Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model

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Objective of the Paper: Within a medium-scale estimated model of the macroeconomy

1. characterize the optimal inflation target

2. characterize optimal monetary stabilization policy

3. characterize implementation of optimal monetary policy

A medium-scale macroeconomic model
(Altig et al., 2005, with minor differences)

- Nominal Frictions:

  1. Sticky product prices

     (Calvo-Yun without indexation)

  2. Sticky nominal wages
     (Calvo-Yun with indexation to lagged price inflation and long-run growth)

  3. Cash-in-advance constraint on wages

     $$m_t^f \geq \nu w_t h_t^d$$

  4. Money demand by households

     Transaction costs: $c_t[1 + \ell(c_t/m_t^h)]$

- Real Rigidities:

1. Monopolistically competitive product markets:

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t,$$

2. Monopolistically competitive labor markets:

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d$$

3. Habit persistence in consumption

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^j - bc_{t-1}^j, h_t^j)$$

4. Investment adjustment costs

$$k_{t+1} = (1-\delta)k_t + i_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right]$$

5. Variable capacity utilization

$$\cdots + \Upsilon_t^{-1}[i_t + a(u_t)k_t] + \ldots$$

- Government Policy Objectives and Instruments

  1. Ramsey optimal stabilization policy

  2. Nominal interest rate implements the monetary policy

  3. Lump-taxes balance the budget

Complete Set of Equilibrium Conditions

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right]$$

$$U_c(t) - b\beta E_t U_c(t+1) = \lambda_t[1 + \ell(v_t) + v_t \ell'(v_t)]$$

$$q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^k u_{t+1} - \frac{a(u_{t+1})}{\Upsilon_{t+1}} + q_{t+1}(1-\delta)\right]$$

$$\begin{aligned}\Upsilon_t^{-1}\lambda_t \;&=\; \lambda_t q_t\left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right)\mathcal{S}'\left(\frac{i_t}{i_{t-1}}\right)\right] \\ &\quad + \beta E_t \lambda_{t+1} q_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t}\right)\end{aligned}$$

$$v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}$$

$$r_t^k = \Upsilon_t^{-1} a'(u_t)$$

$$\begin{aligned}f_t^1 \;&=\; \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t \lambda_t \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d \\ &\quad + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1,\end{aligned}$$

$$f_t^2 = -U_{ht} \left(\frac{w_t}{\tilde{w}_t} \right)^{\tilde{\eta}} h_t^d + \tilde{\alpha} \beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*} \pi_t)^{\tilde{\chi}}} \right)^{\tilde{\eta}} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\tilde{\eta}} f_{t+1}^2$$

$$f_t^1 = f_t^2$$

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$

$$y_t = c_t [1 + \ell(v_t)] + g_t + \Upsilon_t^{-1} [i_t + a(u_t) k_t]$$

$$x_t^1 = \frac{y_t \mathrm{mc}_t}{\tilde{p}_t^{1+\eta}} + \alpha \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_{t+1}}{\tilde{p}_t} \right)^{1+\eta} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}} \right)^{-\eta} x_{t+1}^1$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_{t+1}}{\tilde{p}_t} \right)^{\eta} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}} \right)^{1-\eta} x_{t+1}^2$$

$$\eta x_t^1 = (\eta - 1) x_t^2$$

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)} + (1 - \alpha) \tilde{p}_t^{1-\eta}$$

$$F(u_t k_t, z_t h_t^d) - \psi z_t^* = s_t y_t$$

$$s_t = (1 - \alpha) \tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^{\chi}} \right)^{\eta} s_{t-1}$$

$$\mathsf{mc}_t z_t F_2(u_t k_t, z_t h_t^d) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]$$

$$\mathsf{mc}_t F_1(u_t k_t, z_t h_t^d) = r_t^k$$

$$h_t = \tilde{s}_t h_t^d$$

$$\tilde{s}_t = (1 - \tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}} \left(\frac{\pi_t}{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{s}_{t-1}$$

$$w_t^{1-\tilde{\eta}} = (1 - \tilde{\alpha}) \tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}$$

Long-run Policy Tradeoff

- Price stickiness distortion calls for price stability:

$$\text{inflation} = 0\%$$

- Money demand distortion calls for Friedman rule:

