slides
chapter 13
sovereign default
Motivation

• We do observe a significant amount of borrowing and lending across nations.

• But why do countries pay their international debts?

• There is no supranational authority capable of enforcing international debt contracts.

• Defaulting on international financial contracts appears to have no legal consequences.
• Two reasons typically offered for why countries honor their debts:
  1. Economic sanctions
  2. Reputation

• In this chapter:
  – Empirical regularities about international lending, default, and costs of default.
  
  – Theoretical models of sovereign debt.

  – Quantitative models of sovereign debt.
Section 13.1

Sovereign Default: Empirical Regularities
Empirical Definition of Default
Much of the data on sovereign default is produced by credit rating agencies, especially Standard and Poor’s (S&P).

Entrance To Default Status: S&P defines default as the failure to meet a principal or interest payment on the due date. This includes situations in which the sovereign forces an exchange of old debt for new debt with less-favorable terms or converts debt into a different currency of less value.

Exit From Default Status: S&P considers a country to have emerged from default when it resumes payments of interest and principal (including arrears), or after a debt settlement that leads the rating agency to conclude that no further near-term resolution of creditors’ claims is likely.
### Table 13.1 Frequency And Length of Sovereign Defaults: 1824-2014

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of Defaults 1824-2014</th>
<th>Probability of Default all years</th>
<th>Probability of Default years not in default</th>
<th>Years in State of Default per Default Episode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>5</td>
<td>0.026</td>
<td>0.035</td>
<td>10</td>
</tr>
<tr>
<td>Brazil</td>
<td>7</td>
<td>0.037</td>
<td>0.047</td>
<td>6</td>
</tr>
<tr>
<td>Chile</td>
<td>3</td>
<td>0.016</td>
<td>0.020</td>
<td>14</td>
</tr>
<tr>
<td>Colombia</td>
<td>7</td>
<td>0.037</td>
<td>0.058</td>
<td>10</td>
</tr>
<tr>
<td>Egypt</td>
<td>2</td>
<td>0.010</td>
<td>0.012</td>
<td>11</td>
</tr>
<tr>
<td>Mexico</td>
<td>8</td>
<td>0.042</td>
<td>0.056</td>
<td>6</td>
</tr>
<tr>
<td>Philippines</td>
<td>1</td>
<td>0.005</td>
<td>0.006</td>
<td>32</td>
</tr>
<tr>
<td>Turkey</td>
<td>6</td>
<td>0.031</td>
<td>0.037</td>
<td>5</td>
</tr>
<tr>
<td>Venezuela</td>
<td>10</td>
<td>0.052</td>
<td>0.079</td>
<td>6</td>
</tr>
<tr>
<td>Mean</td>
<td>5.4</td>
<td>0.029</td>
<td>0.039</td>
<td>11</td>
</tr>
</tbody>
</table>

Note. The sample includes only emerging countries with at least one external-debt default or restructuring episode between 1824 and 1999. The 2014 selective default of Argentina with 1 percent of the holdout investors that did not enter the debt restructurings of 2005 and 2010 is not counted as a default event. Source: Own calculations based on Reinhart, Rogoff, and Savastano (2003) table 1 for the period 1824-1999 and USG (2017) table 13.19 for the period 2000-2014.
Observations

• 49 defaults in 9 countries over 191 years. ⇒ average probability of default is 2.9% per year (once per 33 years).

• Suppose we divide not by all years, but only by years not in default, then the average probability of default is 3.9% per year.

• When a country defaults it stays in default status on average for 11 years.
Has The Frequency and Length Of Default Changed Over Time?

Table 13.2 1824-1999 Versus 1975-2014

<table>
<thead>
<tr>
<th>Period</th>
<th>Probability of Default per year</th>
<th>Years in State of Default per Default Episode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1824-1999</td>
<td>0.029</td>
<td>11</td>
</tr>
<tr>
<td>1975-2014</td>
<td>0.040</td>
<td>8</td>
</tr>
</tbody>
</table>

Observation: Defaults have become more frequent but shorter. Important for calibrating default models.
Figure 13.1 Distribution Of The Length Of Default Episodes

Observations on figure
(1) Distribution of length of default is skewed (mean 8 years, median 5).
(2) Distribution looks exponential ⇒ not unreasonable to assume probability of reaccess to credit markets constant over time as in most Eaton-Gersovitz-style models of default.

1975-2014 sample of 147 defaults in 93 defaulting nations
13.1.2 Haircuts
The Size of Defaults

- Most models of default assume that defaults are always on 100% of the debt. In reality, this is not the case.

- A haircut is the loss inflicted to creditors upon restructuring, measured as the decrease in the present value of current and future expected payments.

- A number of studies estimate similar values of haircuts (Sturzenegger and Zettelmeyer, 2008; Cruces and Trebesch, 2013; Benjamin and Wright, 2008):

  - Average Haircut: 40%
  - Standard deviation of haircut: 22%.
Debt to GNP Ratio is Higher Than Usual Prior to Default

Table 13.3 Debt-to-GNP Ratios Among Defaulters: 1970-2000

<table>
<thead>
<tr>
<th>Country</th>
<th>Average Debt-to-GNP Ratio</th>
<th>Debt-to-GNP Ratio in Year of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>37.1</td>
<td>54.4</td>
</tr>
<tr>
<td>Brazil</td>
<td>30.7</td>
<td>50.1</td>
</tr>
<tr>
<td>Chile</td>
<td>58.4</td>
<td>63.7</td>
</tr>
<tr>
<td>Colombia</td>
<td>33.6</td>
<td></td>
</tr>
<tr>
<td>Egypt</td>
<td>70.6</td>
<td>112.0</td>
</tr>
<tr>
<td>Mexico</td>
<td>38.2</td>
<td>46.7</td>
</tr>
<tr>
<td>Philippines</td>
<td>55.2</td>
<td>70.6</td>
</tr>
<tr>
<td>Turkey</td>
<td>31.5</td>
<td>21.0</td>
</tr>
<tr>
<td>Venezuela</td>
<td>41.3</td>
<td>46.3</td>
</tr>
<tr>
<td>Average</td>
<td>44.1</td>
<td>58.1</td>
</tr>
</tbody>
</table>

Source: Own calculations based on Reinhart, Rogoff, and Savastano (2003), tables 3 and 6.
Table 13.4 Interest Rate Spreads Among Defaulters: 1999-2013

<table>
<thead>
<tr>
<th>Country</th>
<th>Average Country Spread</th>
<th>All Years</th>
<th>Not In Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>15.8</td>
<td>7.43</td>
<td>15.8</td>
</tr>
<tr>
<td>Brazil</td>
<td>5.61</td>
<td>5.61</td>
<td>5.61</td>
</tr>
<tr>
<td>Chile</td>
<td>1.44</td>
<td>1.44</td>
<td>1.44</td>
</tr>
<tr>
<td>Colombia</td>
<td>3.70</td>
<td>3.70</td>
<td>3.70</td>
</tr>
<tr>
<td>Egypt</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
</tr>
<tr>
<td>Mexico</td>
<td>3.47</td>
<td>3.47</td>
<td>3.47</td>
</tr>
<tr>
<td>Philippines</td>
<td>3.49</td>
<td>3.49</td>
<td>3.49</td>
</tr>
<tr>
<td>Turkey</td>
<td>4.10</td>
<td>4.10</td>
<td>4.10</td>
</tr>
<tr>
<td>Average</td>
<td>5.5</td>
<td>4.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Notes: The sample includes only emerging countries with at least one external-debt default or restructuring episode between 1824 and 1999. Country spreads are measured using the EMBI Global index, produced by J.P. Morgan, and expressed in percent, and are averages through 2013, with varying starting dates as follows: Argentina 1994; Brazil 1995; Chile 2000; Colombia 1998; Egypt 2002; Mexico 1994; Philippines 1998; Turkey 1997; Venezuela 1994. Start and end dates of default episodes are taken from table 13.9 of USG (2017).
The Spread-Default-Frequency Differential is Positive
After Correcting for the Sample-Mismatch Bias

