Monetary Policy and Multiple Equilibria

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This paper characterizes conditions under which interest-rate feedback rules that set the nominal interest rate as an increasing function of the inflation rate induce aggregate instability by generating multiple equilibria. It shows that these conditions depend not only on the monetary-fiscal regime (as emphasized in the fiscal theory of the price level) but also on the way in which money is assumed to enter preferences and technology. It provides a number of examples in which, contrary to what is commonly believed, active monetary policy gives rise to multiple equilibria and passive monetary policy renders the equilibrium unique. (*JEL* E52, E31, E63)

Recent developments in monetary economics have emphasized the link between the degree to which monetary and fiscal policies respond to endogenous variables such as the inflation rate or the stock of public debt and macroeconomic stability.[1] Perhaps the best-known result in this literature is that if fiscal solvency is preserved under all circumstances, then an active monetary policy, that is, a policy that aggressively fights inflation by raising the nominal interest rate by more than the increase in inflation, stabilizes the real side of the economy by ensuring the uniqueness of equilibrium. At the same time, a passive monetary policy, that is, a policy that underreacts to inflation by raising the nominal interest rate by less than the observed increase in inflation, destabilizes the economy by giving rise to expectations-driven fluctuations.

In this paper, we argue that whether a particular monetary-fiscal regime is conducive to macroeconomic stability in the sense given above depends crucially on the way in which money is assumed to enter preferences and technology. One important point of departure of our theoretical framework from the one used in the recent related literature is our emphasis on the role that the demand for money by firms plays in the monetary-transmission mechanism. Specifically, we follow Stanley Fischer (1974), John B. Taylor (1977), and Guillermo A. Calvo (1979) in assuming that money affects not only preferences but also production possibilities. Introducing a demand for money by firms is motivated by the fact that in industrialized countries, firms hold a substantial fraction of the money supply. For example, in the United States, nonfinancial firms held at least 50-percent more demand deposits than households over the period 1970–1990 (see Casey B. Mulligan [1997] and the references cited therein). Given this empirical regularity, it is surprising that recent theoretical evaluations of monetary-policy rules have restricted attention to the case in which variations in the nominal interest rate affect real variables solely through their effect on aggregate demand.

Our main finding is that regardless of the stance of fiscal policy, an active monetary policy does not necessarily bring about the determinacy of equilibrium. In the context of a flexible-price, money-in-the-utility-function model, we show that the standard result, i.e., equilibrium is unique under active monetary policy, holds in an endowment economy in which consumption

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[1] See, for example, Eric Leeper (1991), Christopher Sims (1994, 1997), Michael Woodford (1994, 1995, 1996), Richard Clarida et al. (1997), and Schmitt-Grohé and Uribe (2000).

and real balances are Edgeworth complements in preferences in the sense that the marginal utility of consumption is increasing in real balances. However, the opposite result, i.e., uniqueness of the equilibrium occurs under passive monetary policy and multiplicity of equilibria occurs under active monetary policy, obtains if consumption and real balances are Edgeworth substitutes. More importantly, the opposite result also obtains in an economy in which money enters in the production function even if real balances and consumption are Edgeworth complements in preferences. Furthermore, we provide examples in which active monetary policy renders the equilibrium indeterminate in models with money in the production function even if the output effects of money are arbitrarily small. This result is of particular interest, for it highlights the fact that the usual implicit assumption that firms' costs of production are unaffected by variations in the nominal interest rate is not inconsequential for the stabilizing properties of alternative interest-rate feedback rules.

In the literature that stresses the desirability of active interest-rate feedback rules, the need for monetary-stabilization policy typically arises because of short-run nominal price rigidities. Therefore, we also analyze economies with sluggish price adjustment. Specifically, we study the role of interest-rate feedback rules for aggregate stability in a model with convex cost of price adjustment like the one developed by Julio J. Rotemberg (1982).[2] We show that, as in the flexible-price case, when money enters in the production function, active monetary policy may render the equilibrium indeterminate regardless of the stance of fiscal policy. A further novel result that emerges from the analysis of sticky-price models in which productivity is affected by the cost of funds is that restricting the evaluation of interest-rate rules to their consequences for local stability can be misleading. In particular, we find that an active monetary-policy stance may appear to be stabilizing because it ensures the uniqueness of equilibrium locally, when in fact it is destabilizing because it gives rise to equilibria in which the economy converges to a cycle.[3]

In our baseline analysis, we assume that the policy maker sets the nominal interest rate as a function of an instantaneous rate of inflation. This is clearly not a practical proposal since instantaneous measures of inflation do not exist. Consequently, any practical inflation measure to which the central bank responds must involve some average of past inflation rates or forecasts of expected future rates of inflation, or both. For example, Taylor's (1993) characterization of recent U.S. monetary policy assumes that the Federal Reserve's operating target for the federal funds rate is a function of average inflation over the previous four quarters. Other authors have argued that actual Federal Reserve policy is best described by a forward-looking interest-rate feedback rule (Athanasios Orphanides, 1997; Clarida et al., 1998). The use of medium-term forecasts of inflation, rather than past inflation, has also been suggested as the ideal intermediate variable for an inflation-targeting regime on theoretical grounds (Lars E. O. Svensson, 1997, 1998). For these reasons, we extend the baseline analysis to allow for feedback rules in which the interest rate depends on past or expected future rates of inflation. We find that our main argument is robust to this extension: even in the presence of backward- or forward-looking components in monetary policy, the deep structural parameters defining preferences and technologies play a crucial role in determining whether a particular monetary-fiscal regime is stabilizing or not. A general pattern that arises, particularly in sticky-price environments, is that if the monetary authority follows an active stance, then a forward-looking component in the intermediate target makes indeterminacy more likely, whereas a backward-looking component makes determinacy more likely.

The remainder of the paper is organized in four sections. Section I studies the macroeconomic effects of interest-rate feedback rules in a flexible-price economy. Section II extends the analysis to an economy with slow adjustment of product prices. Section III investigates the robustness of the results to the introduction of

[2] In an unpublished Appendix to this paper (Benhabib et al., 2000b), we show that our results also apply in a model with staggered price setting à la Calvo (1983) and Tack Yun (1996).

[3] For analyses of other types of global indeterminacy under active interest-rate feedback rules, see Benhabib et al. (2001).

backward- and forward-looking elements in the intermediate target, and Section IV concludes.

I. A Flexible-Price Model

In this section, we study the determinacy of equilibrium under alternative monetary and fiscal policies in a flexible-price model. Our modeling approach differs from the one used in the recent related literature in that we assume that money facilitates firms' production as in Calvo (1979). We depart from Calvo's analysis by considering different monetary-fiscal regimes. Calvo focuses on monetary policies whereby the central bank pegs either the money growth rate or the inflation rate in combination with a fiscal policy that specifies zero public debt at all times implying full monetization of primary deficits. By contrast, as will be explained in detail shortly, we analyze interest-rate feedback rules in combination with fiscal policies in which the real primary surplus is either constant or proportional to the stock of real government liabilities.[4]

A. *The Economic Environment*

Households.—The household's lifetime utility function is given by

$$U = \int_0^\infty e^{-rt} u(c, m^{np})\, dt, \tag{1}$$

where $r > 0$ denotes the rate of time preference, c consumption, $m^{np} \equiv M^{np}/P$ real balances held for nonproduction purposes, M^{np} nominal money balances held for nonproduction purposes, and P the nominal price level. The instant utility function $u(\cdot, \cdot)$ satisfies Assumption 1, which implies that c and m^{np} are normal goods.

ASSUMPTION 1: *$u(\cdot, \cdot)$ is strictly increasing and strictly concave, and satisfies $u_{cc} - u_{cm}u_c/u_m < 0$ and $u_{mm} - u_{cm}u_m/u_c < 0$.*

[4] Taylor (1977) is another example of a study of price-level determination in a model with money in the production function. The two main differences between our analysis and Taylor's is that his model does not allow for optimizing behavior on the part of households and firms and that, like Calvo, he considers different monetary- and fiscal-policy regimes.

We consider two alternative production technologies: (1) output is produced with real balances held by the household for production purposes, $m^p \equiv M^p/P$, where M^p denotes nominal money balances held for production purposes, and (2) output is equal to a positive constant. Formally, the production technology, $y(m^p)$, satisfies either Assumption 2 or Assumption 2′.