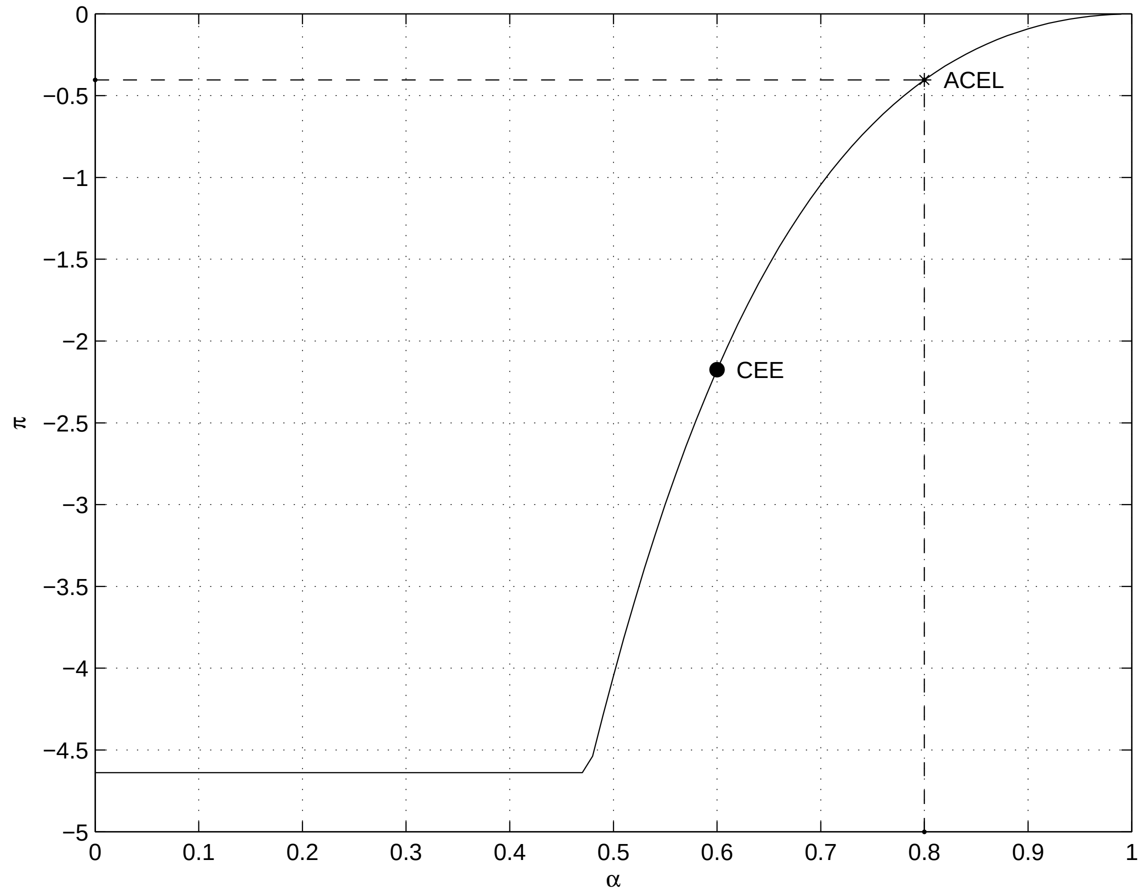
$$\text{Nominal interest rate} = 0\ \%$$

- Wage stickiness distortion plays no role because wages are indexed

Selected Structural Parameters

Parameter	Value	Description
α	0.8	Fraction of firms with nonreoptimized price
$\tilde{\alpha}$	0.69	Fraction of labor markets with nonreoptimized wag
χ	0	Degree of price indexation
$\tilde{\chi}$	1	Degree of wage indexation
ν	0.6	Fraction of wage bill subject to a CIA constraint
β	$1.03^{1/4}$	Subjective discount factor (quarterly)
μ_z^*	$1.018^{1/4}$	Quarterly growth rate of output
θ	0.36	Share of capital in value added
δ	0.025	Depreciation rate (quarterly)
η	6	Price-elasticity of demand for a specific good variet
$\tilde{\eta}$	21	Wage-elasticity of demand for a specific labor varie
b	0.69	Degree of habit persistence

