# Deep Habits: Technical Notes\*

### Morten Ravn<sup> $\dagger$ </sup> Stephanie Schmitt-Grohé<sup> $\ddagger$ </sup> Martín Uribe<sup>§</sup>

October 18, 2004

This note derives in detail the model, its associated equilibrium conditions, the steadystate equilibrium, and the calibration of the fully-fledged version of the additive deep habit model presented in "Deep Habits" (M. Ravn, S. Schmitt-Grohé, and M. Uribe, 2004). *JEL Classification:* D10, D12, D42, E30.

*Keywords:* Habit formation, customer market models, brand switching cost models, time varying markups, business cycles.

## Contents

1	The Model         1.1       Households	<b>1</b> 1 3 4		
<b>2</b>	Stationary Symmetric Equilibrium Conditions			
3	Functional Forms	7		
4	Deterministic Steady State	inistic Steady State 7		
5	Calibration	10		

<sup>‡</sup>Duke University, CEPR, and NBER. Phone: 919 660 1889. E-mail: grohe@duke.edu.

<sup>\*</sup>File Location: c:/data/joint/deep\_habit/rbc/capital/mfiles/habit\_stock/separable

<sup>&</sup>lt;sup>†</sup>London Business School and CEPR. Phone: +44 207 262 5050 ext. 3717. E-mail: mravn@london.edu.

<sup>&</sup>lt;sup>§</sup>Duke University and NBER. Phone: 919 660 1888. E-mail: uribe@duke.edu.

### 1 The Model

#### 1.1 Households

The preferences of household  $j \in [0, 1]$  are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j - v_t, h_t^j),$$
 (1)

where  $v_t$  denotes an exogenous and stochastic preference shock following a univariate autoregressive process of the form

$$v_t = \rho_v v_{t-1} + \epsilon_t^v,$$

with  $\rho_v \in [0, 1)$  and  $\epsilon_t^v$  distributed i.i.d. with mean zero and standard deviation  $\sigma_v$ . This shock is meant to capture innovations to the level of private non-business absorption. The variable  $x_t^j$  is a composite of habit-adjusted consumption of a continuum of differentiated goods indexed by  $i \in [0, 1]$ . Formally,

$$x_t^j = \left[\int_0^1 \left(c_{it}^j - \theta s_{it-1}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)},\tag{2}$$

where  $s_{it-1}$  denotes the stock of external habit in consuming good *i* in period *t*. The stock of external habit is assumed to depend on a weighted average of consumption in all past periods. We assume that habits evolve over time according to the following law of motion

$$s_{it} = \rho s_{it-1} + (1-\rho)c_{it}.$$
(3)

The parameter  $\rho \in [0, 1)$  measures the speed of adjustment of the stock of external habit to variations in the cross-sectional average level of consumption of variety *i*. When  $\rho$  takes the value zero, habit is measured by past consumption. The demands for individual varieties are the solution to the dual problem of minimizing consumption expenditure, given by  $\int_0^1 P_{it} c_{it}^j di$ , where  $P_{it}$  denotes the nominal prices of good *i*, subject to the aggregation constraint (2). The resulting demand for variety *i* is of the form

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t + \theta s_{it-1},\tag{4}$$

where  $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$ , is a nominal price index.

Households are assumed to own physical capital. The capital stock held by household j,

denoted  $k_t^j$ , is assumed to evolve over time according to the following law of motion

$$k_{t+1}^{j} = (1-\delta)k_{t}^{j} + i_{t}^{j}, \tag{5}$$

where  $i_t^j$  denotes investment by household j in period t. Investment is a composite good produced using differentiated goods via the following technology:<sup>1</sup>

$$i_t^j = \left[\int_0^1 \left(i_{it}^j\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}.$$
(6)

For any given levels of  $i_t^j$ , purchases of each variety  $i \in [0, 1]$  in period t must solve the dual problems of minimizing total investment expenditure,  $\int_0^1 P_{it} i_{it}^j di$ , subject to the aggregation constraint (6). The optimal level of  $i_{it}^j$  for  $i \in [0, 1]$  is then given by

$$i_{it}^{j} = \left(\frac{P_{it}}{P_{t}}\right)^{-\eta} i_{t}^{j}.$$
(7)

At the optimum, we have that  $P_{t}i_{t}^{j} = \int_{0}^{1} P_{it}i_{it}^{j}di$ . On aggregate, households demand  $i_{it} \equiv \int_{0}^{1} i_{it}^{j}dj$  units of good *i* for investment purposes. Equation (7) implies that

$$i_{it} = p_{it}^{-\eta} i_t, \tag{8}$$

where  $i_t \equiv \int_0^1 i_t^j dj$ .

