Optimal Bank Reserve Remuneration and Capital Control Policy*

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Abstract

A central prediction of open economy models with a pecuniary externality due to a collateral constraint is that the unregulated economy overborrows relative to what occurs under optimal capital controls. A maintained assumption in this literature is that households borrow directly from foreign lenders. This paper shows that in a more realistic setting in which foreign lending to households is intermediated by domestic banks and in which the government has access to capital controls and interest on bank reserves, the unregulated economy underborrows. Under optimal policy, the central bank injects reserves during recessions. In this way, when the collateral constraint binds, the central bank uncouples household deleveraging from economy-wide deleveraging, which results in a higher average level of external debt. The paper documents that during the 2007-2009 global financial crisis the lending spread in emerging and developed economies displayed a muted response. This fact is consistent with a decline in the demand rather than in the supply of loans and gives credence to models in which the collateral constraint is placed at the level of the nonfinancial sector as opposed to at the level of the bank.

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1 Introduction

A central question in open economy macroeconomics is whether, when left to their own devices, countries overborrow. This question has been largely analyzed in the context of models in which households are subject to a collateral constraint, whereby debt is limited by a fraction of income or the value of an asset (Mendoza, 2002, 2010; Bianchi, 2011; Korinek, 2011; Benigno et al. 2013; Dávila and Korinek, 2018; Jeanne and Korinek, 2019; among others). Collateral constraints of this type create a pecuniary externality because individual agents take as given the prices of objects that they pledge as collateral, but in the aggregate, these prices are determined endogenously. A key result in this literature is that the unregulated economy overborrows from the rest of the world relative to what it would borrow under optimal capital control policy.

A common feature of this class of models is the assumption that private agents can borrow directly from foreign lenders. In reality, however, individual agents seldom borrow directly from foreign lenders. Instead, capital inflows are intermediated by banks operating in domestic markets. An immediate question is whether this simplification has consequences for the main prediction of this class of models. This paper revisits the question of overborrowing in the context of a model that builds on the collateral-constraint framework by adding a bank-intermediation channel. The formulation of the banking channel follows Cúrdia and Woodford (2011).

The paper studies an open economy with a collateral constraint by which household debt is limited by a fraction of income. A banking sector receives deposits from foreign investors and lends them to households. This intermediation activity is costly. Banks can mitigate the cost of originating loans by holding reserves at the central bank. Thus, banks extend loans to households who satisfy the collateral constraint up to a point at which the marginal cost of originating a loan equals the lending spread (the difference between the loan rate and the deposit rate). Similarly, banks hold bank reserves up to a point at which the marginal benefit of holding reserves equals the reserve spread (the difference between the interest rate on reserves and the deposit rate).

As in the related literature on macroprudential policy in open-economy models with collateral constraints, the government can impose capital control taxes. With the introduction of a banking sector the interest rate on reserves emerges as an additional policy instrument that the government may use jointly with capital controls to achieve an allocation that improves upon the one associated with the unregulated competitive equilibrium. Thus, relative to the standard overborrowing model, the present environment features an additional friction, bank intermediation, and an additional policy instrument, interest on bank reserves. The fiscal cost (revenue) generated by capital controls and interest payments on bank reserves is assumed to be financed by income taxes (transfers).

The paper shows that in the environment described above, the government can circumvent the banking friction by an appropriate use of the interest rate on reserves. This result extends one derived by Cúrdia and Woodford (2011) in the context of a closed economy to an open economy with collateral constraints. A novel result of the paper is that the optimal interest-on-reserve policy is able to circumvent not only the banking friction but also the collateral constraint friction. In fact, the optimal bank-reserve remuneration policy achieves the first-best allocation. The first-best allocation is the allocation that solves the problem of maximizing household welfare subject to the economy's resource constraint and to a natural debt limit. That is, the first-best allocation is the competitive equilibrium corresponding to an economy without any financial frictions (banks or collateral constraints). In this equilibrium, the central bank floods the market with reserves, households demand no loans, and the interest rate on loans and deposits equals the world interest rate. Furthermore, capital controls are superfluous, as they are unnecessary to achieve the first-best allocation. The unregulated equilibrium displays underborrowing, because external borrowing is limited by the collateral constraint and the banking friction, whereas in the first-best equilibrium external borrowing is limited only by the natural debt limit.

This underborrowing result is a useful point of reference, but of limited practical interest, as it is unrealistic to believe that with a single instrument the government can implement an equilibrium in which all financial frictions are completely neutralized. To motivate more reasonable outcomes, we follow the closed-economy literature and assume that it is costly for the central bank to create bank reserves. We show that with this additional friction, a social planner with access to capital controls and interest on reserves as the policy instruments can achieve only a constrained optimal allocation. Specifically, now capital controls and interest on reserves no longer allow the social planner to circumvent the intermediation friction and the collateral constraint.

The model is calibrated to an emerging economy. The parameters of the non-banking sector are set along the lines of the existing open-economy literature with collateral constraints. The parameters pertaining to the banking sector are estimated by simulated method of moments to match a number of moments of bank reserves, deposits, bank-lending spreads, and bank operating costs observed in emerging markets.

The central result of the paper is that under plausible parameterizations the economy with banks underborrows. That is, the distribution of external debt in the competitive equilibrium in which the government does not remunerate reserves or taxes capital flows lies to the left of the one associated with an equilibrium in which the interest rate on bank reserves and capital control taxes are set optimally.

The intuition behind this result is that bank reserves act as a cushion between household debt and the country's external debt: Essentially, the bank's balance sheet states that external debt equals the sum of household debt and bank reserves. By an appropriate use of interest on reserves, the government can uncouple episodes of household deleveraging (a decline in household debt) from episodes of economy-wide deleveraging (a decline in external debt). Specifically, when the economy suffers a negative shock that causes the household's collateral constraint to bind, the government steps in and provides liquidity in the form of bank reserves. This intervention allows the economy to continue to borrow from abroad despite a binding constraint at the household level. By contrast, in the economy without banks, whenever households are forced to deleverage by a binding collateral constraint, so is the economy as a whole. As a result, under optimal policy in the economy with banks episodes of binding collateral constraints are less disruptive than in the economy without banks. For this reason, in the economy with banks the government can induce an equilibrium with less precautionary saving, and therefore more debt, than in the economy without banks. Put differently, in the economy with banks the role of the government is to generate an equilibrium in which households are not scared of operating close to the collateral limit, whereas in the standard model without banks, the primary role of the government is exactly the opposite, that is, to discourage households from encountering a binding collateral constraint.

The modeling decision of imposing the collateral constraint in the nonbanking private sector as opposed to in the banking sector has consequences for the predicted behavior of the lending spread around financial crises. When the collateral constraint is imposed at the level of the household, as in the present paper, the lending spread tends to change little during financial crises, because a binding collateral constraint represents a decline in the demand for bank credit. By contrast, when the collateral constraint is imposed at the bank level, the lending spread tends to rise, because in this case a financial crisis represents a fall in the supply of credit. The paper documents that during the global financial crisis of 2007 to 2009 the lending spread in emerging and rich countries displayed a subdued response, which is consistent with a formulation in which the collateral constraint is placed at the level of the nonfinancial sector.

A macroprudential instrument closely related to reserve remuneration is reserve requirements. A natural question is whether the equilibrium outcomes the policy maker can attain with one instrument can also be achieved or improved upon with the other. The paper shows that in the proposed open economy model, there is a clear ranking between these two policy tools: bank reserve remuneration strictly welfare dominates reserve requirements. Intuitively, by paying interest on reserves, the central bank controls the price of this component of the bank's asset side, but allows the quantity (reserves) to be determined endogenously. On the other hand, a reserve requirement with no interest on reserves represents a restriction on both the quantity and the price of bank reserves, therefore reducing the set of real allocations that it can support as competitive equilibria.

This paper is related to two strands of literature, one on closed-economy models with a bank channel and one on open-economy models with collateral constraints in the nonfinancial sector. The banking model follows Cúrdia and Woodford (2011). A similar formulation of the banking sector can be found in Eggertsson et al. (2019). Uribe and Yue (2006) introduce bank intermediation along the lines of the model considered in this paper to create a spread between the domestic and the world interest rates. However, their formulation does not contemplate a role for bank reserves. Open economy models with collateral constraints at the household level are studied in Mendoza (2002), Uribe (2006), Korinek (2011), Bianchi (2011), Benigno et al. (2013), Dávila and Korinek (2018), Jeanne and Korinek (2019), and Schmitt-Grohé and Uribe (2021), among others. The present paper builds upon these two bodies of work by combining a banking sector and a collateral constraint at the household level in the context of an open economy. Finally, the paper is related to a class of models in which the collateral constraint is placed on banks rather than on households. Céspedes and Chang (2020) represents an example of this formulation in the context of an open economy.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes optimal macroprudential policy in the economy with costless bank reserve provision. Section 4 characterizes the competitive equilibrium and the constrained optimal allocation in the economy with costly reserve provision. Section 5 presents the calibration and the numerical algorithm used in the quantitative analysis. Section 6 shows that under plausible calibrations the economy underborrows. Section 7 characterizes the optimal macroprudential policy during sudden stops. Section 8 compares reserve remuneration and reserve requirements. Section 9 concludes.

2 The Model

In this section, we present a model of an open economy in which banks serve as intermediaries between foreign investors, who supply funds, domestic households, who demand bank loans, and the domestic government, who operates a reserve facility. The specification of the bank lending channel follows Cúrdia and Woodford (2011). The banking sector is embedded into a standard open economy model with a flow collateral constraint, whereby household debt is limited by a fraction of income, along the lines of Mendoza (2002), Bianchi (2011), and

Korinek (2011). The collateral constraint introduces a pecuniary externality because the relative price of nontradable goods, which determines the value of collateral, is taken as exogenous by individual borrowers but is endogenous to the economy.

2.1 Banks

We assume that the economy has a large number of identical, perfectly competitive financial intermediaries, which we will refer to as banks. Each period, banks issue loans, l_t, hold reserves, r_t, capture deposits, d_t, and distribute dividends, π_t. Banks face intermediation costs, denoted Γ_t. This cost is meant to capture expenses such as those related to loan monitoring and management. The sequential budget constraint of a bank is

$$\pi_t + l_t + r_t + (1 + i^d_{t-1})d_{t-1} + \Gamma_t = (1 + i^l_{t-1})l_{t-1} + (1 + i^r_{t-1})r_{t-1} + d_t, \tag{1}$$

where i^d_{t-1} is the interest rate paid by the bank on deposits held from period $t-1$ to period t, i^l_{t-1} is the interest rate charged by the bank on loans made in period $t-1$ and due in period t, and i^r_{t-1} is the interest rate the bank earns on reserves deposited at the central bank from period $t-1$ to period t.

The dividend policy of banks is assumed to consist in distributing a fraction ω of the beginning-of-period net worth,

$$\pi_t = \omega\left[(1 + i^l_{t-1})l_{t-1} + (1 + i^r_{t-1})r_{t-1} - (1 + i^d_{t-1})d_{t-1}\right]. \tag{2}$$

The intermediation cost is assumed to depend on the volume of loans and bank reserves,

$$\Gamma_t = \Gamma(l_t, r_t). \tag{3}$$

We introduce the following assumptions about this function:

Assumption 1 (Intermediation Cost Function)**.** *The function* $\Gamma(\cdot,\cdot)$ *satisfies: (i)* $\Gamma(\cdot,\cdot) \geq 0$*; (ii)* $\Gamma_l(\cdot,\cdot) \geq 0$ *and* $\Gamma_r(\cdot,\cdot) \leq 0$*; (iii)* $\Gamma_{ll} \geq 0$, $\Gamma_{rr} \geq 0$, *and* $\Gamma_{ll}\Gamma_{rr} - \Gamma_{rl}^2 \geq 0$*; (iv)* $\Gamma(0,\cdot) = \Gamma_r(0,\cdot) = 0$ *and* $\Gamma(l,\cdot) > 0$ *for* $l > 0$*, and (v) there exists a finite level of reserves,* $\bar{r} > 0$*, such that* $\Gamma_r(\cdot, r) = 0$ *for all* $r \geq \bar{r}$*.*

Assumptions (i)-(iii) are standard. In particular, the assumption that the cost function is nondecreasing in loans is meant to capture administrative and default costs of originating bank credit to the private sector, and the assumption that it is nonincreasing in bank reserves is meant to capture that bank reserves contribute to reducing default risk and possible maturity mismatches between bank liabilities and assets. Assumption (iv) is meant to capture the idea that the central bank has zero default risk, so, aside from interest differentials, it is costless for banks to park funds there, in the form of reserves. As will become apparent shortly, this assumption will play a role in determining the optimal interest-on-reserve policy. Assumption (v) is common in models with a formulation of the banking sector of the type studied here (Cúrdia and Woodford, 2011; Eggertsson et al. , 2019). It says that there exists a satiation level of reserves above which reserves cease to lower the intermediation costs of loans.