Table 13.5

<table>
<thead>
<tr>
<th>Sample for spread</th>
<th>Country Spread (percent)</th>
<th>Default Probability common</th>
<th>1824-2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1994-2013</td>
<td>7.43</td>
<td>6.7</td>
</tr>
<tr>
<td>Brazil</td>
<td>1995-2013</td>
<td>5.61</td>
<td>0</td>
</tr>
<tr>
<td>Chile</td>
<td>2000-2013</td>
<td>1.44</td>
<td>0</td>
</tr>
<tr>
<td>Colombia</td>
<td>1998-2013</td>
<td>3.70</td>
<td>0</td>
</tr>
<tr>
<td>Egypt</td>
<td>2002-2013</td>
<td>2.56</td>
<td>0</td>
</tr>
<tr>
<td>Mexico</td>
<td>1994-2013</td>
<td>3.47</td>
<td>0</td>
</tr>
<tr>
<td>Philippines</td>
<td>1998-2013</td>
<td>3.49</td>
<td>0</td>
</tr>
<tr>
<td>Turkey</td>
<td>1997-2013</td>
<td>4.10</td>
<td>0</td>
</tr>
<tr>
<td>Venezuela</td>
<td>1994-2013</td>
<td>9.23</td>
<td>13.3</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>4.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>

The sample-mismatch bias: Due to data limitations, spread samples are short, starting in the mid 1990s or later for most countries. By contrast, default data is much longer, starting at least in 1975 and going back almost two centuries for some countries. Computing spread default-frequency differentials using samples of different lengths yields quite different results than using samples spanning the same periods.

With risk neutral lenders, spreads should equal default probability. With partial default, and risk neutral lenders, spreads should be smaller than default probabilities. Evidence of spreads in excess of default probability suggests risk aversion on the part of lenders.
Do Countries Default In Bad Time? Yes.

Figure 13.2 Output Around Default Episodes

Note: Annual log-quadratically detrended real GDP per capita. The year of default is normalized to 0. Median over 105 default episodes between 1975 and 2014.
Observations

• Detrended output falls by 6.5 percent in the 3 years leading up to default.

• 75 percent of the default episodes are associated with a contraction in detrended output (i.e., a fall in detrended output between years -1 and 0).

• Relation to Tomz and Wright (2007): Also find that countries default in bad times, but claim that the evidence is weak. Why?
  – In the period of default output is only 1.5 percent below trend. Same true in current data set, but what about the 5% contraction in output between years -3 and -1?
  – Tomz and Wright find that in only 60 percent of the default episodes output is below trend. Same true in current data set. But if one asks how many countries are contracting at the time of default (i.e., displaying a fall in output), the answer is 75%.

• Default comes at the end of a large contraction and the beginning of a growth (but not level) recovery as stressed by Levy-Yeyati and Panizza (2011).
Other Macro Indicators Around Defaults support view that countries default in bad times

Figure 13.3 Consumption, investment, the trade balance, and the real exchange rate around default episodes
Observations

• Consumption contracts by as much as output in the run up to default (about 6 percent).

• The contraction of investment leading up to default is 3 times as large as that of output.

• The trade balance is below average up until the year of default. And in the year of default it experiences a reversal of about 2% of GDP.

• The real exchange rate depreciates significantly in the default year (by over 4%), and then begins to gradually appreciate.
Section 13.2

The Cost of Default:

Empirical Evidence
What are possible costs of default?

1. Use of force?

2. Financial Exclusion? (Reputation)

3. Output Losses?

4. Reductions in trade?
The use of force by one country or a group of countries to collect debt from another country was not uncommon until the beginning of the twentieth century. In 1902, an attempt by Great Britain, Germany, and Italy to collect the public debt of Venezuela by force prompted the Argentine jurist Luis-Maria Drago, who at the time was serving as minister of foreign affairs of Argentina, to articulate a doctrine stating that no public debt should be collected from a sovereign American state by armed force or through the occupation of American territory by a foreign power. The Drago doctrine was approved by the Hague Conference of 1907.
# Exclusion Costs

## Are Defaulters Excluded From Financial Markets?

### And For How Long?

<table>
<thead>
<tr>
<th>Measure</th>
<th>Partial Reaccess (flows &gt; 0)</th>
<th>Full Reaccess (flows &gt; 1%GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Default*</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Issuance of New Debt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Gelos, et al. (2011)*</td>
<td>4.7</td>
<td>8.4</td>
</tr>
<tr>
<td>- Richmond and Dias (2009)**</td>
<td>5.7</td>
<td>8.4</td>
</tr>
<tr>
<td>- Adjusted Richmond and Dias*</td>
<td>13.7</td>
<td>16.4</td>
</tr>
<tr>
<td>- Cruces and Trebesch (2013)**</td>
<td>5.1</td>
<td>7.4</td>
</tr>
<tr>
<td>- Adjusted Cruces and Trebesch*</td>
<td>13.1</td>
<td>15.4</td>
</tr>
<tr>
<td>Average</td>
<td>9.8</td>
<td>15.9</td>
</tr>
</tbody>
</table>

Note. Reaccess is measured in years after the beginning of default (*) or in years after the end of default (**). Averages are taken over single-star lines.
Observations

• Estimates of the average length of the exclusion period is important for the calibration of default models.

• The change in net external debt is an equilibrium outcome. Thus, if a country does not borrow, it could be due to lack of demand, and not necessarily to a supply restriction motivated by punishment. Thus, estimates of exclusion may carry an upward bias.
Determinants of the Exclusion Period

• Frequency of default is not a significant determinant of the length of exclusion (Gelos et al., 2011)

• Defaults that resolve quickly are associated with short exclusion periods (Gelos et al., 2011).

• Excusable defaults (such as those following a natural disaster) are associated with reduced exclusion periods (Richmond and Dias, 2009).

• Restructurings involving higher hair cuts (i.e., higher losses to creditors) are associated with significantly longer periods of capital market exclusion (Cruces and Trebesch, 2013).

• In most theoretical models of default, the length of exclusion is assumed to be random and exogenous.
Output Cost of Default

Theoretical default literature relies on assumption that default leads to output loss. What does the data show?

Standard approach: Growth regressions augmented with default variables (see, for example, Chuan and Sturzenegger, 2005; Borensztein and Panizza, 2009; De Paoli, Hoggarth, and Saporta, 2006; and Levy Yeyati and Panizza, 2011).
Default Dummies in Growth Regressions

Borensztein and Panizza (2009) estimate the equation

\[ \text{Growth}_{it} = \alpha + \beta X_{it} + \gamma \text{Default}_{it} + \sum_{j=0}^{3} \delta_j \text{DefaultB}_{it-j} \]

where \( \text{Growth}_{it} \) is the growth rate of real per capita GDP in country \( i \) between years \( t - 1 \) and \( t \), \( X_{it} \) is a vector of standard controls in growth regressions (initial income, education, population, etc.), \( \text{Default}_{it} \) is a dummy taking the value 1 if country \( i \) is in default in year \( t \) and 0 otherwise, and \( \text{DefaultB}_{it} \) is a dummy taking the value 1 if country \( i \) enters default in year \( t \).
Estimation Results
Using annual data for 83 countries for the period 1972 to 2000, Borensztein and Panizza estimate

\[
\text{Growth}_{it} = \alpha + \beta X_{it} - 1.184 \text{Default}_{it} \\
-1.388 \text{DefaultB}_{it} + 0.481 \text{DefaultB}_{it-1} \\
+ 0.337 \text{DefaultB}_{it-2} + 0.994 \text{DefaultB}_{it-3}
\]

with $\gamma$ and $\delta_0$ significant at confidence level of 5% or less.

This estimate implies that the beginning of a default is associated with a fall in the growth rate of 2.5 percentage points ($= \gamma + \delta_0$).

More importantly, default is associated with a permanent loss of output. This is shown in the next figure, which displays a simulation of the log of output per capita implied by the regression equation (default occurs in period 0).
Simulated Path Of Output After A Default Implied By the Borensztein-Panizza Regression

In producing the figure, long-run growth is set to 1.5%, and exclusion is set to 5 years (median from figure 13.1)
Observations on Default Dummies in Growth Regressions

• After an initial fall, growth gradually regains its long-run level.