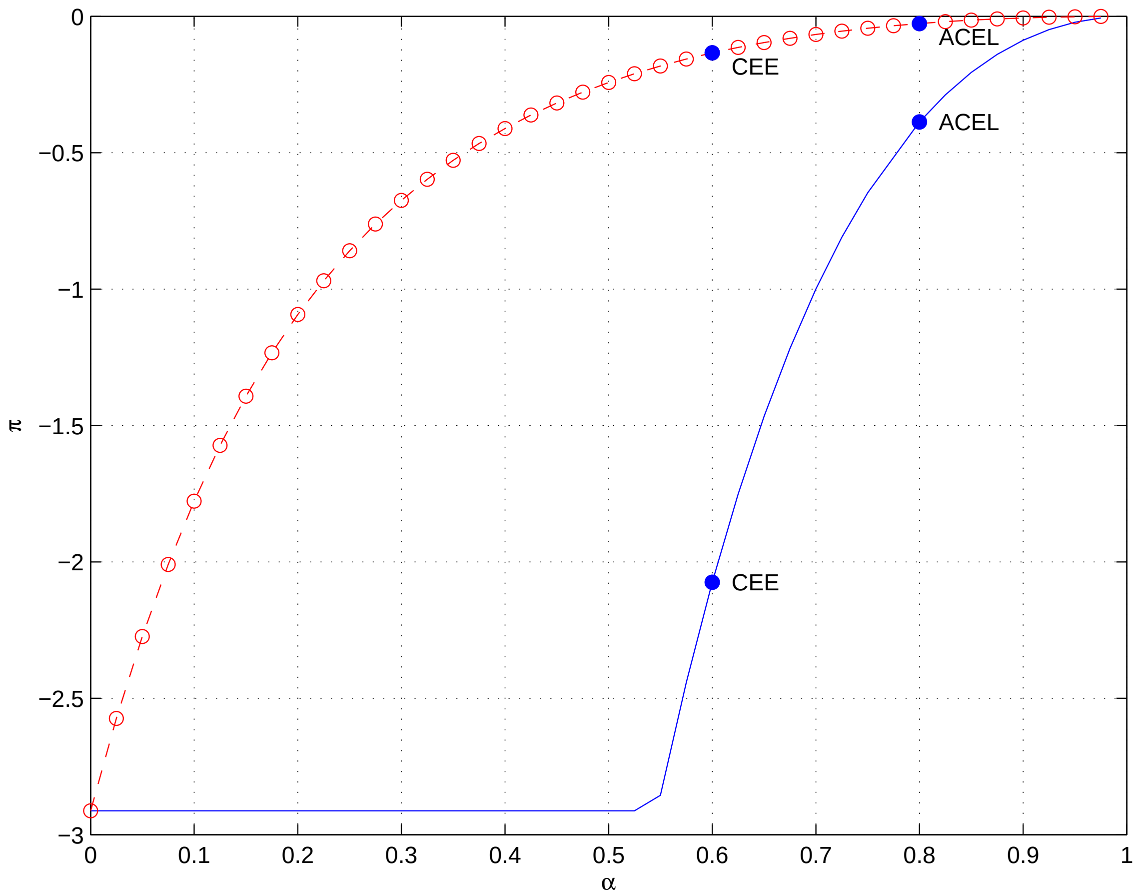
Degree of Price Stickiness and the Optimal Rate of Inflation



- Macro evidence: $0.5 \leq \alpha \leq 0.85$

- Micro evidence: $\alpha \approx 0.3$

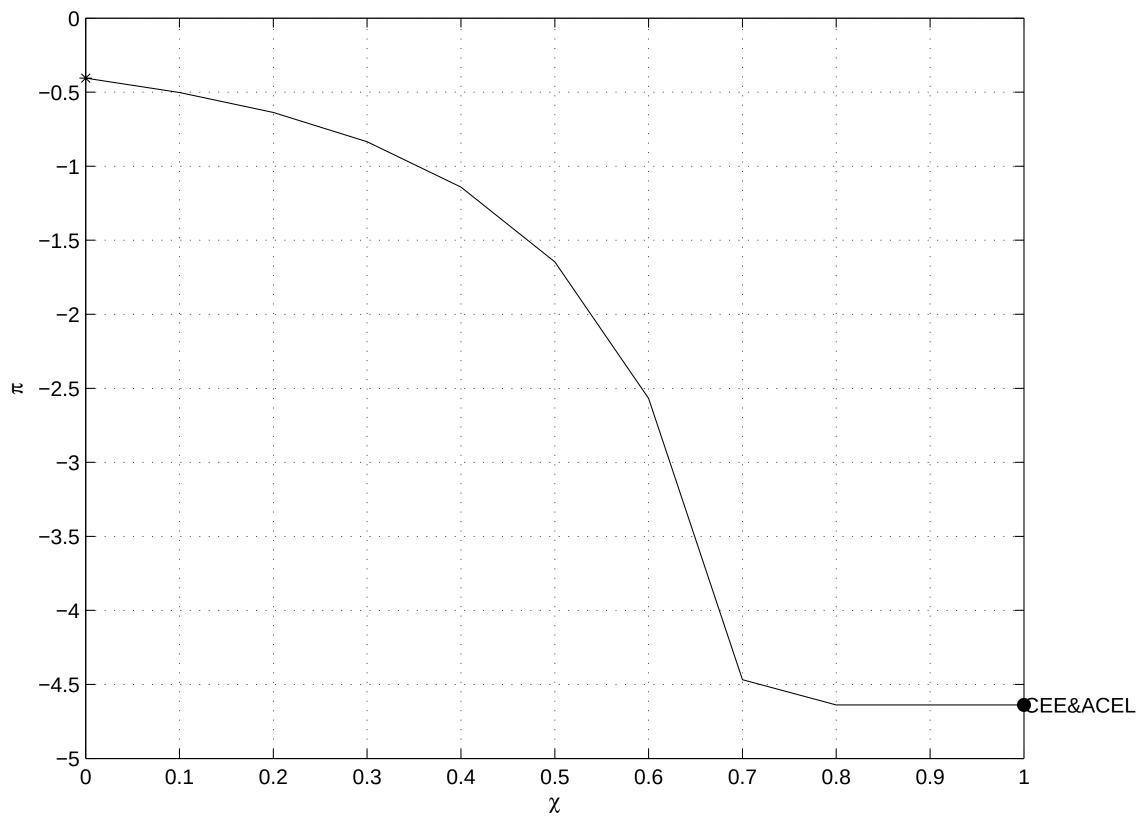
Optimal Distortionary Taxation, Price Stickiness, and the Optimal Rate of Inflation