ASSUMPTION 2: *$y(m^p)$ is positive, strictly increasing, strictly concave, and satisfies $\lim_{m^p\to 0} y'(m^p) = \infty$, and $\lim_{m^p\to\infty} y'(m^p) = 0$.*

ASSUMPTION 2′: *$y(m^p)$ is a positive constant.*

In addition to money, the household can hold nominal bonds, B, which pay the nominal interest rate $R > 0$. Letting $a \equiv (M^{np} + M^p + B)/P$ denote the household's real financial wealth, τ real lump-sum taxes, and $\pi \equiv \dot{P}/P$ the inflation rate, the household's instant budget constraint can be written as

$$\dot{a} = (R - \pi)a - R(m^{np} + m^p) + y(m^p) - c - \tau. \tag{2}$$

The household chooses sequences for c, m^{np}, $m^p \geq 0$ and a so as to maximize (1) subject to (2) and the following no-Ponzi-game condition

$$\lim_{t\to\infty} e^{-\int_0^t [R(s) - \pi(s)]\, ds} a(t) \geq 0, \tag{3}$$

taking as given $a(0)$ and the time paths of τ, R, and π. The optimality conditions associated with the household's problem are

$$u_c(c, m^{np}) = \lambda \tag{4}$$

$$m^p[y'(m^p) - R] = 0 \tag{5}$$

$$\frac{u_m(c, m^{np})}{u_c(c, m^{np})} = R \tag{6}$$

$$\lambda(r + \pi - R) = \dot{\lambda} \tag{7}$$

$$\lim_{t\to\infty} e^{-\int_0^t [R(s) - \pi(s)]\, ds} a(t) = 0, \tag{8}$$

where λ is the Lagrange multiplier associated with the household's instant budget constraint.

Assumption 2 together with equation (5) and $R > 0$ implies that m^p is a strictly decreasing function of R:

$$m^p = m^p(R), \tag{9}$$

with $m^{p\prime} \equiv dm^p/dR < 0$. Alternatively, Assumption 2′, equation (5), and the fact that $R > 0$ imply that $m^p = m^{p\prime} = 0$. Using equation (6) and Assumption 1, m^{np} can be expressed as a function of consumption and the nominal interest rate that is increasing in c and decreasing in R:

$$m^{np} = m^{np}(c, R). \tag{10}$$

The Government.—We assume that monetary policy takes the form of an interest-rate feedback rule whereby the nominal interest rate is set as an increasing function of the inflation rate. Specifically, we assume that

$$R = \rho(\pi), \tag{11}$$

where $\rho(\cdot)$ is continuous, nondecreasing, and strictly positive and there exists at least one $\pi^* > -r$ such that $\rho(\pi^*) = r + \pi^*$. Following Leeper (1991), we refer to monetary policy as active if $\rho' > 1$ and as passive if $\rho' < 1$.

Government purchases are assumed to be zero at all times. Then, the sequential budget constraint of the government is given by $\dot{B} = RB - \dot{M}^{np} - \dot{M}^p - P\tau$, which can be written as

$$\dot{a} = (R - \pi)a - R(m^{np} + m^p) - \tau. \tag{12}$$

Because the nominal value of initial government liabilities, $A(0) > 0$, is predetermined, the initial condition $a(0)$ must satisfy

$$a(0) = \frac{A(0)}{P(0)}. \tag{13}$$

We classify fiscal policies into two categories: Ricardian fiscal policies and non-Ricardian ones. Ricardian fiscal policies are those that ensure that the present discounted value of total government liabilities converges to zero—that is, equation (8) is satisfied—under all possible, equilibrium or off-equilibrium, paths of endogenous variables such as the price level, the money supply, inflation, or the nominal interest rate.

Throughout the paper, we restrict attention to one particular Ricardian fiscal policy that takes the form

$$\tau + R(m^{np} + m^p) = \alpha a, \tag{14}$$

where the sequence α is chosen arbitrarily by the government subject to the constraint that it is positive and bounded below by $\underline{\alpha} > 0$. This policy states that consolidated government revenues, that is, tax revenues plus interest savings from the issuance of money, are always higher than a certain fraction $\underline{\alpha}$ of total government liabilities. A special case of this type of policy is a balanced-budget rule whereby tax revenues are equal to interest payments on the debt, which results when $\alpha = R$ (provided R is bounded away from zero). To see that the fiscal policy given by (14) is Ricardian, let $d \equiv \exp[-\int_0^t (R - \pi)\, ds]$ and $x \equiv da$. The definition of a Ricardian fiscal policy requires that $x \to 0$ as $t \to \infty$. Note that $\dot{x} = d[\dot{a} - (R - \pi)a]$. Using equations (12) and (14), this expression can be written as $\dot{x} = -\alpha x$, which implies that x converges monotonically to zero.

We will also analyze a particular non-Ricardian fiscal policy consisting of an exogenous path for lump-sum taxes

$$\tau = \bar{\tau} > 0. \tag{15}$$

Equilibrium.—In equilibrium the goods market must clear

$$c = y(m^p). \tag{16}$$

Using equations (9)–(11) and (16) to replace m^p, m^{np}, R, and c in equation (4), λ can be expressed as a function of π,

$$\lambda = \lambda(\pi), \tag{17}$$

with

$$\lambda'(\pi) = \rho'[u_{cc}y'm^{p\prime} + u_{cm}(m_c^{np}y'm^{p\prime} + m_R^{np})], \tag{18}$$

where m_c^{np} and m_R^{np} denote the partial derivatives of m^{np} with respect to c and R, respectively. Using this expression, (9)–(11), and (16), equations (7), (8), (12), and (14) can be rewritten as

$$(19) \quad \lambda'(\pi)\dot{\pi} = \lambda(\pi)[r + \pi - \rho(\pi)]$$

$$(20) \quad \dot{a} = [\rho(\pi) - \pi]a - \rho(\pi) \times [m^{np}(y(m^p(\rho(\pi))), \rho(\pi)) + m^p(\rho(\pi))] - \tau$$

$$(21) \quad \lim_{t\to\infty} e^{-\int_0^t [\rho(\pi) - \pi(s)]\, ds} a(t) = 0$$

$$(22) \quad \tau + \rho(\pi)[m^{np}(y(m^p(\rho(\pi))), \rho(\pi)) + m^p(\rho(\pi))] = \alpha a.$$

Definition 1 (Perfect-Foresight Equilibrium in the Flexible-Price Economy): In the flexible-price economy, a perfect-foresight equilibrium is a set of sequences $\{\pi, a, \tau\}$ and an initial price level $P(0) > 0$ satisfying (13), (19)–(21) and either (15) if fiscal policy is non-Ricardian or (22) if fiscal policy is Ricardian, given $A(0) > 0$.

Given an equilibrium sequence for π, equations (9)–(11), (16), and (17) uniquely determine the equilibrium sequences $\{c, m^{np}, m^p, \lambda, R\}$.

B. *Determinacy of Equilibrium Under Alternative Monetary-Fiscal Regimes*

In this subsection, we restrict the analysis to equilibria in which the inflation rate converges asymptotically to a steady-state value, π^*, which is defined as a constant value of π that solves (19), that is, a solution to $r + \pi = \rho(\pi)$. By assumption, π^* exists and is greater than $-r$. It is clear from equations (9)–(11) and (16) that if the equilibrium time path of inflation is unique, then so is the equilibrium real allocation $\{c, m^{np}, m^p\}$ independently of whether the equilibrium price level is unique. Thus, it is useful to introduce the following terminology.

Definition 2 (Real and Nominal Indeterminacy): The equilibrium displays real indeterminacy if there exists an infinite number of equilibrium sequences $\{\pi\}$. The equilibrium exhibits nominal indeterminacy if for any equilibrium sequence $\{\pi\}$, there exists an infinite number of initial price levels $P(0) > 0$ consistent with a perfect-foresight equilibrium.

Under a Ricardian fiscal policy, the set of equilibrium conditions includes equation (22). Given a sequence $\{\pi\}$ satisfying (19) and an arbitrary initial price level $P(0) > 0$, equations (13), (20), and (22) can be used to construct a pair of sequences $\{a, \tau\}$. Because the fiscal policy is Ricardian, the transversality condition (21) is always satisfied. Therefore, under a Ricardian fiscal policy the price level is always indeterminate. If instead the fiscal authority follows the non-Ricardian fiscal policy given in (15), combining (13), (20), and (21) yields

$$(23) \quad \frac{A(0)}{P(0)} = \int_0^\infty e^{-\int_0^t [\rho(\pi) - \pi]\, ds} \{\rho(\pi) \times [m^{np}(y(m^p(\rho(\pi))), \rho(\pi)) + m^p(\rho(\pi))] + \bar{\tau}\}\, ds,$$

which given $A(0) > 0$ and a sequence for π converging to π^* uniquely determines the initial price level $P(0)$.

The above analysis demonstrates that for the class of monetary-fiscal regimes studied in this paper, nominal determinacy depends only on fiscal policy and not on monetary policy—a result that has been emphasized in the recent literature on the fiscal determination of the price level and that we summarize in the following proposition.

PROPOSITION 1: *If fiscal policy is Ricardian, the equilibrium exhibits nominal indeterminacy. Under the non-Ricardian fiscal policy given by* (15), *the equilibrium displays nominal determinacy.*

By contrast, the determinacy of the real allocation is independent of fiscal policy but depends on the stance of monetary policy and on the particular way in which inflation affects production and consumption. To see this, consider solutions to equation (19). If $\lambda'(\pi^*)$ and $1 - \rho'(\pi^*)$ are of opposite sign, any initial inflation rate near the steady-state π^* will give rise to an inflation trajectory that converges to π^*. If, on the other hand, $\lambda'(\pi^*)$ and $1 - \rho'(\pi^*)$

are of the same sign, the only sequence of inflation rates that converges asymptotically to π^* is one in which the inflation rate is constant and equal to π^*. If $\rho'(\pi) = 0$ for all π, then equations (18) and (19) imply that λ and π are constant. Thus, under a pure interest-rate peg the economy exhibits real determinacy.