Using the dividend policy function (2) in period t and period $t+1$ to eliminate, respectively, d_{t-1} and d_t from the bank's sequential budget constraint (1) and combining the resulting expression with the intermediation cost function (3), we can write

$$\frac{\pi_{t+1}}{1+i_t^d} = (1-\omega)\pi_t + \omega\left[\frac{i_t^l - i_t^d}{1+i_t^d} l_t + \frac{i_t^r - i_t^d}{1+i_t^d} r_t - \Gamma(l_t, r_t)\right].$$

For simplicity, following Cúrdia and Woodford (2011), we assume that banks distribute as dividends all of the beginning-of-period net worth, that is, we set $\omega = 1$. We then have that

$$\frac{\pi_{t+1}}{1+i_t^d} = \frac{i_t^l - i_t^d}{1+i_t^d} l_t + \frac{i_t^r - i_t^d}{1+i_t^d} r_t - \Gamma(l_t, r_t). \tag{4}$$

According to this expression, banks distribute at the beginning of period $t+1$ all of the operating profits of period t. Thus, we refer to π_{t+1} as profits or dividends interchangeably.

Banks choose $l_t \geq 0$ and $r_t \geq 0$ to maximize profits, taking as given i_t^l, i_t^d, and i_t^r. By Assumption 1 profits vanish at $l_t = r_t = 0$. Thus, a profit maximizing bank never distributes negative dividends. Profit maximization implies the following first-order conditions with respect to l_t and r_t:

$$\frac{i_t^l - i_t^d}{1+i_t^d} \leq \Gamma_l(l_t, r_t), \quad l_t \geq 0, \quad \left[\frac{i_t^l - i_t^d}{1+i_t^d} - \Gamma_l(l_t, r_t)\right] l_t = 0, \tag{5}$$

and

$$\frac{i_t^r - i_t^d}{1+i_t^d} \leq \Gamma_r(l_t, r_t), \quad r_t \geq 0, \quad \left[\frac{i_t^r - i_t^d}{1+i_t^d} - \Gamma_r(l_t, r_t)\right] r_t = 0. \tag{6}$$

Optimality condition (5) says that when the volume of loans is positive, $l_t > 0$, the marginal net revenue of originating a loan, given by the lending spread $(i_t^l - i_t^d)/(1+i_t^d)$, must equal the marginal cost of originating a loan $\Gamma_l(l_t, r_t)$. Optimality condition (6) has an analogous interpretation.

Since $\Gamma_l(\cdot, \cdot)$ is nonnegative, optimality condition (5) implies that when the volume of loans is positive, the deposit rate is the lower bound of the loan rate. Similarly, because

$\Gamma_r(\cdot,\cdot)$ is nonpositive, optimality condition (6) implies that the deposit rate is the upper bound of the reserve rate.

The sequential budget constraint (1), the dividend rule (2), and the assumption that ω is unity imply that

$$l_t + r_t + \Gamma(l_t, r_t) - d_t = 0.$$

This expression provides the balance sheet of the bank at the end of the period. Deposits are used to fund loans and reserves and to cover intermediation costs.

2.2 Households

Households have preferences for consumption described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{7}$$

where c_t denotes consumption, $\beta \in (0,1)$ is the subjective discount factor, and $u(\cdot)$ is an increasing and strictly concave period utility function. Consumption is a composite of tradable and nontradable goods,

$$c_t = A(c_t^T, c_t^N), \tag{8}$$

where c_t^T and c_t^N denote consumption of tradables and nontradables and $A(\cdot,\cdot)$ is an increasing, concave, and linearly homogeneous aggregator function. Each period, households are endowed with y_t^T units of tradable goods and y_t^N units of nontradable goods, receive dividends π_t from the ownership of banks, pay income taxes at the rate τ_t, and can borrow from banks at the rate i_t^l. Their sequential budget constraint is

$$c_t^T + p_t c_t^N + (1 + i_{t-1}^l) l_{t-1} = (1 - \tau_t)[y_t^T + p_t y_t^N + \pi_t] + l_t, \tag{9}$$

where p_t is the relative price of nontradables in terms of tradables.

Loans face a collateral constraint that depends on the value of income in units of tradable goods as follows,

$$l_t \leq \kappa(y_t^T + p_t y_t^N), \tag{10}$$

where $\kappa > 0$ is a parameter. We use this specification of collateral to be in line with the related literature (e.g., Bianchi, 2011) and for analytical tractability. An alternative plausible but less tractable specification is one in which collateral is proportional to disposable income, including after-tax profits.

Households choose processes c_t, c_t^T, c_t^N, and $l_t \geq 0$ to maximize the lifetime utility function (7) subject to the aggregation technology (8), the sequential budget constraint (9), and the collateral constraint (10), taking as given p_t, i_t^l, π_t, τ_t, y_t^T, and y_t^N. The first-order conditions associated with this problem are

$$u'(A(c_t^T, c_t^N))A_1(c_t^T, c_t^N) = \lambda_t,$$

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t,$$

$$\lambda_t(1-\mu_t) \leq \beta(1+i_t^l)E_t\lambda_{t+1} \quad (= \text{if } l_t > 0),$$

$$\mu_t \geq 0,$$

and

$$\mu_t[\kappa(y_t^T + p_t y_t^N) - l_t] = 0,$$

where $\beta^t\lambda_t$ and $\beta^t\lambda_t\mu_t$ are the Lagrange multipliers associated with the sequential budget constraint (9) and the collateral constraint (10), respectively.

2.3 Foreign Lenders

Banks capture deposits from foreign lenders at the world interest rate i_t^*. Free capital mobility guarantees that

$$i_t^d = i_t^*. \tag{11}$$

2.4 The Government

The government sets the interest rate on reserves, i_t^r, levies income taxes, τ_t, and stands ready to accept any amount of reserves, r_t, offered by banks. The government's budget constraint is

$$\tau_t(y_t^T + p_t y_t^N + \pi_t) + r_t = (1 + i_{t-1}^r)r_{t-1}. \tag{12}$$

We assume that the government does not play Ponzi games.

2.5 Equilibrium

In equilibrium, the market for nontradable goods must clear,

$$c_t^N = y_t^N. \tag{13}$$

The budget constraint of the bank (1), the budget constraint of the household (9), the interest-rate parity condition (11), the budget constraint of the government (12), and the market clearing condition in the nontraded sector (13) imply the following economy-wide resource constraint:

$$c_t^T + (1 + i_{t-1}^*)d_{t-1} = y_t^T - \Gamma(l_t, r_t) + d_t.$$

We are now ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium). *A competitive equilibrium is a set of processes l_t, r_t, d_t, i_t^l, i_t^d, c_t^T, p_t, λ_t, and μ_t satisfying*

$$\frac{i_t^l - i_t^d}{1 + i_t^d} \leq \Gamma_l(l_t, r_t), \quad l_t \geq 0, \quad \left[\frac{i_t^l - i_t^d}{1 + i_t^d} - \Gamma_l(l_t, r_t)\right] l_t = 0, \tag{14}$$

$$\frac{i_t^r - i_t^d}{1 + i_t^d} \leq \Gamma_r(l_t, r_t), \quad r_t \geq 0, \quad \left[\frac{i_t^r - i_t^d}{1 + i_t^d} - \Gamma_r(l_t, r_t)\right] r_t = 0, \tag{15}$$

$$l_t + r_t = d_t - \Gamma(l_t, r_t), \tag{16}$$

$$i_t^d = i_t^*, \tag{17}$$

$$u'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N) = \lambda_t, \tag{18}$$

$$\frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} = p_t, \tag{19}$$

$$\lambda_t(1 - \mu_t) \leq \beta(1 + i_t^l)E_t\lambda_{t+1} \quad (= \text{ if } l_t > 0), \tag{20}$$

$$c_t^T + (1 + i_{t-1}^*)d_{t-1} = y_t^T - \Gamma(l_t, r_t) + d_t, \tag{21}$$

$$\mu_t \geq 0, \tag{22}$$

$$l_t \leq \kappa(y_t^T + p_t y_t^N), \tag{23}$$

and

$$\mu_t[\kappa(y_t^T + p_t y_t^N) - l_t] = 0, \tag{24}$$

for $t \geq 0$, given a reserve remuneration policy i_t^r, exogenous processes i_t^*, y_t^T, and y_t^N, and the initial condition $(1 + i_{-1}^*)d_{-1} > 0$.

Following the open economy literature with collateral constraints, we consider an environment in which agents are impatient in the sense that they discount future period utilities at a higher rate than the one at which the world financial market discounts future payments. Formally, we assume that

$$\beta(1 + i_t^*) < 1.$$

In the related literature this condition is assumed to be strong enough to ensure an equilibrium in which the country is a net external debtor at all times. Throughout the present analysis, we assume that this is indeed the case.

3 Optimal Reserve Remuneration Policy

In this section, we show that the optimal interest-on-reserve policy achieves the first-best allocation. Here we define the first-best allocation as the one that solves the problem of a social planner who is constrained only by the sequential resource constraint and the prohibition to play Ponzi schemes. That is, the social planner is neither subject to the collateral constraint nor to the bank intermediation friction. The following definition provides a formal statement:

Definition 2 (First-Best Allocation)**.** *The first-best allocation is a pair of processes of tradable consumption and foreign deposits, $\tilde{c}_t^T$ and $\tilde{d}_t$, that solves the problem*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(A(c_t^T, y_t^N))$$

subject to the sequential resource constraint

$$c_t^T + (1 + i_{t-1}^*) d_{t-1} = y_t^T + d_t \tag{25}$$

and to a no-Ponzi-game constraint, taking as given the processes i_t^*, y_t^T, and y_t^N, and the initial condition $(1 + i_{-1}^*) d_{-1} > 0$.

The first-order condition of this maximization problem with respect to holdings of foreign deposits, d_t, is the Euler equation

$$\tilde{\lambda}_t = \beta(1 + i_t^*) E_t \tilde{\lambda}_{t+1},$$

with

$$\tilde{\lambda}_t = u'(A(\tilde{c}_t^T, y_t^N)) A_1(\tilde{c}_t^T, y_t^N).$$

To show that the first-best allocation can be supported as a competitive equilibrium by an interest-on-reserve policy, we must show that there exists a process for i_t^r such that equations (14) to (24) are satisfied when evaluated at the processes for consumption and foreign deposits associated with the first-best allocation. To this end, start by setting $c_t^T = \tilde{c}_t^T$ and $d_t = \tilde{d}_t$. Set $l_t = 0$. Then the sequential resource constraint in the competitive equilibrium (21) becomes identical to the resource constraint of the social planner (25), since

$\Gamma(0, r_t) = 0$ by Assumption 1. Next, set $\mu_t = 0$. It follows immediately that equilibrium conditions (22) and (24) are satisfied. To satisfy (18), set $\lambda_t = \tilde{\lambda}_t$. Next, set $i_t^l = i_t^*$, which guarantees that equilibrium condition (20) is satisfied. Pick p_t residually to satisfy (19). Because $l_t = 0$ for all t, the collateral constraint (23) holds. Set $r_t = \tilde{d}_t$. This guarantees the satisfaction of equilibrium condition (16). Equilibrium condition (14) holds because $l_t = 0$, $i_t^l = i_t^d = i_t^*$, and $\Gamma_l(\cdot, \cdot) \geq 0$. Finally, to ensure that (15) is satisfied, set the policy rate i_t^r equal to $i_t^d (= i_t^*)$ and invoke Assumption 1. This completes the proof that the first-best allocation can be supported as a competitive equilibrium by an appropriate interest-on-reserve policy.[1] The following proposition summarizes this result.

Proposition 1 (Optimal Interest-On-Bank-Reserve Policy)**.** *Suppose that Assumption 1 holds. In an economy with equilibrium conditions given by equations* (14) *to* (24)*, the first best allocation, given in Definition 2, can be supported as a competitive equilibrium by the interest-on-reserve policy* $i_t^r = i_t^*$.