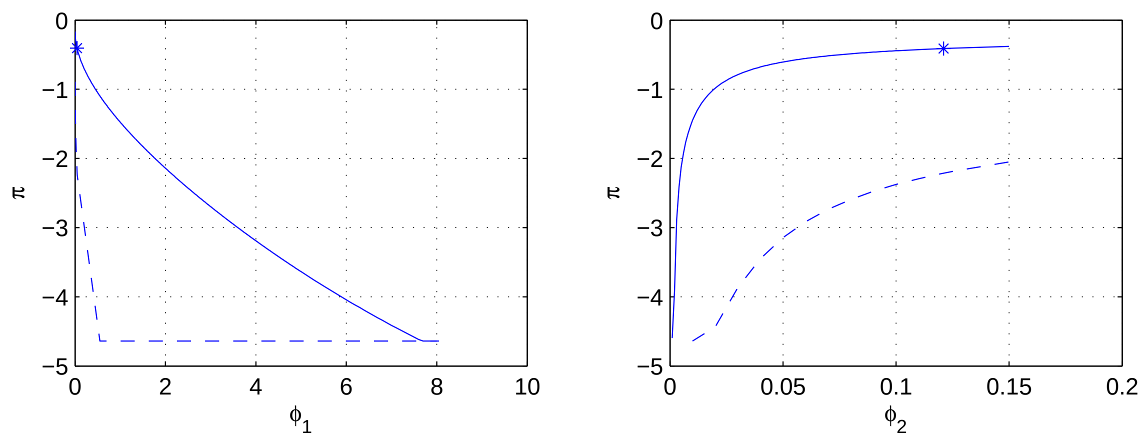
_____ Lump-Sum Taxes -o-o- Optimal Distortionary Taxes

- When lump-sum taxes are unavailable, and instead the government must set distortionary taxes optimally, price stability emerges as a robust Ramsey outcome.

Degree of Price Indexation and the Optimal Rate of Inflation



Money Demand Parameters and the Optimal Rate of Inflation



Money demand by HH:

$$m_t^h = c_t \sqrt{\frac{\phi_1}{\phi_2 + (1 - R_t^{-1})}}$$

Sources of Uncertainty

1. Permanent investment-specific technology shocks

$$\widehat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon}\widehat{\mu}_{\Upsilon,t-1} + \epsilon_{\mu_\Upsilon,t}$$

2. Permanent neutral technology shocks

$$\widehat{\mu}_{z,t} = \rho_{\mu_z}\widehat{\mu}_{z,t-1} + \epsilon_{\mu_z,t}$$

3. Temporary government purchases shocks

$$\ln\left(\frac{\bar{g}_t}{\bar{g}}\right) = \rho_{\bar{g}} \ln\left(\frac{\bar{g}_{t-1}}{\bar{g}}\right) + \epsilon_{\bar{g},t}$$

Fraction of variance explained by exogenous disturbances in the data

Variable	$\mu_{\Upsilon,t}$	$\mu_{z,t}$
Output	0.15	0.13
Consumption	0.12	0.21
Investment	0.15	0.09
Interest Rate	0.16	0.04
Inflation	0.12	0.16
Hours	0.16	0.13

Source: ACEL (2005)

