Deep Habits and Sticky Prices: A Technical Note*

Morten Ravn† Stephanie Schmitt-Grohé‡ Martín Uribe§

January 8, 2004

Abstract

This note applies the deep habit model developed in Ravn, Schmitt-Grohé, and Uribe (2003) to a sticky-price framework. *JEL Classification:* D10, D12, D42, E30.

Keywords: Habit formation, customer market models, switching cost models, time varying markups and business cycles, sticky prices.

*Newer versions of this paper are maintained at http://www.econ.duke.edu/~uribe.

†London Business School and CEPR. Phone: +44 207 262 5050 ext. 3717. E-mail: mravn@london.edu.

‡Duke University, CEPR, and NBER. Phone: 919 660 1889. E-mail: grohe@duke.edu.

§Duke University and NBER. Phone: 919 660 1888. E-mail: uribe@duke.edu.

1 The Model

1.1 Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0,1]$. Each household j has preferences defined over consumption of a continuum of differentiated consumption goods, c^j_{it} indexed by $i \in [0,1]$ and labor effort, h^j_t. Following Abel (1990), preferences feature 'catching up with the Joneses.' However, unlike in the work of Abel, we assume that consumption externalities operate at the level of each individual good rather than at the level of the composite final good. We refer to this variant as 'catching up with the Joneses good by good' or 'deeply rooted habits.' Specifically, we assume that household j derives utility from an object x^j_t defined by

$$x^j_t = \left[\int_0^1 \left(c^j_{it} - \theta c_{it-1}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}, \tag{1}$$

where $c_{it-1} \equiv \int_0^1 c^j_{it-1} dj$ denotes the lagged cross-section average level of consumption of variety i, which the household takes as exogenously given. The parameter θ measures the degree of time nonseparability in consumption of each variety. When $\theta = 0$, we have the benchmark case of time separable preferences. The parameter $\eta > 0$ denotes the intratemporal elasticity of substitution of habit-adjusted consumption across different varieties.

For any given level of x^j_t, purchases of each variety $i \in [0,1]$ in period t must solve the dual problem of minimimizing total expenditure, $\int_0^1 P_{it} c^j_{it} di$, subject to the aggregation constraint (1), where P_{it} denotes the nominal price of a good of variety i at time t. The optimal level of c^j_{it} for $i \in [0,1]$ is then given by

$$c^j_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x^j_t + \theta c_{it-1} \tag{2}$$

where $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$, is the nominal price of the composite consumption good. Note that consumption of each variety is decreasing in its relative price, P_{it}/P_t, increasing in the level of habit-adjusted consumption, x^j_t, and, for $\theta > 0$, increasing in past aggregate consumption of the variety in question. At the optimum, we have that $P_t x^j_t = \int_0^1 P_{it}(c^j_{it} - \theta c_{it-1}) di$.

The utility function of the household is assumed to be of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x^j_t, h^j_t), \tag{3}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, h_t^j denotes the number of hours worked by household j in period t, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, twice continuously differentiable, and strictly concave.

In each period $t \geq 0$, households have access to a risk-free nominal bond, B_t^j, that pays the gross nominal interest rate R_t in period $t+1$. Also, they must pay nominal lump-sum taxes in the amount T_t. Furthermore, households receive pure profits from the ownership of firms, Φ_t^j. Then, the representative household's period-by-period budget constraint can be written as

$$P_t x_t^j + \omega_t + B_t^j + T_t = R_{t-1} B_{t-1}^j + W_t h_t^j + \Phi_t^j, \tag{4}$$

where $\omega_t \equiv \theta \int_0^1 P_{it} c_{it-1} di$ and W_t denotes the nominal wage rate. The household takes both ω_t and W_t as given. In addition, households are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games. Then, the representative household's problem can be stated as consisting in choosing processes x_t^j, h_t^j, and B_t^j so as to maximize the lifetime utility function (3) subject to (4) and a no-Ponzi-game constraint, taking as given the processes for W_t, ω_t, P_t, T_t, R_t, and Φ_t^j and inital asset holdings $R_{-1} B_{-1}^j$.

The first-order conditions associated with the household's problem are (4),

$$-\frac{U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)} = \frac{W_t}{P_t} \tag{5}$$

and

$$U_x(x_t^j, h_t^j) = \beta R_t E_t \left[U_x(x_{t+1}^j, h_{t+1}^j) \frac{P_t}{P_{t+1}} \right] \tag{6}$$

1.2 The Government

Each period $t \geq 0$, the government demands g_t units of a composite good that is made of different varieties according to the relationship $g_t = \left[\int_0^1 g_{it}^{1-1/\eta} di\right]^{1/(1-1/\eta)}$. The variable g_t is assumed to be exogenous and stochastic. The government buys the necessary amounts of intermediate goods so as to minimize the total cost of this purchases, $\int_0^1 P_{it} g_{it} di$, subject to $g_t \geq \left[\int_0^1 g_{it}^{1-1/\eta} di\right]^{1/(1-1/\eta)}$. The resulting demands for intermediate goods are $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$. The resulting expenditure function is simply given by $P_t g_t$. Public spending is assumed to be fully financed via lump-sum taxes. That is,

$$g_t = \tau_t,$$

where $\tau_t \equiv T_t/P_t$. For now, we assume that $g_t = 0$ for all t.

