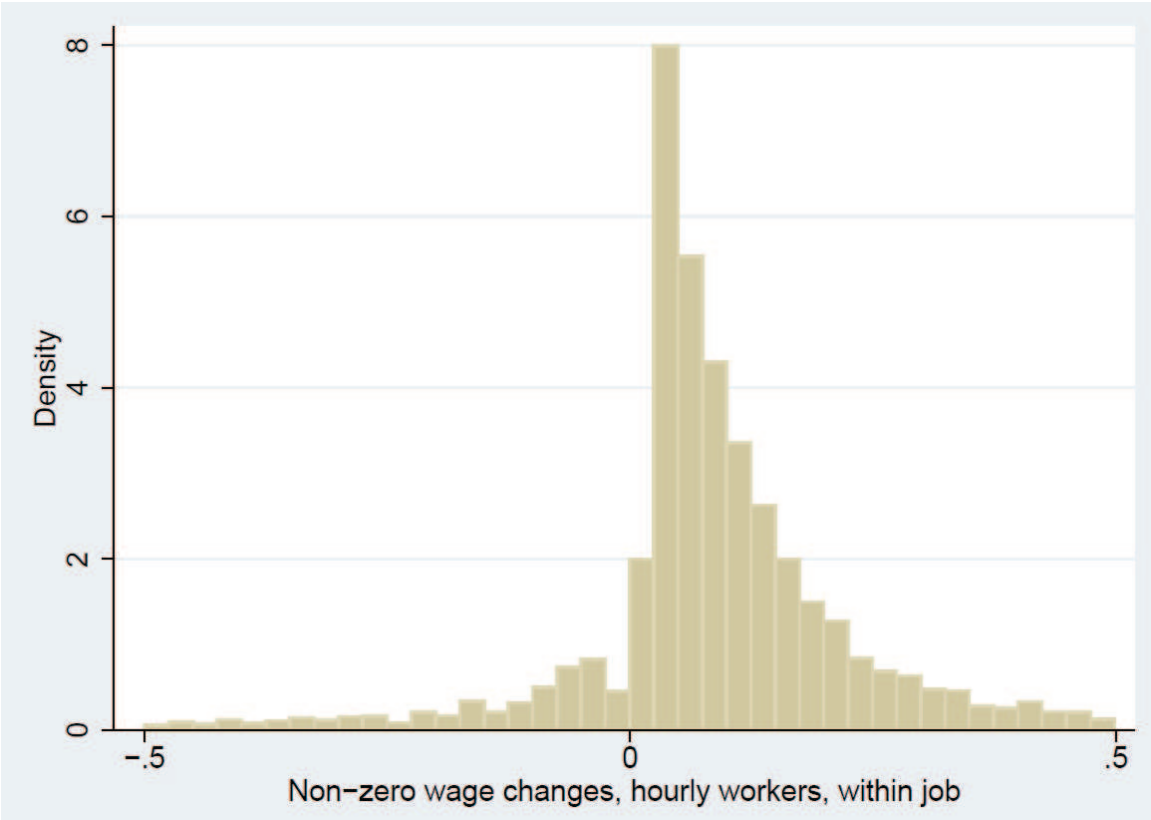
	Interviews One Year apart			
	Adjusted for ME		Raw data	
	Males	Females	Males	Females
Decline	5.1%	4.3%	15.7%	13.6%
Constant	53.7%	49.2%	25.1%	22.8%
Increase	41.2%	46.5%	59.1%	63.6%

Source: Table 4 of Gottschalk (2005). Note: Male and female hourly-paid workers not in school, 18 to 55 at some point during the panel. All nominal-wage changes are within-job wage changes, defined as changes while working for the same employer. SIPP panel data.

- Focus of Gottschalk is on correction for measurement error (ME).
- Large mass at 'Constant' suggests nominal wage rigidity.
- Small mass at 'Decline' suggests downward nominal wage rigidity.

2.) United States 1996-1999, SIPP panel data

Distribution of Non-Zero Nominal Wage Changes

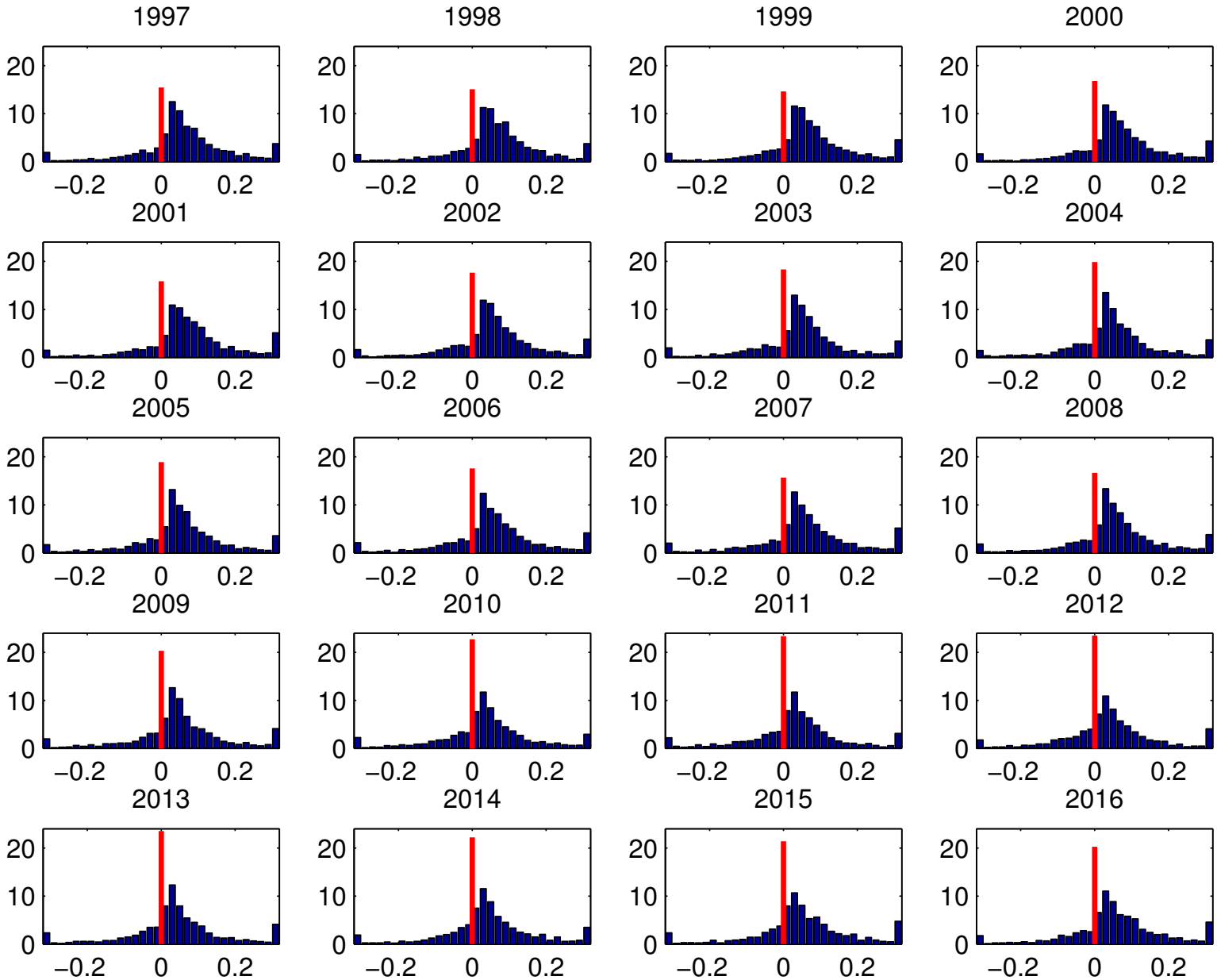


Source: Barattieri, Basu, and Gottschalk (2012). SIPP panel data.

3.) United States, 1997-2016, CPS panel data

Year-over-year log changes in nominal hourly wages of hourly-paid job stayers

Take a look at the next graph. It shows wage change distributions for each year since 1997. The horizontal axis measures the year-over-year percent change in the nominal hourly wage of an hourly-paid jobstayer. The vertical axis measures the share of workers in each bin. The bin size is two percent, with the exception of a wage freeze, which is defined as an exact zero change. Each wage change distribution is based on about 5,000 workers.



Source: Jo, Schmitt-Grohé, and Uribe (2017).

Observations on the figure:

• Large spike at zero wage changes.

• Many more wage increases than wage cuts.

• Fraction of wage freezes is cyclical, rises from 15 percent in 2007 to 20 percent in 2009.

• Much smaller cyclical increase in wage cuts.

4.) United States, CPS and SIPP panel data

Boom: 2007

Bust: 2010

In 2007 the economy was in a boom. Unemployment and inflation were low.

In 2010 the unemployment rate hit the highest point of the recession (10 percent) and inflation was below the target of 2 percent. In addition total factor productivity growth, not shown, was also low, thus making 2010 a year in which one should expect to see more nominal wage cuts than in 2007.

Comparing boom (2007) to bust (2010)

Percent of Workers with Decline, Increase, or No Change in Wages (year-over-year)

	SIPP data		CPS data	
	2007	2010	2007	2010
Decline	14	16	20	25
Constant	31	42	14	21
Increase	55	41	65	54

Source: Jo (2019), Table A13, SIPP data. Table A2, CPS data.

- From boom to bust share of workers with constant wages increases by 11 (SIPP) percentage points.
- From boom to bust share of workers with a nominal wage decline increases by 2 (SIPP) percentage points.
- $\Rightarrow$ in the recession a larger increase in share of workers with no wage change than in the share of workers with wage declines.
- This pattern is consistent with downward nominal wage rigidity.

• This regularity holds not only for the 2007-2010 boom bust episode, but also generally for the period 1979-2017:

Table 6: The spike at zero, the fraction of wage cuts, and raises along the business cycles

	(1) Spike at zero $\Delta W = 0$	(2) Fraction of $\Delta W < 0$	(3) Fraction of $\Delta W > 0$	(4) Spike at zero $\Delta W = 0$	(5) Fraction of $\Delta W < 0$	(6) Fraction of $\Delta W > 0$
1-Epop ratio	0.433	0.200	-0.632	0.616***	0.305*	-0.921***
$(1 - e_t)$	(0.299)	(0.221)	(0.498)	(0.161)	(0.156)	(0.281)
Inflation rate, π_t				-1.181***	-0.674***	1.855***
				(0.122)	(0.145)	(0.218)
				0.616/0.920 = 0.67		
Observations	37	37	37	37	37	37
Adjusted R^2	0.0419	-0.00492	0.0313	0.727	0.331	0.703

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Data source: CPS and author's calculation. Sample Period: 1979-2017 (except 1995). Inflation rate is calculated from CPI-U. This table shows regression results from regressing the spike at zero, the fraction of workers with wage cuts, and raises on 1-epop ratio without and with controlling for inflation. Controlling for inflation, the spike at zero exhibits greater fluctuations compared to the share of workers with wage cuts.

Source. Jo (2019), Table 6. Sample period 1979-2017

• When unemployment $(1 - e)$ rises by 1 percentage point, then the share of workers with no annual wage change increases by 0.6 percentage points, and the share of workers with a wage decline increases by only 0.3 percentage points.

Evidence from the Cross-Section of U.S. States During the 2007-2010 Recession

The next slide, again taken from Jo (2019), compares changes in nominal hourly wages of hourly-paid workers from the CPS over the period 2007 to 2010, the great recession in the United States, across the 50 states and relates it to employment changes over the same period.

The figures shows that

- U.S. states with relatively larger contractions in employment, such as Nevada and Utah, were states with the largest increases in the share of workers experiencing wage freezes.
- overall there is a negative correlation between employment changes and changes in the share of workers with wage freezes across states in the United States.

Again, these empirical regularities are consistent with the existence of downward nominal wage rigidity.

The larger the decline in employment, the greater the increase of workers with constant wages will be

cross section of U.S. states, 2007 to 2010

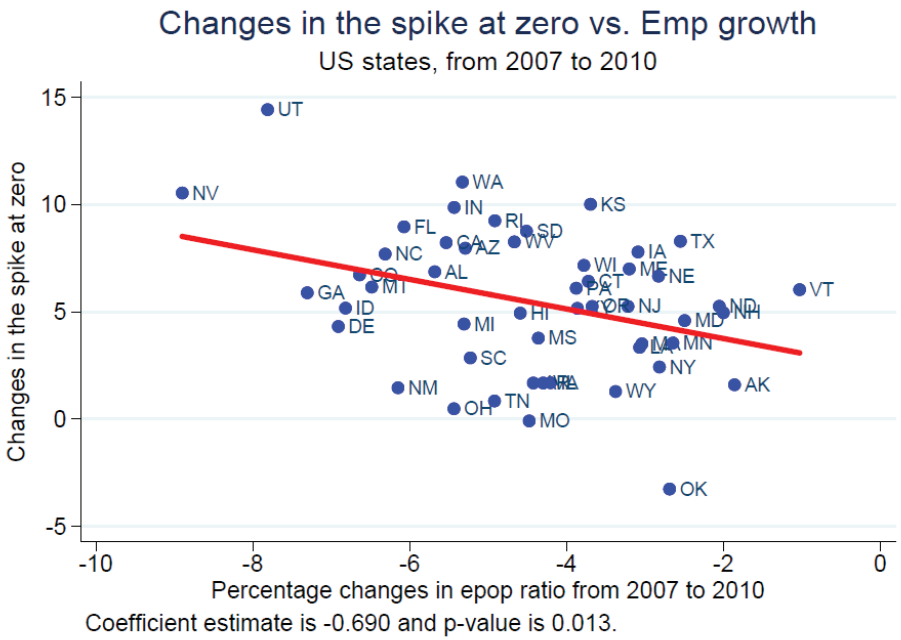


Figure 5: Nominal wage growth rates and changes in the spike at zero vs. employment growth from 2007 to 2010

Data source: CPS and author's calculation. The top panel shows the median nominal wage growth versus employment growth rates from 2007 to 2010 across states. The bottom panel shows the changes in the spike at zero versus employment growth from 2007 to 2010 across states. From 2007 to 2010, the annualized inflation rate was 1.7 percent, and the cumulative inflation was 5 percent.

Source: Figure 5 of Jo (2019).