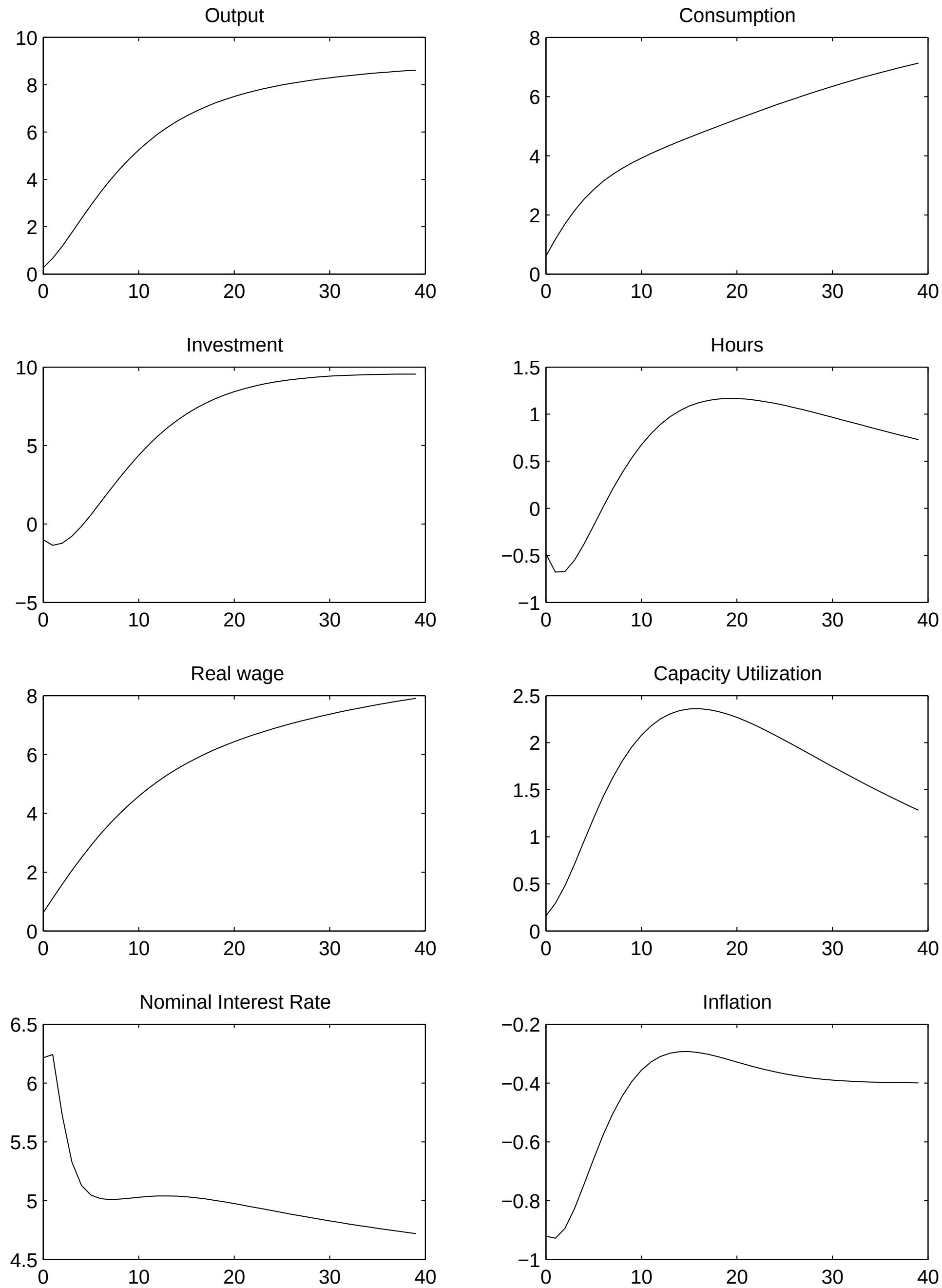
Fraction of variance explained by each of the three exogenous disturbances in the Ramsey equilibrium

Variable	$\mu_{\Upsilon,t}$	$\mu_{z,t}$	g_t
$\ln y_t/y_{t-1}$	0.11	0.44	0.45
$\ln c_t/c_{t-1}$	0.10	0.80	0.10
$\ln I_t/I_{t-1}$	0.61	0.33	0.06
$\ln R_t$	0.21	0.62	0.17
$\ln \pi_t$	0.13	0.83	0.04
$\ln \pi_t^W$	0.37	0.63	0.00
$\ln h_t^d$	0.47	0.44	0.09

Ramsey Optimal Stabilization Policy

Variable	Standard Deviation (in percentage points per year)	
	Baseline	High $\tilde{\alpha}$
Nominal Interest Rate	0.4	0.4
Price Inflation	0.1	0.4
Wage Inflation	1.2	1.0
Output Growth	0.8	0.8
Consumption Growth	0.5	0.5
Investment Growth	1.3	1.5

Ramsey Response To A Neutral Productivity Shock



Note: The size of the initial innovation to the neutral technology shock is one percent, $\ln(\mu_{z,0}/\mu_z) = 1\%$. The nominal interest rate and the inflation rate are expressed in levels in percent per year. Output, wages, investment, and consumption are expressed in cumulative growth rates in percent. Hours and capacity utilization are expressed in percentage deviations from their respective steady-state values.