• However, the level of output never recovers. It remains 5.5% below the no-default trajectory. Taken at face value, the cost of default is enormous.

• Reasons why the output loss estimate can be upwardly biased:
  – The regression doesn’t include lags in the variable Default_{it} and may include insufficient lags in DefaultB_{it}. If they entered with positive signs, these extra lags could capture a catch-up effect.
  – More importantly, output growth and default are both endogenous variables. Not clear who causes whom. If default occurs during recessions, estimated effect of default may be negative even if in reality they are zero.
Output Cost Of Default: A Growth Accounting Approach (Zarazaga, 2012)

Take a look at the behavior of the capital-output ratio around the Argentine defaults of 1982 and 2001:

Observation: Both defaults are associated with a fall in the capital-output ratio. What does this imply for output per worker?
Default and Growth Accounting (continued): Let the production function be
\[ y_t = k_t^{0.4} h_t^{0.6} \Rightarrow \frac{y_t}{h_t} = \left( \frac{k_t}{y_t} \right)^{2/3} \]
In the 2002 default, \( \frac{k_t}{y_t} \) fell from 1.9 to 1.35 in 2007. Therefore,
Percent change in \( \frac{y_t}{h_t} \) equals \[ \left( \frac{1.9}{1.35} \right)^{2/3} - 1 \times 100 = 26\% \]. This means that, on average, output was 13\% \((=26\%/2)\) lower every year between 2002 and 2007. Large output cost of default!

Usual cautionary note: This calculation assumes that all of the change in \( \frac{k_t}{y_t} \) is due to default. But causality may go the other way as well. To the extent that this is the case, the 13\% output loss must be interpreted as an upper bound of the cost of default.
Trade Costs of Default

**Hypothesis:** Countries honor their debts, among other things, to avoid trade sanctions.

Rose (2005) sets out to test this hypothesis by estimating the following gravity model augmented with default variables:

$$\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m ACRED_{ijt-m} + \epsilon_{ijt}$$

**Definitions:** $T_{ijt} =$ real value of trade between countries $i$ and $j$ in year $t$; $X_{ijt} =$ usual gravity variables; $ACRED_{ijt} =$ 1 if one country is a restructuring debtor and the other a negotiating creditor (the affected creditor), 0 otherwise.

**Sample:** 1948-1997 annual, 217 countries.

**Finding:** $\sum_{m=0}^{15} \phi_m = -1.12$. Thus, the cumulative cost of default is over one year of trade over 16 years. It looks like the trade costs of default are enormous.

But is it sanctions or general economic stress around default episodes? Keep reading...
Does Default Disrupt Int’l Trade So Much?

Martínez and Sandleris (2011) estimate the following variant of Rose’s gravity model:

$$\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m ACRED_{ijt-m} + \sum_{m=0}^{M} \gamma_m DEBTOR_{ijt-m} + \epsilon_{ijt},$$

**Definition:** $DEBTOR_{ijt} = 1$ if either country is a restructuring debtor, 0 otherwise.

**Finding:** $\sum_{m=0}^{15} \phi_m = 0.01$ and $\sum_{m=0}^{15} \gamma_m = -0.41$.

**Implication:** It might not be trade sanctions to defaulters, but general economic stress around default episodes.

But if default is punished with collective trade sanctions, not just sanctions by the affected creditors, the variable $DEBTOR_{ijt}$ may not pick up just economic distress, but also the effect of collective punishment.
Controlling for Collective Sanctions To control for collective sanctions, Martínez and Sandleris (2011) estimate the following equation

\[
\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m CRED_{ijt-m} + \sum_{m=0}^{M} \gamma_m DEBTOR_{ijt-m} + \epsilon_{ijt}
\]

**Definition:** \( CRED_{ijt} = 1 \) if one country is a renegotiating debtor and the other is a (not necessarily renegotiating) creditor.

**Finding:** \( \sum_{m=0}^{M} \phi_m \) positive at horizons 0, 5, and 10, and negative at horizons 15 or longer.

**Interpretation:** No sanction effect, unless creditors have very long memories (at least 15 years) and are willing or have reasons to delay punishment.
Martínez and Sandleris III:

\[ \ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m ACRED_{ijt-m} + \sum_{m=0}^{M} \xi_m NACRED_{ijt-m} \]

\[ + \sum_{m=0}^{M} \gamma_m NOTCRED_{ijt-m} + \epsilon_{ijt}. \]

**Definitions:**  
- \( NACRED_{ijt} = 1 \) if one country is a renegotiating debtor and the other a nonrenegotiating creditor, 0 otherwise.  
- \( NOTCRED_{ijt} = 1 \) if one country is a renegotiating debtor and the other is not a creditor.

**Finding:** \( \sum_{m=0}^{M} \xi_m < 0 \), but \( \sum_{m=0}^{M} \phi_m > 0 \) at horizons 0, 5, 10, and negative only at horizon 15.

**Interpretation:** Again, no sanction effect, unless creditors have very long memories (at least 15 years) and are willing or have reasons to delay punishment.
Section 13.3

Default Incentives With

State-Contingent Contracts

[slides incomplete]
Section 13.4

Default Incentives With

Non-State-Contingent Contracts

[slides for this section are not complete]
The Eaton-Gersovitz Model

References: Eaton and Gersovitz (RES, 1981); Arellano (2008).

Some analytical results:

1. Countries default in low income states.

2. The more indebted the country is, the higher the default probability.
Preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \]

Endowment:

\[ y_t \in Y \equiv [y, \bar{y}], \quad \text{i.i.d.} \]

One-period non-statecontingent debt, \( d \).

Value of bad financial standing:

\[ v^b(y) = u(y) + \beta E v^b(y') \]

Value of good financial standing:

\[ v^g(d, y) = \max \{ v^b(y), v^c(d, y) \} \]

Value of continuing to honor debt:

\[ v^c(d, y) = \max_{d'} \left[ u(y + q(d')d' - d) + \beta E v^g(d', y') \right] \]
The Default Set:

\[ D(d) = \{ y \in Y : v^b(y) > v^c(d, y) \} \]

**Proposition 1 (Must pay part of interest, trade surplus)**  
If \( D(d) \neq \emptyset \), then \( t_b = d - q(d')d' > 0 \) for all \( d' \leq \bar{d} \).

**Proposition 2 (Default in bad times)**  
If \( y_1 \in D(d) \) and \( y \leq y_2 < y_1 \), then \( y_2 \in D(d) \).

Compare to figure 13.2 in USG (2017).

**Proposition 3 (Default when debt is high)**  
If \( D(d) \neq \emptyset \), then \( D(d) \) is an interval, \( [\underline{y}, y^*(d)] \), where \( y^*(d) \) is increasing in \( d \) if \( y^*(d) < \bar{y} \).

see table 13.3 of USG (2017)
Default Risk and the Country Premium

World interest rate, $r^*$. Risk neutral lenders.

Participation constraint:

$$1 + r^* = \frac{\text{Prob}\{y' \geq y^*(d')\}}{q(d')}.$$ 

Note the country spread is equal to the probability of default. [Yet, in the data, country spreads typically exceed the probability of default, see tables 13.1, 13.2, 13.4, and 13.5 of USG (2017).]

**Proposition 4 (More debt, higher spread)** The country spread, given by $1/q(d') - (1 + r^*)$ is nondecreasing in the stock of debt.
Section 13.5
Saving and the Breakdown of Reputational Lending

Bulow and Rogoff (1989) show that if defaulters cannot borrow but can save, then no lending can be supported on reputational grounds.

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Section 13.6

Quantitative Analysis of the

The Eaton-Gersovitz Model
Additional features of the default model relative to the analytical example discussed thus far:

- Serially correlated output.
- Nonzero probability of reentry.
- Asymmetric output cost of default.
Section 13.6.1: Serially Correlated Output

\( y_t = \) tradable output measured as sum of GDP in agriculture, forestry, fishing, mining, and manufacturing.


\[
\ln y_t = 0.9317 \ln y_{t-1} + 0.037 \epsilon_t
\]

with \( \epsilon_t \sim N(0, 1) \)

Now probability of default in \( t + 1 \) depends on \( y_t \). Thus \( q_t = q(d_{t+1}, y_t) \).
Section 13.6.2: Nonzero Probability of Reentry

canstant probability of reentry, $\theta$.