The optimal bank reserve remuneration policy $i_t^r = i_t^*$ eliminates households' need to borrow from banks, $l_t = 0$. All of the financial intermediation occurs between banks and the government. Banks take deposits from international lenders and deposit them entirely at the central bank in the form of reserves. This is efficient, because the government does not suffer from repayment problems. In effect, the government borrows from banks at the world interest rate and transfers resources to the private sector via income taxes or subsidies, as needed. From the point of view of the household, these taxes or subsidies are exogenous. In equilibrium households endogenously become hand-to-mouth agents. They have access to bank loans at the world interest rate and have collateral to back them, but nevertheless choose not to use this credit facility. Cúrdia and Woodford (2011) show, in the context of a closed economy, that, as in the present environment, the optimal reserve remuneration policy allows agents to completely circumvent the bank intermediation friction. Here, the optimal policy also allows agents to completely circumvent the financial friction arising from the collateral constraint, which makes it feasible to attain the first-best allocation.

Can capital control taxes also support the first-best allocation? The answer is no. Consider the economy described thus far, but assume that the government has access to an additional policy instrument, namely, capital control taxes, denoted τ_t^c. Capital controls introduce a wedge between the world interest rate and the rate effectively paid by domestic banks on deposits. Specifically, the deposit rate and the world interest rate are linked by

[1] This is not the only competitive equilibrium that can support the first-best allocation. It is possible to show that the same processes for all endogenous variables except for $(1 + i_t^l)$, which can be set at any value in the interval $[1 + i_t^*, (1 + i_t^*)(1 + \Gamma_l(0, r_t))]$, also supports the first-best allocation.

the relationship

$$1 + i_t^d = (1 + \tau_t^c)(1 + i_t^*). \tag{26}$$

The capital control tax rate can take positive or negative values. When $\tau_t^c < 0$, the government subsidizes capital inflows and when $\tau_t^c > 0$, it taxes them.

The government is assumed to rebate any revenues or outlays associated with capital control taxes to private households through income taxes or subsidies. Thus, the government's budget constraint (12) now becomes

$$\tau_t(y_t^T + p_t y_t^N + \pi_t) + r_t + \tau_{t-1}^c(1 + i_{t-1}^*)d_{t-1} = (1 + i_{t-1}^r)r_{t-1}. \tag{27}$$

A competitive equilibrium then includes the same conditions listed in Definition 1, except that the interest-parity condition (17) is replaced by (26):

Definition 3 (Competitive Equilibrium with Capital Controls)**.** *A competitive equilibrium with capital controls is a set of processes l_t, r_t, d_t, i_t^l, i_t^d, c_t^T, p_t, λ_t, and μ_t satisfying* (14)-(16), (18)-(24), *and* (26), *for $t \geq 0$, given a reserve remuneration policy i_t^r, a capital control tax rate τ_t^c, exogenous processes i_t^*, y_t^T, and y_t^N, and the initial condition $(1 + i_{-1}^*)d_{-1} > 0$.*

In general, given an arbitrary bank-reserve remuneration policy i_t^r, there does not exist a capital control policy τ_t^c that supports the first-best allocation given in Definition 2. To see this, consider an example in which $i_t^r < i_t^*$ for some t. Comparing the resource constraints in the competitive equilibrium and in the social planner's problem, equations (21) and (25), it is clear that the first-best consumption process $c_t^T = \tilde{c}_t^T$ is sustainable only if $d_t = \tilde{d}_t$ and $\Gamma(l_t, r_t) = 0$ for all t. Then, by Assumption 1, $l_t = 0$ for all t. That is, the capital control policy must induce households to voluntarily choose not to borrow. It follows that the collateral constraint (23) is always slack and $\mu_t = 0$, so that equilibrium conditions (22)-(24) are satisfied. The Lagrange multiplier λ_t and the relative price of nontradables p_t are uniquely pinned down residually by equilibrium conditions (18) and (19). Because μ_t is zero, the Euler equation (20) implies that $i_t^l \geq i_t^*$. The fact that $l_t = 0$ implies, by equilibrium condition (16), that $r_t = \tilde{d}_t$. It remains to show that the bank efficiency conditions (14) and (15) are satisfied in equilibrium. Because $r_t > 0$, the first expression in (15) must hold with equality, and by Assumption 1 $\Gamma_r(0, r_t) = 0$. Thus we have that $i_t^d = i_t^r$. This expression and (26) uniquely determine τ_t^c as $1 + \tau_t^c = (1 + i_t^r)/(1 + i_t^*)$. Then, in the period in which we assumed that $i_t^r < i_t^*$, we have that $\tau_t^c < 0$. At the same, since $1 + i_t^d = (1 + i_t^*)(1 + \tau_t^c)$, and $l_t = 0$, the first expression in (14) becomes $(1 + i_t^l)/(1 + i_t^*) \leq [1 + \Gamma_l(0, r_t)](1 + \tau_t^c)$. Combining this expression with the fact that $i_t^l \geq i_t^*$ yields $1 \leq (1 + i_t^l)/(1 + i_t^*) \leq [1 + \Gamma_l(0, r_t)](1 + \tau_t^c)$. This expression is in general at odds with the fact that τ_t^c must be negative. A case in point is when the intermediation cost function satisfies $\Gamma_l(0, r_t) = 0$, which is consistent with assumptions made in much of the related literature. We then have the following proposition:

Proposition 2 (Failure of Capital Controls to Support the First-Best Allocation)**.** *Given an arbitrary bank-reserve remuneration policy, i_t^r, in general, the optimal capital control policy cannot achieve the first-best allocation given in Definition 2.*

By arguments similar to those we just employed to establish Proposition 2, one can show that if the bank-reserve remuneration policy i_t^r satisfies $i_t^r > i_t^*$ for all t, then an appropriate choice of capital controls, namely, $1 + \tau_t^c = (1 + i_t^r)/(1 + i_t^*) > 1$, can support the first-best allocation as a competitive equilibrium. This gives rise to the following corollary:

Corollary 1 (Capital Controls and First-Best Allocation)**.** *For any arbitrary bank-reserve remuneration policy i_t^r satisfying $i_t^r > i_t^*$ for all t, the capital control policy $1 + \tau_t^c = (1 + i_t^r)/(1 + i_t^*)$ supports the first-best allocation given in Definition 2 as a competitive equilibrium.*

The condition under which the optimal capital control policy can support the first-best allocation, namely, $i_t^r > i_t^*$ for all t, is not the most compelling one. The reason is that if $i_t^r > i_t^*$, then, in the absence of capital controls, banks can exploit an arbitrage opportunity (consisting in taking deposits at the rate i_t^* and lending them at the rate i_t^r), which renders infinite profits. Thus, the reserve remuneration policy $i_t^r > i_t^*$ is untenable as a competitive equilibrium in the absence of capital controls. Put differently, in this case there is room for capital controls only because the reserve remuneration policy is inconsistence with equilibrium in their absence.

4 Costly Provision of Bank Reserves

As explained earlier, the reason why in the model studied thus far the optimal bank-reserve remuneration policy i_t^r achieves the first-best allocation is that in effect the central bank borrows at the world interest rate (by setting $i_t^r = i_t^*$) and passes these funds to households in a nondistorting fashion and in an amount just enough to make them not want to borrow. To prevent the possibility of a perfect circumvention of all financial frictions in the economy by an appropriate interest-on-reserve policy, we now assume that the central bank faces a cost of producing bank reserves. Specifically, we assume that generating bank reserves in the amount r_t entails a cost given by $\Gamma^r(r_t)$, where the function $\Gamma^r(\cdot)$ is assumed to have the following properties:

Assumption 2 (Bank-Reserve Cost Function)**.** *The function $\Gamma^r(\cdot)$ satisfies: (i) $\Gamma^r(\cdot)$ is increasing and convex; and (ii) $\Gamma^r(0) = 0$ and $\Gamma^r(r) > 0$ for $r > 0$.*

In equilibrium this assumption simply introduces a term in the economy-wide resource constraint that subtracts from the endowment of tradables. Specifically, the competitive equilibrium is identical to the one given in Definition 3, except that equation (21) is replaced by

$$c_t^T + (1+i_{t-1}^*)d_{t-1} + \Gamma(l_t, r_t) + \Gamma^r(r_t) = y_t^T + d_t. \tag{28}$$

We then have the following definition:

Definition 4 (Competitive Equilibrium with Costly Provision of Bank Reserves)**.** *A competitive equilibrium in the economy with costly provision of bank reserves is a set of processes* l_t*,* r_t*,* d_t*,* i_t^l*,* i_t^d*,* c_t^T*,* p_t*,* λ_t*, and* μ_t *satisfying* (14)-(16)*,* (18)-(20)*,* (22)-(24)*,* (26)*, and* (28) *for* $t \geq 0$*, given a reserve remuneration policy* i_t^r*, a capital control tax rate* τ_t^c*, exogenous processes* i_t^**,* y_t^T*, and* y_t^N*, and the initial condition* $(1+i_{-1}^*)d_{-1} > 0$*.*

In this new environment, the first-best allocation cannot be supported by an interest-on-reserve policy or by a capital control policy or by a combination thereof. To see this, note that, by the resource constraint (28), the only way in which the first-best consumption process can be supported without playing a Ponzi scheme is with $\Gamma(l_t, r_t) = \Gamma^r(r_t) = 0$. This condition, in turn, implies that $l_t = r_t = 0$. The bank's balance sheet (16) then implies that $d_t = 0$, that is, that the economy is in financial autarky forever and, consequently, households consume their endowment, $c_t^T = y_t^T - (1+i_{-1}^*)d_{-1}I(t=0)$, for all t, which in general is different from $\tilde{c}_t^T$.

A combination of bank-reserve remuneration and capital control policies can achieve the following constrained optimal allocation:

Definition 5 (Constrained Optimal Allocation with Costly Reserve Provision)**.** *The constrained optimal allocation is a set of processes* $\hat{c}_t^T$*,* $\hat{d}_t$*,* $\hat{l}_t \geq 0$*, and* $\hat{r}_t \geq 0$ *that solves the problem*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(A(c_t^T, y_t^N))$$

subject to (16)*,* (28)*, and*

$$l_t \leq \kappa \left[y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right], \tag{29}$$

taking as given the processes i_t^**,* y_t^T*, and* y_t^N*, and the initial condition* $(1+i_{-1}^*)d_{-1} > 0$*.*

To see that the constrained optimal allocation can be supported by an appropriate combination of bank-reserve remuneration policy i_t^r and capital control policy τ_t^c, in the definition of a competitive equilibrium given in Definition 4 set $\mu_t = 0$, which ensures that conditions (22) and (24) hold, λ_t to satisfy (18), p_t to satisfy (19), i_t^l to satisfy the Euler equation (20), i_t^d to satisfy (14), i_t^r to satisfy (15), and τ_t^c to satisfy (26).

It can be shown that the optimization problem in Definition 5 implies that when the collateral constraint (29) is slack, given d_t, the social planner chooses l_t and r_t to minimize the resource cost of loan and reserve provision subject to the bank's balance sheet. Specifically, when the collateral constraint is slack, l_t and r_t solve the problem

$$\min_{\{l_t, r_t\}} \Gamma(l_t, r_t) + \Gamma^r(r_t)$$

subject to (16). When l_t and r_t are both positive, the first-order condition associated with this problem is

$$\frac{\Gamma_l(l_t, r_t)}{1 + \Gamma_l(l_t, r_t)} = \frac{\Gamma_r(l_t, r_t) + \Gamma^{r\prime}(r_t)}{1 + \Gamma_r(l_t, r_t)}. \tag{30}$$

Roughly speaking, this optimality condition says that when the collateral constraint is slack, the social planner equates the private marginal cost of originating loans to the central bank's marginal cost of reserve provision net of the private bank's marginal benefit of holding reserves. Under relatively weak conditions, namely, $\Gamma_{lr}(l_t, r_t) < 0$, $1 + \Gamma_r(l_t, r_t) > 0$, and $\Gamma^{r\prime}(r_t) < 1$, this optimality condition implies that l_t and r_t move in the same direction. This means that when the planner's collateral constraint is slack, movements in the desired level of external debt, d_t, are achieved by moving the volume of loans and bank reserves in the same direction, so that d_t, l_t, and r_t comove positively. As it will be apparent in the quantitative analysis, the picture is quite different when the collateral constraint binds for the planner. In such circumstances, l_t and r_t move in different directions, reflecting the fact that the central bank substitutes reserves for loans to implement desired changes in the level of external debt, without violating the collateral constraint.

A byproduct of efficiency condition (30) is that, under the weak condition $\Gamma_l(0, r_t) = 0$, the volume of loans must be strictly positive at all times ($l_t > 0$ $\forall t$) in the constrained optimal equilibrium. To see this, suppose, on the contrary, that $l_t = 0$ for some t. Then the collateral constraint is slack so that equation (30) is valid. But if $l_t = 0$, then the left-hand side of (30) is nil. At the same time, the balance sheet condition (16) implies that $r_t = d_t > 0$. This, in turn, implies that the right-hand side of (30) is strictly positive, since $\Gamma_r(0, r_t) = 0$ by Assumption 1 and $\Gamma^{r\prime}(r_t) > 0$ by Assumption 2. Thus, equation (30) does not hold with equality, which is a contradiction.