... but no such negative association when inflation is high

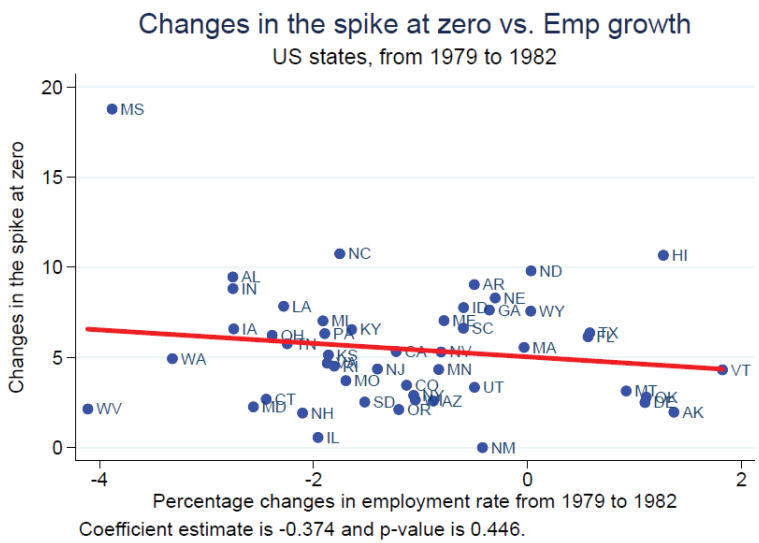


Figure 6: Nominal wage growth and changes in the spike at zero vs. employment growth from 1979 to 1982

Data source: CPS and author's calcuation. The top panel shows the median nominal wage growth with respect to employment growth rates from 1979 to 1982 across states. The bottom panel shows the change in the spike at zero with respect to employment growth from 1979 to 1982 across states. From 1979 to 1982, the average of annualized inflation rate was 9.5 percent and the cumulative inflation was 28.5 percent.

Source: Figure 6 of Jo (2019).

- 28.5% cummulative inflation from 1979-1982.
- Slope coefficient insignificant.

5.) Micro Evidence On Downward Nominal Wage Rigidity From Other Developed Countries

- Canada: Fortin (1996).

- Japan: Kuroda and Yamamoto (2003).

- Switzerland: Fehr and Goette (2005).

- Industry-Level Data: Holden and Wulfsberg (2008), 19 OECD countries from 1973 to 1999.

B.) Evidence From Informal Labor Markets

• Are nominal wages downwardly flexible in in informal labor markets, where labor unions, wage legislation, or regulation play, if any, a small role?

• Kaur (2019) addresses this issue by examining the behavior of nominal wages, employment, and rainfall in casual daily agricultural labor markets in rural India (500 districts from 1956 to 2008).

• Finds asymmetric nominal wage adjustment:

— W_t increases in response to positive rainfall shocks

— W_t failure to fall, labor rationing, and unemployment are observed in response to negative rain shocks.

• Inflation (uncorrelated with local rain shocks) tends to moderate rationing and unemployment during negative rain shocks, suggesting downward rigidity in nominal rather than real wages.

C.) Evidence From the Great Depression in the U.S.

• How do nominal wages behave during extraordinary contractions?

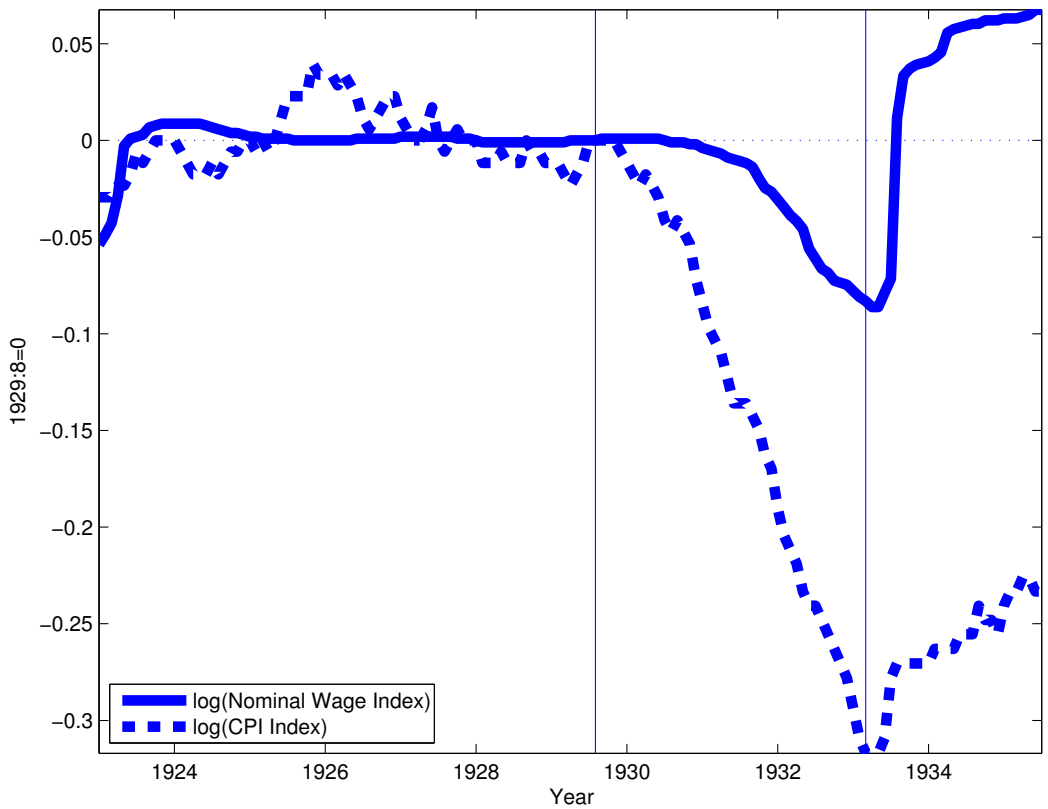
• The next slide shows the nominal wage rate and the consumer price index in the United States from 1923:1-1935:7.

• Between 1929 and 1931 the U.S. economy experienced an enormous contraction in employment of 31%.

• Nonetheless, during this period nominal hourly wages fell by 0.6% per year, while consumer prices fell by 6.6% per year. See the figure on the next slide.

• A similar pattern is observed during the second half of the Depression. By 1933, real wages were 26% higher than in 1929, in spite of a highly distressed labor market.

Figure 9.8 Nominal Wage Rate and Consumer Prices, United States 1923:1-1935:7



Solid line: natural logarithm of an index of manufacturing money wage rates. Broken line: logarithm of the consumer price index.

D.) Evidence From the Great Depression In Europe

• Countries that left the gold standard earlier recovered faster than countries that remained on gold.

— Left Gold Early (sterling bloc): United Kingdom, Sweden, Finland, Norway, and Denmark.

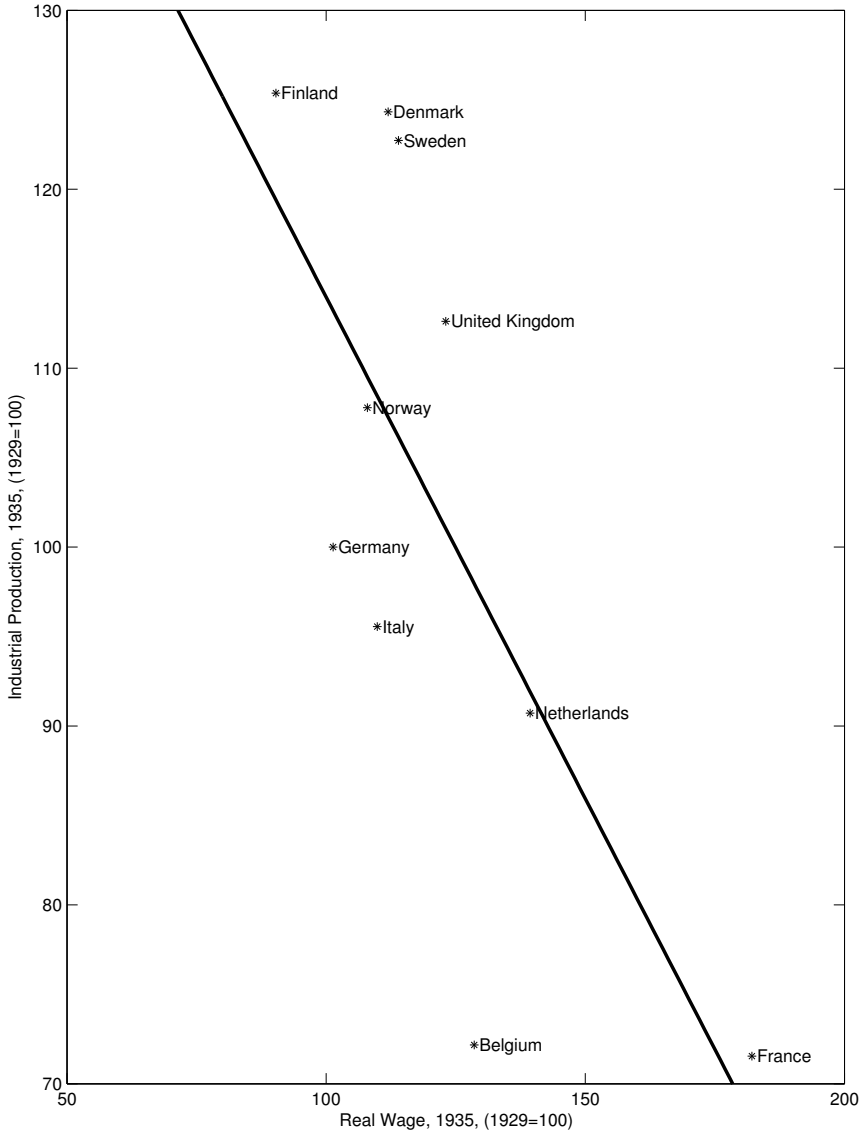
— **Countries That Stuck To Gold (gold bloc):** France, Belgium, the Netherlands, and Italy.

• The gold standard is akin to a currency peg. A peg not to another currency, but to gold.

• When the sterling-bloc left gold, they effectively devalued, as their currencies lost value against gold.

• The figure on the next slide shows that between 1929 and 1935 the sterling-bloc experienced less real wage growth and a larger increase in industrial production than the gold bloc.

Changes In Real Wages and Industrial Production, 1929-1935



E.) Evidence From Emerging Countries

• Argentina pegged the peso at a 1-to-1 rate to the dollar between 1991 and 2001.

• Starting in 1998, the economy was buffeted by a number of large negative shocks (weak commodity prices, large devaluation in Brazil, large increase in country premium).

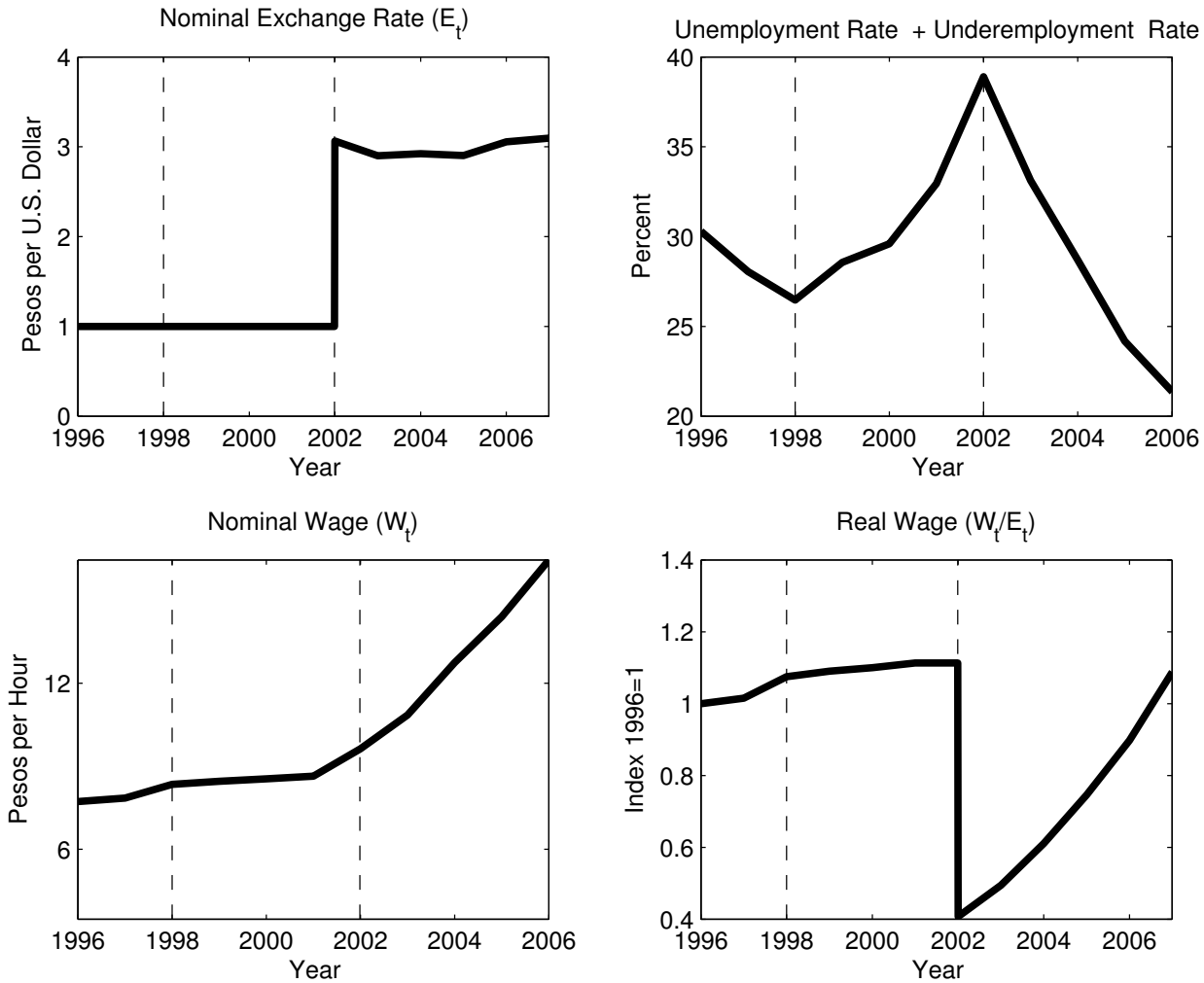
• Not surprisingly, between 1998 and 2001, unemployment rose sharply. See the figure on the next slide.

• Nonetheless, nominal wages remained remarkably flat.

• This evidence is consistent with downward nominal wage rigidity, and suggests that γ, the parameter governing downward wage rigidity in the model, is about 1.

•Why $\gamma \approx 1$? The slackness condition $(\bar{h} - h_t)(W_t - \gamma W_{t-1})$ (recall $\epsilon_t = 1$ between 1991 and 2001), implies that if unemployment is growing, then wages must grow at the gross rate γ. Argentine wages were flat $\Rightarrow \gamma \approx 1$.

Argentina 1996-2006



Implied Value of γ: Around unity.

Evidence From Peripheral Europe (2008-2011)

• The next slide shows the unemployment rate and nominal wage growth between 2008:Q1 and 2011:Q2 in 12 European countries that were either in the eurozone or pegging to the euro.

• Between 2008 and 2011, all countries in the periphery of Europe experienced increases in unemployment; Some very large increases.

• In spite of extreme duress in the labor market, nominal hourly wages experienced increases in most countries and modest declines in only a few.

• The slide following the table explains how to use the information in the table to infer a range for γ.