Prob. to be excluded exactly 1 quarter: $\theta$

Prob. to be excluded exactly 2 quarters: $\theta(1 - \theta)$

$\vdots$

Prob. to be excluded exactly $j$ quarters: $\theta(1 - \theta)^{j-1}$

$\Rightarrow$ expected length of exclusion is $\frac{1}{\theta}$ quarters.

[Table 13.6 of USG (2017), typical exclusion period is 4.7-13.7 years.]
Section 13.6.3: The Output Cost of Default

The endowment received by households, $\tilde{y}_t^T$, is given by

$$\tilde{y}_t = \begin{cases} 
  y_t & \text{if the country is in good standing} \\
  y_t - L(y_t) & \text{if the country is in bad standing}
\end{cases},$$

where $L(y_t)$ is an output loss function assumed to be positive and nondecreasing.

[Why? Because exclusion is not enough punishment to support empirically realistic amounts of debt, see section 13.6.11 of USG (2017).]
Section 13.6.4: The Model

Value of continuing to participate in financial markets

\[ v^c(d, y) = \max_{d'} \left\{ u(y + q(d', y)d' - d) + \beta E_y v^g(d', y') \right\} \]

Value of being in bad financial standing

\[ v^b(y) = u(y - L(y)) + \beta \theta E_y v^g(0, y') + \beta (1 - \theta) E_y v^b(y') \]

Value of being in good financial standing

\[ v^g(d, y) = \max \left\{ v^c(d, y), v^b(y) \right\} \]

Participation Constraint

\[ 1 + r^* = \frac{\text{Prob}_y \{ v^c(d', y') \geq v^b(y') \}}{q(d', y)} \]

Country interest rate: \( 1 + r \equiv \frac{1}{q(d', y)} \)

Country Spread = \( r - r^* \)
Section 13.6.5: Calibration and Functional Forms

Preferences:

\[ u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}. \]

Output Cost of Default:

\[ L(y_t) = \max\{0, a_0 + a_1 y_t + a_2 y_t^2\}. \]
Calibration

- Time unit is a quarter.

- Parameter values:
  \( \sigma = 2 \).
  
  \( \theta = 0.0385 \) probability of reentry.
  
  \( r^* = 1\% \) per quarter.
  
  \( \beta = 0.85 \).
  
  \( a_1 = -0.36 \); parameter of output loss function.
  
  \( a_2 = 0.4403 \); parameter of output loss function.

(\( \beta, a_1, a_2 \)) set to match debt to GDP of 60% per quarter; 2.6 defaults per century; output cost of default is 7% per year of autarky.
Asymmetric Output Cost of Default

\[ y_t - L(y_t) \text{ (flat)} \]
\[ y_t - L(y_t) \text{ (quadratic)} \]
Section 13.6.6: Computation

Discretization of State Space

\( n_y = 200; \) Number of output grid points (equally spaced in logs)

\( n_d = 200; \) Number of debt grid points (equally spaced)

\([y, \overline{y}] = [0.6523, 1.5330];\) 4.2 standard deviations

\([d, \overline{d}] = [0, 1.5];\)

Transition probability matrix computed using Schmitt-Grohé and Uribe (2009) interactive procedure, see tpm.m
Section 13.6.7:

Quantitative Predictions of the Eaton-Gersovitz Model
Selected First and Second Moments: Data and Model Predictions

<table>
<thead>
<tr>
<th></th>
<th>Default frequency</th>
<th>E(d/y)</th>
<th>E(r - r*)</th>
<th>σ(r - r*)</th>
<th>corr(r - r*, y)</th>
<th>corr(r - r*, tb/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
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<td>58.0</td>
<td>7.4</td>
<td>2.9</td>
<td>-0.64</td>
<td>0.72</td>
</tr>
<tr>
<td>Model</td>
<td>2.6</td>
<td>59.0</td>
<td>3.5</td>
<td>3.2</td>
<td>-0.54</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2014. The variable \( d/y \) denotes the quarterly debt-to-GDP ratio in percent, \( r-r^* \) denotes the country premium, in percent per year, \( y \) denotes (quarterly detrended) output, and \( tb/y \) denotes the trade-balance-to-GDP ratio. The symbols \( E, \sigma, \) and \( corr \) denote, respectively, the mean, the standard deviation, and the correlation. In the theoretical model, all moments (other than default frequency) are conditional on the country being in good financial standing. Replication file statistics_model.m in sovereign_default.zip.

Predicted default frequency while in good standing is 3.2% (which is close to the country spread of 3.5%).
Observations on the Table

- $r$ is countercyclical; when it rains it pours; this feature should increase consumption volatility relative to output;

- Model explains only half of observed country premium (3.5 versus 7.4 percent).
By construction, model can either match the default frequency or
the spread but not both, because it implies that the spread is equal
to the default probability:

$$r - r^* \approx \ln \left[ \frac{1}{q(d', y)(1 + r^*)} \right]$$

$$= \ln \left[ \frac{1}{\text{Prob}\{\text{repayment in } t + 1 \text{ given information in } t\}} \right]$$

$$= \ln \left[ \frac{1}{1 - \text{Prob}\{\text{default in } t + 1 \text{ given information in } t\}} \right]$$

$$\approx \text{Prob}\{\text{default in } t + 1 \text{ given information in } t\}.$$
Q: Can partial default explain the spread-default frequency differential? Partial default:

\[ q(d', y) = \frac{\lambda \text{Prob}_y\{v^c(d', y') < v^b(y')\} + \text{Prob}_y\{v^c(d', y') \geq v^b(y')\}}{1 + r^*} \]

This implies that

\[ r - r^* \approx (1 - \lambda) \text{Prob}\{\text{default in } t + 1 \text{ given information in } t\} \]

Now spread smaller than default frequency. Thus, allowing for partial default only widens the spread-default frequency differential.
Q: Does making foreign lenders risk averse close the spread-default frequency differential.

A: Lizarazo (2013), yes. However, as we will show shortly, this result relies on the assumption that default has large negative wealth effects on the foreign lender. If defaulting country is small so that wealth of foreign lender is not affected by default, then allowing for risk aversion, does not help much to close the spread-default frequency differential. (Section 13.9 of USG, 2017)
## Data and Model Predictions: Standard Business-Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(c)/\sigma(y)$</th>
<th>$\sigma(tb/y)/\sigma(y)$</th>
<th>$\text{corr}(c,y)$</th>
<th>$\text{corr}(tb/y,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emerging Countries</td>
<td>1.23</td>
<td>0.69</td>
<td>0.72</td>
<td>-0.51</td>
</tr>
<tr>
<td>Argentina</td>
<td>1.11</td>
<td>0.48</td>
<td>0.75</td>
<td>-0.87</td>
</tr>
<tr>
<td>Model</td>
<td>1.22</td>
<td>0.57</td>
<td>0.88</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

Note. Data moments for emerging countries and Argentina are taken from chapter 1, tables 1.6 and 1.9, respectively. The symbols $c$ and $y$ denote the log deviation from trend, $tb/y$ denotes the trade-balance-to-output ratio, and $\sigma$ and $\text{corr}$ denote, respectively, standard deviation and correlation. Replication file for model predictions simu.m in sovereign_default.zip.
Observations on the table:

• model captures excess volatility of consumption. why? in good standing interest rate is countercyclical exacerbating consumption adjustment to negative shocks; in bad standing $c = y$. So overall consumption more volatile than output.

• Model explains sign but not size of $corr(tb/y, y)$. In the absence of default risk this model would predict $tb/y$ to be procyclical. Recall: finance temporary income shocks, and adjust to permanent ones.