To understand the conditions under which the model displays real indeterminacy, it is instructive to consider the following two polar cases. Consider first the case in which preferences are separable in consumption and money ($u_{cm} = 0$) and money is productive (Assumption 2 holds). In this case, equation (18) implies that $\lambda' = \rho' u_{cc} y' m^{p\prime} > 0$, so that the model displays real indeterminacy if $1 - \rho'(\pi^*) < 0$, that is, if monetary policy is active, and is unique if $1 - \rho'(\pi^*) > 0$, that is, if monetary policy is passive. The intuition behind this result is as follows. Suppose firms initially hold more real balances for production purposes than in the steady state. This will happen only if the nominal interest rate is below its steady-state level. By the interest-rate feedback rule, the inflation rate has to be below its steady-state value as well. If monetary policy is active, the decline in the inflation rate is accompanied by a decline in the real interest rate, $R - \pi$, which in turn induces negative consumption growth. Since in equilibrium consumption equals output, and output is an increasing function of real balances, real balances for production purposes will be expected to decline. Therefore, the initial increase in real balances is reversed and the resulting trajectory is consistent with equilibrium. If, on the other hand, monetary policy is passive, the decline in the inflation rate is associated with a rise in the real interest rate, and thus consumption will be expected to grow, moving output and real balances even further away from the steady state. Such a trajectory for real balances would not remain bounded in a neighborhood around the steady state and thus would not be consistent with an equilibrium in which inflation converges to π^*. This result is summarized in the following proposition.

PROPOSITION 2: *Suppose preferences are separable in consumption and money* ($u_{cm} = 0$) *and money is productive* (*Assumption* 2 *holds*), *then if monetary policy is active* ($\rho'(\pi^*) > 1$), *the equilibrium displays real indeterminacy, whereas if monetary policy is passive* ($\rho'(\pi^*) < 1$), *then the only perfect-foresight equilibrium in which the real allocation converges to the steady state is the steady state itself.*

Consider now the case in which money is not productive, that is, Assumption 2′ holds. In this case, equation (18) implies that $\lambda' = \rho' u_{cm} m_R^{np}$, which is positive if $u_{cm} < 0$, that is, if consumption and money are Edgeworth substitutes, and is negative if $u_{cm} > 0$, that is, if consumption and money are complements. Thus the economy displays real indeterminacy if monetary policy is active and consumption and money are substitutes, or if monetary policy is passive and consumption and money are complements.[5] The intuition behind this indeterminacy result is as follows. Consider the case that monetary policy is passive and $u_{cm} > 0$. Suppose that real balances for nonproductive purposes are increased above their steady-state level. Because the money demand function of the household is decreasing in the nominal interest rate and consumption is constant, it follows that the nominal interest rate has to be below its steady-state level. At the same time, passive monetary policy implies that the decline in the nominal interest rate is associated with an increase in the real interest rate. In response to the increase in the real interest rate, agents will lower the growth rate of the marginal utility of consumption. With consumption constant and $u_{cm} > 0$, this requires that the growth rate of real balances be negative. Thus real balances will return to their steady level and this trajectory is consistent with equilibrium. The next two propositions summarize these results.

PROPOSITION 3: *Suppose that money is not productive* (*Assumption* 2′ *holds*) *and consumption and money are Edgeworth substitutes* ($u_{cm} < 0$). *Then, if monetary policy is active* ($\rho'(\pi^*) > 1$), *the real allocation is indetermi-*

[5] As is well known, there exists an exact correspondence between the equilibrium conditions of the economy with $y' = 0$ and $u_{cm} > 0$ and those of the cash-in-advance economy with cash and credit goods developed by Robert E. Lucas, Jr. and Nancy Stokey (1987). Therefore, in the (continuous-time version of the) Lucas-Stokey model, the real allocation is indeterminate under passive monetary policy.

TABLE 1—REAL INDETERMINACY IN THE FLEXIBLE-PRICE MODEL

	Nonproductive money ($y' = 0$)			Productive money ($y' > 0$)		
Monetary policy	$u_{cm} > 0$	$u_{cm} < 0$	$u_{cm} = 0$	$u_{cm} > 0$	$u_{cm} < 0$	$u_{cm} = 0$
Passive ($\rho'(\pi^*) < 1$)	I	D	D	A	D	D
Active ($\rho'(\pi^*) > 1$)	D	I	D	A	I	I

Notes: The notation is D, determinate; I, indeterminate; A, ambiguous. (Under A the real allocation may be determinate or indeterminate depending on specific parameter values.)

nate, and if monetary policy is passive ($\rho'(\pi^*) < 1$), *then the only perfect-foresight equilibrium in which the real allocation converges to the steady state is the steady state itself.*

PROPOSITION 4: *Suppose that money is not productive* (*Assumption* 2′ *holds*) *and consumption and money are Edgeworth complements* ($u_{cm} > 0$). *Then, if monetary policy is passive* ($\rho'(\pi^*) < 1$), *the real allocation is indeterminate, and if monetary policy is active* ($\rho'(\pi^*) > 1$), *then the only perfect-foresight equilibrium in which the real allocation converges to the steady state is the steady state itself.*

Table 1 summarizes the combinations of preference, technology, and monetary-policy specifications under which real indeterminacy arises in the flexible-price model. The second row of the table highlights the main result of this section, namely, that, contrary to what is often asserted, real indeterminacy may arise under active monetary policy. Most existing studies restrict attention to the case in which money is not productive ($y' = 0$) and money and consumption either are complements in preferences or enter the utility function in a separable fashion ($u_{cm} \geq 0$). As a result these studies have arrived at the potentially misleading conclusion that an active monetary policy stabilizes the economy by bringing about real determinacy.

An assumption of our analysis is that in setting the nominal interest rate the central bank responds to an instantaneous measure of inflation. This assumption is clearly unrealistic, for such a measure of inflation does not exist in practice. In Section III we try to capture the notion that a realistic measure of the inflation rate to which the central bank responds involves some degree of time averaging by assuming that the policy instrument is a function of a geometric average of all past inflation rates.

Before closing this subsection, we wish to point out that there are additional reasons to those spelled out here, why the results on local uniqueness under active monetary policy should be interpreted with caution. For if there exists a steady-state π^* with $\rho'(\pi^*) > 1$, then since $\rho(\cdot)$ is assumed to be nondecreasing and nominal interest rates are bounded below by zero, there must also exist a steady state at which inflation is below target and ρ' is less than one. This steady state is indeterminate precisely under the assumptions that assure local determinacy at the steady-state π^*. Benhabib et al. (2001) demonstrate the existence of equilibrium paths connecting these two steady states. Benhabib et al. (2000a) characterize trajectories leading from the active to the passive steady states as liquidity traps and provide a number of fiscal strategies that keep the economy from falling into such traps without requiring a change in monetary policy.

C. *Comparison with Discrete-Time Models*

In discrete-time models, preference and technology specifications also play an important role in determining whether active interest-rate feedback rules are stabilizing. In making a sensible comparison between discrete- and continuous-time models it is important to note that in discrete time the current rate of inflation, π_t, is given by the change in the price level between periods $t - 1$ and t, whereas in continuous-time specifications, π_t is the instantaneous rate of inflation given by the right-hand-side derivative of the log of the price level. The discrete-time counterpart of the right-hand-side derivative of the log of the price level is best approximated by the percentage change in the price level between periods t and $t + 1$. Thus, the natural discrete-time version of the monetary-policy rule (11) is given by a forward-looking rule of the form $R_t = \rho(P_{t+1}/P_t)$. In addition to this technical reason for considering the case of forward-looking rules, there is also an empirical motivation. Studies by Orphanides (1997) and Clarida et al. (1998) have shown that over the past two decades, forward-looking interest-rate feedback rules have represented a better description of actual monetary policy in the G-3 economies than backward-looking rules.