At the beginning of each period  $t \ge 0$ , household j rents its stock of capital to firms at the rate  $u_t$ . Households are assumed to have access to complete contingent claims markets. Let  $r_{t,t+j}$  denote the stochastic discount factor such that  $E_t r_{t,t+j} z_{t+j}$  is the period-t price of a random payment  $z_{t+j}$  in period t + j. In addition, households are assumed to be entitled to the receipt of pure profits from the ownership of firms,  $\Phi_t^j$ . Then, the representative household's period-by-period budget constraint can be written as

$$x_t^j + i_t^j + \omega_t + E_t r_{t,t+1} d_{t+1}^j = d_t^j + w_t h_t^j + \Phi_t^j + u_t k_t^j,$$
(9)

<sup>&</sup>lt;sup>1</sup>Note that we do not assume any habit in the production of investment goods. However, if we were to reinterpret our catching-up-with-the-Joneses habit model as a switching costs model, then one may plausibly argue that in fact, the aggregate investment good should depend not only on the current level of purchases of differentiated investment goods but also on their respective past levels. Alternatively, one could assume that there is habit formation in investment as well. For example, Giannoni and Woodford (2003) introduce superficial habit formation into an otherwise standard Neo-Keynesian aggregate-supply aggregate-demand model and interpret the object that is subject to habit formation not simply as private consumption but as total (private) aggregate demand. They justify the assumption of habit in the investment demand by saying that it should be understood as a proxy for adjustment costs in investment expenditure that imply an inertial response in the rate of investment spending.

where  $\omega_t \equiv \theta \int_0^1 P_{it}/P_t s_{it-1} di$ . The variable  $w_t$  denotes the real wage rate. In addition, households are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games. Household j's problem can then be stated as consisting in choosing processes  $x_t^j$ ,  $h_t^j$ ,  $i_t^j$ ,  $d_{t+1}^j$ , and  $k_t^j$ , so as to maximize the lifetime utility function (1) subject to (5), (9), and a borrowing constraint that prevents it from engaging in Ponzi-type schemes, given processes  $v_t$ ,  $\omega_t$ ,  $w_t$ ,  $r_{t,t+1}$ ,  $u_t$ , and  $\Phi_t^j$ .

The optimality conditions associated with this problem are (5), (9), a transversality condition, and

$$-\frac{U_h(x_t^j - v_t, h_t^j)}{U_x(x_t^j - v_t, h_t^j)} = w_t$$
$$U_x(x_t^j - v_t, h_t^j) = \beta E_t U_x(x_{t+1}^j - v_{t+1}, h_{t+1}^j) [1 - \delta + u_{t+1}]$$
$$U_x(x_t^j - v_t, h_t^j) r_{t,t+1} = \beta U_x(x_{t+1}^j - v_{t+1}, h_{t+1}^j)$$

#### 1.2 The Government

Each period  $t \ge 0$ , nominal government spending is given by  $P_t g_t$ . We assume that real government expenditures, denoted by  $g_t$ , are exogenous, stochastic, and follow a univariate first-order autoregressive process of the form

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g,$$

where the innovation  $\epsilon_t^g$  distributes i.i.d. with mean zero and standard deviation  $\sigma_g$ . The government allocates spending over individual varieties of goods,  $g_{it}$ , so as to maximize the quantity of a composite good produced with differentiated varieties of goods according to the relationship

$$x_t^g = \left[\int_0^1 (g_{it} - \theta s_{it-1}^g)^{1-1/\eta} di\right]^{1/(1-1/\eta)}$$

The variable  $s_{it}^g$  denotes the government's stock of habit in good *i*, and is assumed to evolves over time according to the following expression

$$s_{it}^g = \rho s_{it-1}^g + (1-\rho)g_{it}.$$
(10)