Monetary policy takes the form of an interest-rate rule of the form

$$R_t - R^* = \alpha_\pi(\pi_t - \pi^*) + \alpha_y \left(\frac{y_t - y^*}{y^*}\right) + \epsilon_t,$$

where ϵ_t is an exogenous, mean-zero, iid shock.

1.3 Firms

Intermediate goods are produced by monoplistic firms. Each good $i \in [0, 1]$ is manufactured using labor as an input via the following production technology:

$$y_{it} = z_t h_{it}^\alpha, \tag{7}$$

where y_{it} denotes output of good i, h_{it} denotes labor, and z_t denotes an aggregate technology shock. In the aggregate, households demand $c_{it} \equiv \int_0^1 c_{it}^j dj$ units of good i for consumption purposes. Equation (2) implies that

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t + \theta c_{it-1} \tag{8}$$

where $x_t \equiv \int_0^1 x_t^j dj$. Firms are price setters and are assumed to stand ready to satisfy demand at the announced prices. Formally, firm i must satisfy

$$y_{it} \geq c_{it}. \tag{9}$$

Firm i's profits in period t are given by

$$\Phi_t \equiv \left[P_{it}c_{it} - W_t h_{it} - \frac{\zeta}{2} P_t \left(\frac{P_{it}}{P_{it-1}} - \tilde{\pi}_t\right)^2\right]. \tag{10}$$

We introduce sluggish price adjustment by assuming that firms face quadratic price adjustment cost following Rotemberg (1982). A firm incurs price adjustment costs whenever the change in P_{it} deviates from some benchmark measure of inflation $\tilde{\pi}_t$. The parameter $\zeta \geq 0$ measures the degree of price stickiness in the model. If $\zeta = 0$, then prices are perfectly flexible and if $\zeta > 0$, then prices are sticky. The larger is ζ, the more sticky are prices. We will consider two alternative specifications for $\tilde{\pi}_t$. Under the first specification, price adjustment costs arise whenever P_{it} grows at a rate different from the steady-state inflation rate, π^*, that is, $\tilde{\pi}_t = \pi^*$ for all t. This specification is the one typically assumed in the related literature.[1] Under the second specification $\tilde{\pi}_t = \pi_{t-1}$, where $\pi_t \equiv P_t/P_{t-1}$ denotes the inflation rate. This formulation implies that it is costly for firms to adjust the current product price at a rate other than the previous period's inflaiton. This specification introduces price indexation into the model.

The firm's problem consists in choosing processes P_{it}, y_{it}, h_{it} so as to maximize the present discounted value of profits,

$$E_0 \sum_{t=0}^{\infty} q_t \Phi_t^i, \tag{11}$$

subject to (7), (8), and (9), given processes q_t, W_t, z_t, P_t, $\tilde{\pi}_t$ and x_t. The variable q_t is a pricing kernel determining the period-zero utility value of one unit of the composite good delivered in a particular state of period t. It follows from the household's problem that $q_t P_t \equiv \beta^t U_x(x_t, h_t)$.

The Lagrangean of firm i's problem can be written as

$$\begin{aligned} \mathcal{L} \;\; = \;\; & E_0 \sum_{t=0}^{\infty} q_t \left\{ P_{it} c_{it} - W_t h_{it} - P_t \frac{\zeta}{2} \left(\frac{P_{it}}{P_{it-1}} - \tilde{\pi}_t \right)^2 \right. \\ & \left. + P_t \gamma_t \left(z_t h_{it} - c_{it} \right) + \nu_t P_t \left[\left(\frac{P_{it}}{P_t} \right)^{-\eta} x_t + \theta c_{it-1} - c_{it} \right] \right\} \end{aligned}$$

The associated first order conditions are:

$$\nu_t = \theta E_t \left[\frac{q_{t+1} P_{t+1}}{q_t P_t} \nu_{t+1} \right] + \frac{P_{it}}{P_t} - \frac{W_t}{z_t P_t}$$

$$\zeta E_t \left[\frac{q_{t+1} P_{t+1}}{q_t P_t} \left(\frac{P_{it+1}}{P_{it}} - \tilde{\pi}_{t+1} \right) \frac{P_{it+1}}{P_{it}} \frac{P_t}{P_{it}} \right] = \zeta \frac{P_t}{P_{it-1}} \left(\frac{P_{it}}{P_{it-1}} - \tilde{\pi}_t \right) - c_{it} + \eta \nu_t \frac{P_t}{P_{it}} (c_{it} - \theta c_{it-1})$$

[1] Often it is assumed in addition that $\pi^* = 1$, so that unless prices are constant over time, firms must pay price adjustment costs.