Table 9.2 Unemployment, Nominal Wages, and γ Evidence from the Eurozone

Country	Unemployment Rate 2008Q1 (in percent)	Unemployment Rate 2011Q2 (in percent)	Wage Growth $\frac{W_{2011Q2}}{W_{2008Q1}}$ (in percent)	Implied Value of γ
Bulgaria	6.1	11.3	43.3	1.028
Cyprus	3.8	6.9	10.7	1.008
Estonia	4.1	12.8	2.5	1.002
Greece	7.8	16.7	-2.3	0.9982
Ireland	4.9	14.3	0.5	1.0004
Italy	6.4	8.2	10.0	1.007
Lithuania	4.1	15.6	-5.1	0.996
Latvia	6.1	16.2	-0.6	0.9995
Portugal	8.3	12.5	1.91	1.001
Spain	9.2	20.8	8.0	1.006
Slovenia	4.7	7.9	12.5	1.009
Slovakia	10.2	13.3	13.4	1.010

Note. W is an index of nominal average hourly labor cost in manufacturing, construction, and services, including the public sector (except for Spain). Source: Schmitt-Grohé and Uribe (JPE, 2016)

How To Infer γ From European Data

As explained in the analysis of the Argentine Convertibility Plan, the slackness condition of the model, $(W_t - \gamma W_{t-1})(\bar{h} - h_t)$, implies that if unemployment increases form one period to the next, then nominal wages must be growing at the rate γ: $\frac{W_t}{W_{t-1}} = \gamma$.

How to calculate γ:

$$\gamma = \left(\frac{W_{2011:Q2}}{W_{2008:Q1}}\right)^{\frac{1}{13}}$$

Subtract 0.6% per quarter to adjust for foreign inflation and long-run growth (because they are not explicitly incorporated in the model) to obtain the estimate:

$$\gamma \in [\mathbf{0.99}, \mathbf{1.022}]$$

Quantitative Analysis

(Sections 9.5-9.8 and 9.10)

Replication files: `usg_dnwr.zip` available online with the materials for this chapter.

Functional Forms

Assume a CRRA form for preferences, a CES form for the aggregator of tradables and nontradables, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}$$

$$F(h) = h^{\alpha},$$

with $\sigma, \xi, a, \alpha > 0$.

9.5 The case of Equal Intra- and Intertemporal Elasticities of Substitution

Consider the case

$$\xi = \frac{1}{\sigma}$$

Why this case is of interest:

• It makes the determination of the equilibrium levels of debt, d_t, and consumption of tradables, c_t^T, independent of the level of activity in the nontraded sector (see the next slide). As a result, the welfare consequences of exchange-rate policy or nominal wage rigidity are fully attributable to their effect on unemployment, and not on their effect on the accumulation of external debt.

• It facilitates the computation of equilibrium, as the equilibrium dynamics of d_t and c_t^T can be computed separately from the equilibrium dynamics of h_t, w_t, c_t^N, and p_t.

• As we will argue shortly, $\sigma = 1/\xi = 2$ is empirically plausible.

Debt and Tradable Consumption when $\xi = \frac{1}{\sigma}$

In this case,

$$U(A(c_t^T, c_t^N)) = \frac{a{c_t^T}^{1-\sigma} + (1-a){c_t^N}^{1-\sigma} - 1}{1-\sigma},$$

which is separable in c_t^T and c_t^N. Then d_t and c_t^T solve

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}$$

$$(c_t^T)^{-\sigma} = \beta(1+r_t)E_t(c_{t+1}^T)^{-\sigma}$$

This subsystem is independent of h_t, w_t, p_t, and c_t^N.

It can be cast as a Bellman equation problem, which facilitates the quantitative analysis (more on the next slide).

9.6 Approximating Equilibrium Dynamics Under Optimal Exchange-Rate Policy when $\xi = 1/\sigma$

Equilibrium processes $\{c_t^T, d_{t+1}\}$ solve the Bellman equation problem

$$v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \left\{ U(A(c_t^T, F(\bar{h}))) + \beta E_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1}) \right\} \tag{1}$$

$$\text{subject to } c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}; \qquad \text{and } d_{t+1} \leq \bar{d}. \tag{2}$$

Approximate by value function iteration over a discretized state space (y_t^T, r_t, d_t). Use 21 values for y_t^T and 11 for r_t (the estimated joint process (y_t^T, r_t) is given below). Use 501 equally spaced points for d_t between 1 and 8.34.

Given approximated solutions to d_{t+1} and c_t^T, all other variables of the model can be backed out:
$h_t = \bar{h}$,
$p_t = \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))}$,
$w_t = p_t F'(\bar{h})$,
and a chosen member of the family $\epsilon_t \geq \gamma w_{t-1}/w_t$.

Approximating Equilibrium Under Currency Pegs with $\xi = 1/\sigma$

When $\xi = 1/\sigma$, the solution for d_{t+1} and c_t^T is the same as for the optimal exchange-rate policy, since c_t^T is independent of it.

The determination of w_t requires knowledge of past real wages. So, w_{t-1} becomes a relevant endogenous state variable. This is different from the equilibrium under optimal exchange-rate policy.

Given w_{t-1} and c_t^T, the solution for w_t can be found from solving a static problem (and hence is simple to solve): First, conjecture that $h_t = \bar{h}$, and obtain $w_t = \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$. If $w_t \geq \gamma w_{t-1}$, this is the solution. Otherwise, $w_t = \gamma w_{t-1}$, and h_t solves $w_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t)$.

Discretization of w_{t-1} grid: 500 points between 0.25 and 6, equally spaced in logs.

Approximating Equilibrium when $\xi \neq 1/\sigma$

Optimal Exchange-Rate Policy: As before, the solution is separable. First, the processes d_{t+1} and c_t^T solve the Bellman equation problem (1)-(2). Then, all other variables (w_t, p_t, c_t^N) are then easily backed out.

Currency Peg: The the solution is no longer separable. All equilibrium conditions must be solved jointly and becomes more complicated. Schmitt-Grohé and Uribe (JPE, 2016) develop an algorithm to handle this case.

The Driving Process:

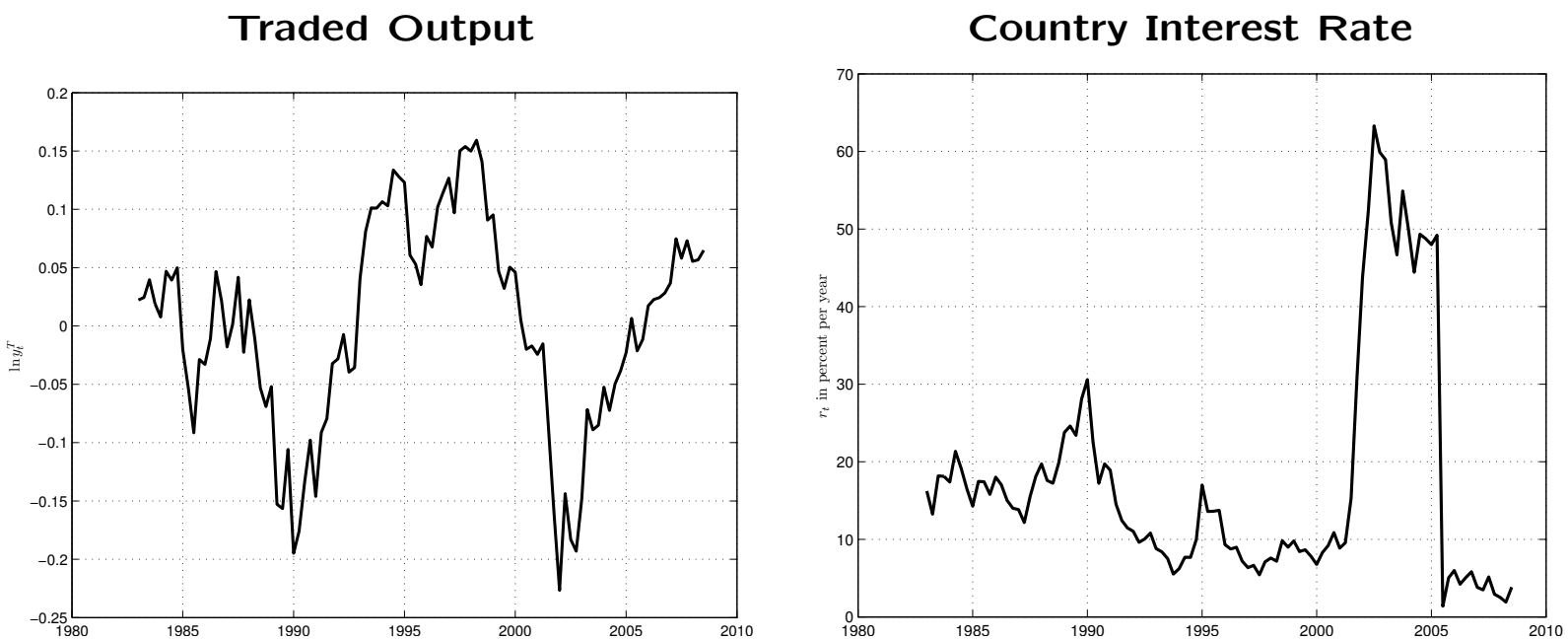
Data: Argentine data over the period 1983:Q1—2001:Q3. Exclude the period 2001:Q4 to present, because of the default episode in 2002 (no default in the model).

Empirical Measure of y_t^T: sum of GDP in agriculture, manufacturing, fishing, forestry, and mining. Quadratically detrended.

Empirical Measure of r_t: Sum of Argentine EMBI+ plus 90-day Treasury-Bill rate minus a measure of U.S. expected inflation.

The following slide displays the two time series.

Tradable Output and Country Interest Rate
Argentina 1983:Q1 to 2001:Q3



Estimate the AR(1) system

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t,$$

OLS Estimate of the Driving Process

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \qquad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix};$$

$$r = 0.0316 \text{ (3.16\% per quarter)}.$$

Some Unconditional Summary Statistics

Statistic	y^T	r
Std. Dev.	12%	6%yr
Serial Corr.	0.95	0.93
Corr(y_t^T, r_t)	-0.86	
Mean	1	12%yr

Observations:

(1) High volatility of both y_t^T and r_t;

(2) negative correlation between y_t^T and r_t (when it rains it pours);

(3) High mean country interest rate.

Calibration

Parameter	Value	Description
γ	0.99	Degree of downward nominal wage rigidity
σ	2	Inverse Intertemp. elast. of subst.
y^T	1	Steady-state tradable output
$\bar{h}$	1	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Intratemp. elast. of subst.
α	0.75	Labor share in nontraded sector
β	0.9635	Quarterly subjective discount factor

Note: $\sigma = 2$ is widely used in business cycle analysis, and $\xi = 0.5$ is within the range of values estimated for emerging countries (see the survey by Akinci, 2011). Consequently, the restriction $\xi = 1/\sigma$ is quite compelling on empirical and computational grounds.

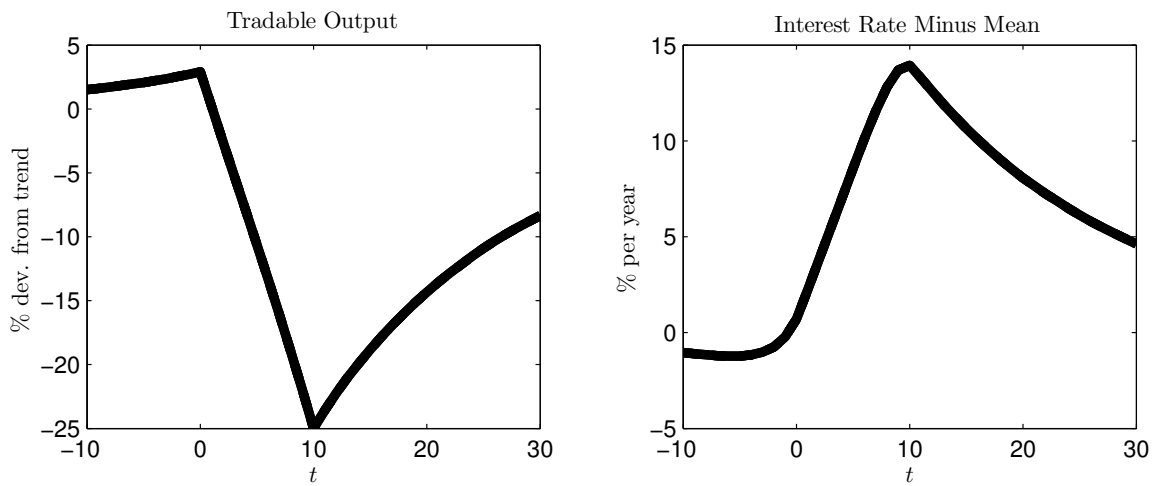
9.8.3 Crisis Dynamics Under A Currency Peg and Under Optimal Exchange-Rate Policy

We are interested in characterizing quantitatively the response of the model economy to large contractions like the ones observed in Argentina in 2001 and in the periphery of Europe in 2008. In Argentina, for instance, traded output fell by 2 standard deviations in a period of two and a half years (10 quarters). Accordingly, we use the following operational definition of an external crisis.

Definition of an External Crisis. A crisis is a situation in which in period t tradable output, y_t^T, is at or above average, and 10 quarters later, in period $t + 10$, it is at least two standard deviations below trend.

The Typical External Crisis: Simulate the model for 20 million periods. Extract all windows of time in which y_t^T conforms to the definition of a crisis. For each variable of interest, average all windows and subtract its unconditional mean (i.e., the mean taken over the 20 million observations).

Figure 9.11 Sources of an External Crisis



Note. Replication file plot_ir.m in usg_dnwr.zip.

Comments: (1) Because y_t^T and r_t are negatively correlated, the collapse in y_t^T coincides with a sharp increase in the country interest rate. (2) The response of y_t^T and r_t are exogenous to the model, so this plot is independent of the exchange-rate policy. The next slide displays the response of the endogenous variables.

Which Optimal Exchange-Rate Policy to Pick?

From the family of optimal exchange-rate policies, we pick

$$\epsilon_t = \frac{w_{t-1}}{\omega(c_t^T)}$$

Properties of this policy:

(1) It implies that the nominal wage rate, W_t, and the nominal price of nontradables, P_t^N, are constant at all times. Note: It fully stabilizes the (factor) price that suffers from nominal rigidity.

(2) It induces zero inflation and zero devaluation on average.

(3) From (2), we have that the assumed optimal exchange-rate policy delivers devaluations ($\epsilon > 1$) and revaluations ($\epsilon_t < 1$) over the business cycle. Specifically, the government devalues during downturns and revalues during booms. After presenting the dynamics of external crises predicted by the DNWR model, we will provide some empirical support for property (3).