We begin by showing that, as in the continuous-time case, in the discrete-time model without output effects of money ($y' = 0$), small perturbations of the specification of preferences around $u_{cm} = 0$—the case typically studied in the related literature—can reverse the stability properties of a given interest-rate feedback rule. Consider first the case of preferences that are separable in consumption and real balances ($u_{cm} = 0$). In the continuous-time model such a specification implies by equation (18) that λ is constant. It then follows that π, R, and m^{np} are also constant, and therefore the only equilibrium real allocation is the steady state. The same result obtains in a discrete-time model under a forward-looking feedback rule. To see this, consider the loglinearized version of the equilibrium conditions of a discrete-time endowment economy with money in the utility function:

$$\rho_{cm}(\hat{m}_t - \hat{m}_{t+1}) = \hat{R}_t - \hat{\pi}_{t+1},$$

$$\hat{m}_t = -\varepsilon_{mR}\hat{R}_t,$$

$$\hat{R}_t = \varepsilon_\rho \hat{\pi}_{t+1},$$

where $\hat{m}_t$ is the log deviation of real balances from steady state and $\hat{R}_t$ and $\hat{\pi}_t$ are the log deviations of the gross nominal interest rate and gross inflation from steady state. The first equation represents an Euler equation, where we used the fact that consumption is constant over time. The parameter ρ_{cm} denotes the steady-state elasticity of the marginal utility of consumption with respect to real balances. The second equation is the liquidity preference function with $\varepsilon_{mR} > 0$ denoting the interest elasticity of money demand. The third equation is the interest-rate feedback rule, where ε_ρ denotes the elasticity of the feedback rule with respect to the intermediate target. When preferences are separable in consumption and real balances, then $\rho_{cm} = 0$ and the above three expressions collapse to $0 = (\varepsilon_\rho - 1)\hat{\pi}_{t+1}$, which implies that $\pi_t = \pi^*$ for all $t > 0$, provided that ε_ρ is different from unity. It follows that the nominal interest rate as well as real balances are uniquely determined and constant. Therefore, the model displays real determinacy.[6]

When preferences are nonseparable in consumption and real balances ($\rho_{cm} \neq 0$), the above three equations can be reduced to the following first-order difference equation in inflation

$$\hat{\pi}_{t+2} = \left[1 + \frac{\varepsilon_\rho - 1}{\varepsilon_\rho \rho_{cm} \varepsilon_{mR}}\right]\hat{\pi}_{t+1}.$$

If consumption and real balances are Edgeworth complements ($\rho_{cm} > 0$), the equilibrium is determinate when monetary policy is active ($\varepsilon_\rho > 1$) and is indeterminate when monetary policy is passive but not too irresponsive to inflation.[7] By contrast, if consumption and real balances are Edgeworth substitutes ($\rho_{cm} < 0$), the equilibrium is determinate when monetary policy is passive and is indeterminate when monetary policy is active. If $0 < -\rho_{cm}\varepsilon_{mR} < \frac{1}{2}$, a condition that arises at low degrees of substitutability between consumption and real balances or at a low interest elasticity of money demand or both, the equilibrium becomes determinate for relatively highly active feedback rules.[8] These results are identical to those obtained under continuous time.

We now show that in discrete-time models,

[6] However, because π_0 is not pinned down, the model displays nominal indeterminacy unless fiscal policy is non-Ricardian.

[7] Specifically, if $1 > \varepsilon_\rho > 1/(1 + 2\rho_{cm}\varepsilon_{mR})$, the equilibrium is indeterminate.

[8] Specifically, this change in the stability properties of the equilibrium system takes place when the elasticity of the feedback rule with respect to inflation exceeds the critical value $\overline{\varepsilon_\rho} \equiv 1/(1 + 2\rho_{cm}\varepsilon_{mR}) > 1$. Although in this case the real allocation is locally unique, real instability may still be present due to the emergence of attracting two-period cycles. The possibility of two-period cycles arises because at $\varepsilon_\rho = \overline{\varepsilon_\rho}$ the system displays a flip bifurcation. For certain parameter specifications the cycles appear for $\varepsilon_\rho > \overline{\varepsilon_\rho}$ and are therefore attracting (see J. Guckenheimer and P. Holmes, 1983 Ch. 3).

as in continuous-time specifications, small perturbations in the elasticity of output with respect to the nominal interest rate around zero also can fundamentally alter the stabilizing properties of a particular interest-rate feedback rule. Specifically, consider a discrete-time environment in which money is productive (Assumption 2 holds) and preferences are separable in consumption and real balances ($u_{cm} = 0$). In this case, the loglinearized equilibrium conditions consist of the following two equations:

$$\rho_{cc}(\hat{y}_{t+1} - \hat{y}_t) = \varepsilon_\rho \hat{\pi}_{t+1} - \hat{\pi}_{t+1}$$

$$\hat{y}_t = -\varepsilon_{yR}\varepsilon_\rho \hat{\pi}_{t+1},$$

where $\rho_{cc} > 0$ is the elasticity of the marginal utility of consumption, $\varepsilon_{yR} \geq 0$ is the elasticity of output with respect to the nominal interest rate and where $\hat{y}_t$ denotes the log deviation of output from steady state. The first expression is a standard Euler equation and the second represents the reduced-form production function describing a negative relationship between output and expected inflation. Combining the two loglinearized equilibrium conditions we obtain the following first-order difference equation

$$\hat{\pi}_{t+2} = \left[1 + \frac{1 - \varepsilon_\rho}{\rho_{cc}\varepsilon_{yR}\varepsilon_\rho}\right]\hat{\pi}_{t+1}.$$

If money is productive ($\varepsilon_{yR} > 0$), then a monetary-policy stance that is neither passive nor active ($\varepsilon_\rho = 1$) represents a bifurcation point of the system. For $\varepsilon_\rho < 1$, that is, when monetary policy is passive, the coefficient on $\hat{\pi}_{t+1}$ is greater than one. This implies that the only perfect-foresight equilibrium that converges asymptotically to the steady-state π^* is $\pi_t = \pi^*$ for all $t > 0$. For $\varepsilon_\rho > 1$, that is, when monetary policy is active, the coefficient on $\hat{\pi}_{t+1}$ is less than one in absolute value, implying indeterminacy of the perfect-foresight real allocation. Consequently, as in the continuous-time case, the real allocation is uniquely determined when monetary policy is passive and is indeterminate when monetary policy is active.[9]

To summarize, the above discussion shows that when the interest-rate feedback rule is assumed to be forward looking, then the basic results obtained under continuous time are robust to specifying time as a discrete variable. In particular, the stability properties of a given monetary-policy stance change discontinuously around the particular specification for preferences and technologies typically assumed in the related literature, namely, $u_{cm} = 0$ and $y' = 0$.

It is important to emphasize that if the interest-rate feedback rule is assumed to be backward looking in the sense that the nominal interest rate depends not on P_{t+1}/P_t but on P_t/P_{t-1}, then, for productivity effects of money that are small relative to the interest sensitivity of aggregate demand ($\rho_{cc}\varepsilon_{yR} < ½$), the real allocation is always determinate under active monetary policy, as is the case when money does not affect production possibilities.[10] This conclusion is of interest because it shows that the results obtained by previous authors using backward-looking specifications of the interest-rate feedback rule (e.g., Leeper, 1991), are robust to small perturbations in the technology specification assumed in that literature ($y' = 0$).[11] On the other hand, the real allocation is always indeterminate under active monetary policy when productivity effects of money are sufficiently large ($\rho_{cc}\varepsilon_{yR} \geq 1$).

Table 2 summarizes the results obtained under the assumption that time is discrete.

II. A Sticky-Price Model

The flexible-price economy analyzed in the previous section conveys the main message of

[9] However, a difference with the continuous-time model arises when productivity effects of money are small relative to the interest sensitivity of aggregate demand. In particular, if $\rho_{cc}\varepsilon_{yR} < ½$, then a second bifurcation point emerges at $\varepsilon_\rho = \overline{\varepsilon_\rho} \equiv 1/(1 - 2\rho_{cc}\varepsilon_{yR}) > 1$. For mildly active monetary policy ($1 < \varepsilon_\rho < \overline{\varepsilon_\rho}$) equilibrium is still indeterminate but for a more aggressive stance ($\varepsilon_\rho > \overline{\varepsilon_\rho}$), the real allocation becomes locally determinate. For certain parameterizations, real instability is present even if $\varepsilon_\rho > \overline{\varepsilon_\rho}$ due to the emergence of two-period cycles (see footnote 8). Note that the second bifurcation point does not exist if $\rho_{cc}\varepsilon_{yR} > ½$.

[10] These continuity results regarding the conditions for determinacy as the productivity effects of money become negligible is similar to the one obtained by Woodford (1998) in the context of a monetary economy converging to a cashless limit.

[11] We thank a referee for drawing our attention to this result.

TABLE 2—REAL INDETERMINACY IN THE DISCRETE-TIME FLEXIBLE-PRICE MODEL

	Forward-looking rule: $R_t = \rho(P_{t+1}/P_t)$				Backward-looking rule: $R_t = \rho(P_t/P_{t-1})$			
	$y' = 0$			$y' > 0$	$y' = 0$			$y' > 0$
Monetary policy	$u_{cm} > 0$	$u_{cm} < 0$	$u_{cm} = 0$	$u_{cm} = 0$	$u_{cm} > 0$	$u_{cm} < 0$	$u_{cm} = 0$	$u_{cm} = 0$
Passive ($\varepsilon_\rho < 1$)	I	D	D	D	I	I	I	I
Active ($\varepsilon_\rho > 1$)	D	I	D	I	D	D	D	D

Notes: The notation is as in Table 1. The results shown in the table correspond to small deviations of u_{cm}, y', and $\varepsilon_\rho - 1$ from zero. For a more general characterization of equilibrium, see footnotes 7, 8, and 9.

the paper in a direct and transparent way. However, most of the literature devoted to evaluating the stabilizing properties of interest-rate feedback rules includes as a central theoretical element the presence of nominal rigidities. Consequently, in this section we consider a model with sluggish price adjustment. Following Rotemberg (1982), we introduce price stickiness by assuming that the household-firm unit operates in imperfectly competitive product markets and dislikes changing the price it charges for the goods it produces. The implications of the Rotemberg model for the relation between the stance of monetary policy and aggregate stability are likely to carry over to economic environments with alternative sources of nominal price rigidities. For example, in an unpublished Appendix to this paper (Benhabib et al., 2000b) we show that the equilibrium conditions associated with the model developed here are locally identical to those of a model with Calvo-Yun price staggering. Models with price stickiness are a step closer to the data than flexible-price frameworks in that they allow for a source of nonneutrality of monetary-policy shocks. However, this type of new Keynesian feature per se is not enough to overcome a number of well-known empirical shortcomings of standard optimizing macroeconomic models. One limitation that has attracted some attention recently is the lack of inflation persistence. Later in the paper, we seek to introduce a low-frequency component in the nominal interest rate and inflation by introducing lags in the interest-rate feedback rule.