• Model predicts countercyclical interest rate, this makes savings, and hence the trade balance itself countercyclical.
Section 13.6.8:

Dynamics Around A Typical Default Episode
Notes: Lines display medians of 25-quarter windows centered on default episodes occurring in an artificial time series of 1 million quarters. The default date is normalized to 0. Replication file typical_default_episode.m in sovereign_default.zip.
Observations on the figure

• When does a country default? After a sudden deep contraction in output. From at mean to 1.3 std below mean in just 3 quarters.
• model can explain that default coincides with end of contraction and beginning of recovery.
• consumption falls by more than output (no consumption smoothing!). Why, because spreads increase from 3 to 6 percent.
• debt fails to increase prior to default. \(d/y\) little changed until period of default. in the data, we see \(d/y\) increasing.
Section 13.6.9:

Goodness of Approximation of the Eaton-Gersovitz Model
## Approximating the Eaton-Gersovitz Model: Accuracy Tests

<table>
<thead>
<tr>
<th>Grid Points</th>
<th>Default frequency</th>
<th>Default Correlation</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_y$</td>
<td>$n_d$</td>
<td>$E(d/y)$</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td>2.6</td>
</tr>
<tr>
<td>Model*</td>
<td>200</td>
<td>200</td>
<td>2.65</td>
</tr>
<tr>
<td>Model</td>
<td>25</td>
<td>200</td>
<td>2.30</td>
</tr>
<tr>
<td>Model</td>
<td>400</td>
<td>200</td>
<td>2.63</td>
</tr>
<tr>
<td>Model</td>
<td>200</td>
<td>400</td>
<td>2.65</td>
</tr>
<tr>
<td>Model</td>
<td>400</td>
<td>400</td>
<td>2.65</td>
</tr>
</tbody>
</table>

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2014. The variable $d/y$ denotes the quarterly debt-to-GDP ratio in percent, $r - r^*$ denotes the country premium, in percent per year, $y$ denotes (quarterly detrended) output, and $tb/y$ denotes the trade-balance-to-GDP ratio. The symbols $E$ and $\sigma$ denote, respectively, the mean and the standard deviation. The symbols $n_y$ and $n_d$ denote the number of grid points for the endowment and debt, respectively. In the theoretical model, all moments are conditional on the country being in good financial standing. Theoretical moments were computed by running the Matlab script statistics_model.m after appropriately adjusting the number of grid points in eg.m.

*Baseline grid specification.

- Hatchondo, Martínez, and Sapriza (2010) find that the numerical solution of the Eaton-Gersovitz model deteriorates significantly when the endowment grid is coarsely specified. Correlation of country premium with output falls from -0.54 to -0.28 when $n_y$ is reduced from 200 to 25. Std of premium is 1 percentage point higher under coarser grid.
Section 13.6.10 and 13.6.11:

The Quantitative Importance of Output Costs of Default

and

The Quantitative Irrelevance of Exclusion
Reputation or Direct Output Costs? — The Quantitative Irrelevance of Exclusion

Consider no output cost of default, \( L(y) = 0 \). Leaving financial autarky as the only cost of default.

What happens? The debt distribution becomes degenerate. For all values of \( y \) debt is equal to zero. That is, the model cannot support any debt in equilibrium. This means that the benefit of having access to financial markets must always be smaller than the benefit of defaulting.

Therefore, the current model is not really a reputational model of default but instead a sanctions model of default.
But does reputation play any role at all? To answer this question, let’s compare the predictions of the model with output costs of default and with and without financial exclusion post default.

The model then becomes:

\[ v^{gc}(d, y) = \max_{d'} \left\{ u(y + q^g(d', y)d' - d) + \beta E_y v^g(d', y') \right\}, \]

The value of defaulting, \( v^d(y) \)

\[ v^d(y) = \max_{d'} \left\{ u(y - L(y) + q^b(d', y)d') + \beta \theta E_y v^g(d', y') + \beta (1 - \theta) E_y v^b(d', y') \right\} \]

The value of being in good financial standing

\[ v^g(d, y) = \max\{v^{gc}(d, y), v^d(y)\}. \]

\[ q^g(d', y) = \frac{\text{Prob}_y\{v^{gc}(d', y') \geq v^d(y')\}}{1 + r^*}, \]
The value of being in bad financial standing and continuing to service the debt, $v^{bc}$,
\[
v^{bc}(d, y) = \max_{d'} \{ u(y - L(y) + q^b(d', y)d' - d) + \beta \theta E_y v^g(d', y') + \beta (1 - \theta) E_y v^b(d', y') \}.
\]

The value of being in bad financial standing
\[
v^b(d, y) = \max\{v^{bc}(d, y), v^d(y)\}
\]

The reason why $v^b(d, y)$ may not always be equal to $v^d(y)$ is that a country in bad standing may have assets $d < 0$, in which case it will never default.

The price of debt in periods of bad financial standing, $q^b(d', y)$
\[
q^b(d', y) = \frac{\theta \text{Prob}_y\{v^{gc}(d', y') \geq v^d(y')\} + (1 - \theta) \text{Prob}_y\{v^{bc}(d', y') \geq v^d(y')\}}{1 + r^*}
\]
The Quantitative Irrelevance of Exclusion: Selected First and Second Moments

<table>
<thead>
<tr>
<th></th>
<th>Default frequency</th>
<th>E(d/y)</th>
<th>E(r − r*)</th>
<th>σ(r − r*)</th>
<th>corr(r − r*, y)</th>
<th>corr(r − r*, tb/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>2.6</td>
<td>58.0</td>
<td>7.4</td>
<td>2.9</td>
<td>-0.64</td>
<td>0.72</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Baseline</strong></td>
<td>2.6</td>
<td>59.0</td>
<td>3.5</td>
<td>3.2</td>
<td>-0.54</td>
<td>0.81</td>
</tr>
<tr>
<td><strong>No Exclusion</strong></td>
<td>3.0</td>
<td>53.1</td>
<td>4.1</td>
<td>3.6</td>
<td>-0.61</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**Observations on the table:**

- The model in which default is not punished by exclusion from financial markets behaves remarkably similar to the baseline model.

- Specifically, the no-exclusion model can support about the same amount of debt than the model with exclusion. The mean debt to output ratio in times of good financial standing is 53 percent per quarter in the no-exclusion model, slightly below the value of 59 percent predicted by the baseline model.
• The average country premium in times of good standing is predicted to be 4.1 percent in the no-exclusion model compared to 3.5 percent in the baseline case. The default frequency conditional on good standing is 3.7 percent. (In this model, up to first order the spread-default frequency differential is zero.)

• The volatility of the country premium and the correlation of the country premium with output and the trade balance to output ratio are also little changed.

• The model predicts that on average the country defaults three times per century compared to a default frequency of 2.6 times per century predicted by the baseline model.

⇒ We conclude that exclusion from credit markets plays a negligible role for the quantitative performance of the Eaton-Gersovitz model. The main mechanism supporting debt in equilibrium is the output loss associated with default!
Section 13.6.13:

The Role of Discounting
Varying the subjective discount factor, $\beta$

If $\beta \uparrow$, then agents discount the future less. All else constant the cost of default (exclusion as well as output loss) has a higher present discounted value. This should make countries default less often. Lower default frequencies then immediately imply lower country premia. And with more ability to repay, debt should increase. By this argument $\beta \uparrow \Rightarrow d \uparrow$.

Yet, $\beta$ does not only affect the present value of the costs of default, it also changes the desired level of debt. In a model without default, the more patient agents are the lower is the level of debt, thus, by this argument $\beta \uparrow \Rightarrow d \downarrow$.

Which force dominates in equilibrium? The next table shows that the first force does.
Varying $\beta$

<table>
<thead>
<tr>
<th>Default frequency</th>
<th>$E(d/y)$</th>
<th>$E(r - r^*)$</th>
<th>$\sigma(r - r^*)$</th>
<th>corr($r - r^*, y$)</th>
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</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 0.85^*$</td>
<td>2.6</td>
<td>59.0</td>
<td>3.5</td>
<td>-0.54</td>
<td>0.81</td>
</tr>
<tr>
<td>$\beta = 0.90$</td>
<td>1.4</td>
<td>71.4</td>
<td>1.6</td>
<td>-0.52</td>
<td>0.78</td>
</tr>
<tr>
<td>$\beta = 0.95$</td>
<td>0.4</td>
<td>87.8</td>
<td>0.5</td>
<td>-0.51</td>
<td>0.71</td>
</tr>
</tbody>
</table>

* indicates the baseline calibration.