The question we wish to address next is how the constrained optimal equilibrium compares to the laissez-faire equilibrium ($i_t^r = \tau_t^c = 0$, $\forall t$). To this end, we turn to a quantitative characterization of the model's dynamics under these two policy regimes.

5 Functional Forms, Calibration, and Computation

The period utility function takes the CRRA form

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma},$$

with $\sigma > 0$. The aggregator function of tradable and nontradable consumption takes the CES form

$$c_t = A(c_t^T, c_t^N) \equiv \left[a {c_t^T}^{1-1/\xi} + (1-a) {c_t^N}^{1-1/\xi} \right]^{1/(1-1/\xi)}, \tag{31}$$

with $\xi > 0$ and $a \in (0,1)$.

The financial intermediation cost function of banks takes the form

$$\Gamma(l_t, r_t) = A l_t^{1+\alpha}[1 + \phi(r_t - \bar{r})^2 I(r_t < \bar{r})], \tag{32}$$

with $A, \alpha, \phi, \bar{r} > 0$. The operating-cost function of the central bank takes the form

$$\Gamma^r(r_t) = B r_t^{1+\alpha}. \tag{33}$$

The specification of the two cost functions assumes that the volume elasticity of the cost of originating loans, $1+\alpha$, is the same for the central bank and the commercial bank. The purpose of this assumption is to economize on parameters. It implies that the administrative and monitoring costs of loans are similarly sensitive to the scale of operation. Because the coefficients A and B can in principle be different from each other and are determined to match actual data, the assumed parameterization allows for the total, the average, and the marginal intermediation costs to differ across the two types of bank. The specifications of the cost functions $\Gamma(l, r)$ and $\Gamma^r(r)$ satisfy Assumptions 1 and 2, respectively.

The calibration of the parameters of the model is summarized in Table 1. The time unit is meant to be one year. Following Bianchi (2011), we set $\sigma = 2$, $a = 0.31$, $\xi = 0.83$, and the world interest rate i_t^* at a constant value of 4 percent per annum. We also follow Bianchi (2011) in setting the degree of relative impatience (i.e., the difference between the subjective and market rates of discount). In the present model, this factor is given by $\beta(1 + i^l)$, where i^l denotes the average interest rate on bank loans. Bianchi's model does not include bank intermediation, so loans to households originate directly from foreign lenders. Therefore, relative impatience in that environment is $\beta(1 + i^*)$. Bianchi sets β at 0.91 and i^* at 0.04, which gives a coefficient of relative impatience of 0.9464. We set i^l to 9.19 percent to match a median lending spread of 4.99 percent observed in emerging countries (see Table 2 and further discussion of this data below). In the the unregulated economy ($i_t^r = \tau_t^c = 0$), the

Table 1: Calibration

Parameter	Value	Description
		Structural Parameters
σ	2	Inverse of intertemporal elasticity of consumption
a	0.31	Parameter of CES aggregator
ξ	0.83	Elasticity of substitution between tradables and nontradables
i^*	0.04	World interest rate
β	0.8667	Subjective discount factor
κ	0.3205	Parameter of collateral constraint
A	0.0089	Parameter of intermediation cost function $\Gamma(l,r)$
α	1.8104	Parameter of the intermediation cost functions $\Gamma(l,r)$ and $\Gamma^r(r)$
ϕ	6.7983	Parameter of intermediation cost function $\Gamma(l,r)$
$\bar{r}$	0.5848	Parameter of intermediation cost function $\Gamma(l,r)$
B	2.6852	Parameter of intermediation cost function $\Gamma^r(r)$
		Discretization of State Space
n_{y^T}	13	Number of grid points for $\ln y_t^T$, equally spaced
n_{y^N}	13	Number of grid points for $\ln y_t^N$, equally spaced
n_d	800	Number of grid points for d_t, equally spaced
$\left[\ln \underline{y}^T, \ln \overline{y}^T\right]$	[-0.1093, 0.1093]	Range for logarithm of tradable output
$\left[\ln \underline{y}^N, \ln \overline{y}^N\right]$	[-0.1328, 0.1328]	Range for logarithm of nontradable output
$[\underline{d}, \overline{d}]$	[0.4, 1.05]	Debt range unregulated economy

Note. The time unit is a year.

Table 2: Empirical Moments Used in the Calibration

	Moment	Formula	Observed	Predicted
(1)	Lending spread	$\frac{i^l - i^d}{1+i^d}$	0.0499	0.0509
(2)	Reserve-to-deposit ratio	$\frac{r}{d}$	0.0644	0.0712
(3)	Debt-to-output ratio	$\frac{d}{y^T + py^N}$	0.2900	0.2992
(4)	Intermediation-cost-to-deposit ratio	$\frac{\Gamma(l,r)}{d}$	0.0175	0.0165
(5)	Central-bank-operating-cost-to-reserve ratio	$\frac{\Gamma^r(r)}{r}$	0.0205	0.0228
(6)	Frequency of binding collateral constraint		0.0500	0.0530

Note. Lines 1, 2, and 5 are cross-country medians of cross-time medians. The definition of an emerging country and the countries included follow Uribe and Schmitt-Grohé (2017). Data Sources: Lending spread, IFS; reserve-to-deposit ratio, Bankscope data for commercial banks; debt-to-output ratio, Bianchi (2011); intermediation-cost-to-deposit ratio, Philippon (2015) and Bazot (2018); central-bank-operating-cost-to-reserve ratio, Bankscope data for commercial banks.

lending spread is equal to $(i^l - i^*)/(1 + i^*)$. The implied value of β is therefore 0.8667.

The remaining parameters, A, α, ϕ, $\bar{r}$, B, and κ, pertain to the financial side of the economy. They are calibrated by simulated method of moments to jointly match six moments for which the unregulated model economy can produce precise predictions: (1) The lending spread $((i^l - i^d)/(1 + i^d))$. Using data from the IMF's International Financial Statistics (IFS), we estimate the average value of this spread across emerging countries to be 4.99 percent per year. Recalling that the world interest rate is set at 4 percent, this estimate implies that in the model economy banks lend to the domestic private sector at a rate more than twice as high as the rate at which they borrow from international lenders. (2) The reserve-to-deposit ratio (r/d). We estimate this ratio using data on commercial banks from Bankscope. On average, banks in emerging countries hold 6.44 percent of their deposits in the form of reserves at the central bank. This ratio is more than three times the one observed in rich countries (1.98 percent). (3) The debt-to-output ratio $(d/(y^T + py^N))$. In the model, d_t is both the amount of bank deposits and the country's net foreign debt position. This is because, being a representative-agent economy, the model does not feature deposits by domestic agents (all households are borrowers). For this reason, one must take a stance on whether to calibrate the ratio $d/(y^T + py^N)$ to match the observed deposit-to-output ratio or to match the observed net-foreign-debt-to-output ratio. We pick the latter option to keep in line with calibrations in the related literature. Specifically, following Bianchi (2011), we set d to be 29 percent of output.[2] (4) The bank-operating-cost-to-deposit ratio ($\Gamma(l,r)/d$). We set this ratio to 0.0175. This calibration lies in the middle of the range estimated by Philippon (2015) and Bazot (2018), who estimate that bank unit costs range from 1.5 and 2 percent in the United States (first author) and in a set of 20 countries (second author). The sample in Bazot (2018) contains mostly developed countries. However, it includes four countries that during the sample period of this study, 1970-2014, can be considered emerging economies, namely, China, Portugal, South Korea, and Spain. The average unit cost across these four countries is 1.99 percent, which is close to the value assigned in the calibration. (5) The central-bank-operating-cost-to-reserve ratio ($\Gamma^r(r)/r$). Using data from Bankscope, we estimate this ratio to be on average 2.05 percent in emerging countries. This value is slightly lower than the one observed across rich countries, 2.55 percent in the same database. (6) The probability of a binding collateral constraint. We set this moment to 5 percent (or one financial crisis every 20 years on average), which is a value within the range used in the related literature.

Table 2 reports the six targeted moments and the corresponding predictions of the unregulated economy. The relevant steady state for the calibration is the stochastic steady state rather than the deterministic one, as in the latter the collateral constraint binds at all times. For this reason the calibration is computationally demanding, and exact matches are not possible in general. However, as a comparison of the last two columns of Table 2 suggests, the match is quite close.

The computation of the unregulated and constrained competitive equilibria employs a discretized state space. Specifically, it uses 13 equally spaced points for each of the exogenous driving forces, $\ln y^T$ and $\ln y^N$. The two exogenous variables are assumed to follow a bivariate AR(1) process. The parameters of this process are set to the values used in Bianchi (2011). The transition probability matrix of the vector $(\ln y^T, \ln y^N)$ is computed using the simulation approach described in Schmitt-Grohé and Uribe (2014). The endogenous state, external debt d, is discretized using 800 equally spaced points. Table 1 provides more details.

The unregulated competitive equilibrium, that is, the equilibrium without interest on reserves or capital controls ($i_t^r = \tau_t^c = 0$), is approximated using an Euler equation iteration procedure over the discretized state space. The constrained optimal competitive equilibrium, that is, the equilibrium in which the policymaker optimally sets i_t^r and τ_t^c, is computed using a value function iteration approach over the discretized state space.

Unlike in a version of the present model without a banking channel (e.g., Bianchi, 2011; Schmitt-Grohé and Uribe, 2021), the computation of the constrained optimal equilibrium

[2] This argument does not apply to the calibration of the reserve-to-deposit ratio, r/d, because both r and d are expected to be proportionally affected by the presence of domestic deposits.

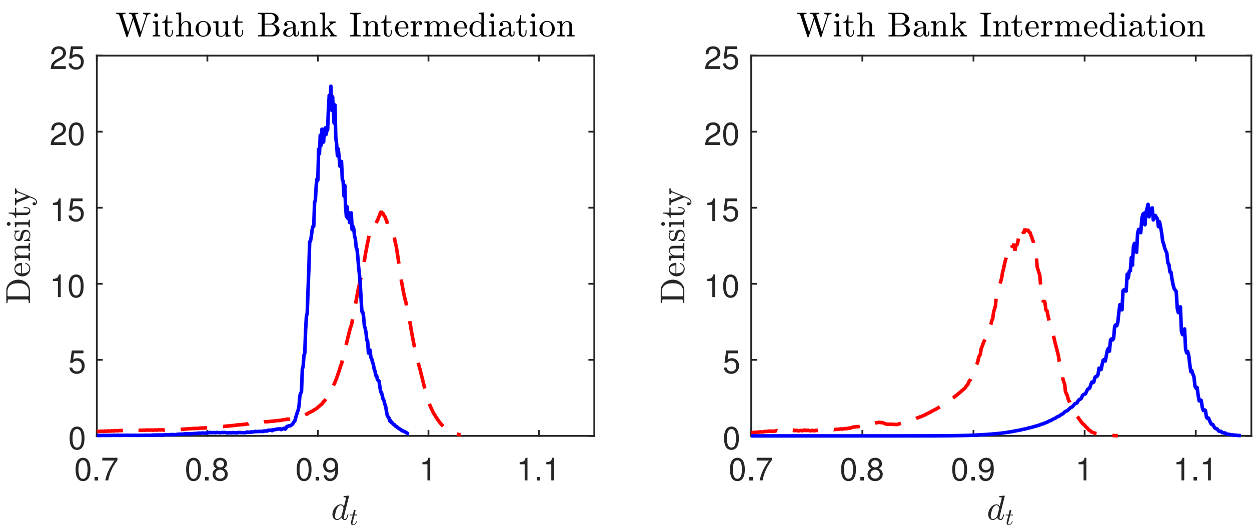
turns out to be more involved than that of the unregulated economy. The reason is that in the unregulated economy, given a value for d_t, the volume of loans and reserves, l_t and r_t, are determined by solving equations (15) and (16), evaluated at $i_t^r = 0$ and $i_t^d = i_t^*$, which is a relatively simple numerical problem. With r_t and l_t in hand, consumption of tradables, c_t^T, is found residually by solving the sequential resource constraint (28). This consumption level and the associated candidate value for d_t are feasible if the equilibrium collateral constraint (29) is satisfied, which is also a simple condition to check. By contrast, the social planner is not constrained by equation (15), since she can pick i_t^r. As a result, given d_t, the values of l_t, r_t, and c_t^T are jointly determined as the solution to the problem of maximizing c_t^T subject to (16), (28), and (29). Thus, the social planner solves a static optimization problem for each candidate choice of d_t and for each state (y_t^T, y_t^N, d_{t-1}). In turn, this static optimization problem is nested in the dynamic optimization problem of choosing the debt policy function, d_t.