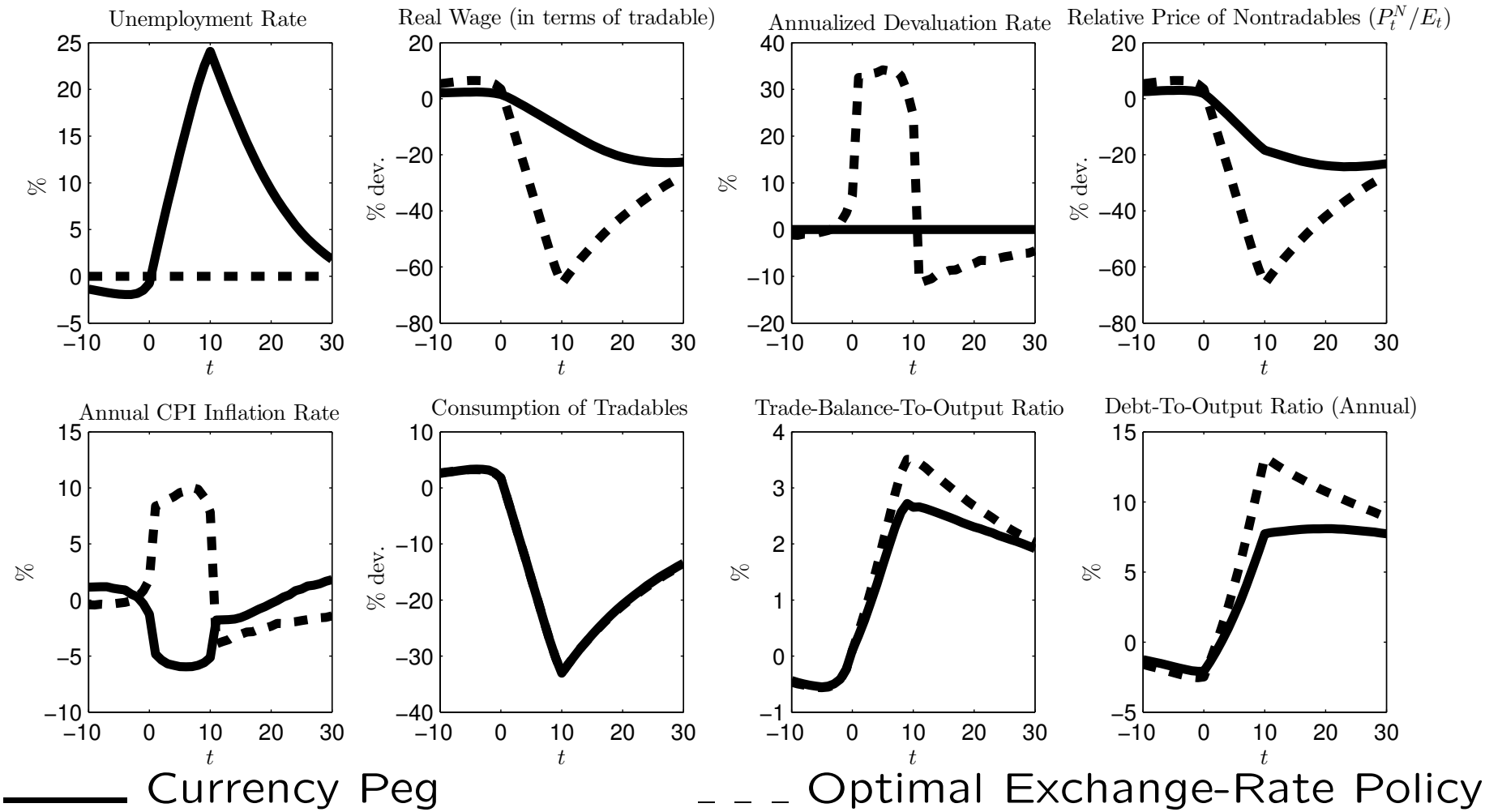
External Crisis under

Alternative Exchange-rate Policies with Downward Nominal Wage Rigidity

We are now ready to present the predicted equilibrium dynamics during a typical crisis under a currency peg and under the optimal exchange rate policy we selected.

The next slide shows with solid lines the response of the economy under a currency peg and with broken lines the response under the assumed optimal exchange-rate policy.

Pegs Amplify Negative External Shocks



Note. Replication file plot_ir.m in usg_dnwr.zip.

Observations

• Large contraction in c^T, driven primarily by the rise in the country interest rate. The trade balance, $y_t^T - c_t^T$, actually improves in spite of the fact that y^T falls sharply. The response of c^T is independent of exchange-rate policy, because $\xi = 1/\sigma$.

• **Currency Pegs:** large increase in unemployment (25%), because the real wage does not fall sufficiently (stays 40% above the full-employment real wage). Firms don't cut prices because labor cost remains high. As a result, consumers don't switch expenditures away from tradables and toward nontradables.

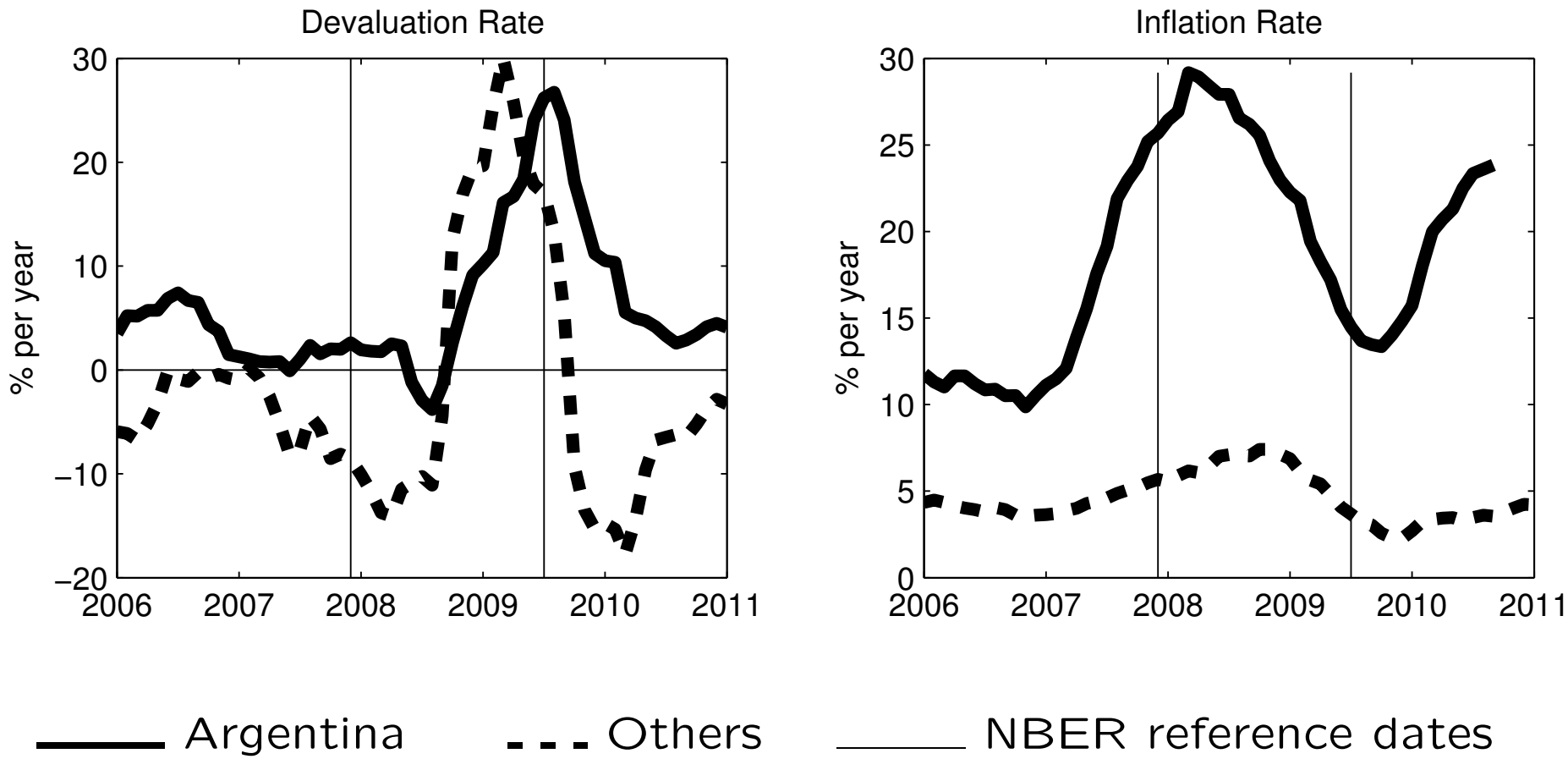
• **Optimal Exchange-Rate Policy:** It cannot avoid the contraction in the tradable sector. But it can prevent the external crisis to spread to the nontradable sector. In fact, it preserves full employment throughout the crisis (this result was also established analytically earlier in this chapter). Large devaluations of around 30% per year for 2.5 years. Consistent with devaluations post Convertibility in Argentina. Devaluations bring the real wage down ($\mathcal{E} \uparrow \Rightarrow w = W/\mathcal{E} \downarrow$), fostering employment and allowing the real exchange rate to depreciate ($p = P^N/\mathcal{E} \downarrow$). Real depreciation facilitates expenditure switch toward nontradables.

9.8.4 Devaluations and Revaluations in Reality

Do countries devalue during crises and revalue when the crisis is over, and why? Look at the next graph. It displays the devaluation rate and the inflation rate for two sets of Latin American countries during the global crisis of 2008. One set is Argentina, and the other set consists of Chile, Colombia, Mexico, Peru, and Uruguay.

During the global financial crisis, all countries devalued significantly. However, during the recovery, all countries but Argentina revalued their currencies. The countries that revalued experienced lower inflation than Argentina.

Devaluation and Inflation In Latin America: 2006-2011

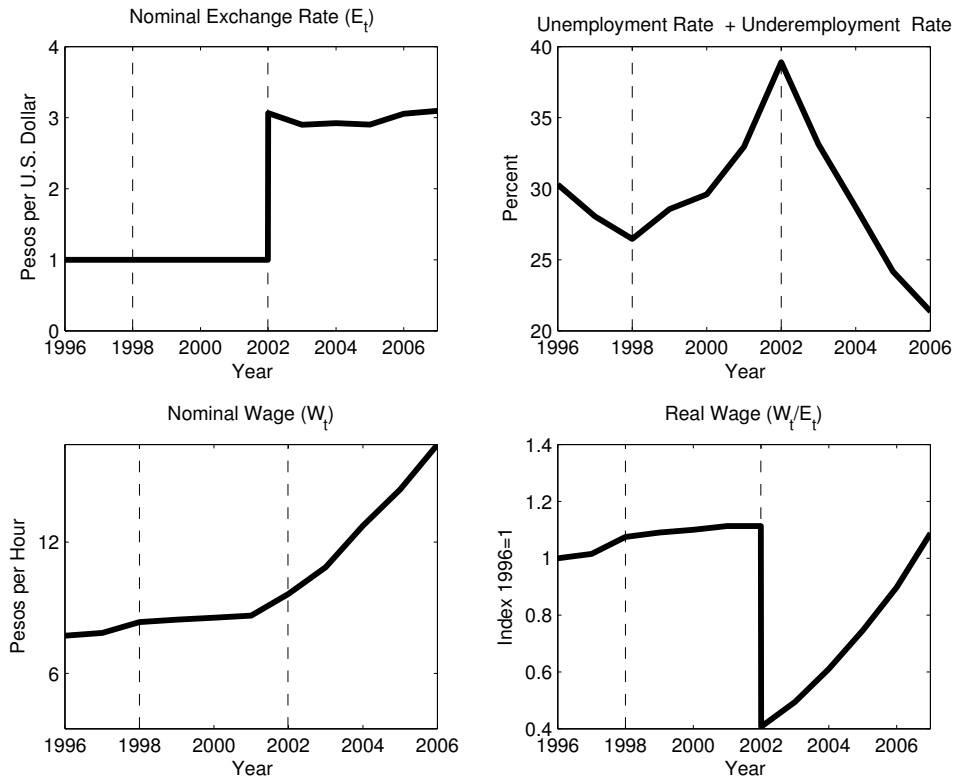


Are devaluations expansionary as predicted by the DNWR model?

To address this issue, we take another look at two episodes of exiting a currency peg:

- Ending Convertibility: Argentina 1996-2006
- Exiting the Gold Standard: Europe 1929-1935

Argentina Post Convertibility 1996-2006



Observations:• Argentine peso pegged to the U.S. dollar from April 1991 to December 2001. (Convertibility Plan) • Since 1998 severe recession, subemployment rate reaches 35 percent. • but no nominal wage decline in 1998-2001; • December 2001 Argentina devalues by 250%; • leads to large decline in the real wage; • labor market conditions then improved quickly. • sizable fall in real wages right after the Dec 2001 devaluation suggests that the 1998-2001 period was one of censored wage deflation. • nominal wage growth after devaluation suggests upward flexibility of nominal wages;

Overall, predicted expansionary effects of devaluation are consistent with dynamics in Argentina post Convertibility

Memo: Average annual CPI inflation 1998-2001: -0.86%

Exiting the Gold Standard: Europe 1929 to 1935

• Friedman and Schwartz (1963) observe that countries that left gold early (the sterling bloc) enjoyed more rapid recoveries than countries that stayed on gold longer (the gold bloc).

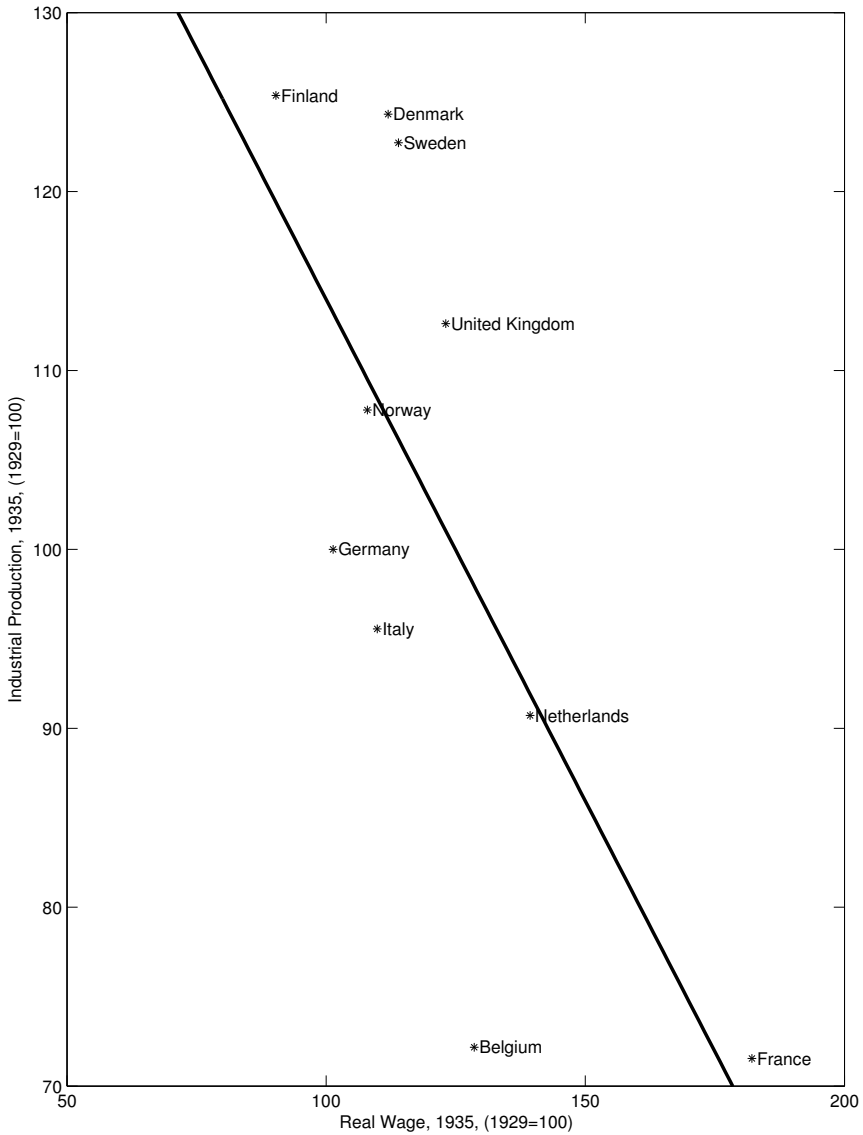
• Sterling bloc: United Kingdom, Sweden, Finland, Norway, Denmark.