The larger is $\beta$, all else constant, the more debt can be supported, which is the exact opposite of what happens under commitment.

Intuition, higher $\beta$ increases the present value of the default cost.
Section 13.6.15:

Varying the Persistence of the Output Process
Varying $\rho$ holding constant the variance of $y$.

<table>
<thead>
<tr>
<th>Default frequency</th>
<th>E(d/y)</th>
<th>$E(r - r^*)$</th>
<th>$\sigma(r - r^*)$</th>
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</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>0.1</td>
<td>274.1</td>
<td>0.1</td>
<td>-0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>$\rho = 0.5$</td>
<td>0.2</td>
<td>176.5</td>
<td>0.2</td>
<td>-0.68</td>
<td>0.67</td>
</tr>
<tr>
<td>$\rho = 0.75$</td>
<td>0.7</td>
<td>104.3</td>
<td>0.8</td>
<td>-0.57</td>
<td>0.66</td>
</tr>
<tr>
<td>$\rho = 0.85$</td>
<td>1.5</td>
<td>74.0</td>
<td>1.7</td>
<td>-0.52</td>
<td>0.73</td>
</tr>
<tr>
<td>$\rho = 0.9317^*$</td>
<td>2.6</td>
<td>59.0</td>
<td>3.5</td>
<td>-0.54</td>
<td>0.81</td>
</tr>
<tr>
<td>$\rho = 0.95$</td>
<td>2.8</td>
<td>59.7</td>
<td>3.8</td>
<td>-0.58</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho = 0.97$</td>
<td>2.8</td>
<td>67.3</td>
<td>3.7</td>
<td>-0.62</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Note. $^*$ = baseline value.

- the higher is $\rho$ the lower is debt and the higher is the default frequency. Why? Recall that the output cost kicks in only for high levels of output. When $\rho = 0$, it is quite likely that output in the near future is high and hence the country has to pay a high default cost. Thus, the country chooses a low default frequency. And with infrequent defaults the country can support more debt.
Section 13.7:

The Welfare Cost of Lack of Commitment
• If agents could commit to repay, then debt would be 100 times larger, 65.88 versus 0.6133, conditional on good standing, or 0.5091 unconditionally.

Why? Agents are very impatient, $\beta(1 + r) = 0.8585 < 1$

• Lack of commitment manifests itself as an endogenous borrowing constraint.

• And lack of commitment is welfare reducing.

How much would agents be willing to pay to be able to commit to repay?
Value function under commitment:

\[ v^{\text{com}}(d, y) = \max_{d'} \left\{ u(y + d'(1 + r^*)^{-1} - d) + \beta E_y v^{\text{com}}(d', y') \right\} \]

With \(d') in hand, we can find equilibrium consumption from

\[ c^{\text{com}} = y + d'/ (1 + r^*) - d \]

The welfare costs of lack of commitment, \(\Lambda(d, y)\):

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{\text{com}}}{1 - \sigma} \right)^{1-\sigma} - 1 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 + \Lambda(d_0, y_0)) c_t^{\text{nocom}} \right]^{1-\sigma} - 1. \]

Solving for \(\Lambda(d, y)\) yields

\[ \Lambda(d, y) = \left[ \frac{v^{\text{com}}(d, y)(1 - \sigma)(1 - \beta) + 1}{v^{\text{nocom}}(d, y)(1 - \sigma)(1 - \beta) + 1} \right]^{\frac{1}{1-\sigma}} - 1, \]

Note that the welfare cost of lack of commitment are state dependent, that is, \(\Lambda\) is a function of the current state \((d, y)\).
Compute $v^{\text{no com}}(d, y)$ and $v^{\text{com}}(d, y)$ and then take unconditional expectations using the ergodic distribution of the state $(d, y)$ associated with the economy displaying lack of commitment.

The unconditional mean of the welfare cost of lack of commitment is **273 percent**, that is,

$$100 \times E\Lambda (d, y) = 273$$

This is an enormous value. It means that the consumption stream of an individual living in the economy without commitment must almost **quadruplicate** in order for him to be as well off as living in the economy with commitment to repay debts.
The totality of this welfare cost is due to the transitional dynamics of switching from no commitment to commitment. Along this transition, debt increases from a mean of 0.50 to a mean of 65.88, and consumption declines from a mean of 0.9873 to a mean of 0.36. Of course, along this transition consumption is temporarily much higher than 0.9873.

The next figure displays the typical transition from the lack of commitment economy to the commitment economy.
• The typical transition path is the mean of 10,000 transition paths each starting at a pair \((d, y)\) drawn from the ergodic distribution under lack of commitment.

• The transitional dynamics are the key determinant of the welfare gains of commitment. Consider a naive approach to welfare evaluation consisting in computing the unconditional welfare in each economy separately. Because in the stationary state average consumption in the commitment economy is one third as high as consumption in the no-commitment economy, one would erroneously conclude that lack of commitment is welfare improving.
Section 13.8: Decentralization Of The Eaton-Gersovitz Model

The Eaton-Gersovitz model is cast in terms of a social planner’s problem. A benevolent government aims to maximize the lifetime welfare of households. In doing so, it chooses how much to borrow, when to default, and how much the household should consume each period. The household itself makes no relevant decision. It does not participate in financial markets, nor in goods markets, but passively consumes the goods it receives from the government each period.

New environment: Private households participate in credit markets and choose optimally how much to consume and save each period. Government retains only the decisions to default and to conduct fiscal policy.
A central question that we will address is whether there exist fiscal instruments that the government can use to induce households to undertake borrowing and consumption decisions that mimic the social planner’s.

This exercise is known as the decentralization of the social planner’s equilibrium.

- **Externality:** The reason why the government may need fiscal instruments to alter the behavior of private households is that while the former internalizes that the interest rate faced by the country in international financial markets depends on its net external debt position, the latter does not. Individual households are too small to affect with their borrowing the country’s credit conditions.

- **Capital Controls** By applying fiscal distortions, the government makes its borrowing decisions and the private sector’s coincide. We will show that the social planner’s equilibrium can be decentralized via capital controls.
The Decentralized Eaton-Gersovitz Economy
(The exposition is based on Na, Schmitt-Grohé, Uribe, and Yue, 2014.)

13.8.1 Households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

subject to

\[
c_t + d_t = (1 + \tau^y_t)\tilde{y}_t + (1 - \tau^d_t)q^d_t d_{t+1}
\]

(1)

\(\tau^y_t\) = is an income subsidy (tax if negative)
\(\tau^d_t\) = is a tax (subsidy if negative) on debt
FOC wrt debt: \(u'(c_t)(1 - \tau^d_t)q^d_t = \beta E_t u'(c_{t+1})\)
13.8.2 The Government

Default decisions are assumed to be made by the government. Each period the country can be either in good financial standing or in bad financial standing. If it is in good financial standing, it can choose to honor its international debts or default. Let $I_t$ be a binary variable taking the value 1 if the country is in good standing in period $t$ and chooses to honor its debt and 0 if it is in bad standing. If the country defaults in period $t$, it immediately acquires bad financial standing and $I_t$ takes the value 0. If the country is in bad standing in period $t$, it regains good standing in period $t + 1$ with constant and exogenous probability $\theta$, and maintains its bad financial standing with probability $1 - \theta$.