6 Underborrowing

A well-known result that arises in a version of the present model without a bank intermediation channel is that the unregulated economy overborrows (e.g., Bianchi, 2011). Specifically, when households can borrow directly from foreign lenders, the equilibrium density function of external debt is located to the right of the one associated with the constrained optimal allocation. The economy without banks is a special case of the one analyzed here in which $r = 0$ and $\Gamma(l, r) = \Gamma^r(r) = 0$, for all l. The left panel of Figure 1 plots the equilibrium distribution of debt in the economy without banks for the unregulated and constrained optimal cases. All parameters of the model other than those pertaining to the bank and central bank cost functions take the values shown in Table 1. The resulting economy is identical to the one analyzed in Bianchi (2011) except for the values of β and κ, which are slightly different (0.8667 versus 0.91 and 0.3205 versus 0.32, respectively). The debt density under optimal capital control policy lies to the left of the one associated with the unregulated economy. Thus, the plot shows that the model without banks reproduces the standard overborrowing result.

The picture is quite different when household borrowing from foreign lenders is intermediated by banks and the government has access to an additional policy instrument, namely, the interest rate on bank reserves, i_t^r. The result is shown in the right panel of Figure 1. Now the distribution of debt in the unregulated economy lies clearly to the left of the corresponding distribution under the constrained optimal equilibrium, suggesting that in the economy with a banking channel the unregulated economy underborrows.

Figure 1: Unconditional Distributions of Debt With and Without a Bank Intermediation Channel



Notes. The left panel corresponds to an economy without a bank intermediation channel and the right panel to an economy with a bank intermediation channel. Parameters take the values shown in Table 1 when applicable. The debt densities associated with constrained optimal allocations are shown with solid lines and debt densities of unregulated economies with broken lines. The figure shows that in the absence of a banking channel there is overborrowing, whereas in the presence of the bank intermediation friction there is underborrowing.

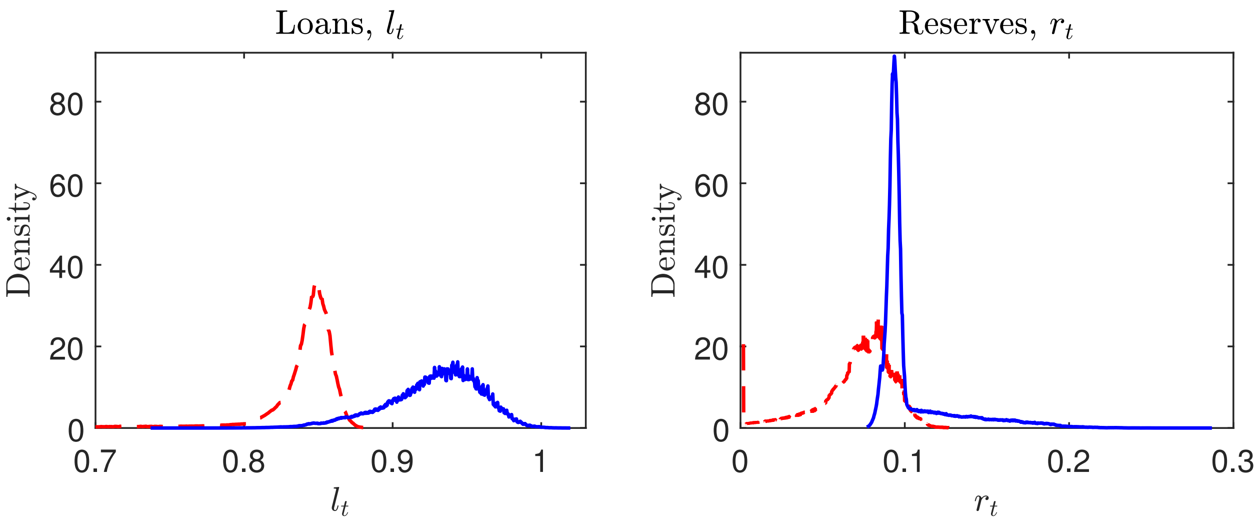
The intuition for why the economy with a banking channel underborrows is that interest on reserves turns out to be a powerful policy tool to dampen the negative macroeconomic consequences of credit crunches at the household level. Essentially, bank reserves introduce a cushion between debt and private loans. This can be seen from the balance sheet of the bank, equation (16), which, up to the resource cost of producing loans, says that $l_t + r_t = d_t$. So even if l_t is restricted by the collateral constraint, the government can achieve a desired level of external debt by making up the shortfall in loans with reserve creation.

Suppose, for example, that the economy faces a negative tradable endowment shock. Since the endowment process is mean reverting, the intertemporal approach to the current account dictates that the economy should finance the negative shock by borrowing from international lenders (i.e., by running a current account deficit). However, the fall in output tightens the collateral constraint, making banks reluctant to extend loans to households. In this case, the central bank can induce an increase in bank reserves by offering a higher interest rate on this type of financial asset. Thus, through an increase in bank reserves, the economy as a whole can increase external debt, d_t, even though l_t is impeded to expand by the binding collateral constraint. This is evident from Figure 2, which displays the distributions of loans and reserves in the economy with banks in the unregulated and constrained optimal equilibria. Under optimal policy the economy has a larger volume of loans and bank reserves. It is noteworthy that the optimal distribution of bank reserves has a fat right tail. This characteristic plays the role of avoiding episodes of large macroeconomic deleveraging (falls in d_t) when the household's collateral constraint binds. By contrast, in the unregulated economy the distribution of bank reserves not only lacks a fat right tail but displays a mass concentration at zero.

Examination of the predicted frequency of a binding collateral constraint confirms the interpretation that the role of bank reserves in the constrained optimal equilibrium is to attenuate the negative macroeconomic consequences of a binding collateral constraint. The collateral constraint binds 5 percent of the time in the unregulated economy and 27 percent of the time in the constrained optimal one. By contrast, in the environment without banks, the collateral constraint binds 14 percent of the time in the unregulated economy and never under the constrained optimal allocation.[3] Intuitively, in the absence of bank reserves, there is no difference between household debt and external debt. As a result a binding collateral constraint forces macroeconomic deleveraging, which is welfare decreasing. For this reason, the primary concern of the social planner is to reduce the likelihood of a binding borrowing

[3]The predicted frequency of a binding collateral constraint in the unregulated economy without banks is unrealistically high. Recall, however, that the calibration of the present model is chosen to deliver a realistic frequency of a binding collateral constraint in the unregulated economy with banks.

Figure 2: Unconditional Distributions of Loans and Reserves



Note. The densities associated with the constrained optimal allocation are shown with a solid line and the densities of the unregulated economy with a broken line.

constraint. In spite of this significant elevation in the frequency of a binding collateral constraint, households are better off in the constrained optimal equilibrium, precisely because the government can induce a path of bank reserves that acts as a shock absorber, or as a cushion between d_t and l_t. A fall in collateral that causes deleveraging in the private sector (a fall in l_t) need not result in deleveraging of the economy as a whole (a fall in d_t), because the government can fill in the gap between d_t and l_t by creating reserves through an appropriate interest-on-reserve policy.

7 Optimal Capital Controls and Bank-Reserve Remuneration During Sudden Stops

To understand how the social planner manages a sudden stop in the presence of an intermediation friction, we examine equilibrium dynamics in the unregulated economy and in the constrained optimal equilibrium around a typical episode in which the collateral constraint binds in the unregulated economy. To this end, we simulate the unregulated economy for 1 million periods and extract all windows of eleven years containing a binding collateral constraint in the middle. This yields 53,019 sudden stop episodes, which is consistent with the calibrated frequency of a binding collateral constraint of 5 percent (see Table 2). For each variable, we compute the average across the sudden stop episodes. The result is shown with broken lines in Figures 3 and 4. The period in which the collateral constraint binds is normalized to 0, so time runs from period -5 to period 5.

To compare the sudden stop dynamics in the unregulated economy with those in the economy with optimal capital-control and reserve-remuneration policies, for each sudden stop episode in the former economy, we compute the equilibrium dynamics implied by the constrained optimal equilibrium assuming that in period -5 (five years prior to the sudden stop) the unregulated and regulated economies have the same level of debt. We then hit the regulated economy with the same sequence of endowment shocks that buffeted the unregulated economy between periods -5 and 5. The results are shown with solid lines in Figures 3 and 4.

In the unregulated economy, the typical sudden stop occurs when the economy suffers a string of negative shocks to the endowments of tradable and nontradable goods. Both endowments fall by more than 8 percent between periods -5 and 0. By construction, the path of the two endowments is the same in the regulated and unregulated economies. Given the relative price of nontradables, the fall in output causes a decline in the value of collateral. When the collateral constraint binds in the unregulated economy, consumption of tradables falls sharply, because the economy, forced to deleverage, runs a large trade balance surplus to repay part of its external debt. The fall in aggregate demand depresses the relative price of nontraded goods, that is, the real exchange rate depreciates. The fall in the relative price of nontradables, further tightens the collateral constraint, a phenomenon known as a Fisherian deflation.

By contrast, in the economy with optimal capital controls and optimal bank-reserve remuneration, the contraction in the demand for nontradables and the real depreciation are milder. The reason is not that in the constrained optimal equilibrium the collateral constraint does not bind. In fact, in the regulated economy households are often borrowing constrained before, during, and after the sudden stop (see the bottom right panel of Figure 4). Instead, the reason why in the regulated economy aggregate demand and the real exchange rate are less affected by the contraction in the endowments is that under the optimal macroprudential policy the economy as a whole continues to have access to international credit. This is apparent from Figure 4. Specifically, the bottom left panel shows that debt falls sharply in the unregulated economy but is little changed in the regulated one. In fact, in the regulated economy debt increases slightly during the entire episode. This is because, although the level of debt in period -5 is the same in the unregulated and regulated economies, the unconditional average level of debt is higher in the latter than in the former. So over the entire time window, the regulated economy is transitioning to a higher level of debt.

Contrary to what happens with external debt, at the household level the sudden stop causes deleveraging (a decline in loans) in both, the unregulated and the regulated economies (top left panel of Figure 4). The social planner manages to avoid aggregate deleveraging in spite of seizable deleveraging at the household level by raising the stock of reserves held by banks (recall that up to the resource cost of loans, the balance sheet of the bank states that $d_t = l_t + r_t$). In the unregulated economy, the decline in loans is accompanied by a decline in bank reserves, which exacerbates macroeconomic deleveraging. Thus, a key difference between the sudden stop in the unregulated and regulated economies lies in the behavior of bank reserves. The expansion of reserves renders the sudden stop in the regulated economy relatively painless.