Implementing the Ramsey equilibrium with an interest rate rule

$$\widehat{R}_t = \alpha_\pi \widehat{\pi}_t + \alpha_W \widehat{\pi}_t^W + \alpha_y \Delta \ln y_t + \alpha_R \widehat{R}_{t-1}$$

Pick the 4 policy coefficients, $(\alpha_\pi, \alpha_W, \alpha_y, \alpha_R)$, so as to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t)$$

Result:

α_π	5.0
α_W	1.6
α_y	-0.1
α_R	0.4

Welfare Cost Measure

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^a - bc_{t-1}^a), h_t^a) = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda^c)(c_t^r - bc_{t-1}^r), h_t^r)$$

The Optimal Operational Rule

	Baseline	Taylor Rule
Policy Coefficients		
α_π	5.0	1.5
α_{π^W}	1.6	–
α_y	-0.1	0.5
α_R	0.4	–
Welfare Costs		
in percent of c_t	0.001	0.14
in 2006 dollars	\$0.23	\$41.81

- The welfare cost of the optimal rule is almost zero.
- The optimal response to output is zero.
- Welfare gains from interest rate smoothing are negligible.

The Wage Phillips Curve

ACEL model:

$$\widehat{\pi}_t^W - \widehat{\pi}_{t-1} = \beta(E_t \widehat{\pi}_{t+1}^W - \widehat{\pi}_t) - \gamma \widehat{\mu}_t^W$$

This paper:

$$\widehat{\pi}_t^W - \widehat{\pi}_{t-1} = \beta(E_t \widehat{\pi}_{t+1}^W - \widehat{\pi}_t) - (1 + \tilde{\eta})\gamma \widehat{\mu}_t^W$$

$$\gamma = \left(\frac{1}{1 + \tilde{\eta}}\right)\left(\frac{(1 - \tilde{\alpha})(1 - \tilde{\alpha}\beta)}{\tilde{\alpha}}\right)$$

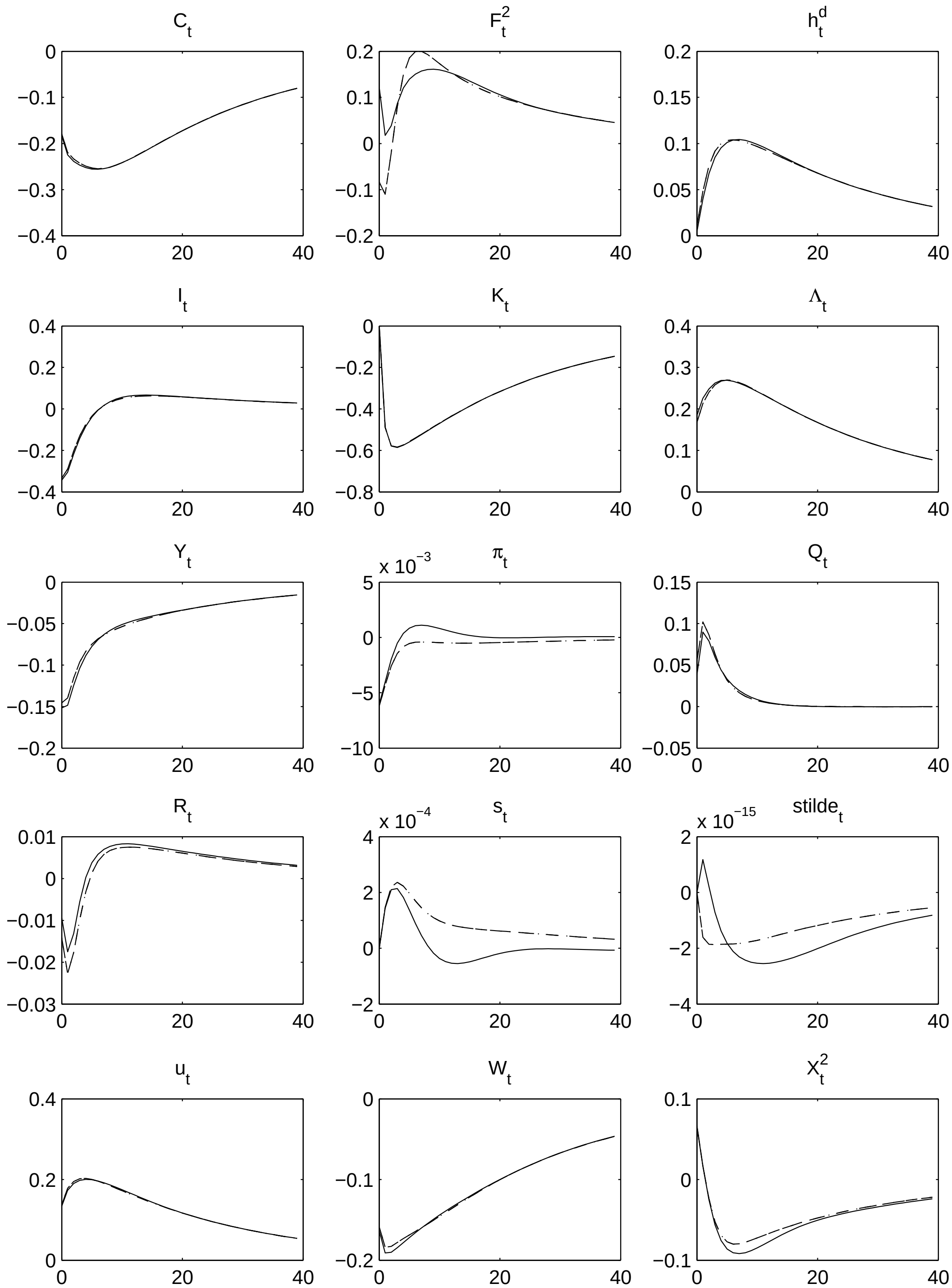
Higher wage stickiness: $\tilde{\alpha} = 0.9$