When the country is in bad financial standing, it is excluded from international financial credit markets and is therefore unable to borrow or lend internationally. We then have that

$$(1 - I_t)d_{t+1} = 0.$$  \hspace{1cm} (2)$$
In periods in which the country is in bad standing \((I_t = 0)\), the government confiscates any payments of households to foreign lenders and returns the proceeds to households via income subsidies. The government also uses the income subsidy to rebate the proceeds from the debt tax. The resulting sequential budget constraint of the government is then given by

\[
\tau_t y_t = \tau_d t q_t d_{t+1} + (1 - I_t) d_t. \tag{3}
\]

Let \(q_t\) denote the price of debt charged by foreign lenders to domestic borrowers during periods in which the government maintains good financial standing. As before, the price of debt, \(q_t\), must satisfy the condition that the expected return of lending to the domestic country equal the opportunity cost of funds. Formally,

\[
q_t = \frac{\text{Prob}\{I_{t+1} = 1 | I_t = 1\}}{1 + r^*}. \tag{4}
\]

This expression can be equivalently written as

\[
I_t \left[ q_t - \frac{E_t I_{t+1}}{1 + r^*} \right] = 0.
\]
13.8.3 Competitive Equilibrium

Because all domestic households are identical, there is no borrowing or lending among them. This means that in equilibrium the household’s net asset position equals the country’s net foreign asset position. This in turn implies that the debt tax, $\tau_t^d$, can be interpreted as a capital control tax. Because when the country is in bad standing external debt is nil, the value of $\tau_t^d$ in periods of bad standing is immaterial. Without loss of generality, we set $\tau_t^d = 0$ when $I_t = 0$, that is,

$$(1 - I_t)\tau_t^d = 0.$$  \hspace{1cm} (5)

As before, the endowment received by the household, $\tilde{y}_t$, is given by

$$\tilde{y}_t = \begin{cases} 
  y_t & \text{if } I_t = 1 \\
  y_t - L(y_t) & \text{otherwise}
\end{cases}.$$  \hspace{1cm} (6)
In any period $t$ in which the country is in good financial standing, the domestic price of debt, $q^d_t$, must equal the price of debt offered by foreign lenders, $q_t$, that is

$$I_t(q^d_t - q_t) = 0.$$  \hfill (7)

Combining (1)-(3), (6), and (7) yields the following market-clearing condition

$$c_t = y_t - (1 - I_t)L(y_t) + I_t[q_t d_{t+1} - d_t].$$
A competitive equilibrium is a set of stochastic processes \(\{c_t, d_{t+1}, q_t, q_t^d\}\) satisfying

\[
c_t = y_t - (1 - I_t)L(y_t) + I_t[q_t d_{t+1} - d_t],
\]

\[\tag{8}
(1 - I_t)d_{t+1} = 0,
\]

\[\tag{9}
(1 - \tau_t^d)q_t^d u'(c_t) = \beta E_t u'(c_{t+1}),
\]

\[\tag{10}
I_t(q_t^d - q_t) = 0,
\]

\[\tag{11}
I_t \left[ q_t - \frac{E_t I_t + 1}{1 + r^*} \right] = 0.
\]

given processes \(\{y_t, \tau_t^d, I_t\}\) and the initial condition \(d_0\).
Proposition 5 (Competitive Equilibrium When $\tau^d_t$ Is Unrestricted)

When the government can choose $\tau^d_t$ freely, stochastic processes $\{c_t, d_{t+1}, q_t\}$ can be supported as a competitive equilibrium if and only if they satisfy

$$c_t = y_t - (1 - I_t)L(y_t) + I_t[q_t d_{t+1} - d_t], \quad (8)$$

$$ (1 - I_t)d_{t+1} = 0, \quad (9)$$

and

$$I_t \left[ q_t - \frac{E_t I_{t+1}}{1 + r^*} \right] = 0, \quad (12)$$

given processes $\{y_t, I_t\}$ and the initial condition $d_0$. 
Proof:

\[
c_t = y_t - (1 - I_t)L(y_t) + I_t[q_t d_{t+1} - d_t]
\]

\[
(1 - I_t)d_{t+1} = 0
\]

\[
(1 - \tau_t^d)q_t^d u'(c_t) = \beta E_t u'(c_{t+1}),
\]

\[
I_t(q_t^d - q_t) = 0,
\]

\[
I_t \left[ q_t - \frac{E_t I_{t+1}}{1+r^*} \right] = 0
\]

given \( I_t \). Pick \( q_t^d \) and \( \tau_t^d \) to satisfy the non-boxed equations.
In the Eaton-Gersovitz model the government lacks commitment to honor past promises regarding debt payments or defaults. The lack of commitment opens the door to time inconsistency. For this reason, attention is restricted to Markov perfect equilibria, which are time consistent equilibria in which the equilibrium policy functions are time-invariant functions of the pay-off relevant state variables of the competitive equilibrium of the economy in period $t$.

The states appearing in the conditions of the competitive equilibrium listed in Proposition 5 are the endowment, $y_t$, and the stock of net external debt, $d_t$. Thus, in a Markov perfect equilibrium the optimal default decision in period $t$ is a time invariant function of $y_t$ and $d_t$. 

When $I_t = 1$, under the assumption of a Markov perfect equilibrium equation (12) becomes

$$q_t = \frac{E_t I(y_{t+1}, d_{t+1})}{1 + r^*}$$

which we can write as

$$q_t = q(y_t, d_{t+1})$$

Using this expression to eliminate $q_t$ from (8), we have that the constraint set faced by the benevolent government in the decentralized economy are the same as those in the centralized economy. It follows that the allocation must be identical to the one characterized in the Eaton-Gersovitz problem studies in Section 13.6.
Note. The solid line displays the median of 25-quarter windows centered around default episodes occurred in an artificial time series of 1 million quarters. The default date is normalized to 0. The dotted line displays the unconditional median.
Observations:

The government increases capital controls sharply in the three quarters prior to the default from 9 to 17 percent. This increase in capital control taxes increases the effective interest rate faced by households. In this way, the government makes private agents internalize the increased sensitivity of the interest rate premium with respect to debt as the debt crisis nears. The debt elasticity of the country premium is larger in the run up to the default because foreign lenders understand that the lower is output the higher is the incentive to default, as the output loss that occurs upon default, $L(y_t)$, decreases in absolute and relative terms as $y_t$ falls.

This capital control tax is implicitly present in every default model à la Eaton-Gersovitz. Analyzing the decentralized version of the model, as we did in this section, makes its presence explicit.
If the government does not have access to capital control taxes, then the model with decentralized borrowing and centralized default is no longer isomorphic to the social planner’s allocation. Kim and Zhang (2012) study this case. Absent the capital control tax, the model features a pecuniary externality. Why? Because household’s fail to internalize that their borrowing decisions affect the rate at which they can borrow.

Kim and Zhang find that the country interest rate increases unambiguously, but that, perhaps surprisingly, the model may or may not lead to overborrowing.
Section 13.9

Risk Averse Lenders
Motivation to allow for risk averse lenders

- Observed country spreads tend to be larger than observed default probabilities.
- Average spread-default-frequency differential greater 200 basis points.
- By contrast EG model predicts a negative or zero spread-default-frequency differential.
- Possible solution: introduced risk averse lenders into the EG model.
- Why will that help? Spread will then be the sum of default probability and compensation for default risk.
The EG model with risk averse lenders

Preferences of foreign lenders

\[ E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t u(\tilde{c}_t), \]

\( \tilde{\beta} \) = foreign lender’s subjective discount factor

\( \tilde{c}_t \) = consumption of foreign lenders

\( u(\tilde{c}) = \frac{\tilde{c}^{1-\tilde{\sigma}} - 1}{1 - \tilde{\sigma}} \), foreign lenders period utility function

\( \tilde{\sigma} \) = foreign lender’s coefficient of relative risk aversion.
Pricing kernel

\[ m' \equiv \bar{\beta} \left( \frac{\tilde{c}'}{\bar{c}} \right)^{-\tilde{\sigma}} \]

- Assume that the emerging country is **too small to affect the pricing kernel in the rest of the world.**

- Alternatively, one could assume that the emerging country is large enough to affect the world’s pricing kernel. Lizarazo (2013) pursues this alternative.
Let $g' \equiv \frac{\tilde{c}'}{\tilde{c}}$

Assume an AR(1) process: $\ln \left( \frac{g'}{\bar{g}} \right) = \rho_g \ln \left( \frac{g}{\bar{g}} \right) + \mu'$

The foreign-lender’s optimality condition is

$$ q = E\{m'I'|I = 1, y, g, d'\} $$

$q =$ price of the emerging country’s debt
$y =$ emerging country’s endowment
$d =$ emerging country’s debt
$I =$ indicator function that takes the value 1 if the emerging country is in good standing in the current period and chooses to honor its debt and 0 otherwise.
Pricing equation for risk free bond, $r^*$.

$$1 = (1 + r^*) E\{m' | g\}$$

$\rightarrow r^*$ is also a random variable.