• Gold bloc: France, Belgium, the Netherlands, and Italy.

• Eichengreen and Sachs (1986) observe that real wages behaved differently in countries that left gold early (ie devalued) and in countries that stayed on gold longer (ie stayed on the peg). Take a look at the next figure, which is redrawn from Eichengreen and Sachs.

Changes In Real Wages and Industrial Production, 1929-1935

Figure 9.15

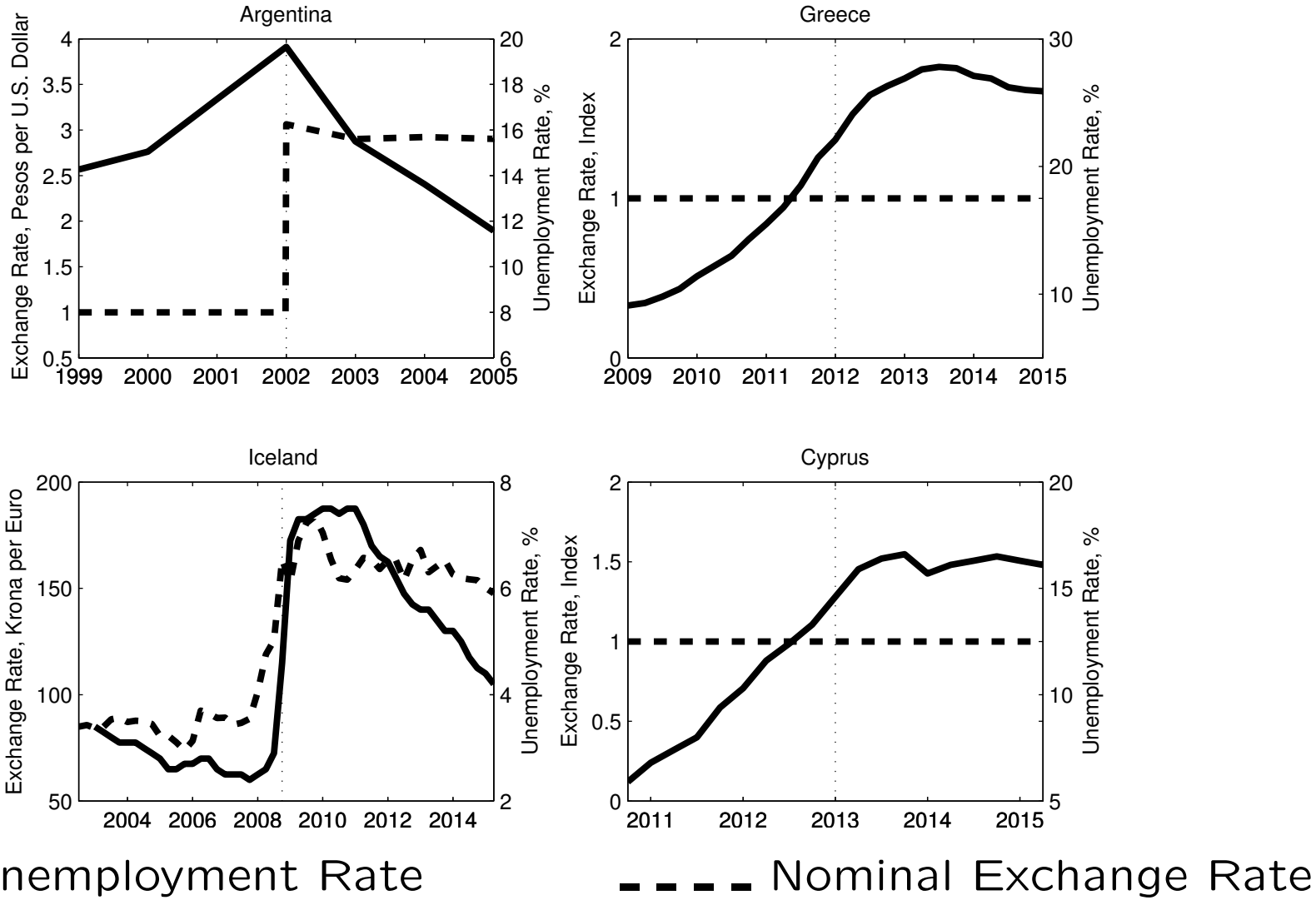


• The horizontal axis measures the 1935 real wage, defined as the nominal wage deflated by the wholesale price index; • The vertical axis measures industrial production in 1935; • Real wages and industrial production are normalized to 100 in 1929; • The solid line is the fitted regression line: $IP = 170.0 - 0.561 \times$ Real Wage.

• relative to their respective 1929 levels real wages in 1935 in the sterling bloc countries were lower than real wages in the gold bloc countries. • And industrial production in the sterling bloc countries in 1935 exceeded their respective 1929 levels whereas industrial production in the gold bloc countries was below their respective 1929 levels.

• Taken together, these two facts suggest that in the great depression years nominal wages were downwardly rigid in Europe and that abandoning a peg during a recession can be expansionary.

Default, Devaluation, and Unemployment:



Note. Vertical line indicates the year of default. Own calculations based on data from INDEC (Argentina), EuroStat, and the Central Bank of Iceland.

9.10 The Welfare Costs of Currency Pegs

Find the compensation, measured as percent increase in the stream of consumption in the peg economy, denoted $\Lambda(s_t)$, that makes agents indifferent between living under a peg or under the optimal exchange-rate policy, given the current state $s_t = (y_t^T, r_t, d_t, w_{t-1})$. This compensation is implicitly given by

$$E\left\{\sum_{j=0}^{\infty}\beta^j U\left(c_{t+j}^{\text{PEG}}\left(1+\frac{\Lambda(s_t)}{100}\right)\right)\middle| s_t\right\} = E\left\{\sum_{j=0}^{\infty}\beta^j U\left(c_{t+j}^{\text{OPT}}\right)\middle| s_t\right\},$$

Solve for $\Lambda(s_t)$ to obtain

$$\Lambda(s_t) = 100\left\{\left[\frac{v^{OPT}(y_t^T, r_t, d_t)(1-\sigma)+(1-\beta)^{-1}}{v^{PEG}(y_t^T, r_t, d_t, w_{t-1})(1-\sigma)+(1-\beta)^{-1}}\right]^{1/(1-\sigma)} - 1\right\},$$

where $v^{OPT}(y_t^T, r_t, d_t) \equiv E\left\{\sum_{j=0}^{\infty}\beta^j U\left(c_{t+j}^{\text{OPT}}\right)\middle| s_t\right\}$
and $v^{PEG}(y_t^T, r_t, d_t, w_{t-1}) \equiv E\left\{\sum_{j=0}^{\infty}\beta^j U\left(c_{t+j}^{\text{PEG}}\right)\middle| s_t\right\}$,
with the expectation taken over the distribution of s_t in the peg economy. The welfare cost of a peg, $\Lambda(s_t)$, is a random variable as it is a function of the state in period t, s_t. When $\sigma = 1/\xi$ the only state variable that is policy dependent is w_{t-1} and c_t^T is policy independent. Thus, when $\sigma = 1/\xi$ the only source of welfare loss of suboptimal exchange rate policy stems from the dynamics of c_t^N.

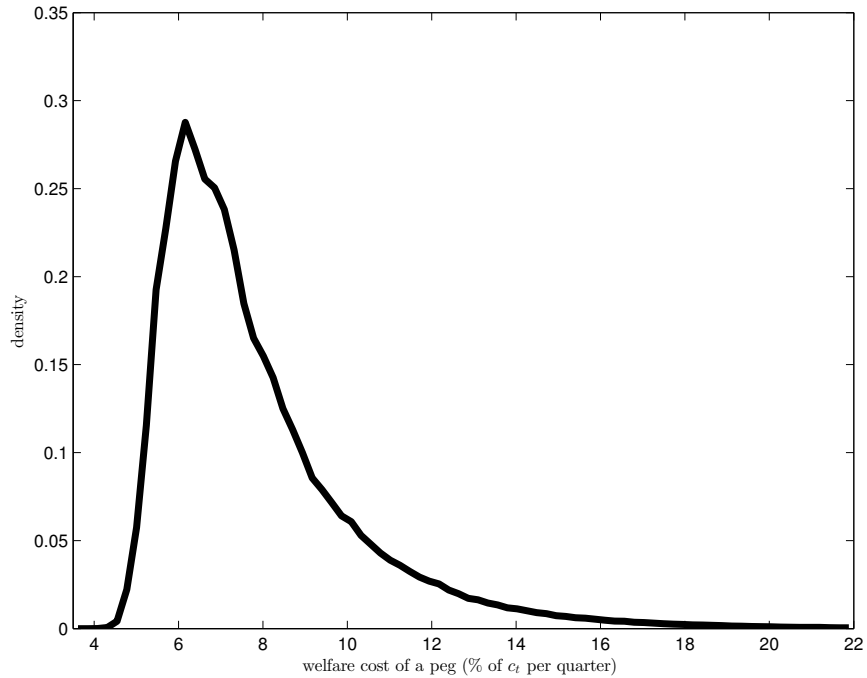
Table 9.4 The Welfare Costs of Currency Pegs

	Welfare Cost		Unemployment
Model	Mean	Median	Mean Rate
Baseline ($\gamma = 0.99$)	7.8	7.2	11.7

Note. The welfare cost of a currency peg is expressed in percent of consumption. Welfare costs are computed over the distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$ induced by the peg economy. Replication files: simu_welf.m (welfare cost) and simu.m (unemployment) in usg_dnwr.zip.

Observation: Large welfare costs of currency pegs. All of the cost is explained by lost consumption of nontradables due to unemployment in that sector.

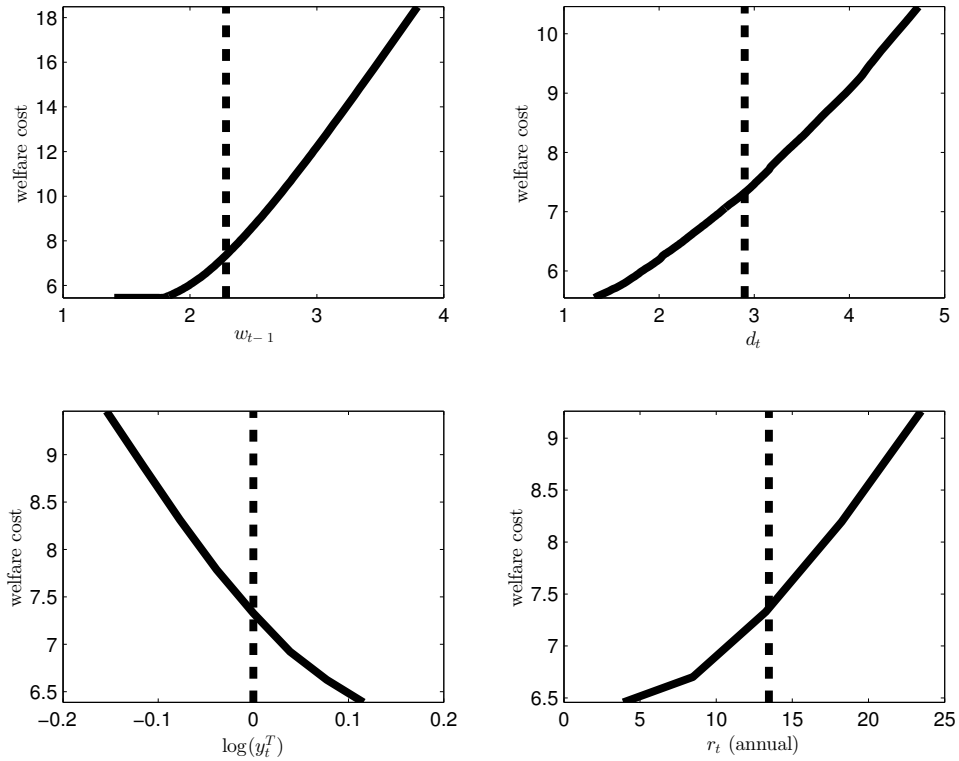
Figure 9.16 Probability Density Function of the Welfare Cost of Currency Pegs



Replication file plot_welf.m in usg_dnwr.zip.

Observation: The distribution of welfare costs of pegs is highly skewed to the right, suggesting the existence of initial states, $(y_t^T, r_t, d_t, w_{t-1})$, in which pegs are highly costly in terms of unemployment. The next slide identifies such states.

Figure 9.17 Welfare Cost of Currency Pegs and the Initial State



Note. In each plot, all states except the one shown on the horizontal axis are fixed at their unconditional mean values. The dashed vertical lines indicate the unconditional mean of the state displayed on the horizontal axis (under a currency peg if the state is endogenous). Replication file plot_welf.m in usg_dnwr.zip.

Observation: Currency pegs are more costly the higher the initial past wage, the higher the initial stock of external debt, the lower the initial endowment of tradables, and the higher the initial country interest rate.

Alternative Parameterizations and Model Specifications

9.10 Varying the Degree of Wage Rigidity

9.11 Symmetric Wage Rigidity

9.13 Endogenous Labor Supply

9.14 Production in the Traded Sector

9.15 Product Price Rigidity

Varying the Degree of Downward Wage Rigidity

Model	Welfare Cost Mean	Welfare Cost Median	Unempl. Rate
Baseline ($\gamma = 0.99$)	7.8	7.2	11.7
Lower Downward Wage Rigidity			
$\gamma = 0.98$	5.7	5.3	8.9
$\gamma = 0.97$	3.5	3.3	5.6
$\gamma = 0.96$	2.8	2.7	4.6
Higher Downward Wage Rigidity			
$\gamma = 0.995$	14.3	13.0	19.5

Observation: Sizable welfare costs and unemployment even for highly flexible wages, e.g., $\gamma = 0.96$. Recall, $\gamma = 0.96$ means that wages can fall frictionlessly by 16% per year.

9.11 Symmetric Wage Rigidity

Is more wage flexibility always welfare increasing?

Not always. We have just seen that the welfare costs of a currency peg increase as the degree of downward wage rigidity, γ, increases. So the answer here is Yes.

We now consider a different way of increasing wage rigidity, namely, bi-directional wage rigidity:

$$\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma$$

We will see that this increase in wage rigidity is welfare enhancing.