Optimal Rule coefficients:

α_π	0.4
α_W	1.9
α_y	0.1
α_R	2.3

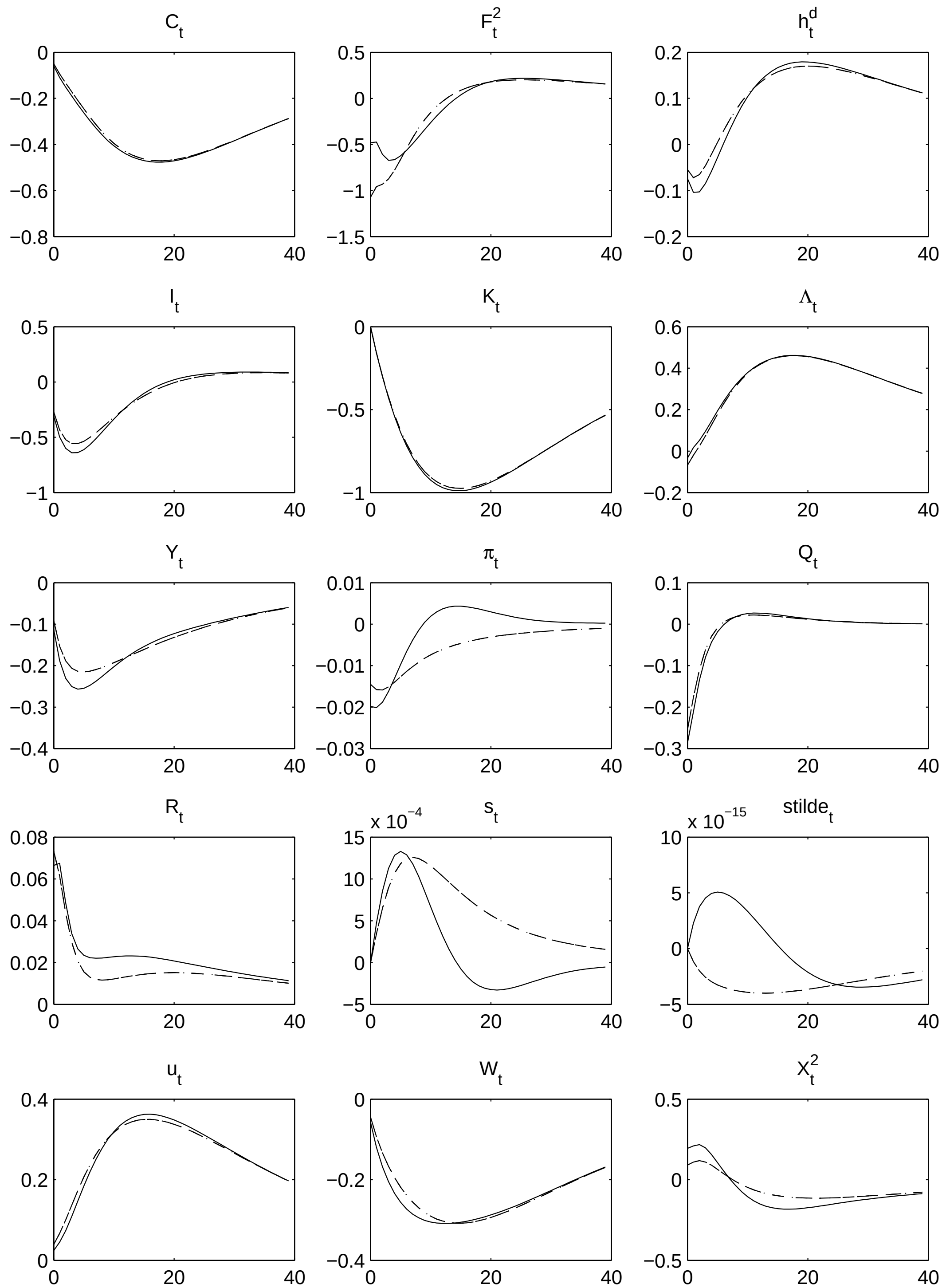
Welfare costs in percent of c_t: 0.008
in 2006 dollars: $2.50