If $\tilde{\sigma} = 0$, $m = \tilde{\beta}$, which is the case of risk neutrality studies earlier. In this case, $r^* = \tilde{\beta}^{-1} - 1$. Thus, the present formulation nests the baseline Eaton-Gersovitz model with risk-neutral lenders studied in section 13.6 as a special case.
Derive the country premium:

Let $1 + r =$ gross country interest rate

$$1 + r \equiv \frac{1}{q}; \quad \text{country premium} \equiv \frac{1 + r}{1 + r^*}$$

If debtor is in good financial standing ($I = 1$)

$$\frac{1 + r}{1 + r^*} = \frac{E\{m'|g\}}{E\{m'I'|y, g, d'\}}$$

Use $E\{m'I'|y, g, d'\} = \text{Cov}(m', I'|y, g, d') + E\{m'|g\}E\{I'|y, g, d'\}$

$$\frac{1 + r}{1 + r^*} = \frac{E\{m'|g\}}{\text{Cov}(m', I'|y, g, d') + E\{m'|g\}E\{I'|y, g, d'\}}.$$
Recalling that $E\{m'|g\} = 1/(1 + r^*) > 0$ and that $E\{I'|y, g, d'\} = 1 - \text{Prob}\{\text{default in } t + 1|y, g, d'\}$, we can write the (gross) country interest rate premium as

$$\frac{1 + r}{1 + r^*} = \frac{1}{(1 + r^*)\text{Cov}(m', I'|y, g, d') + 1 - \text{Prob}\{\text{default in } t + 1|y, g, d'\}}$$

Taking logs on both sides of this expression yields

$$r - r^* \approx \text{Prob}\{\text{default in } t + 1|y, g, d'} - (1 + r^*)\text{Cov}(m', I'|y, g, d')$$

which implies that the country premium will exceed the probability of default if and only if the conditional covariance between the pricing kernel and the decision to repay is negative.
Finally, averaging over all states in which the emerging country is in good financial standing and chooses to repay \((I = 1)\), we obtain

\[
E(r - r^*) > \text{Prob}\{\text{default in } t + 1\} \text{ if and only if } \text{Cov}(m', I') < 0.
\]

According to this expression, the more negative the covariance between the world pricing kernel and the decision to repay, the larger is the average spread-default-frequency differential. Although the world pricing kernel \(m\) is an exogenous stochastic process, its covariance with the decision to repay, \(I\), need not be nil. The reason is that \(m\) determines the world interest rate, and therefore it affects the emerging country’s cost of external funds and its decision to default or repay.

We find that in the numerical analysis \(\text{Cov}(m', I')\) is negative 32 percent of the time and thus positive 68 percent of the time. The unconditional mean of \(\text{Cov}(m', I')\) is \(1e-5\). This means that this channel will not generate a positive spread-default-frequency differential.

The next table documents this in detail.
Predictions of the Eaton-Gersovitz Model with Risk-Averse Lenders

<table>
<thead>
<tr>
<th>Default Frequency</th>
<th>All Periods</th>
<th>Periods in Good Standing</th>
<th>E(d/y)</th>
<th>E(r - r*)</th>
<th>σ(r - r*)</th>
<th>corr(r - r*, y)</th>
<th>corr(r - r*, tb/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ĉ</td>
<td>0</td>
<td>2.6</td>
<td>3.2</td>
<td>59.0</td>
<td>3.5</td>
<td>3.2</td>
<td>-0.54</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.8</td>
<td>3.4</td>
<td>58.3</td>
<td>3.6</td>
<td>3.5</td>
<td>-0.54</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.7</td>
<td>3.3</td>
<td>58.0</td>
<td>3.6</td>
<td>3.4</td>
<td>-0.55</td>
</tr>
</tbody>
</table>

Note. The variable $d/y$ denotes the quarterly debt-to-output ratio in percent, $r - r^*$ denotes the country premium, in percent per year, $y$ denotes (quarterly detrended) output, and $tb/y$ denotes the trade-balance-to-output ratio. The symbols $E$, $σ$, and corr denote, respectively, the mean, the standard deviation, and the correlation. All moments are conditional on the country being in good financial standing. Theoretical moments were computed by running the Matlab script statistics_modelral.m.
The key message of the table is that the assumption of risk-averse foreign lenders has quantitatively negligible effects on the predictions of the Eaton-Gersovitz model.

The assumption of risk-averse foreign lenders does not change the prediction of a near-zero spread-default-frequency differential.

The insensitivity of the endogenous variables of the model to changes in foreign risk aversion occurs even though the volatility of the world interest rate (not shown in the table) increases significantly with $\tilde{\sigma}$.

Specifically, the standard deviation of $r^*$ is 0, 1.2, and 3.0 percent per year for $\tilde{\sigma}$ equal to 0, 2, and 5, respectively.

Why is it then that this sizable increase in the volatility of the world interest rate does not affect the domestic economy? We conjecture that the reason is that this is an economy with highly impatient
agents, who can borrow much less than what they would like to borrow under commitment. As a result, the present model behaves quite similarly to one in which the agent is up against a borrowing constraint most of the time. In such a setting, the price of credit is little allocative, and hence variation therein do not affect much consumption or borrowing decisions. This result may change in a setting with default and more patient consumers.
Section 13.10

Long-Term Debt and Default

[slides not yet written]
Section 13.11

Debt Renegotiation

[slides not yet written]
Section 13.12

Default and Monetary Policy
In this section we

— introduce nominal rigidities into the Eaton-Gersovitz model.
— analyze optimal default and devaluation policy.
— document that defaults are accompanied by devaluations.
— study the consequences of suboptimal monetary policy on default.

This section is based on Na, Schmitt-Grohé, Uribe, and Yue (2014).
The Twin Ds

There exists a strong empirical link between sovereign default and large devaluations.

- Reinhart (2002) examines data for 58 countries over the period 1970 to 1999 and finds that:
  - The unconditional probability of a large devaluation in any 24-month period is 17%.
  - The probability of a large devaluation conditional on the 24-month period containing a default is 84%.

- Reinhart refers to this phenomenon as the **Twin Ds**.

Next we show an additional (ie not highlighted by Reinhart) aspect of the Twin D phenomenon, namely that the default is **not** associated with a permanent increase in the devaluation rate.
Figure 13.14: Excess Devaluation Around Default, 1975-2013

Observations on the figure

• the exchange rate is defined so that an increase in the exchange rate is a devaluation of the domestic currency.

• exchange rate depreciates 40 percent more if window contains default episode than if it does not.

• post devaluation level of exchange rate stabilizes, hence devaluations around default are more akin to a change in the level of the nominal exchange rate than to a switch to a higher rate of depreciation.

Let’s look at some recent defaults in specific countries.
Figure 13.15: The Twin Ds: Six Recent Examples

![Graphs showing exchange rates for Argentina, Uruguay, Ukraine, Russia, Paraguay, Ecuador over years 1996 to 2008.]

Nominal Exchange Rate

Default Date

Note: Exchange rates are nominal dollar exchange rates, annual average, first observation normalized to unity.
Observations on the figure.

In each of these 6 examples, we plot the level of the nominal exchange rate. That is, we don’t show excess devaluation, but simply the level.

All 6 cases conform to the regularity that we wish to highlight, namely, that the devaluation that accompanies the default looks more like a one-time change in the level of the exchange rate than a switch to a permanently higher devaluation rate.
Argentina 1996-2006


Vertical Line 2002, default and devaluation.
Na et al. paper:

– develops a model that explains the Twin Ds phenomenon as an optimal policy outcome.

● Main Elements

– Imperfect enforcement of debt contracts.

– Downward nominal wage rigidity.

● Intuition

– Under the optimal policy, default occurs during large recessions.
– A contracting demand for labor puts downward pressure on real wages.
– A large devaluation reduces the real value of wages, thereby preventing unemployment.
– The devaluation is necessary to bring real wages down because nominal wages are downwardly rigid.