The Welfare Costs of Pegs: Symmetric Wage Rigidity

($\gamma = 0.99$)

	Welfare Cost Mean	Unempl. Rate
Downward only: $\frac{W_t}{W_{t-1}} \geq \gamma$	7.8	11.7
Upward and downward: $\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma$	3.3	5.2

• Welfare costs under symmetric rigidity, while still large, are half that under downward wage rigidity. Thus greater wage flexibility is welfare decreasing. Why? Symmetric wage rigidity alleviates the peg-induced externality (we saw this theoretically).

• To the extent that downward wage rigidity is the case of greatest empirical relevance, this result suggests that models with upward and downward wage rigidity underestimate the welfare costs of currency pegs.

9.13 Endogenous Labor Supply

Thus far, we have assumed that households supply $\bar{h}$ units of labor inelastically. How should an endogenous labor supply affect the main predictions of the model?

• Now negative income shocks (e.g., $y^T \downarrow$), may cause an increase in labor supply, elevating the level of involuntary unemployment.

• Thus far, the model included only one type of non-work activity, namely involuntary unemployment. Now there will be two, involuntary unemployment (or involuntary lesure) and voluntary leisure (or voluntary unemployment). How do voluntary and involuntary unemployment enter in the utility function? Are they substitutes or complements? How substitutable are they? This margin will make a difference in determining the welfare costs of currency pegs.

With these questions in mind, let's now introduce a labor choice more formally.

Assume that the period utility function is increasing in consumption, c_t, and leisure, ℓ_t,

$$U(c_t, \ell_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{{\ell_t}^{1-\theta} - 1}{1-\theta}$$

The desired demand for leisure, (or desired supply of labor) is a notional object that results from assuming that the household can work as many hours as it wishes at the going real wage:

$$\varphi(\ell_t^v)^{-\theta} = w_t \lambda_t,$$

where ℓ_t^v denotes voluntary leisure. Let $\bar{h}$ denote the total endowment of hours per period. The desired (or voluntary) labor supply is given by

$$h_t^v = \bar{h} - \ell_t^v.$$

Let h_t denote actual hours worked.

A no-forced-labor condition:

$$h_t^v \geq h_t,$$

Closing the labor market

$$(h_t^v - h_t)\left(w_t - \gamma\frac{w_{t-1}}{\epsilon_t}\right) = 0.$$

The above four equations replace the conditions $h_t \leq \bar{h}$ and $(\bar{h} - h_t)(w_t - \gamma w_{t-1}/\epsilon_t) = 0$ of the economy with inelastic labor supply. The rest of the equilibrium conditions are unchanged.

Involuntary unemployment, or, synonymously, involuntary leisure, denoted u_t, is given by

$$u_t = h_t^v - h_t$$

Policy evaluation requires addressing an important question:

How should voluntary and involuntary leisure enter in the period utility function?

One possibility is to assume that ℓ_t^v and u_t are perfect substitutes. In this case, the second argument of the utility function becomes $\ell_t = \ell_t^v + u_t$. Is this assumption empirically realistic?

The existing empirical literature seems to reject this assumption:

• Krueger and Mueller (2012): the unemployed enjoy leisure activities to a lesser degree than the employed and on a typical day report higher levels of sadness than the employed.

• Winkelmann and Winkelmann (1998): Unemployment has a large non-pecuniary detrimental effect on life satisfaction.

• Krueger and Mueller (2012): Unemployed spend 101 minutes more per day on job search than employed (not surprising). However, job search generates the highest feeling of sadness after personal care out of 13 time-use categories.

$\Rightarrow$ Better specification appears to be : $\ell_t = \ell_t^v + \delta u_t$

with $\delta < 1$. Will consider three values, 1, 0.75, and 0.5.

Endogenous Labor Supply And The Welfare Costs of Currency Pegs

	Welfare Cost		Unemployment
Parameterization	Mean	Median	Rate
Baseline (inelastic labor supply)	7.8	7.2	11.7
Endogenous Labor Supply $\ell_t = \ell_t^v + \delta u_t$			
$\delta = 0.5$	16.5	15.2	30.9
$\delta = 0.75$	8.2	7.5	30.9
$\delta = 1$	1.7	1.5	30.9

Observations: (1) Unemployment larger under endogenous labor supply specification. (2) Welfare cost of peg larger or smaller depending on preferences about involuntary leisure, δ.

9.15 Product Price Rigidity

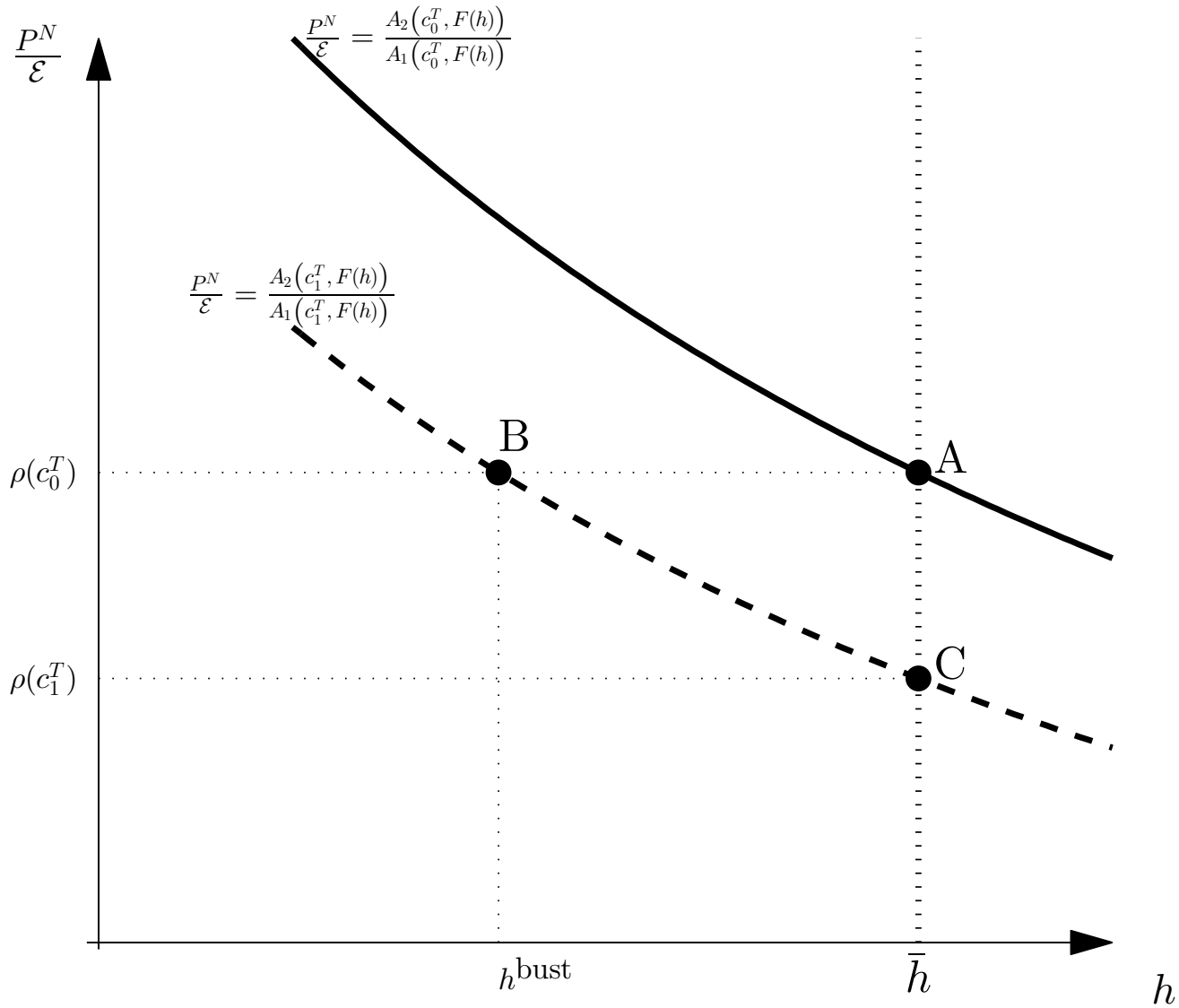
Assume now that nominal wages are fully flexible and that instead prices are sticky. Consider downward nominal price ridigity and symmetric price rigidity.

$$\text{downward price rigidity:} \qquad \frac{P_t^N}{P_{t-1}^N} \geq \gamma_p$$

$$\text{symmetric price rigidity:} \qquad \frac{1}{\gamma_p} \geq \frac{P_t^N}{P_{t-1}^N} \geq \gamma_p$$

Calibrate models as before, but set $\gamma = 0$ and $\gamma_p = 0.99$.

Figure 9.18 Adjustment to a Negative External Shock with Downward Price Rigidity Under A Currency Peg



$c_1^T < c_0^T; \qquad \gamma_p = 1$

Observations: When c^T falls from c_0^T to c_1^T, full employment would occur if prices could decline to $\rho(c_1^T)$, point C in the figure. But because of downward nominal price rigidity, $P_t^N/\mathcal{E}$ remains at $\rho(c_0^T)$. At this price households only demand $F(h^{bust})$ units of nontradables and the economy suffers of unemployment due to week demand.

Optimal policy calls for a devaluation that lowers $P_t^N/\mathcal{E}_t$ down to $\rho(c_1^T)$ and restores full employment. Hence contractions continue to be devaluatory!

The economy continues to suffer from a peg induced externality. Increases in P_t^N during booms should be limited to avoid unemployment during the recession phase of the cycle.

Price Rigidity And The Welfare Costs of Currency Pegs

Parameterization	Welf Cost	Unempl Rate
Baseline (wage rigidity, $\gamma = 0.99$ and $\gamma_p = 0.$)	7.8	11.7
Nominal Price Rigidity ($\gamma = 0$, $\gamma_p = 0.99$)		
Downward Price Rigidity, $P_t^N / P_{t-1}^N \geq \gamma_p$	9.9	14.1
Symmetric Price Rigidity, $1/\gamma_p \geq P_t^N / P_{t-1}^N \geq \gamma_p$	4.4	6.6
Calvo Price Rigidity, $\theta = 0.7$	3.6	N/A

Note. The welfare cost of a currency peg is expressed in percent of consumption per quarter. Unemployment rates are expressed in percent.

Observations. The welfare costs of pegs under downward price rigidity are as large as under downward wage rigidity. The welfare costs under symmetric price rigidity are only about half as large as under downward price rigidity. It follows that adding upward price rigidity ameliorates the peg induced externality, just like adding upward wage rigidity amerliorates it in the model model with wage rigidity.

The last line, labeled Calvo pricing, pertains to an economy with Calvo-type price rigidity in the nontraded sector. The probability of not being able to change the nominal price is $\theta = 0.7$ per period. The calibration of the shocks is the same as in the baseline model. The Calvo model is one of symmetric price rigidity and so it is not surprising that the welfare costs of pegs are most similar to those associated with the economy with the bi-directional price rigidity studied earlier.

Summary of Theoretical results:

• **Analytical results** — The combination of a currency peg and downward nominal wage rigidity creates an externality that amplifies the severity of contractions.

— The model predicts that the average rate of unemployment is increasing in the degree of aggregate volatility.

— The DNWR model predicts significant amplification of booms-bust cycles under suboptimal monetary policy (and in particular under currency pegs.).

• Quantitative Results:

— The costs of currency pegs due to downward nominal wage rigidity are large,

· in terms of welfare, 4 to 10% of consumption per period.

· and in terms of unemployment, 10 to 30%.

— These results are robust to a variety of changes parameter values and model specifications, including endogenous labor supply, bidirectional nominal rigidity, and product price rigidity.

— The welfare costs of currency pegs are higher the higher the past real wage, the higher the level of external debt, the lower the level of tradable output, and the higher the country interest rate.

9.12 The Mussa Puzzle

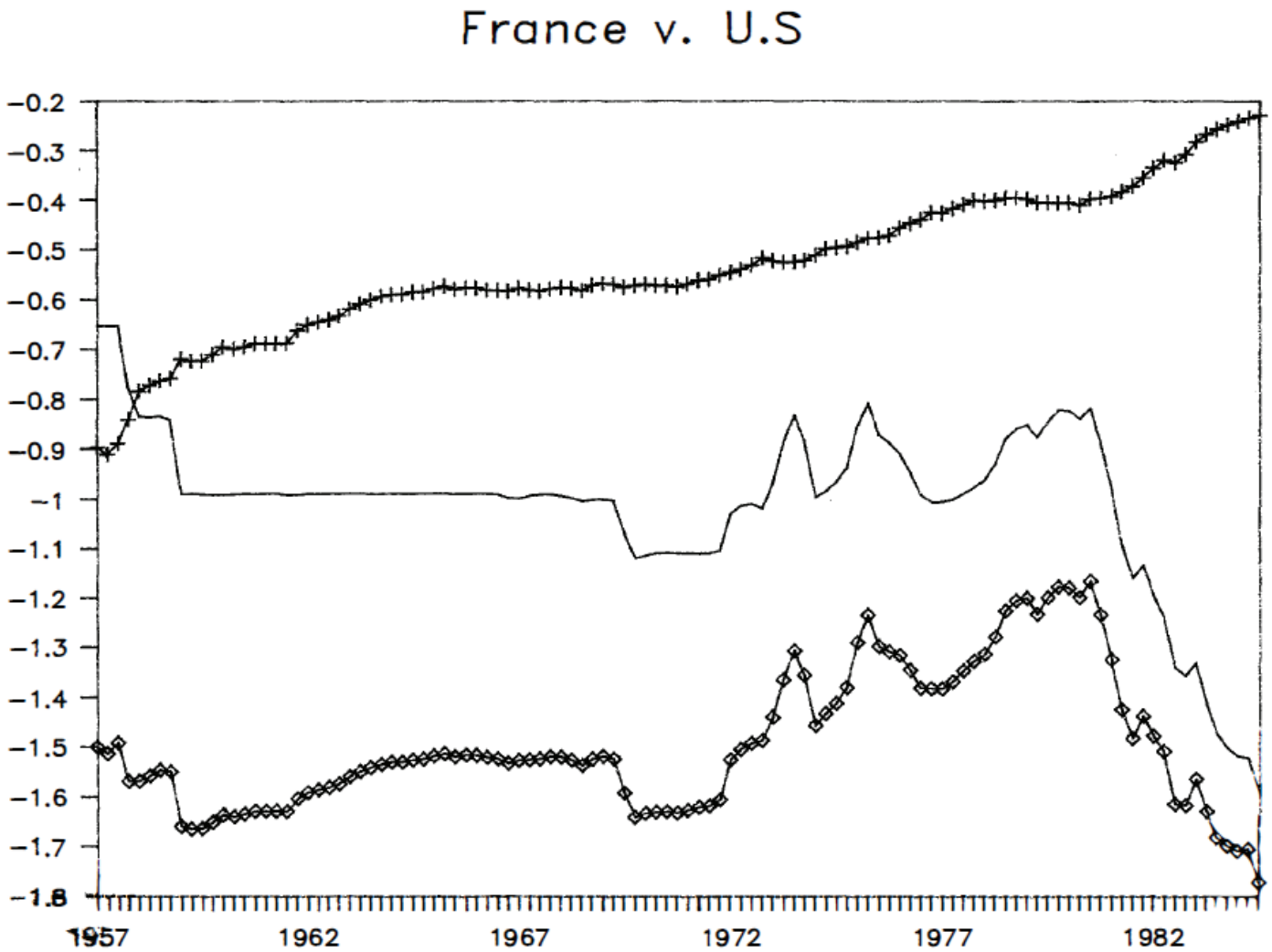
Mussa (1986) is an empirical paper that provides evidence against the "nominal exchange rate regime neutrality" hypothesis.