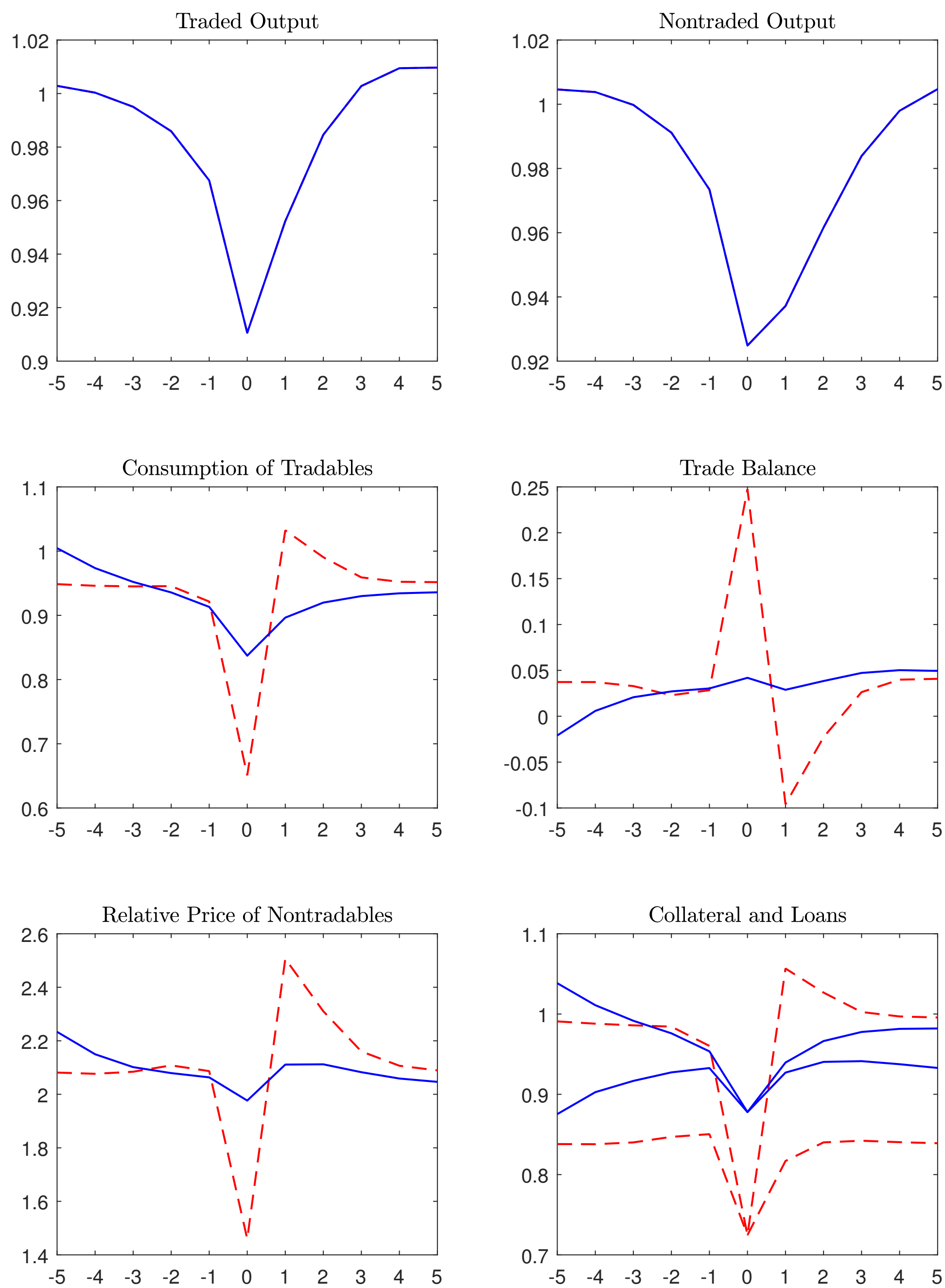
For the constrained optimal allocation to be supported as a competitive equilibrium, the social planner must create incentives, via an appropriate choice of interest on reserves and capital controls, for banks to choose the optimal quantities of bank loans and bank reserves. These incentives are materialized in the reserve spread , $(i_t^r - i_t^d)/(1 + i_t^d)$, and the lending spread, $(i_t^l - i_t^d)/(1 + i_t^d)$. In the unregulated equilibrium the reserve spread is constant at all times because central bank reserves are unremunerated and because the deposit rate equals the world interest rate, as the government imposes no capital controls. In the regulated economy, the reserve spread increase sharply during the sudden stop (middle-right panel of Figure 4), which incentivizes banks to elevate their reserve holdings. In the unregulated economy, the lending spread displays a remarkable stability over the sudden stop episode (top-right panel of Figure 4). The reason is that the sudden stop represents a decline in the demand for loans rather than a decline in the supply of loans. The next subsection expands this intuition. In the regulated economy, by contrast, it falls significantly, luring banks to reallocate funds away from loans (which are in low demand) and toward reserves.

The next subsection presents a heuristic partial-equilibrium explanation of the of the adjustment in the loan and reserve markets during a sudden stop.

7.1 Adjustment of the Banking Sector in a Sudden Stop: A Partial Equilibrium Explanation

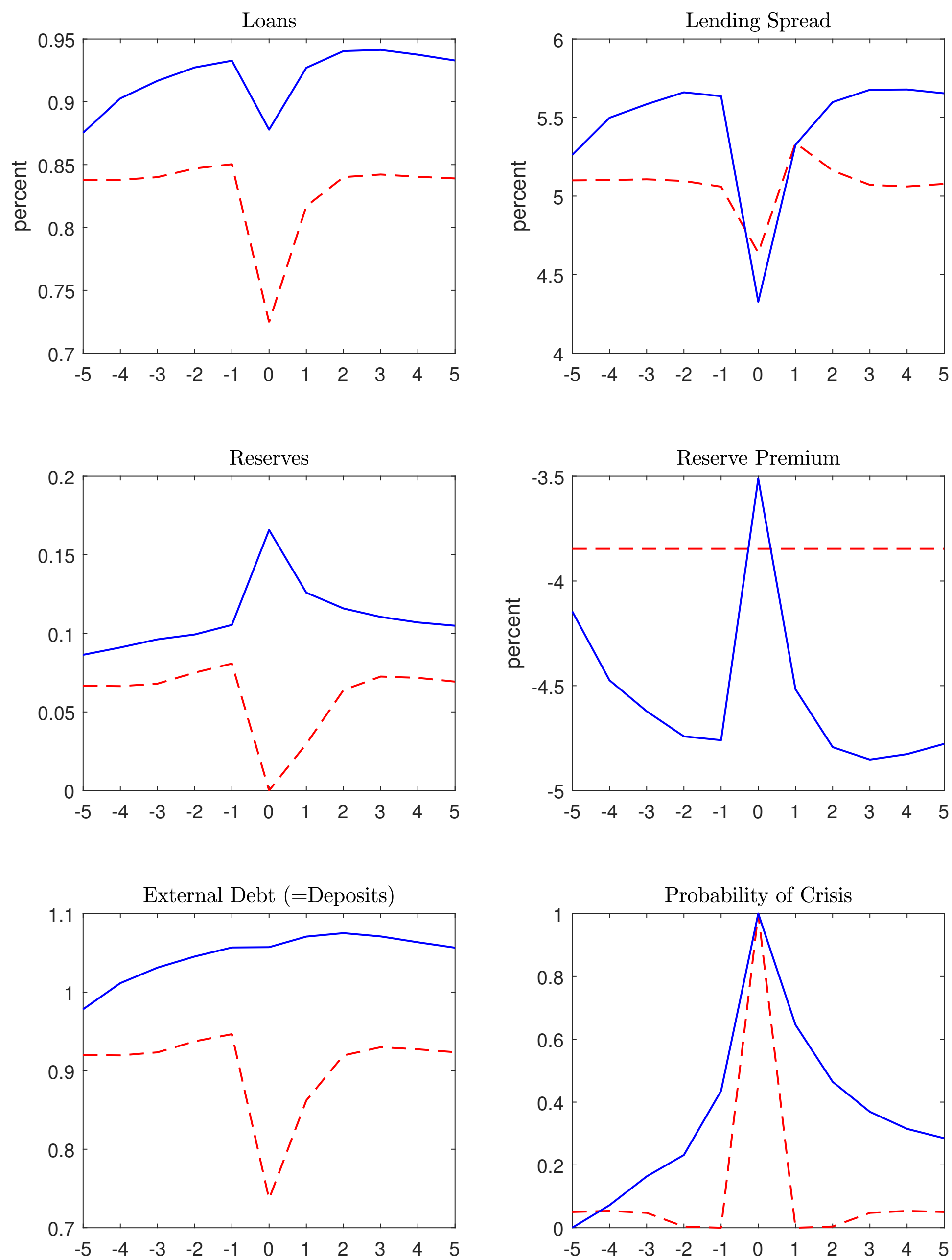
To understand the behavior of quantities and prices of loans and bank reserves during a sudden stop consider the following graphical explanation. The left panel of Figure 5 depicts the loan market. The supply of loans is given by the marginal cost of bank intermediation, $\Gamma_l(l, r)$ (efficiency condition (14)). Holding bank reserves constant, the loan supply schedule is increasing in l. When the collateral constraint is slack, the demand for loans is downward sloping and stems from the household's Euler equation. The higher is the interest on loans, the lower the demand for loans will be, as households have an increased incentive to postpone

Figure 3: The Typical Sudden Stop Episode



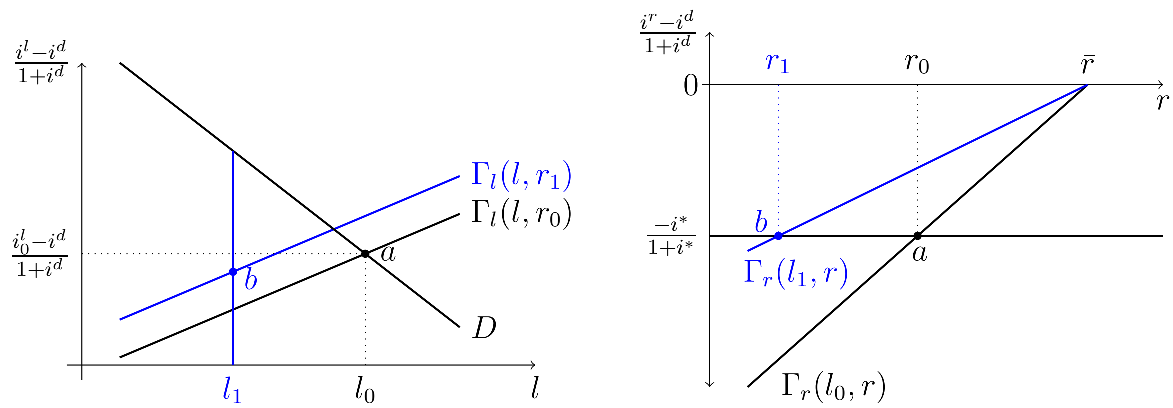
Note. The sudden stop associated with the constrained optimal allocation is shown with a solid line and the sudden stop associated with the unregulated economy with a broken line.

Figure 4: The Typical Sudden Stop Episode (cont.)



Note. The sudden stop associated with the constrained optimal allocation is shown with a solid line and the sudden stop associated with the unregulated economy with a broken line.

Figure 5: The Loan and Reserve Markets During a Sudden Stop in the Unregulated Economy



consumption. The initial equilibrium is at point a, where the supply and demand for loans intersect. The volume of loans is l_0 and the lending spread is $(i_0^l - i^d)/(1+i^d)$ (recall that in the unregulated equilibrium i^d is constant and equal to i^*).

Consider now the market for bank reserves, which is depicted in the right panel of Figure 5. The supply of reserves is perfectly elastic at the constant spread $-i^d/(1+i^d)$, as the central bank stands ready to supply any amount of reserves to private banks at a zero interest rate. Holding the volume of loans constant, the demand for bank reserves is given by the marginal benefit of reserve holdings by private banks, $\Gamma_r(l_0, r)$ (efficiency condition (15)). The demand schedule is upward sloping in the range $0 < r < \bar{r}$, which is the relevant one for the present analysis. Equilibrium in the bank reserve market occurs at point a, where the demand for reserves meets the (horizontal) supply of reserves.

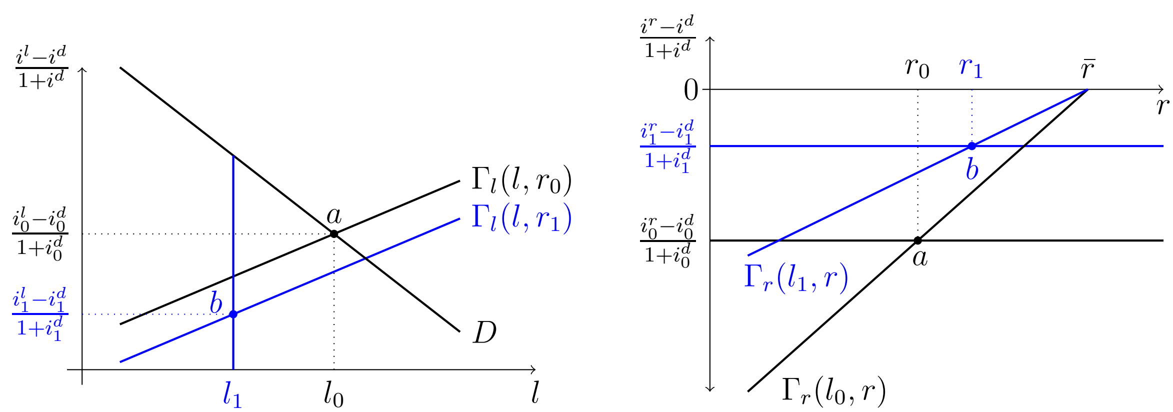
Consider now the effect of a sudden stop on the markets for loans and reserves. Suppose that the economy suffers a negative endowment shock that makes the collateral constraint bind, forcing households to deleverage. Suppose that the volume of loans demanded after the negative shock is $l_1 < l_0$. In the market for loans, this is represented by a kink in the demand for loans. For simplicity, we assume that the new demand for loans is given by the original one for $l < l_1$. At $l = l_1$, the new demand schedule is vertical. In the reserve market, the fall in the volume of loans shifts the demand schedule up and to the left from $\Gamma_r(l_0, r)$ to $\Gamma_r(l_1, r)$ (recall that $\Gamma_{rl}(l, r) < 0$). The supply of reserves is unchanged. The new equilibrium is at point b. The equilibrium level of reserves falls from r_0 to $r_1 < r_0$. In turn, the fall in the stock of bank reserves shifts the loan supply schedule up and to the left from $\Gamma_l(l, r_0)$ to $\Gamma_l(l, r_1)$. The new equilibrium in the loan market is at point b. (For expositional convenience we describe these effects as occurring sequentially, but in fact they occur simultaneously.)