We justify our specification of the aggregator function for government consumption by assuming that private households value government spending in goods in a way that is separable from private consumption and leisure and that households derive habits on consumption of government provided goods. The government's problem consists in choosing  $g_{it}$ ,  $i \in [0, 1]$ , so as to maximize  $x_t^g$  subject to the budget constraint  $\int_0^1 P_{it}g_{it} \leq P_tg_t$ , taking as given the initial condition  $g_{it} = g_t$  for t = -1, all *i*. In solving this maximization problem, the government takes as given the effect of current public consumption on the level of next period's composite good—i.e., habits in government consumption are external. Conceivably, government habits could be treated as internal to the government even if they are external to their beneficiaries, namely, households. This, alternative, however, is analytically less tractable. The case of no habits in government consumption results from setting  $\theta = 0$  in the above aggregator function for public goods. We believe that this is not the case of greatest interest under our maintained assumption that government spending on goods is valued by habit-forming private agents.

The resulting demand for each differentiated good  $i \in [0, 1]$  by the public sector is

$$g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t^g + \theta s_{it-1}^g, \tag{11}$$

where

$$x_t^g = g_t - \theta \int_0^1 \frac{P_{it}}{P_t} s_{it-1}^g di$$

Public spending is assumed to be fully financed by lump-sum taxation.

#### 1.3 Firms

Each good  $i \in [0, 1]$  is manufactured using labor and capital as inputs via the following production technology:

$$y_{it} = A_t F(k_{it}, h_{it}) - \phi, \qquad (12)$$

where  $y_{it}$  denotes output of good *i*,  $k_{it}$  and  $h_{it}$  denote services of capital and labor, and  $\phi$  denotes fixed costs of production. The presence of fixed costs introduces increasing returns to scale in the production technology. We include fixed costs to ensure that profits are relatively small on average as is the case for the U.S. economy in spite of equilibrium markups of price over marginal cost significantly above zero. The variable  $A_t$  denotes an aggregate technology shock. We assume that the logarithm of  $A_t$  follows a first-order autoregressive process

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a, \tag{13}$$

where  $\epsilon_t^a$  is a white noise disturbance with standard deviation  $\sigma_a$ .

Firms are price setters. In exchange, they must stand ready to satisfy demand at the announced prices. Formally, firm i must satisfy

$$A_t F(k_{it}, h_{it}) - \phi \ge c_{it} + i_{it} + g_{it}.$$
(14)

where  $c_{it}$ ,  $i_{it}$ , and  $g_{it}$  are given by equations (4), (8), and (11), respectively.

Firm *i*'s problem consists in choosing processes  $p_{it}$ ,  $c_{it}$ ,  $g_{it}$ ,  $i_{it}$ ,  $h_{it}$ , and  $k_{it}$ , so as to maximize the present discounted value of profits, which is given by

$$E_0 \sum_{t=0}^{\infty} r_{0,t} \left[ p_{it} (c_{it} + i_{it} + g_{it}) - w_t h_{it} - u_t k_{it} \right],$$
(15)

subject to (3), (4), (8), (10), (11), and (14), given processes  $r_{0,t}$ ,  $w_t$ ,  $u_t$ ,  $A_t$ ,  $x_t^g$ , and  $x_t$  and given  $c_{i-1}$  and  $g_{i-1}$ . The Lagrangian associated with firm *i*'s optimization problem can be written as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} r_{0,t} \left\{ p_{it} c_{it} + p_{it}^{1-\eta} i_t + p_{it} g_{it} - w_t h_{it} - u_t k_{it} \right. \\ \left. + \kappa_t \left[ A_t F(k_{it}, h_{it}) - \phi - c_{it} - p_{it}^{-\eta} i_t - g_{it} \right] \right. \\ \left. + \nu_t \left[ p_{it}^{-\eta} x_t + \theta s_{it-1} - c_{it} \right] + \lambda_t \left[ \rho s_{it-1} + (1-\rho) c_{it} - s_{it} \right] \right. \\ \left. + \nu_t^g \left[ p_{it}^{-\eta} x_t^g + \theta s_{it-1}^g - g_{it} \right] + \lambda_t^g \left[ \rho s_{it-1}^g + (1-\rho) g_{it} - s_{it}^g \right] \right\},$$