A. *The Economic Environment*

The Household-Firm Unit.—Assume an economy populated by a continuum of household-firm units indexed by j, each of which produces a differentiated good Y^j and faces a demand function $Y^d d(P^j/P)$, where Y^d denotes the level of aggregate demand, P^j the price firm j charges for its output, and P the aggregate price level. Such a demand function can be derived by assuming that households have preferences over a composite good that is produced from differentiated intermediate goods via a Dixit-Stiglitz production function. The function $d(\cdot)$ is assumed to satisfy $d(1) = 1$ and $d'(1) < -1$. The restriction imposed on $d'(1)$ is necessary for the firm's problem to be well defined in a symmetric equilibrium. The production of good j is assumed to take real money balances, m^{pj}, as the only input

$$Y^j = y(m^{pj}),$$

where $y(\cdot)$ satisfies Assumption 2.

The household's lifetime utility function is assumed to be of the form

$$(24)\quad U^j = \int_0^\infty e^{-rt}\left[u(c^j, m^{npj}) - \frac{\gamma}{2}\left(\frac{\dot{P}^j}{P^j} - \pi^*\right)^2\right] dt,$$

where c^j denotes consumption of the composite good by household j, $m^{npj} \equiv M^{npj}/P$ denotes real money balances held by household j for nonproductive purposes, M^{npj} denotes nominal money balances, and $\pi^* > -r$ denotes the steady-state inflation rate. The utility function $u(\cdot, \cdot)$ satisfies Assumption 1, and the parameter γ, measuring the degree to which household-firm units dislike to deviate in their price-setting behavior from the long-run level of aggregate price inflation, is positive. The household's instant budget constraint and no-Ponzi-game restriction are

$$(25)\quad \dot{a}^j = (R - \pi)a^j - R(m^{npj} + m^{pj}) + \frac{P^j}{P} y(m^{pj}) - c^j - \tau$$

and

$$(26)\quad \lim_{t\to\infty} e^{-\int_0^t [R(s) - \pi(s)]\, ds} a^j(t) \geq 0.$$

In addition, firms are subject to the constraint that given the price they charge, their sales are demand determined

$$(27)\quad y(m^{pj}) = Y^d d\left(\frac{P^j}{P}\right).$$

The household chooses sequences for c^j, m^{npj}, m^{pj}, $P^j \geq 0$, and a^j so as to maximize (24) subject to (25)–(27) taking as given $a^j(0)$, $P^j(0)$, and the time paths of τ, R, Y^d, and P. The Hamiltonian of the household's optimization problem takes the form

$$e^{-rt}\Bigg\{ u(c^j, m^{npj}) - \frac{\gamma}{2}\left(\frac{\dot{P}^j}{P^j} - \pi^*\right)^2 + \lambda^j\Bigg[(R - \pi)a^j - R(m^{npj} + m^{pj}) + \frac{P^j}{P} y(m^{pj}) - c^j - \tau - \dot{a}^j\Bigg] + \mu^j\left[Y^d d\left(\frac{P^j}{P}\right) - y(m^{pj})\right]\Bigg\}.$$

The first-order conditions associated with c^j, m^{npj}, m^{pj}, a^j, and P^j and the transversality condition are, respectively,

$$(28)\quad u_c(c^j, m^{npj}) = \lambda^j$$

$$(29)\quad u_m(c^j, m^{npj}) = \lambda^j R$$

$$(30)\quad \lambda^j\left[\frac{P^j}{P} y'(m^{pj}) - R\right] = \mu^j y'(m^{pj})$$

$$(31)\quad \dot{\lambda}^j = \lambda^j(r + \pi - R)$$

$$(32)\quad \lambda^j \frac{P^j}{P} y(m^{pj}) + \mu^j \frac{P^j}{P} Y^d d'\left(\frac{P^j}{P}\right) = \gamma r(\pi^j - \pi^*) - \gamma\dot{\pi}^j$$

$$(33)\quad \lim_{t\to\infty} e^{-\int_0^t [R(s) - \pi(s)]\, ds} a^j(t) = 0,$$

where $\pi^j \equiv \dot{P}^j/P^j$. Combining equations (28) and (29), the demand for real balances for non-production purposes can be expressed as

$$(34)\quad m^{npj} = m^{np}(c^j, R),$$

which by Assumption 1 is increasing in c^j and decreasing in R.

Equilibrium.—In a symmetric equilibrium all household-firm units choose identical sequences for consumption, asset holdings, and prices. Thus, $c^j = c$, $m^{pj} = m^p$, $m^{npj} = m^{np}$, $a^j = a$, $P^j = P$, $\lambda^j = \lambda$, $\mu^j = \mu$, and $\pi^j = \pi$. In addition, the goods markets clear and the no-Ponzi-game restriction holds with equality, that is, equations (8) and (16) are part of the equilibrium conditions. Using (11), (16), and (34) to eliminate m^{np}, c, and R in (28) yields

$$(35)\quad u_c(y(m^p), m^{np}(y(m^p), \rho(\pi))) = \lambda.$$

Equation (35) together with Assumption 1 implies that m^p can be expressed as a function of π and λ that is decreasing in λ and decreasing (increasing) in π if $u_{cm} > 0$ (<0). Formally,[12]

$$(36)\quad m^p = m^p(\lambda, \pi),$$

[12] Differentiating equation (35), it follows that $m^p_\lambda = [u_{mm} - (u_m/u_c)u_{cm}]/[y'(u_{cc}u_{mm} - u^2_{cm})]$. The concavity of the instant utility function and the normality of consumption imply, respectively, that the denominator of this expression is positive and the numerator negative. Also, $m^p_\pi = -m^p_\lambda u_{cm} m^{np}_R \rho'$, which is of the opposite sign of u_{cm}.

where $m^p_\lambda < 0$, $m^p_\pi u_{cm} < 0$ if $u_{cm} \neq 0$, and $m^p_\pi = 0$ if $u_{cm} = 0$. Let $\eta \equiv d'(1) < -1$ denote the equilibrium price elasticity of the demand function faced by the individual firm. Using (11), (16), (30), and (36) to eliminate m^p, m^{np}, μ, R, and c from equations (8), (12), (14), (31), and (32) yields

$$\dot{\lambda} = \lambda[r + \pi - \rho(\pi)] \tag{37}$$

$$\gamma\dot{\pi} = \gamma r(\pi - \pi^*) - y(m^p(\lambda, \pi)) \times \lambda\left[1 + \eta\left(1 - \frac{\rho(\pi)}{y'(m^p(\pi, \lambda))}\right)\right] \tag{38}$$

$$\dot{a} = [\rho(\pi) - \pi]a - \rho(\pi) \times [m^{np}(y(m^p(\lambda, \pi)), \rho(\pi)) + m^p(\lambda, \pi)] - \tau \tag{39}$$

$$0 = \lim_{t\to\infty} e^{-\int_0^t [\rho(\pi) - \pi]\, ds} a(t) \tag{40}$$

$$\tau = -\rho(\pi)[m^{np}(y(m^p(\lambda, \pi)), \rho(\pi)) + m^p(\lambda, \pi)] + \alpha a. \tag{41}$$

Definition 3 (Perfect-Foresight Equilibrium in the Sticky-Price Economy): In the sticky-price economy, a perfect-foresight equilibrium is a set of sequences $\{\lambda, \pi, \tau, a\}$ satisfying (37)–(40) and either (15) if the fiscal regime is non-Ricardian or (41) if the fiscal regime is Ricardian, given $a(0)$.

Given the equilibrium sequences $\{\lambda, \pi, \tau, a\}$, the corresponding equilibrium sequences $\{c, m^{np}, m^p, R\}$ are uniquely determined by (11), (16), (34), and (36).

B. *Determinacy of Equilibrium Under Ricardian Fiscal Policy*

In this case, the equilibrium conditions include equation (41). Given a pair of sequences $\{\pi, \lambda\}$, equations (39) and (41) can be used to construct time paths for a and τ. Because the fiscal policy is Ricardian, the sequences $\{\pi, a\}$ satisfy the transversality condition (40). Thus any pair of sequences $\{\lambda, \pi\}$ satisfying (37) and (38) can be supported as a perfect-foresight equilibrium.