Comparing the initial equilibrium, points a in both panels, with the equilibrium after the shock, points b, suggests that the sudden stop causes an unambiguous fall in both the volume of loans and the stock of bank reserves. The effect on the lending spread, however, is ambiguous. The contraction in the demand for loans tends to push the lending spread down, but the contraction in bank reserve holdings tends to push it up. This intuition is consistent with the relatively stable path displayed by the lending spread in the unregulated economy over the sudden stop episode in the calibrated model (top right panel of Figure 4).

The intuitive explanation for why the lending spread does not display a hike during a sudden stop is that the decline in the equilibrium volume of loans is a consequence of a contraction in the demand for loans by private households. By contrast, in models in which the collateral constraint is placed at the level of the bank as opposed to at the level of the household, a sudden stop represents a contraction in the supply of loans and hence is associated with an increase in the lending spread (as, for example, in the model of Céspedes and Chang, 2020). Subsection 7.2 examines the observed behavior of the lending spread during the global financial crisis to shed light on whether the contraction in credit was primarily driven by supply or demand forces.

The situation is quite different when the government intervenes. The adjustment to a negative endowment shock is illustrated in Figure 6. Initially, the markets for loans and bank

Figure 6: The Loan and Reserve Markets of the Regulated Economy During a Sudden Stop



reserves are in equilibrium at point a. The equilibrium levels of loans and bank reserves are l_0 and r_0 and the lending and reserve spreads are $(i^l_0 - i^d_0)/(1 + i^d_0)$ and $(i^r_0 - i^d_0)/(1 + i^d_0)$. As in the unregulated economy, the sudden stop causes a kink in the demand schedule for loans at $l_1 < l_0$ (left panel), and a shift up and to the left in the demand schedule for bank reserves from $\Gamma_r(l_0, r)$ to $\Gamma_r(l_1, r)$ (right panel). Now, unlike in the unregulated economy, to avoid a collapse in the bank-reserve market, the government increases the banks' incentive to hold reserves by raising the reserve spread from $(i_0^r - i_0^d)/(1 + i_0^d)$ to $(i_1^r - i_1^d)/(1 + i_1^d) > (i_0^r - i_0^d)/(1 + i_0^d)$. Thus, the horizontal supply of reserves shifts up in a parallel fashion. If the increase in the reserve spread is large enough, the new equilibrium level of reserves can be larger than before the sudden stop. This is the case illustrated in the right panel of Figure 6, where at the new equilibrium, given by point b, the level of bank reserves is $r_1 > r_0$. The central bank has an incentive to act aggressively because, to avoid a contraction in the level of external debt, the fall in the volume of loans must be compensated by an increase in the holdings of bank reserves. In the loan market, the increase in the stock of reserves shifts the loan supply schedule down and to the right from $\Gamma_l(l, r_0)$ to $\Gamma_l(l, r_1)$. The new equilibrium is at point b, where the lending spread has fallen from $(i_0^l - i_0^d)/(1 + i_0^d)$ to $(i_1^l - i_1^d)/(1 + i_1^d)$.

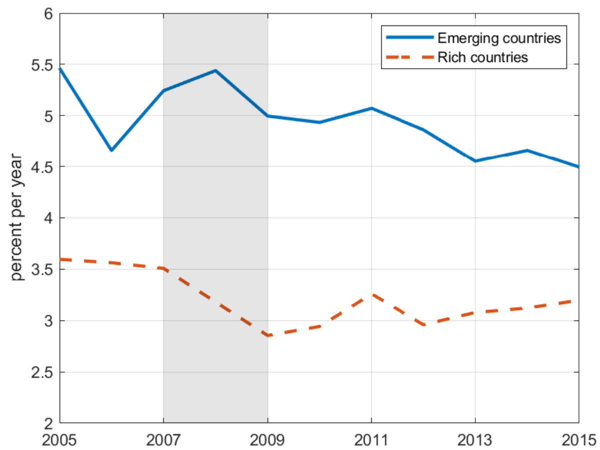
In sum, the intuition derived from Figure 6 is that if the government intervention raises the reserve spread, then the sudden stop is associated with a fall in the volume of loans, an increase in the stock of bank reserves, and a fall in the lending spread. These qualitative effects are consistent with the predictions of the calibrated model under optimal reserve remuneration and capital control policies shown in Figure 4.

7.2 Evidence on Lending Spreads During Financial Crises

In this subsection, we present empirical evidence on the behavior of lending spreads in rich and emerging countries during the global financial crisis of 2007 to 2009. Specifically, Figure 7 displays the cross-country median of the lending spread, $(i_t^l - i_t^d)/(1 + i_t^d)$, for a balanced panel of 9 rich countries and 32 emerging countries from 2005 to 2015. (The sample of countries is dictated by data availability in the IFS database.) The shaded area marks the global financial crisis of 2007 to 2009. In the group of rich countries (broken line), the lending spread fell during the crisis by more than 60 basis points. In the group of emerging countries (solid line), lending spreads rose slightly in the first year of the crisis (less than 20 basis points) but fell in the second year of the crisis. Overall, by 2009 lending spreads in emerging countries were about 25 basis points below their 2007 levels. Thus, the data suggests that in rich countries as well as in emerging countries lending spreads failed to increase during the global financial crisis of 2007 to 2009.

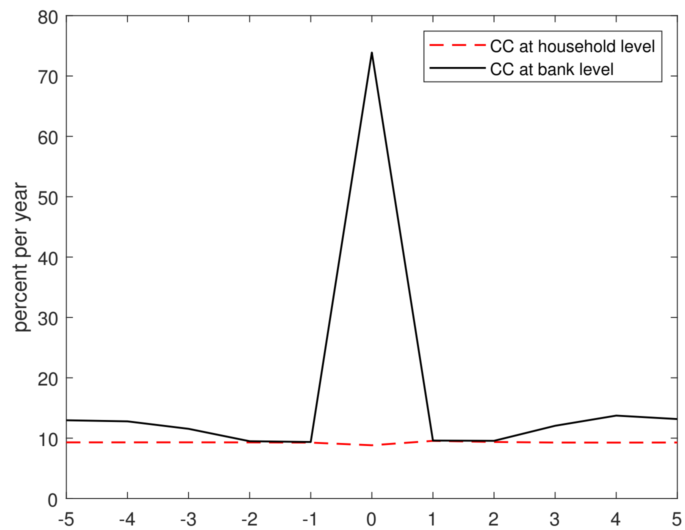
This empirical evidence is therefore consistent with the hypothesis that the collapse in the loan market that occurred during the crisis was driven by a disruption in the demand for loans. In the context of the theoretical model presented in this paper, such a disruption is captured by a binding borrowing constraint at the household level. If instead the model were to feature a lending limit at the bank level, a financial crisis would trigger an increase in the lending spread. For example, a minimal departure from the present theoretical framework

Figure 7: Lending Spreads around the Global Financial Crisis in Emerging and Rich Countries



Notes. The lending spread, $(i_t^l - i_t^d)/(1+i_t^d)$, is computed as the median of the annual lending spread across a group of emerging and rich countries, respectively. The classification of countries follows Uribe and Schmitt-Grohé (2017). Countries with populations smaller than 1 million or with missing data over the period 2005-2015 were excluded. The 32 emerging countries included are: Albania, Algeria, Argentina, Bahrain, Bolivia, Botswana, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Dominican Republic, Egypt, Greece, Guatemala, Hungary, Iran, Jordan, South Korea, Malaysia, Mexico, Namibia, New Zealand, Panama, Paraguay, Peru, Portugal, Spain, Thailand, Trinidad and Tobago, Uruguay, and Venezuela. The 9 rich countries included are: Australia, Canada, Hong Kong, Ireland, Italy, Japan, Singapore, Switzerland, and United States. The data source is IMF, International Financial Statistics, the measure for the loan rate, i_t^l, is the series FILR_PA and the measure for the deposit rate, i_t^d, is the series FIDR_PA. Shading indicates the global financial crisis of 2007 to 2009.

Figure 8: Behavior of the Lending Rate around a Sudden Stop in the Unregulated Economy



Notes. CC stands for collateral constraint. The figure shows that in a sudden stop, the lending rate increases sharply when the collateral constraint enters at the bank level, but falls slightly when the collateral constraint enters at the household level.

is one in which the collateral constraint (10) is placed at the level of the bank as opposed to at the level of the household. Appendix A shows that in this case, under laissez faire, the real allocation is the same as in the case when the collateral constraint is placed at the household level but for a higher value of the lending rate, i_t^l. In particular, when the collateral constraint is slack, then the lending rate is the same under both formulations. However, when the collateral constraint is binding, then the lending rate is higher in the economy with the collateral constraint at the bank level. This difference can be quantitatively significant. Figure 8 displays the behavior of the lending rate around the typical sudden stop under laissez faire in the economies with the collateral constraint at the bank and household levels. The figure is produced using the parameter values shown in Table 1. When the collateral constraint binds (period 0) the interest rate sky rockets to 73 percent per year in the economy with a collateral constraint at the bank level, but remains flat (in fact falls by 0.45 percentage points) in the economy with the collateral constraint at the household level.

The main message conveyed by Figure 8 is that the observed behavior of the lending rate can provide information on whether disruptions in financial markets stem from the financial or the nonfinancial sector. Being able to make this distinction is not inconsequential for policymakers. Historically, the presumption that a key symptom of a financial crisis is a sharp increase in the lending rate has led to the misdiagnosis of major financial crises. A case in point is the Great Depression of 1929 to 1933. This episode shared with the global financial crisis of 2007 to 2009 a lack of a spike in the lending rate. Rockoff (2021) shows that the call money rate—the interest rate charged by banks to stock brokers on collateralized loans—actually fell between June 1930 and June 1931. According to this author, the lack of a hike in this interest rate was a key reason why Oliver M. W. Sprague, the major authority on financial crises at the time and an economic advisor of the Bank of England and the Roosevelt administration, failed to recognize a financial crisis in the economic developments that unfolded during this period. Rockoff further speculates that "had he [Sprague] diagnosed a banking panic and called for an aggressive response by the Federal Reserve, it might have made a difference; but he did not."

8 Non-Equivalence of Reserve Remuneration and Reserve Requirements

In many emerging markets central banks do not remunerate reserves but instead impose reserve requirements. Here we ask whether the constrained optimal allocation with reserve remuneration and capital control taxes can also be supported by an appropriate combination of reserve requirements and capital controls. More generally, we ask whether reserve remuneration welfare dominates reserve requirements as a macroprudential tool. The analysis that follows establishes that in the present theoretical framework this is indeed the case, that is, the policy maker can achieve a better outcome by using a combination of interest on reserves and capital controls than by using a combination of reserve requirements and capital controls.

Suppose the central bank does not pay interest on bank reserves, $i_t^r = 0$, but imposes a reserve requirement

$$r_t \geq \delta_t d_t,$$

where $\delta_t \in [0, 1)$ is a policy instrument. In addition, the government continues to have access to capital control taxes. Combining the reserve requirement with the bank's balance sheet constraint (16) yields

$$r_t \geq \delta_t [l_t + r_t + \Gamma(l_t, r_t)]. \tag{34}$$

Then the problem of a bank consists in choosing $l_t \geq 0$ and $r_t \geq 0$ so as to maximize profits,

$$\frac{i_t^l - i_t^d}{1 + i_t^d} l_t + \frac{i_t^r - i_t^d}{1 + i_t^d} r_t - \Gamma(l_t, r_t),$$

subject to the reserve requirement (34), taking as given i_t^l, i_t^r, i_t^d, and δ_t. Letting η_t denote the Lagrange multiplier on (34), the first-order conditions of the bank's problem are

$$r_t \geq \delta_t[l_t + r_t + \Gamma(l_t, r_t)], \quad \eta_t \geq 0, \quad \eta_t \{r_t - \delta_t[l_t + r_t + \Gamma(l_t, r_t)]\} = 0 \tag{35}$$

$$\frac{i_t^l - i_t^d}{1 + i_t^d} \leq \Gamma_l(l_t, r_t) + \eta_t \delta_t[1 + \Gamma_l(l_t, r_t)], \quad l_t \geq 0, \quad l_t \left\{ \frac{i_t^l - i_t^d}{1 + i_t^d} - \Gamma_l(l_t, r_t) - \eta_t \delta_t[1 + \Gamma_l(l_t, r_t)] \right\} = 0 \tag{36}$$

and

$$\frac{-i_t^d}{1 + i_t^d} - \Gamma_r(l_t, r_t) - \eta_t \left[1 - \delta_t - \delta_t \Gamma_r(l_t, r_t)\right] = 0, \tag{37}$$

where the last first-order condition uses the fact that reserves are unremunerated ($i_t^r = 0$).