Methodology: compare variations in nominal and real exchange rates in quarterly data from 13 industrialized countries during the fixed exchange rate period (1957-1970) with variations in the floating exchange rate period (1973-1984)

Observables:
$\mathcal{E}_t =$ nominal exchange rate vis-á-vis the U.S. dollar
$P_t^* =$ U.S. Consumer Price Index
$P_t =$ Domestic Consumer Price Index
$RER_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ real exchange rate

Take a look at the next figure. It shows 3 series: the natural logarithms of the dollar French franc nominal and real exchange rates as well as the log of the relative CPIs.

FIGURE 1



Solid line: log of nominal exchange rate, $\ln \mathcal{E}_t$; plus-line: log of CPI ratio, $\ln(P_t^*/P_t)$; diamond line: log of real exchange rate, $\ln RER_t \equiv \ln(\mathcal{E}_t P_t^*/P_t)$. Source: Mussa 1986, figure 1.

Comments on the figure

Quarterly data, 1957:Q2 to 1984:Q3.

Fixed exchange rate: Bretton Woods, 1957:Q2-1970:Q4.
French franc was pegged to the U.S. dollar.

Floating exchange rate: Post-Bretton Woods, 1973:Q1-1984:Q3.

Selection of sample period: Why a gap in the sample period? Mussa argues that 1970Q4 to 1973Q1 is a transition period. The float starts in March 1973. Why does Mussa start his sample in 1957, this was afterall not the beginning of the Bretton Woods agreement? Mussa justifies the start date by saying that this is when the IFS tapes start having the raw data. Why does Mussa end the sample 1983Q3? This must be the most recent observation when he wrote the paper.

• The graph shows that during the Bretton Woods period (1957-1970) the nominal exchange rate was basically constant. In the post-Bretton Woods period (1973-1984) it fluctuates quite a bit. This confirms treating these two periods as having a different nominal exchange rate regime.

• The striking feature of the graph is that the real exchange rate mimics the behavior of the nominal exchange rate throughout.

• The flipside of this observation is that the times series properties of relative prices, P_t^*/P_t, remained remarkable constant across the two periods.

• Mussa argues that under the 'nominal exchange rate regime neutrality' hypothesis one should not observe such a high correlation between nominal and real exchange rates.

Second Moments

Mussa documents three facts:
1.) The standard deviation of the real deprecation rate is smaller during the peg era.
2.) The standard deviations and serial correlations of the nominal and real depreciation rates are similar to each other in each exchange rate regime.
3.) The volatility of national CPI inflation is about the same during the fixed and the floating regime sample period.
Let

$$\epsilon_t^{RER} \equiv \ln RER_t - \ln RER_{t-1}$$

$$\epsilon_t \equiv \ln \mathcal{E}_t - \ln \mathcal{E}_{t-1}$$

$$\epsilon_t^p \equiv \ln P_t^*/P_t - \ln P_{t-1}^*/P_{t-1}$$

Illustrate these 3 facts for France

France	1957Q2-1970Q4	1973Q1-1984Q3
var(ϵ_t^{RER})	5.258	23.590
var(ϵ_t)	8.197	24.275
corr($\epsilon_t^{RER}, \epsilon_{t-1}^{RER}$)	0.1150	0.3376
corr($\epsilon_t, \epsilon_{t-1}$)	0.1776	0.4317
var(ϵ_t^p)	1.543	0.540

Taken from Table 1.4 of Mussa (1986)

Theoretical Implications of the Mussa Puzzle

• The Mussa facts are referred to as a puzzle because they suggest that relative prices depend on the behavior of nominal prices. This is inconsistent with flexible-price models in the RBC tradition, possibly augmented with a demand for money, which represented the predominant paradigm around the time of Mussa's writing (e.g., Stockman, 1988).

• The Mussa puzzle suggests a role for nominal rigidities, which gained renewed popularity since the mid 1990s. An early analysis of the Mussa puzzle through the lens of a sticky-price model is Monacelli (2004).

• Here, we wish to address two questions:

(a) Can nominal rigidity explain the Mussa Puzzle?

(b) If so, do the Mussa facts emerge under *any* flexible exchange rate policy? This question is relevant, because there exists only one fixed exchange rate regime, but an infinite number of floating exchange rate regimes.

Preview of answers: (a) We will show that the model with downward nominal wage rigidity studied in this chapter predicts the Mussa facts. (b) The Mussa facts do not emerge under any floating exchange-rate policy. In particular, they do obtain under the optimal exchange-rate policy, but fail to obtain under others. This suggests that the post-Bretton-Woods floating regime was associated with dynamics consistent with optimal exchange-rate policy.

Table 9.5 Real and Nominal Exchange Rates Under Fixed And Floating Exchange-Rate Regimes as Predicted by the DNWR Model

	Peg	Float	
		Optimal	Anti optimal
std(ϵ_t^{RER})	12.0	32.5	5.2
std(ϵ_t)	0	45.2	44.0
corr($\epsilon_t^{RER}, \epsilon_{t-1}^{RER}$)	0.18	-0.04	-0.04
corr($\epsilon_t, \epsilon_{t-1}$)	–	-0.04	0.95
corr($\epsilon_t^{RER}, \epsilon_t$)	–	0.99	-0.15
std(π_t)	13.2	13.2	44.3

Note. Standard deviations are expressed in percent per year. The optimal floating exchange-rate policy is given by $\epsilon_t = w_{t-1}/\omega(c_t^T)$, and the suboptimal floating exchange-rate policy is given by $\epsilon_t = \omega(c_t^T)/w_{t-1}$.

Observations on the Table

• In line with Mussa's first fact, the predicted standard deviation of the real depreciation rate is much larger under the optimal floating exchange rate policy than under the peg (first line of the table).

• In line with Mussa's second fact, under the optimal flexible exchange rate regime, the nominal and real exchange rates have similar standard deviation and first-order serial correlations, and are highly positively contemporaneously correlated (lines 2-5 of the table).

• In line with Mussa's third fact, the predicted volatility of CPI inflation is the same under the peg and the optimal floating regime (last line of the table).

Was Post-Bretton-Woods Exchange-Rate Policy Optimal?

• Often, empirical studies classify exchange-rate regimes into fixed or floating, and then derive stylized facts associated with each regime. This practice is problematic because in reality there is an infinite number of floating exchange-rate regimes, not just one, which can induce different dynamics of nominal and real variables.

• As an illustration, consider the 'anti optimal' exchange rate policy:

$$\epsilon_t = \frac{\omega(c_t^T)}{w_{t-1}}.$$

which is the inverse of the optimal policy used in the previous table and studied earlier in the quantitative analysis.

• The last column of the previous table shows that under the anti optimal floating exchange-rate policy, the model fails to capture all three of the Mussa facts.

• Seen through the lens of the DNWR model, it follows that Mussa's facts can be interpreted as suggesting that during the early post-Bretton-Woods period, overall, the dynamics of inflation and nominal and real exchange rates were consistent with the optimal exchange rate policy.

9.16

Staggered Price Setting:

The Calvo Model

The Calvo model:

— is a very popular framework in monetary economics

— assumes staggered price setting

— was proposed by Calvo (1983) and later refined by Woodford (1996) and Yun (1996)

— assumes that price rigidity is bidirectional, that is, the upward and downward adjustment of nominal prices is sluggish

Differences to the models of nominal rigidities studied earlier in the chapter:

(1) firms are assumed to satisfy demand even if the price is below marginal cost. An implication of this assumption is that in the Calvo model the labor supply must be wage elastic for price stickiness to have first-order effects.

(2) Calvo model assumes imperfect competition in product markets. This assumption allows firms be price setters and to have nonzero finite demand even when their prices differ from those of their closest competitors.

(3) wages are flexible and there is no involuntary unemployment. [We assume this here to have just one rigidity. Of course, Calvo-type nominal wage rigidity could be added to the model as is often done in the DSGE literature.]

9.16.1 Households

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - V(h_t)] \tag{9.41}$$

subject to

$$c_t = A(c_t^T, c_t^N) \tag{9.42}$$

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t d_t = P_t^T y_t^T + W_t h_t + \Phi_t + T_t + \frac{\mathcal{E}_t d_{t+1}}{1 + r_t} \tag{9.43}$$

Assume law of one price holds for tradables and that foreign price is unity

$$P_t^T = \mathcal{E}_t$$

Optimality conditions of the household's problem are:

(9.42)-(9.43) and

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1}$$

$$V'(h_t) = \lambda_t \frac{W_t}{P_t^T}$$

9.16.2 Firms Producing Final Nontraded Goods

Production technology:

$$y_t^N = \left[\int_0^1 \left(a_{it}^N\right)^{1-\frac{1}{\mu}} di\right]^{\frac{1}{1-\frac{1}{\mu}}}, \tag{9.45}$$

y_t^N = output of the final nontraded good
a_{it}^N = quantity of intermediate goods of type $i \in [0,1]$ used in the production of the final nontraded good
$\mu > 1$ elasticity of substitution across varieties

Environment: perfect competition

Firm profits:

$$P_t^N y_t^N - \int_0^1 P_{it}^N a_{it}^N di,$$

Profit maximization implies intermediate input demand of the form

$$a_{it}^N = y_t^N \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu} \tag{9.46}$$

• demand for the intermediate good of variety i is increasing in the level of final output, y_t^N, and decreasing in the relative price of the variety in terms of the final good, with a price elasticity of $-\mu$.

Using this expression to eliminate a_{it}^N from the Dixit-Stiglitz aggregator (9.45) yields

$$P_t^N = \left[\int_0^1 (P_{it}^N)^{1-\mu} di \right]^{\frac{1}{1-\mu}} \tag{9.47}$$

• price of the final nontraded good is increasing and homogeneous of degree one in the price of the intermediate nontraded goods

• zero profits

9.16.3 Firms Producing Nontraded Intermediate Goods

Production technology for variety i

$$y_{it}^N = h_{it}^\alpha; \quad \alpha \in (0, 1] \tag{9.48}$$

Environment: Monopolistic competition, firms are price setters

Production is demand determined. This means that, given the posted price for intermediate good of variety i, P_{it}^N, firms must set production to ensure that all customers are served, that is,

$$y_{it}^N = a_{it}^N \tag{9.49}$$

Profits of intermediate good producers

$$\Phi_{it} = P_{it}^N a_{it}^N - (1-\tau) W_t h_{it}$$

τ = a labor subsidy to offset distortions introduced by imperfect competition. Its presence facilitates the characterization of optimal monetary policy, as it results in a model with a single distortion, namely the one stemming from price rigidity. That is, monetary policy here will not be called up to correct distortions stemming from imperfect competition.

Price setting problem of producer of variety i

Use (9.46), (9.48), and (9.49) to eliminate a_{it}^N, h_{it}, and y_{it}^N, respectively, from the expression for profits, Φ_{it}, in period t :

$$P_{it}^N y_t^N \left(\frac{P_{it}^N}{P_t^N}\right)^{-\mu} - (1-\tau) W_t y_t^{N\frac{1}{\alpha}} \left(\frac{P_{it}^N}{P_t^N}\right)^{-\frac{\mu}{\alpha}}.$$

Under price flexibility, the optimal pricing decision consists in choosing P_{it}^N to maximize the above expression.

Under price stickiness, the pricing problem is different because firms, by assumption, cannot reoptimize prices every period.

With probability $\theta \in (0,1)$ a firm cannot reset its price in period t and must charge the same price as in the previous period, and with probability $1-\theta$ it can adjust the price freely. Consider the pricing decision of a firm that can reoptimize its price in period t.

Let $\tilde{P}_{it}^N$ denote the price chosen in t. Then, with probability θ, the price will continue to be $\tilde{P}_{it}^N$ in period $t+1$. With probability θ^2, the price will continue to be $\tilde{P}_{it}^N$ in period $t+2$, and so on. In general, with probability θ^s the price will continue to be $\tilde{P}_{it}^N$ in period $t+s$.

The original Calvo (1983) formulation assumes that $\tilde{P}_{it}^N$ is set following an ad hoc rule of thumb. The innovation of the New Keynesian literature (see, for example Yun, 1996 or Woodford 2003) is to assume that the firm picks $\tilde{P}_{it}^N$ in a profit-maximizing fashion.

The present discounted value of profits associated with $\tilde{P}_{it}^N$ is given by

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta^s \left[\tilde{P}_{it}^N y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} - (1-\tau) W_{t+s} {y_{t+s}^N}^{\frac{1}{\alpha}} \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\frac{\mu}{\alpha}} \right],$$

$Q_{t,t+s}$ = state-contingent nominal discount factor that converts nominal payments in period $t+s$ into a nominal payment in period t.

The firm picks $\tilde{P}_{it}^N$ to maximize the present discounted value of profits. The FOC is:

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta^s y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} \left\{ \frac{\mu - 1}{\mu} \tilde{P}_{it}^N - \frac{1}{\alpha}(1-\tau) W_{t+s} \left[y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} \right]^{\frac{1-\alpha}{\alpha}} \right\} = 0. \tag{9.50}$$

$\frac{\mu-1}{\mu} \tilde{P}_{it}^N =$ marginal revenue in period $t+s$

$\frac{1}{\alpha}(1-\tau) W_{t+s} \left[y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} \right]^{\frac{1-\alpha}{\alpha}} = \frac{(1-\tau)W_{t+s}}{\alpha (h_{it+s}^N)^{\alpha-1}}$ marginal cost in period $t+s$.

$y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} =$ quantity sold in period $t+s$

• price set in period t equates the present discounted values of marginal revenues and marginal costs weighted by the level of production

• Notice that the first-order condition (9.50) has only one firm specific variable, namely, $\tilde{P}_{it}^N$. It follows that any firm i regardless of its history will charge the same price in period t if it gets the change to reoptimize the price. This feature of the Calvo-style model greatly facilitates aggregations.

9.16.4 Aggregation and Equilibrium

Aggregating inflation in the non-traded sector

Use the fact that every firm that can change the price in period t will choose the same price in the definition of the price index for nontradables, equation (9.47) becomes

$$(P_t^N)^{1-\mu} = \int_0^1 (P_{it}^N)^{1-\mu} di = \theta {P_{t-1}^N}^{1-\mu} + (1-\theta)\left(\tilde{P}_t^N\right)^{1-\mu}.$$

Dividing both sides by $(P_t^N)^{1-\mu}$ yields

$$1 = \theta(\pi_t^N)^{\mu-1} + (1-\theta)(\tilde{p}_t^N)^{1-\mu},$$

where $\pi_t^N \equiv P_t^N / P_{t-1}^N$ denotes the gross rate of inflation of nontradables, and $\tilde{p}_t^N \equiv \tilde{P}_t^N / P_t^N$ denotes the relative price of reoptimized prices in terms of final nontraded goods.