Perfect-Foresight Equilibria Converging to the Steady State.—Consider first perfect-foresight equilibria in which $\{\lambda, \pi\}$ converge to a steady-state $\{\lambda^*, \pi^*\}$. The steady-state values λ^* and π^* are defined as constant values of λ and π that solve (37) and (38). Thus, π^* is a solution to $r + \pi^* = \rho(\pi^*)$, which by assumption exists though need not be unique. Given a π^*, the steady-state value of real balances for production purposes, m^{p*}, is given by the solution to $y'(m^{p*}) = \eta/(1 + \eta)R^*$, where $R^* = \rho(\pi^*)$ is the steady-state value of the nominal interest rate. By Assumption 2, m^{p*} exists and is positive and unique for a given π^*. Finally, λ^* is given by $\lambda^* = u_c(c^*, m^{np}(c^*, R^*)) > 0$, where $c^* = y(m^{p*})$ denotes the steady-state level of consumption. In a neighborhood around (λ^*, π^*), the equilibrium paths of λ and π can be approximated by the solutions to the following linearization of (37) and (38) around $\{\lambda^*, \pi^*\}$

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\pi} \end{pmatrix} = A\begin{pmatrix} \lambda - \lambda^* \\ \pi - \pi^* \end{pmatrix}, \tag{42}$$

where

$$A = \begin{bmatrix} 0 & u_c(1 - \rho') \\ A_{21} & A_{22} \end{bmatrix}$$

$$A_{21} = -\frac{u_c c^* \eta R^* y'' m^p_\lambda}{\gamma y'^2} > 0$$

$$A_{22} = r + \frac{u_c c^* \eta}{\gamma}\left[\frac{\rho'}{y'} - \frac{R^*}{y'^2} y'' m^p_\pi\right].$$

If at the particular steady state considered monetary policy is passive ($\rho'(\pi^*) < 1$), the determinant of A, given by $-A_{21}u_c(1 - \rho')$, is negative, implying that A has one positive real root and one negative real root. Since both λ and π are jump variables, it follows that there exists a neighborhood around the steady state such that for any initial $\lambda(0)$ there exists a $\pi(0)$ in that neighborhood such that the trajectories of λ and π implied by (42) will converge asymptotically to the steady state. The following proposition summarizes this result.

PROPOSITION 5: *If fiscal policy is Ricardian and monetary policy is passive* ($\rho'(\pi^*) < 1$), *then there exists a continuum of perfect-foresight equilibria in which* π *and* λ *converge asymptotically to the steady-state* (π^*, λ^*).

Under active monetary policy ($\rho'(\pi^*) > 1$) the determinant of A is positive and hence the real parts of its eigenvalues have the same sign. If the trace of A, given by A_{22}, is negative, then the real parts of the roots are negative, which implies that near the steady state there exists an infinite number of perfect-foresight equilibria converging to the steady state. If, on the other hand, the trace of A is positive, both eigenvalues have positive real parts, and therefore the only perfect-foresight equilibrium converging to the steady state is the steady state itself. We formally state these results in the following proposition.

PROPOSITION 6: *If fiscal policy is Ricardian and monetary policy is active* ($\rho'(\pi^*) > 1$), *then, if* $A_{22} > 0$ (<0), *there exists a unique* (*a continuum of*) *perfect-foresight equilibria in which* π *and* λ *converge to the steady-state* (π^*, λ^*).

To illustrate that the equilibrium can be either determinate or indeterminate under active monetary policy consider the simple case that the instant utility function is separable in consumption and money and logarithmic in consumption, so that $u_c c^* = 1$. In this case, the trace of A is given by[13]

$$\text{trace}(A) = r + \frac{(1+\eta)\rho'}{\gamma R^*}. \tag{43}$$

Let $\bar{\rho}' \equiv -[(rR^*\gamma)/(1+\eta)]$ denote the value of ρ' at which the trace vanishes. Clearly, $\bar{\rho}'$ may be greater or less than one. If $\bar{\rho}' \leq 1$, then the equilibrium is indeterminate for any active monetary policy. We highlight this result in the following corollary.

COROLLARY 1: *Suppose fiscal policy is Ricardian and preferences are separable in consumption and real balances and logarithmic in consumption. If* $\bar{\rho}' \equiv -\frac{rR^*\gamma}{1} + \eta$ *is less than or equal to one, then there exists a continuum of perfect-foresight equilibria in which* π *and* λ *converge to the steady-state* (π^*, λ^*) *for any active monetary policy. The indeterminacy is of order two.*

If $\bar{\rho}' > 1$, then for values of $\rho' \in (1, \bar{\rho}')$ the trace of A is positive, and the only equilibrium paths $\{\lambda, \pi\}$ converging to the steady state are ones in which λ and π are constant and equal to their steady-state values. For values of $\rho' > \bar{\rho}'$ the trace of A is negative and the perfect-foresight equilibrium is indeterminate.[14]

To facilitate comparison to recent studies on the macroeconomic effects of alternative interest-rate feedback rules in environments with sluggish price adjustment, continue to assume that the instant utility function is loglinear in consumption, for this is the standard assumption in the existing literature. In this case, equations (37) and (38) are qualitatively equivalent to the IS and New Keynesian aggregate supply equations arising from a Calvo-Yun-type sticky-price model in which money does not enter the production function, such as Woodford (1996), Ben Bernanke and Woodford (1997), or Clarida et al. (1997), with one important exception: in our model the aggregate supply equation features an ambiguous partial derivative of $\dot{\pi}$ with respect to π given by $r + \frac{(1+\eta)\rho'}{\gamma R^*}$ whereas in the models just cited this derivative is unambiguously positive and equal to r. The ambiguity in the sign of the partial derivative of $\dot{\pi}$ with

[13] In deriving this expression we used the facts that when $u_{cm} = 0$, $m^p_\pi = 0$ and that in the steady-state $y' = R^*\eta/(1+\eta)$.

[14] In the context of a discrete-time, flexible-price, cash-in-advance economy with cash and credit goods, Schmitt-Grohé and Uribe (2000) obtain a similar result, namely, the perfect-foresight equilibrium is indeterminate for passive and very active monetary policy and is determinate for moderately active policies. Clarida et al. (1997) also find an indeterminacy region for highly active interest-rate feedback rules in a discrete-time model with Calvo-Yun price staggering, with money not affecting production possibilities, and forward-looking monetary policy. It is noteworthy that in a continuous-time version of the their model, active monetary policy always ensures local determinacy.

respect to π is neither due to the way we introduce price stickiness nor is it due to our assumption that time is continuous. In fact, one can show that in a continuous-time version of the Calvo-Yun model without money in the production function, the partial derivative of $\dot{\pi}$ with respect to π is unambiguously positive and equal to r, just like in its discrete-time counterpart. Rather the difference in the sign of the partial derivative arises because of our assumption that money affects productivity.

If the partial derivative of $\dot{\pi}$ with respect to π is positive, then A_{22} is positive and by Propositions 5 and 6 the equilibrium is locally indeterminate under passive monetary policy and is locally determinate under active monetary policy. In this case our findings coincide with those reported in the studies cited above. On the other hand, if the partial derivative of $\dot{\pi}$ with respect to π is negative, that is, $A_{22} < 0$, then equilibrium is indeterminate not only under passive, but also under active, monetary policy. This result is important because it calls into question the policy recommendation implicit in the analysis of previous papers that active monetary policy is stabilizing. The existing literature has arrived at this conclusion by assuming that the only supply-side effects of nominal shocks are brought about through sluggish adjustment of product prices ignoring direct effects of changes in the opportunity cost of holding money on the productivity of firms.

Periodic Perfect-Foresight Equilibria.—So far we have restricted attention to perfect-foresight equilibria in which $\{\lambda, \pi\}$ converge asymptotically to $\{\lambda^*, \pi^*\}$. We now investigate the existence of perfect-foresight equilibria in which λ and π converge asymptotically to a deterministic cycle. Consider an economy with preferences given by $u(c, m^{np}) = (1 - s)^{-1}c^{1-s} + V(m^{np})$, $s > 0$; technology given by $y(m^p) = (m^p)^\alpha$, $0 < \alpha < 1$; and a smooth interest-rate feedback rule, $\rho(\pi) > 0$, which for π in the neighborhood of π^* takes the form $\rho(\pi) = R^* + a(\pi + r - R^*)$, $a > 0$, $R^* > 0$.[15] Consider the steady-state inflation rate $\pi^* = R^* - r$. In this case the trace of A is given by

$$\text{trace } (A) = r + \frac{\eta a}{\alpha\gamma}\left(\frac{\eta}{1+\eta}\frac{R^*}{\alpha}\right)^{(1-\alpha s)/(\alpha-1)}.$$

Let $\bar{a} \equiv \frac{-r\alpha\gamma}{\eta}(\frac{\eta}{1+\eta}\frac{R^*}{\alpha})^{(1-\alpha s)/(\alpha-1)}$ denote the value of a at which the trace of A is equal to zero. Consider parameter configurations for which $\bar{a} > 1$. As a crosses $\bar{a}$ from below, the real parts of the two complex roots of A change sign from positive to negative. This is the standard case of a Hopf bifurcation, which implies that generically (i.e., if the system is nonlinear), there will exist a family of cycles for a either in a left or in a right neighborhood of $\bar{a}$.[16] Furthermore, if the cycle is to the left of $\bar{a}$ where the steady state is unstable (i.e., the bifurcation point is supercritical), the cycle will be attracting. The implication is that if the bifurcation is supercritical, then there exist values of a less than $\bar{a}$ for which any trajectory $\{\lambda, \pi\}$ that starts out in the neighborhood of $\{\lambda^*, \pi^*\}$ will converge to a cycle, so that the equilibrium is indeterminate. The following proposition provides simple conditions under which a supercritical Hopf bifurcation exists.