Definition 6 (Competitive Equilibrium with Reserve Requirements and Capital Controls)**.** *A competitive equilibrium with reserve requirements and capital controls is a set of processes* c_t^T*,* d_t*,* p_t*,* i_t^l*,* i_t^d*,* λ_t*,* μ_t*,* l_t*,* r_t*, and* η_t *satisfying* (16)*,* (18)*-*(20)*,* (22)*-*(24)*,* (26)*,* (28)*, and* (35)*-*(37) *for* $t \geq 0$*, given a reserve requirement policy* δ_t*, a capital control tax rate* τ_t^c*, exogenous processes* i_t^**,* y_t^T*, and* y_t^N*, and the initial condition* $(1 + i_{-1}^*)d_{-1} > 0$*.*

The proof that the best competitive equilibrium with reserve remuneration strictly welfare dominates the best competitive equilibrium with reserve requirements proceeds as follows: Proposition B2 in Appendix B shows that the constrained optimal real allocation defined in Definition 5 cannot be supported as a competitive equilibrium with reserve requirements. Since the constraints in Definition 5 are all equilibrium conditions of the economy with reserve requirements, it follows that the constrained optimal allocation defined in Definition 5 strictly welfare dominates the best competitive equilibrium with reserve requirements. Finally, since the real allocation in Definition 5 is identical to the real allocation of the best competitive equilibrium with reserve remuneration, it follows that the latter strictly welfare dominates the best competitive equilibrium with reserve requirements.

9 Conclusion

This paper studies optimal bank reserve remuneration policy in an open economy with a bank intermediation friction and a pecuniary externality arising from a collateral constraint on private borrowing.

If the central bank can costlessly provide bank reserves, then the optimal bank reserve remuneration policy achieves the first-best allocation. Under the optimal policy the interest rate on bank reserves and the interest rate on loans equal the world interest rate and the market for bank reserves is satiated. In this case, interest on bank reserves dominates capital controls as a macroprudential instrument, as the latter achieves only a second best allocation.

With costly bank reserve provision, the first-best allocation cannot be attained and both interest on reserves and capital controls jointly play a relevant macroprudential role. Under plausible calibrations, unlike in models without a bank lending channel, the unregulated equilibrium displays underborrowing in the sense that external debt, private borrowing, and bank reserves are all lower than under the constrained optimal allocation. Under the optimal allocation, episodes of a binding collateral constraint are significantly more frequent than in the unregulated equilibrium, but they are not associated with sudden stops, reflecting the fact that the economy can maintain access to the international financial market even when domestic households are forced to deleverage.

An important policy issue in macroprudential banking policy is whether reserve requirements and interest on bank reserves are equivalent policy instruments. In the context of the open economy model studied in this paper, this is not the case. Reserve remuneration strictly dominates reserve requirements in a welfare sense. The reason is that reserve remuneration controls the price of reserves but lets its quantity be determined endogenously. By contrast, reserve requirements without interest on reserves amounts to fixing both the quantity and the price of reserves.

Appendix

A Loan Constraint at the Level of the Bank

In this appendix, we present a variant of the model in which the collateral constraint, given in equation (10), is placed at the level of the bank instead of at the level of the household. We show that the competitive equilibrium in this modification of the model is the same as the competitive equilibrium of the baseline version of the model presented in the body of the paper with the exception of the loan rate, i_t^l, which is strictly higher when the collateral constraint binds.

A.1 Banks

As before the problem of the bank consists in maximizing profits, (4). However, now the bank must satisfy the loan limit constraint (10). Formally, banks pick l_t and r_t so as to maximize

$$\frac{i_t^l - i_t^d}{1+i_t^d} l_t + \frac{i_t^r - i_t^d}{1+i_t^d} r_t - \Gamma(l_t, r_t)$$

subject to

$$l_t \leq \kappa \left[y_t^T + p_t y_t^N \right],$$

taking as given i_t^l, i_t^d, i_t^r, y_t^T, p_t, and y_t^N. Let $\mu_t^B \geq 0$ denote the Lagrange multiplier on the bank's loan limit. Then the first-order conditions associated with the optimal choice of l_t and μ_t^B are

$$\frac{i_t^l - i_t^d}{1+i_t^d} \leq \Gamma_l(l_t, r_t) + \mu_t^B, \quad l_t \geq 0, \quad \left[\frac{i_t^l - i_t^d}{1+i_t^d} - \Gamma_l(l_t, r_t) - \mu_t^B \right] l_t = 0, \tag{A1}$$

$$\mu_t^B \geq 0, \tag{A2}$$

$$l_t \leq \kappa \left(y_t^T + p_t y_t^N \right), \tag{A3}$$

and

$$\mu_t^B \left[\kappa \left(y_t^T + p_t y_t^N \right) - l_t \right] = 0. \tag{A4}$$

Comparing the present setting with the one in which the collateral constraint is placed at the household level, we have that equation (A1) replaces equilibrium condition (14) and equations (A2)-(A4) replace equilibrium conditions (22)-(24). The first-order condition with respect to r_t continues to be equation (15), and the balance sheet constraint of the bank continues to be (16).

A.2 Households

Now the household does not face the collateral constraint (23). Thus, its problem consists in choosing c_t, c_t^T, c_t^N, and l_t to maximize the utility function (7) subject to the aggregator (8), the sequential budget constraint (9), and some borrowing limit that prevents it from engaging in Ponzi schemes, taking as given p_t, i_t^l, π_t, τ_t, y_t^T, and y_t^N. The first-order conditions of this problem are (18), (19), and

$$\lambda_t \leq \beta(1+i_t^l)E_t\lambda_{t+1} \quad (= \text{ if } l_t > 0). \tag{A5}$$

The difference between this optimization problem and that in which the household does face a collateral constraint is that condition (A5) replaces condition (20).

A.3 Competitive Equilibrium

The assumption that the borrowing limit is placed at the level of the bank rather than at the level of the household affects neither the interest rate parity condition (26) nor the economy wide resource constraint (28). A competitive equilibrium in the economy with the collateral constraint at the bank level can then be defined as follows:

Definition A1 (Competitive Equilibrium in the Economy with the Collateral Constraint at the Bank Level). *A competitive equilibrium in the economy with costly provision of bank reserves and a collateral constraint at the level of the bank is a set of processes l_t, r_t, d_t, i_t^l, i_t^d, c_t^T, p_t, λ_t, and μ_t^B satisfying* (15), (16), (18), (19), (26), (28), *and* (A1)-(A5), *for $t \geq 0$, given a reserve remuneration policy i_t^r, a capital control tax rate τ_t^c, exogenous processes i_t^*, y_t^T, and y_t^N, and the initial condition* $(1+i_{-1}^*)d_{-1} > 0$.

A.4 Equivalence Result

We wish to show that given a reserve remuneration and capital control policy, i_t^r and τ_t^c, the competitive equilibrium allocation is the same in the baseline economy (with the collateral constraint imposed at the household level) and the present variant in which the collateral constraint is imposed at the bank level, except for the equilibrium value of the loan rate, i_t^l.

Consider processes for l_t, r_t, d_t, i_t^l, i_t^d, c_t^T, p_t, λ_t, and μ_t that constitute a competitive equilibrium of the economy with the collateral constraint at the household level, that is, processes that satisfy Definition 4. We wish to show that this allocation, with the exception of i_t^l, also satisfies Definition A1. Since (15), (16), (18), (19), (26), and (28), belong to both Definition 4 and Definition A1, and do not feature i_t^l, what needs to be shown is that equilibrium conditions (A1)-(A5) hold.

Let $i_t^{l,B}$ denote the equilibirum lending rate in the economy with the collateral constraint at the bank level. Suppose in a given date and state the collateral constraint is slack, $l_t < \kappa(y_t^T + p_t y_t^N)$, so that $\mu_t = 0$. Set $\mu_t^B = \mu_t = 0$ and $i_t^{l,B} = i_t^l$. Then (A1)-(A5) are satisfied. Next, consider the case that the collateral constraint binds in the baseline allocation, $l_t = \kappa(y_t^T + p_t y_t^N)$. Note that in this case $l_t > 0$. Set

$$\mu_t^B = \frac{\mu_t}{1-\mu_t}\frac{(1+i_t^l)}{(1+i_t^d)},$$

which implies that μ_t^B is strictly positive. The facts that μ_t^B is positive and that the collateral constraint holds with equality ensure that (A2)-(A4) hold. It remains to show that (A1) and (A5) are satisfied. Pick $i_t^{l,B}$ so that it satisfies the left-hand side expression of (A1) with equality. This then implies that the middle and the right-hand side expression of (A1) also hold and yields

$$1 + i_t^{l,B} = (1+i_t^d)(1+\Gamma_l(l_t, r_t)) + \mu_t \frac{\lambda_t}{\beta E_t \lambda_{t+1}}.$$

Now use the left expression of (14) holding with equality to replace $(1+i_t^d)(1+\Gamma_l(l_t, r_t))$ with $1+i_t^l$, and replace $(1+i_t^l)$ in turn with (20) holding with equality, which yields $(1+i_t^l) = (1-\mu_t)\frac{\lambda_t}{\beta E_t \lambda_{t+1}}$. Substituting these expressions in the above displayed equation, we obtain

$$(1+i_t^{l,B}) = \frac{\lambda_t}{\beta E_t \lambda_{t+1}}.$$

It follows that (A5) is satisfied, which is what we set out to show. Finally, to see that $i_t^{l,B} \geq i_t^l$ use the facts that $\frac{\lambda_t}{\beta E_t \lambda_{t+1}} = (1+i_t^l)/(1-\mu_t)$ and that $0 \leq \mu_t < 1$.

We therefore have established the following proposition:

Proposition A1 (Equivalence of Equilibrium with Collateral Constraints at the Bank or Household Level)**.** *Suppose the set of processes l_t, r_t, d_t, i_t^l, i_t^d, c_t^T, p_t, and λ_t is a competitive equilibrium in the economy with a collateral constraint at the household level (i.e., satisfies Definition 4). Then this set of processes also represents a competitive equilibrium of the economy with the collateral constraint at the bank level (i.e., satisfies Definition A1) except for the loan rate, i_t^l, in states in which the collateral constraint binds. In these states the loan rate is strictly larger in the economy with the collateral constraint at the bank level than in the economy with the collateral constraint at the household level.*

B Non-Equivalence of Reserve Requirements and Reserve Remuneration

Proposition B2. *[Non-Equivalence of Interest on Reserves and Reserve Requirements] In general, the constrained optimal allocation of Definition 5 does not satisfy the competitive equilibrium conditions of the economy with reserve requirements and capital controls listed in Definition 6.*

Proof. Consider the processes $\hat{c}^T_t$, $\hat{r}_t$, $\hat{l}_t$, and $\hat{d}_t$ that solve the optimization problem in Definition 5. By construction, these processes satisfy the bank's balance sheet (16) and the economy's sequential resource constraint (28). Set λ_t and p_t to satisfy equilibrium conditions (18) and (19). The collateral constraint (23) then is satisfied by construction. Consider a date in which $l_t > 0$ and the collateral constraint is slack.[4] Then, $\mu_t = 0$, which implies that equilibrium conditions (22) and (24) are satisfied. The interest rate on loans, i^l_t, is then determined residually by the Euler equation (20) holding with equality. It remains to check whether (26) and (35)-(37) also hold. Because $l_t > 0$, the left expression of (36) holds with equality. If $\eta_t = 0$, then this expression and (37) form a system of two equations in one unknown, i^d_t, which is in general inconsistent. If, on the other hand, η_t is different from zero, then the left expression in (35) holds with equality. This expression, the left expression in (36) holding with equality, and (37) represent a system of three equations in three unknowns, δ_t, i^d_t, and η_t. There are no guarantees, however, that the solution to this system will yield a non-negative value for the Lagrange multiplier on the reserve requirement, η_t.[5] □

[4] One can show that $l_t > 0$ at all times under the relatively weak assumption $\Gamma_l(0, r_t) = 0$, which is satisfied by the functional forms used in the quantitative analysis (see Section 5). The quantitative analysis further shows that under the assumed calibration in the constrained optimal allocation the collateral constraint is slack 65 percent of the time.

[5] For example, the real allocation in the calibrated economy of Section 6 implies values of η_t ranging from -0.03 to 0.05.

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