1.4 Equilibrium

Because households are identical, consumption and labor supplies are invariant across them. We constrate attention to symmetric equilibria where all firms charge the same price. Therefore, we can drop the subscript i and the superscript j from all variables. The equilibrium conditions are then given by

$$R_t - R^* = \alpha_\pi(\pi_t - \pi^*) + \alpha_y \left(\frac{y_t - y^*}{y^*}\right) + \epsilon_t,$$

$$\tilde{\pi}_t = \delta\pi^* + (1-\delta)\pi_{t-1}$$

$$y_t = z_t h_t$$

$$x_t = c_t - \theta^s c_{t-1}$$

$$U_x(x_t, h_t) = \beta R_t E_t \left[U_x(x_{t+1}, h_{t+1})\frac{1}{\pi_{t+1}}\right] \tag{12}$$

$$w_t = -\frac{U_h(x_t, h_t)}{U_x(x_t, h_t)}$$

$$\nu_t = \theta^d \beta E_t \left[\frac{U_x(x_{t+1}, h_{t+1})}{U_x(x_t, h_t)}\nu_{t+1}\right] + 1 - \frac{w_t}{z_t}$$

$$\zeta\beta E_t \left[\frac{U_x(x_{t+1}, h_{t+1})}{U_x(x_t, h_t)}\left(\pi_{t+1} - \tilde{\pi}_{t+1}\right)\pi_{t+1}\right] = \zeta\pi_t\left(\pi_t - \tilde{\pi}_t\right) - c_t + \eta\nu_t(c_t - \theta^d c_{t-1})$$

$$c_t + \frac{\zeta}{2}\left(\pi_t - \tilde{\pi}_t\right)^2 = z_t h_t$$

1.5 Steady State

We use the following specificaiton for the single-period utility function

$$U(x,h) = \frac{[x(1-h)^{\gamma}]^{1-\sigma}}{1-\sigma}$$

Then, the steady-state equilibrium conditions are:

$$R = R^*$$

$$\pi^* = \beta R^*$$

$$\nu^* = \frac{1}{\eta(1-\theta^d)}$$

$$w^* = 1 - \nu^*(1-\beta\theta^d)$$

$$h^* = \frac{w^*}{w^* + \gamma(1-\theta^s)}$$

$$z^* = 1$$

$$c^* = h^*$$

$$y^* = h^*$$

$$x^* = c^*(1-\theta^s)$$

$$\tilde{\pi}^* = \pi^*$$

1.6 Equilibrium Under Superficial Habit, $\theta^d = 0$, $\theta^s > 0$

$$R_t - R^* = \alpha_\pi(\pi_t - \pi^*) + \alpha_y \left(\frac{y_t - y^*}{y^*} \right) + \epsilon_t,$$

$$\begin{gathered} x_t = c_t - \theta^s c_{t-1} \\ U_x(x_t, h_t) = \beta R_t E_t \left[U_x(x_{t+1}, h_{t+1}) \frac{1}{\pi_{t+1}} \right] \\ w_t = -\frac{U_h(x_t, h_t)}{U_x(x_t, h_t)} \\ \zeta E_t \left[\frac{U_x(x_{t+1}, h_{t+1})}{U_x(x_t, h_t)} \left(\pi_{t+1} - \tilde{\pi}_{t+1}\right) \pi_{t+1} \right] = \zeta \pi_t \left(\pi_t - \tilde{\pi}_t\right) - c_t + \eta c_t \left(1 - \frac{w_t}{z_t}\right) \\ c_t + \frac{\zeta}{2} \left(\pi_t - \tilde{\pi}_t\right)^2 = z_t h_t \end{gathered} \tag{13}$$

1.7 Equilibrium Under Flexible Prices and Deep Habit

$$R_t - R^* = \alpha_\pi(\pi_t - \pi^*) + \alpha_y \left(\frac{y_t - y^*}{y^*}\right) + \epsilon_t,$$

$$x_t = c_t - \theta^s c_{t-1}$$

$$w_t = -\frac{U_h(x_t, h_t)}{U_x(x_t, h_t)}$$

$$U_x(x_t, h_t) = \beta R_t E_t \left[U_x(x_{t+1}, h_{t+1})\frac{1}{\pi_{t+1}}\right] \tag{14}$$

$$\nu_t = \theta^d \beta E_t \left[\frac{U_x(x_{t+1}, h_{t+1})}{U_x(x_t, h_t)}\nu_{t+1}\right] + 1 - \frac{w_t}{z_t}$$

$$\nu_t = \frac{c_t}{\eta(c_t - \theta^d c_{t-1})}$$

$$c_t = z_t h_t$$

2 Findings

2.1 Flexible Prices, temporary technology shocks

Consider the case that $\zeta = 0$ and that $\rho_z = 0$. In this case the deep habit model has a unique equilibrium only for $\theta^d < .43$. The markup in the superficial model is constant. In response to a purely temporary techno shock the deep habib model experiences a sharp decline in the current markup followed by a persistent increase in the markup. Accordingly output rises sharply on impact and then converges fast to the steady state. Real wages rise and then fall below steady state in the first period after the shock.