PROPOSITION 7: *Consider an economy with preferences given by* $u(c, m^{np}) = (1 - s)^{-1}c^{1-s} + V(m^{np})$, $s > 0$; *technology given by* $y(m^p) = (m^p)^\alpha$, $0 < \alpha < 1$; *and monetary policy given by a smooth interest-rate feedback rule,* $\rho(\pi) > 0$, *which for* π *in the neighborhood of* π^* *takes the form* $\rho(\pi) = R^* + a(\pi + r - R^*)$, $a > 0$, $R^* > 0$. *Let fiscal policy be Ricardian and let the parameter configuration satisfy* $\bar{a} \equiv \frac{-r\alpha\gamma}{\eta}(\frac{\eta}{1+\eta}\frac{R^*}{\alpha})^{(1-\alpha s)/(\alpha-1)} > 1$ *and* $1 < s < 1/\alpha$. *Then there exists an infinite number of active monetary policies satisfying* $a < \bar{a}$ *for each of which the perfect-foresight equilibrium is indeterminate and* π *and* λ *converge asymptotically to a deterministic cycle.*

PROOF:

See the unpublished Appendix to this paper (Benhabib et al., 2000b).

[15] Because a linear rule defined for all possible values of π would yield negative nominal interest rates for some π, we do not require the linear specification to hold globally.

[16] The Hopf bifurcation theorem postulates the existence of a family of cycles, which in the pure linear system pile up at the bifurcation value $\bar{a}$ and create a center: any nonlinearity will spread them out to either a left or a right neighborhood of $\bar{a}$. Generically, the amplitude of the cycle varies continuously with $a - \bar{a}$ and is zero at $a = \bar{a}$.

The determinacy results obtained under Ricardian fiscal policies and active monetary policy are of particular relevance to the current debate on optimal monetary policy. It is often argued (typically in the context of discrete-time models) that under fiscal policies which guarantee the solvency of the government, a moderately active monetary policy, that is, a policy such that $\rho'(\pi^*) > 1$ but below a certain threshold, is stabilizing in the sense that it ensures nominal and real determinacy (Bernanke and Woodford, 1997; Clarida et al., 1997; Rotemberg and Woodford, 1997). However, Propositions 6 and 7 show that even moderately active monetary policies may not eliminate the possibility of real indeterminacy in a sticky-price economy, and Corollary 1 gives sufficient conditions for indeterminacy under any active monetary policy. The reason why the sources of instability stressed in this section have been overlooked in the related literature is twofold. First, existing studies have focused on the limiting case in which the nominal interest rate does not affect the cost of production. Second, the majority of previous studies has focused on the dynamics arising from small fluctuations around a particular steady state that are expected to converge asymptotically to the steady state. Thus, by their very nature, analysis of this type are unable to detect equilibria involving bounded fluctuations converging asymptotically to a limit cycle.

C. *Determinacy of Equilibrium Under Non-Ricardian Fiscal Policy*

Suppose now that the government follows the non-Ricardian fiscal policy described in equation (15), that is, a fiscal policy whereby the time path of real lump-sum taxes is exogenous. Using (15) to replace τ in equation (39) yields

$$\dot{a} = [\rho(\pi) - \pi]a - \rho(\pi) \times [m^{np}(y(m^p(\lambda, \pi)), \rho(\pi)) + m^p(\lambda, \pi)] - \bar{\tau}. \tag{44}$$

Perfect-Foresight Equilibria Converging to the Steady State.—As before, we initially restrict attention to equilibria in which $\{\lambda, \pi\}$ converge to a steady-state (λ^*, π^*). It is clear from equation (44) that sequences $\{\lambda, \pi\}$ that converge to (λ^*, π^*) will in general be associated with sequences for a that grow asymptotically at the rate $\rho(\pi^*) - \pi^* = r$, thus violating the transversality condition (40). As a consequence, equations (40) and (44) impose restrictions on the set of sequences $\{\lambda, \pi\}$ that are consistent with a perfect-foresight equilibrium of the type we are considering. Specifically, only sequences $\{\lambda, \pi\}$ converging to the steady-state (λ^*, π^*) that imply [via equation (44)] a sequence for a that converges to a constant value constitute a perfect-foresight equilibrium. Thus, one can analyze the dynamic properties of the model by restricting attention to a linear approximation of the equilibrium conditions (37), (38), and (44), which can be written as

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\pi} \\ \dot{a} \end{pmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ \boldsymbol{\varepsilon} & r \end{bmatrix} \begin{pmatrix} \lambda - \lambda^* \\ \pi - \pi^* \\ a - a^* \end{pmatrix}, \tag{45}$$

where A is defined in (42) and $\boldsymbol{\varepsilon}$ is a one-by-two vector whose elements are the steady-state derivatives of $R(m^{np} + m^p)$ with respect to λ and π.

Since the Jacobian in (45) is quasi-diagonal, its three eigenvalues are given by the two eigenvalues of the matrix $\mathbf{A}$ and $r > 0$. Because a is the only nonjump variable of the system, there exist multiple equilibria converging to the steady state if and only if both roots of $\mathbf{A}$ have negative real parts. Since—as pointed out above—$\rho' > 1$ is a necessary and sufficient conditions for both eigenvalues of $\mathbf{A}$ to be of the same sign, the possibility of multiple equilibrium paths $\{\lambda, \pi\}$ converging asymptotically to the steady state can only arise under active monetary policy. Under passive monetary policy the matrix $\mathbf{A}$ has exactly one negative eigenvalue, therefore, there exists a unique equilibrium converging to the steady state. Finally, if all eigenvalues of $\mathbf{A}$ have positive real parts, which will be the case if monetary policy is active and A_{22} is positive, there exists no equilibrium converging to the steady state. These results are summarized in the following propositions.

PROPOSITION 8: *If fiscal policy is non-Ricardian and monetary policy is passive* ($\rho'(\pi^*) < 1$), *then there exists a unique perfect-*

foresight equilibrium in which $\{\lambda, \pi\}$ *converge asymptotically to the steady-state* (π^*, λ^*).

PROPOSITION 9: *If fiscal policy is non-Ricardian and monetary policy is active* $(\rho'(\pi^*) > 1)$, *then if* $A_{22} > 0$ (<0), *there exists no* (*a continuum of*) *perfect-foresight equilibria in which* $\{\lambda, \pi\}$ *converge asymptotically to the steady-state* (π^*, λ^*).

The result contained in Proposition 8 is similar to the one obtained in Woodford (1996) in the context of a discrete-time Calvo-Yun-type sticky-price model without money in the production function. What distinguishes our findings from previous studies is the result that the equilibrium may be locally indeterminate under non-Ricardian fiscal policy (Proposition 9). As pointed out above, in a continuous-time version of Woodford (1996), the trace of $\mathbf{A}$ is positive and equal to r, so that at least one eigenvalue of $\mathbf{A}$ is always positive. Thus, in such a model indeterminacy can never arise under non-Ricardian fiscal policy. What precludes indeterminacy in that class of models is the assumption that costs of production are unaffected by movements in the nominal interest rate.

Periodic Perfect-Foresight Equilibria.—In the case that monetary policy is active and both eigenvalues of $\mathbf{A}$ are positive, there may exist bounded equilibria that converge to a stable cycle around the steady state. Note that for the system (37), (38), and (44) the dynamics of $\{\lambda, \pi\}$ are independent of a, and thus the analysis of periodic equilibria of the previous section still applies. For example, under the preference and technology specification of the economy described in Proposition 7, if cycles exist, any initial condition for (λ, π) in the neighborhood of the steady state will converge to a cycle. To assure that a does not explode, however, we must restrict ourselves to a one-dimensional manifold in $\{\lambda, \pi\}$. This follows because while cycles restricted to the $\{\lambda, \pi\}$ plane are attracting, in the three-dimensional space the cycle in $\{\lambda, \pi, a\}$ will have only a two-dimensional stable manifold: initial values of λ and π will have to be chosen to assure that the triple $\{\lambda, \pi, a\}$ converges to the cycle and a remains bounded.

Table 3 summarizes the results of this section. It shows the combinations of fiscal and monetary policies for which the real allocation is indeterminate in the sticky-price model. The last two rows of the table illustrate a central point of the paper, namely that active monetary policy need not guarantee real determinacy.

TABLE 3—REAL INDETERMINACY IN THE STICKY-PRICE MODEL

	Fiscal policy	
Monetary policy	Ricardian	Non-Ricardian
Passive $(\rho'(\pi^*) < 1)$	I	D
Active $(\rho'(\pi^*) > 1)$		
$A_{22} < 0$	I	I
$A_{22} > 0$	I or D	I or NE

Note: The notation is D, determinate; I, indeterminate; NE, no perfect-foresight equilibrium exists.