Aggregate output in the nontraded sector and total hours worked

Total hours worked, h_t, is given by

$$h_t = \int_0^1 h_{it} di.$$

Use (9.48) to eliminate h_{it} $h_t = \int_0^1 y_{it}^{1/\alpha} di = \int_0^1 \left[y_t^N \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu} \right]^{1/\alpha} di = y_t^{N\,1/\alpha} \int_0^1 \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu/\alpha} di$. Let

$$s_t \equiv \int_0^1 \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu/\alpha} di.$$

The variable s_t measures price dispersion. If all i varieties sell for the same price, then $s_t = 1$, otherwise, $s_t \geq 1$. Using the definition of s_t in the above expression yields

$$y_t^N = s_t^{-\alpha} h_t^{\alpha}.$$

This expression shows that price dispersion acts as a negative productivity shock. The higher is price dispersion, the lower will be the aggregate level of output associated with a given level of employment.

When prices are fully flexible, all firms charge the same price ($P_{it}^N = P_t^N$), so $s_t = 1$ and $y_t^N = h_t^{\alpha}$. It follows that s_t represents a measure of output loss due to the presence of price rigidity.

A competitive equilibrium is a set of processes c_t^T, π_t^N, p_t, h_t, λ_t, w_t, $\tilde{p}_t^N$, y_t^N, s_t, d_{t+1}, $pvmc_t$, and $pvmr_t$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}, \tag{9.52}$$

$$\lambda_t = U'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N), \tag{9.53}$$

$$\frac{\lambda_t}{1+r_t} = \beta E_t \lambda_{t+1}, \tag{9.54}$$

$$\frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} = p_t, \tag{9.56}$$

$$V'(h_t) = \lambda_t w_t, \tag{9.57}$$

$$y_t^N = s_t^{-\alpha} h_t^{\alpha}, \tag{9.58}$$

$$s_t = \theta s_{t-1}(\pi_t^N)^{\mu/\alpha} + (1-\theta)(\tilde{p}_t^N)^{-\mu/\alpha}, \tag{9.59}$$

$$1 = \theta(\pi_t^N)^{\mu-1} + (1-\theta)(\tilde{p}_t^N)^{1-\mu}, \tag{9.60}$$

$$pvmr_t = pvmc_t, \tag{9.61}$$

$$pvmc_t = \frac{1-\tau}{\alpha}(y_t^N)^{\frac{1}{\alpha}} w_t (\tilde{p}_t^N)^{-\frac{\mu}{\alpha}} + \beta\theta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t^N}{\tilde{p}_{t+1}^N}\frac{1}{\pi_{t+1}^N}\right)^{-\frac{\mu}{\alpha}} pvmc_{t+1}, \tag{9.62}$$

$$pvmr_t = \frac{\mu-1}{\mu} y_t^N p_t (\tilde{p}_t^N)^{1-\mu} + \beta\theta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t^N}{\tilde{p}_{t+1}^N}\frac{1}{\pi_{t+1}^N}\right)^{1-\mu} pvmr_{t+1}, \tag{9.63}$$

$$p_t = p_{t-1}\frac{\pi_t^N}{\epsilon_t}, \tag{9.64}$$

given exogenous processes y_t^T and r_t, initial conditions d_0 and s_{-1}, an exchange-rate policy ϵ_t, and a subsidy policy τ.

Following the standard in the literature, we assume that the government sets the labor subsidy so as to offset the distortion in the labor market created by the presence of imperfect competition in product markets, that is,

$$\tau = \frac{1}{\mu}. \tag{9.65}$$

9.16.7 The Open Economy New-Keynesian Phillips Curve

Perform a log-linerization around the deterministic steady state with $\pi^N = 1$ and letting $\widehat{x}_t \equiv \ln x_t/x$, where x denotes the non-stochastic steady state variable of x_t, combine (9.61) (9.62) (9.63). This yields the famous New-Keynesian Phillips Curve relating current inflation to expected future inflation, π^N_{t+1}, and marginal costs, mc_t.

$$\boxed{\widehat{\pi}^N_t = \beta E_t \widehat{\pi}^N_{t+1} + \kappa \widehat{mc}_t} \quad \text{with} \quad \kappa = \left[\frac{(1-\theta)(1-\beta\theta)}{\theta\left(\frac{\mu}{\alpha} + 1 - \mu\right)}\right] > 0. \tag{9.69}$$

The log-deviation of aggregate marginal cost in the nontraded sector from steady-state is given by

$$\widehat{mc}_t \equiv \left(\frac{1}{\alpha} - 1\right)\widehat{y}^N_t + \widehat{w}_t - \widehat{p}_t$$

9.16.6 Optimal Monetary Policy

With this subsidy policy in place, and if there is no initial price dispersion, $s_{-1} = 1$, the policy of full price stabilization in the nontraded sector

$$\boxed{\pi_t^N = 1}$$

is consistent with the solution to the social planner's problem of the flexible price economy. Hence optimal monetary policy fully stabilizes prices in the sector with nominal rigidities.

What exchange rate supports the optimal monetary policy?

Use the equilibrium condition (9.64) to obtain

$$\epsilon_t = \frac{p_{t-1}}{p_t}.$$

It follows that any shock that causes a real exchange rate depreciation (i.e., causes p_t to fall) in the flexible-price equilibrium—such as a contraction in tradable output y_t^T or an increase in the world interest rate r_t^*—, must be accompanied by a devaluation. (For example, this says that during the global financial crisis when borrowing conditions on world capital market worsened, countries should have devalued.)

Thus, the present model shares two important predictions with the models of downward nominal wage rigidity or downward nominal price rigidity studied earlier in this chapter:

- stabilization of the nominal price of nontradables is optimal.
- contractions are devaluatory.

9.16.8 Crisis Dynamics in the Calvo Model

Numerical Solution technique: use perturbation instead of global methods. They are applicable here as we do not have occasionally binding constraints (no downward nominal wage rigidity). But perturbation methods force us to induce stationarity in a different way than earlier in the chapter. Instead of borrowing limit plus uncertainty, following Schmitt-Grohé and Uribe (2003), we assume that

$$r_t = r_t^* + \psi \left[e^{d_{t+1} - \overline{d}} - 1 \right], \tag{9.70}$$

where r_t is now an endogenous variable and r_t^*, the world interest rate, is an exogenous variable.

Functional Forms and Calibration

As in DNWR model, assume

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}},$$

$$F(h) = h^{\alpha},$$

$$V'(h) = \varphi(\bar{h} - h)^{-\chi},$$

Calibration of the Calvo Model

Parameter	Value	Description
θ	0.7	Probability of no price change in nontraded sector
μ	6	Elasticity of subst. across intermediate nontradables
σ	2	Inverse of intertemporal elasticity of consumption
β	1.0316^{-1}	Quarterly subjective discount factor
φ	1.11	Preference parameter
χ	1	Preference parameter
$\bar{h}$	3	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Elasticity of substitution between tradables and nontradables
α	0.75	Labor share in nontraded sector
ψ	0.0000335	Parameter of debt-elastic interest rate
$\bar{d}$	2.9014	Parameter of debt-elastic interest rate
y^T	1	Steady-state tradable output
r^*	0.0316	Steady-state interest rate (quarterly)

Note. The time unit is a quarter.

Observations on the Calibration:

• The values assigned to σ, a, ξ, α, y^T, and r^* are same as in DNWR baseline model.

• The values assigned to φ, χ, and $\bar{h}$ are same as in DNWR model with endogenous labor supply.

This will make the predictions comparable ... hopefully.

2 new parameters relative to DNWR model:

$\theta =$, the probability of not being able to change the price in a given period

Any empirical evidence on frequency of nominal price changes of non-tradables in emerging countries? Very little. Maybe a reason for this scarcity is given by the fact that for much of the postwar period emerging countries experienced relatively high levels of inflation. But this is changing now, as even more and more emerging markets are experiencing low inflation.

An exception to the dirth of evicence is Gagnon (2009). He uses Mexican micro consumer-price data and reports the monthly frequency of nominal price changes in nonregulated services in Mexico slightly below 10 percent over two low-inflation periods, one preceding and the other following the 1994 'Tequila' crisis.

Gagnon's evidence suggests a quarterly probability of no price change of nontradables of 70 percent. Accordingly, we set θ equal to 0.7.

$\psi =$ parameter governing the debt elasticity of the interest rate.

We calibrate ψ to match the unconditional standard deviation of (percentage deviations from trend of) tradable consumption of 18.5 percent implied by the models studied earlier in this chapter. Those models do not incorporate a debt-elastic interest rate. Instead, stationarity is induced by setting $\beta(1+r) < 1$ and approximating the equilibrium with global methods. The implied value of ψ is

$$\psi = 0.0000335$$

Driving Forces:

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t^*}{1+r^*} \end{bmatrix} = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix} \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}^*}{1+r^*} \end{bmatrix} + \Gamma \epsilon_t, \tag{9.31}$$

with

$$\Sigma_\epsilon = I; \qquad \Gamma\Gamma' = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \qquad r^* = 0.0316,$$

This is the same process as the one used in the DNWR model.

As for the DNWR model, we wish to characterize dynamics in a during a large external crisis

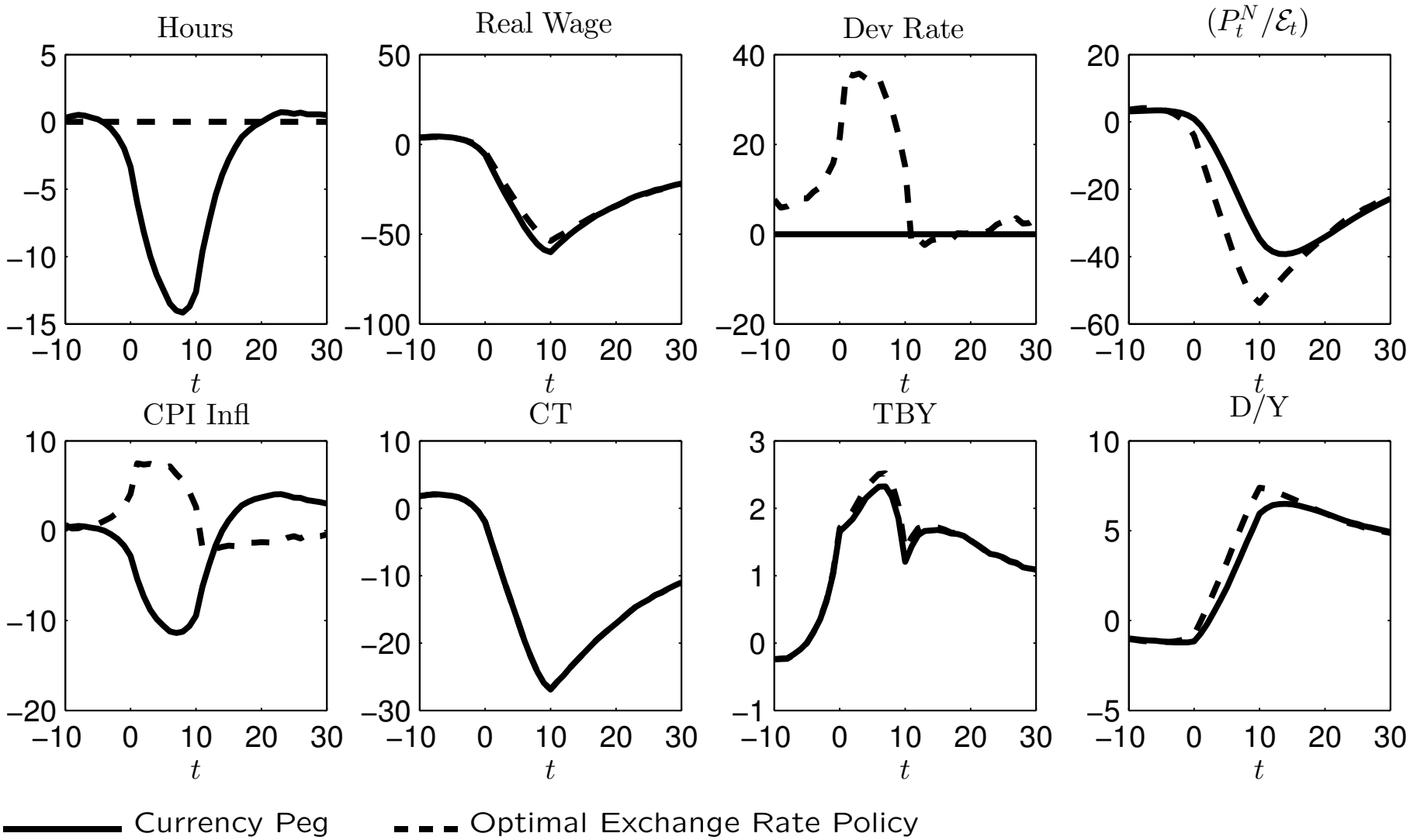
— under a currency peg

and

— under optimal monetary policy

Recall the definition of large external crisis: $\widehat{y}_0^T \geq 0$ in quarter 0 and $\widehat{y}_{10}^T \leq 2\sigma_{y^T}$.

Crisis Dynamics in the Calvo Model: The Role Of Exchange-Rate Policy



Note. Replication file typical_crisis.m in calvo.zip available online with the materials for this chapter.

Observations on the figure:

• Under optimal monetary policy employment is unaffected by the large external shocks. This prediction of the Calvo model is akin to the full-employment result obtained under optimal exchange-rate policy in the DNWR model.

• There are large devaluations throughout the crisis in the order of 30 to 40 percent per year. The devaluations allow the price of nontradables, which is rigid in nominal terms, to decline in real terms, that is, they allow the real exchange rate to depreciate.

• The real depreciation induces an expenditure switch away from tradables and toward nontradables.

• The primary role of the optimal policy is to prevent the external crisis from spreading to the nontraded sector. By contrast, under the currency peg, employment falls by about 15 percent. Why, price stickiness prevents the real exchange rate to depreciate.

• Because preferences are separable in tradable and nontradable consumption ($\sigma = 1/\xi$), the response of c_t^T is identical (down 25%) under the peg and the optimal monetary policy.

• The dynamic responses predicted by the Calvo and DNWR models are largely similar with <u>one important exception</u>:

• Under a peg, the real wage declines in the Calvo model but not in the DNWR model. This is so because nominal wages are flexible and firms, rationed by weak demand, cut their labor demand, which drives the market clearing wage down.

• This prediction is counterfactual. During crises in fixed-exchange-rate economies or in monetary unions (see, for instance, the cases of Argentina 1998-2001 and the periphery of Europe 2008-2011) private sector real wages failed to fall, see figures 9.1, 9.xxx, and 9.yyy