Ramsey And Optimized Responses To An Investment-Specific Productivity Shock



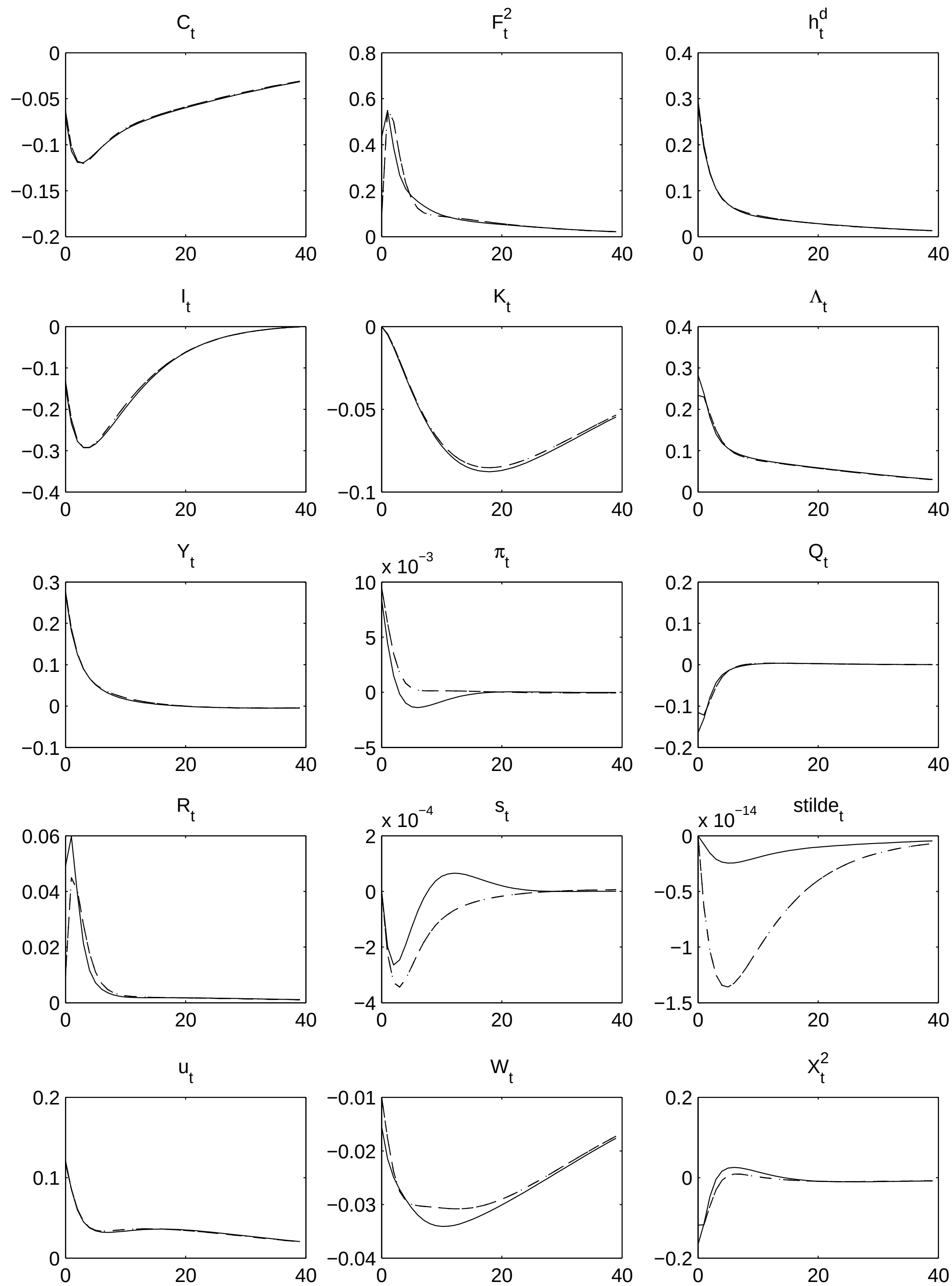
___ Ramsey Policy   _ _ _ _ Optimized Rule

Ramsey And Optimized Responses To A Neutral Productivity Shock



__ Ramsey Policy _ _ _ _ Optimized Rule

Ramsey And Optimized Responses To A Government Purchases Shock



__ Ramsey Policy   _ _ _ _ Optimized Rule

Conclusions

- This paper characterizes optimal monetary policy in a rich DSGE that mimics well U.S. postwar business cycles.

- The optimal rate of inflation is negative. Thus, it is puzzling that inflation targets in inflation targeting countries are positive.

- The zero bound on nominal interest rates does not justify positive inflation targets.

- Under the optimal policy the variance of inflation is near zero, whereas the variance of wage inflation is about 1 percentage point.

- The Ramsey equilibrium is well approximated by a simple interest rate feedback rule that is active, moderately inertial, and does not respond to output growth.