III. Backward- and Forward-Looking Feedback Rules

The interest-rate feedback rule given in equation (11) can be generalized by allowing the nominal interest rate to depend not only on current, but also either on past or on future, expected rates of inflation. Such a generalization is of interest because the inflation measure to which the central bank responds is likely to involve time averages of either past or forecasted rates of inflation. Indeed, Taylor's (1993) seminal paper emphasizes past averaging by including four quarters of lagged inflation in his specification of the feedback rule. On the other hand, authors such as Orphanides (1997) and Clarida et al. (1998) have argued, on econometric grounds, that central-bank behavior in the major industrialized countries is primarily forward looking.

Consider the following backward-looking feedback rule

$$(46) \quad R = \rho(q\pi + (1 - q)\pi^p); \qquad \rho' > 0;\ q \in [0, 1],$$

where π^p is a weighted average of past rates of inflation and is defined as

$$(47) \quad \pi^p = b \int_{-\infty}^{t} \pi(s)e^{b(s-t)}\, ds; \qquad b > 0.$$

Note that when $q = 1$, the feedback rule given in

(46) collapses to a feedback rule linking the nominal interest rate solely to the current instantaneous rate of inflation, which is the case studied in the previous sections. On the other hand, if $q = 0$, the feedback rule becomes purely backward looking. Similarly, consider a forward-looking feedback rule of the form

$$R = \rho(q\pi + (1-q)\pi^f);$$
$$\rho' > 0;\ q \in [0, 1],$$

where π^f is a weighted average of expected future rates of inflation and is defined as

$$\pi^f = d\int_t^{\infty} \pi(s)e^{-d(s-t)}\,ds; \qquad d > 0.$$

Again, in the extreme case in which $q = 1$, the feedback rule collapses to the purely contemporaneous rule given in (11), while in the case in which $q = 0$, the operating target depends only on expectations of future inflation. In this section, we restrict our analysis to the case $q = 0$, that is, to the case in which the feedback rule is either purely backward looking or purely forward looking. The more general case $q \in (0, 1)$ is studied in an unpublished Appendix to this paper (Benhabib et al., 2000b).

The equilibrium conditions of the flexible-price economy with backward-looking monetary policy are the same as those pertaining to the flexible-price model under a contemporaneous interest-rate feedback rule with the only difference that equation (11) is replaced by equations (46) and (47).

Using equations (9), (10), and (16) to replace m^p, m^{np}, and c in equation (4), λ can be expressed as a function of R, $\lambda = \lambda(R)$. It follows that in equilibrium, the evolution of the nominal interest rate is given by

$$\lambda'(R)\dot{R} = \lambda(R)[r + \pi - R],$$

where

$$\lambda'(R) = [u_{cc}y'm^{p\prime} + u_{cm}(m_c^{np}y'm^{p\prime} + m_R^{np})].$$

Differentiating (47) with respect to time yields

$$\dot{\pi}^p = b(\pi - \pi^p).$$

Using the above two expressions together with the backward-looking feedback rule (46), we obtain the following differential equation describing the equilibrium evolution of π^p:

$$\dot{\pi}^p = A[r + \pi^p - \rho(\pi^p)],$$

where

$$A \equiv \frac{b}{b\rho'\lambda'/\lambda - 1}.$$

Because π^p is a predetermined variable, the equilibrium cannot be locally indeterminate. This is an important difference, from the case of a purely contemporaneous rule, in which multiple equilibria can arise under both active and passive feedback rules. The equilibrium exists locally if $A(1 - \rho') < 0$ and fails to exist locally if $A(1 - \rho') > 0$. The results are summarized in Panel A of Table 4. If the monetary authority puts a large weight on inflation rates observed in the distant past (b small), then a passive monetary-policy stance ($\rho' < 1$) is stabilizing, for the equilibrium exists and is unique, while a more aggressive stance ($\rho' > 1$) is potentially disruptive, as it eliminates all equilibria locally. It is important to note that when the central bank is highly backward looking, the local stability properties of the interest-rate feedback rule are independent of preference and technology parameters. On the other hand, if the central bank relies heavily on recent observations of the inflation rate in setting the nominal interest rate (b large), then preferences and technology parameters play an important role for real stability, as is the case under a purely contemporaneous feedback rule. In particular, an active monetary stance is associated with no equilibrium locally for parameterizations typically used in the related literature ($u_{cm} \geq 0$ and $y' = 0$) but ensures determinacy of equilibrium when firms' costs of production are affected by interest-rate variations ($y' > 0$) or when consumption and real balances are Edgeworth substitutes ($u_{cm} < 0$).

If the interest-rate feedback rule is purely forward looking, then the equilibrium condi-

TABLE 4—BACKWARD- AND FORWARD-LOOKING INTEREST-RATE RULES IN THE FLEXIBLE-PRICE MODEL

(A) Backward-Looking Rules
$R = \rho(\pi^p)$
$\pi^p = b \int_{-\infty}^{t} \pi(s)e^{b(s-t)}\,ds;\ b > 0$

Monetary policy	Nonproductive money ($y' = 0$) $u_{cm} = 0$	Nonproductive money ($y' = 0$) $u_{cm} > 0$	Nonproductive money ($y' = 0$) $u_{cm} < 0$	Productive money ($y' > 0$) $u_{cm} = 0$
Passive ($\rho' < 1$)				
Low weight on past (b large)	D	D	NE	NE
High weight on past (b small)	D	D	D	D
Active ($\rho' > 1$)				
Low weight on past (b large)	NE	NE	D	D
High weight on past (b small)	NE	NE	NE	NE

(B) Forward-Looking Rules
$R = \rho(\pi^f)$
$\pi^f = d \int_{t}^{\infty} \pi(s)e^{d(t-s)}\,ds;\ d > 0$

Monetary policy	Nonproductive money ($y' = 0$) $u_{cm} = 0$	Nonproductive money ($y' = 0$) $u_{cm} > 0$	Nonproductive money ($y' = 0$) $u_{cm} < 0$	Productive money ($y' > 0$) $u_{cm} = 0$
Passive ($\rho' < 1$)				
Low weight on future (d large)	D	I	I	I
High weight on future (d small)	D	D	I	I
Active ($\rho' > 1$)				
Low weight on future (d large)	I	D	D	D
High weight on future (d small)	I	I	D	D

Notes: The notation is D, determinate; I, indeterminate; NE, no equilibrium exists locally. The results shown on the table correspond to small deviations of ρ' from 1.

tions reduce to the following single differential equation in the nonpredetermined variable π^f:

$$\dot{\pi}^f = B[r + \pi^f - \rho(\pi^f)],$$

where

$$B \equiv \frac{d}{d\rho'\lambda'/\lambda + 1}.$$

Because π^f is nonpredetermined, at least one equilibrium always exists locally. The equilibrium is locally indeterminate if $B(1 - \rho') < 0$ and is locally determinate if $B(1 - \rho') > 0$. Panel B of Table 4 provides an overview of the results. We find that, as in the case of purely contemporaneous feedback rules, preference and technology specifications are a key determinant of the stabilizing properties of monetary policy. It is also noteworthy that when the policy maker responds to long-run forecasts of inflation (d small), then an active stance renders the equilibrium indeterminate under the conventional parameterization ($u_{cm} \geq 0$ and $y' = 0$) but guarantees uniqueness of the real allocation under the alternative parameterization that allows for productive effects of money.

Finally, we offer a brief summary of the results arising from introducing forward- and backward-looking feedback rules in economies with nominal rigidities, leaving the details of the analysis to the unpublished Appendix to this paper (Benhabib et al., 2000b). When nominal prices are sticky, the pattern that arises is that if monetary policy is active, the introduction of a backward-looking component in monetary policy makes determinacy more likely, whereas a forward-looking component makes indeterminacy more likely.

IV. Conclusion

Given the usual degree of controversy surrounding macroeconomic policy recommendations, the degree of consensus that has emerged regarding the desirability of active monetary policy (in the spirit of Taylor, 1993) for aggregate stability is remarkable. As testified by a recent conference volume on monetary-policy rules edited by Taylor (1999), researchers have arrived at this conclusion following very different modeling approaches, from nonoptimizing reduced-form models (Andrew Levin et al., 1999) to optimizing dynamic general-equilibrium models (Rotemberg and Woodford, 1999). One important assumption maintained in the existing literature is that firms' costs of production are unaffected by variations in the nominal interest rate.

In this paper, we show that the implications of particular interest-rate feedback rules for aggregate stability depend crucially on the precise way in which money affects aggregate demand and supply. In particular, we find that if firms' productivity is affected by changes in the opportunity cost of holding money, then the result that active monetary policy is stabilizing may not hold. In some of the examples provided in this paper, active monetary policy leads to indeterminacy even if the cost effects of interest-rate variations are arbitrarily small. The results of this paper, therefore, suggest that it is desirable for future research aimed at evaluating the stabilizing properties of alternative monetary-policy rules to take into account the supply-side channel for the transmission of nominal interest-rate variations.

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