The first-order conditions associated with the firm's problem are equations (3), (4), (8), (10), (11), (14), and (taking derivatives of the Lagrangian with respect to  $c_{it}$ ,  $s_{it}$ ,  $g_{it}$ ,  $s_{it}^{g}$ ,  $h_{it}$ ,  $k_{it}$ , and  $p_{it}$  in this order)

$$p_{it} - \nu_t - \kappa_t + \lambda_t (1 - \rho) = 0$$

$$\theta E_t r_{t,t+1} \nu_{t+1} + \rho E_t r_{t,t+1} \lambda_{t+1} = \lambda_t$$

$$p_{it} - \nu_t^g - \kappa_t + \lambda_t^g (1 - \rho) = 0,$$
  

$$\theta E_t r_{t,t+1} \nu_{t+1}^g + \rho E_t r_{t,t+1} \lambda_{t+1}^g = \lambda_t^g$$
  

$$\kappa_t = \frac{w_t}{A_t F_h(k_{it}, h_{it})}$$
  

$$\kappa_t = \frac{u_t}{A_t F_k(k_{it}, h_{it})}$$
  

$$c_{it} + (1 - \eta) p_{it}^{-\eta} i_t + g_{it} + \eta \kappa_t p_{it}^{-\eta - 1} i_t - \eta \nu_t p_{it}^{-\eta - 1} x_t - \eta \nu_t^g p_{it}^{-\eta - 1} x_t^g = 0$$

Of particular interest in our analysis is the equilibrium behavior of the markup of prices over marginal costs. Noting that the marginal cost is given by  $w_t/[A_tF_h(k_t, h_t)]$ , and that in equilibrium the relative price of any variety of goods in terms of the composite habit-adjusted consumption good is unity, we have that the markup is given by is

$$\mu_t = \frac{A_t F_h(k_t, h_t)}{w_t}.$$
(16)

This expression shows that the markup acts as a time varying wedge between the wage rate and the marginal product of labor. Our focus will be in determining the cyclicality of this wedge in response to various aggregate disturbances.

## 2 Stationary Symmetric Equilibrium Conditions

$$\frac{U_h(c_t - \theta_{s_{t-1}} - v_t, h_t)}{U_x(c_t - \theta_{s_{t-1}} - v_t, h_t)} = w_t \tag{17}$$

$$U_x(c_t - \theta s_{t-1} - v_t, h_t) = \beta E_t U_x(c_{t+1} - \theta s_t - v_{t+1}, h_{t+1})[1 - \delta + u_{t+1}]$$
(18)

$$A_t F(k_t, h_t) - \phi = c_t + i_t + g_t$$
(19)

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{20}$$

$$\frac{c_t + g_t + i_t}{\eta} = \nu_t (c_t - \theta^d s_{t-1}) + \nu_t^g (g_t - \theta^g s_{t-1}^g) + i_t (1 - \frac{w_t}{A_t F_h(k_t, h_t)})$$
(21)

$$\frac{1 - \nu_t - \frac{w_t}{A_t F_h(k_t, h_t)}}{\rho - 1} = \beta E_t \frac{U_x(c_{t+1} - \theta s_t - \nu_{t+1}, h_{t+1})}{U_x(c_t - \theta s_{t-1} - \nu_t, h_t)} \left\{ \theta^d \nu_{t+1} + \rho \frac{1 - \nu_{t+1} - \frac{w_{t+1}}{A_{t+1} F_h(k_{t+1}, h_{t+1})}}{\rho - 1} \right\}$$
(22)

$$\frac{1 - \nu_t^g - \frac{w_t}{A_t F_h(k_t, h_t)}}{\rho - 1} = \beta E_t \frac{U_x(c_{t+1} - \theta s_t - v_{t+1}, h_{t+1})}{U_x(c_t - \theta s_{t-1} - v_t, h_t)} \left\{ \theta^g \nu_{t+1}^g + \rho \frac{1 - \nu_{t+1}^g - \frac{w_{t+1}}{A_{t+1} F_h(k_{t+1}, h_{t+1})}}{\rho - 1} \right\}$$
(23)

$$\frac{F_h(k_t, h_t)}{F_k(k_t, h_t)} = \frac{w_t}{u_t} \tag{24}$$

$$s_t = \rho s_{t-1} + (1 - \rho)c_t \tag{25}$$

$$s_t^g = \rho s_{t-1}^g + (1-\rho)g_t \tag{26}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a \tag{27}$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{28}$$

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g \tag{29}$$

This is a system of 13 nonlinear, stochastic, difference equations in 13 unknowns. We look for a stationary solution to this system.

## 3 Functional Forms

Consistent with the estimation strategy we adopt in "Deep Habits," we assume that the period utility index is separable in consumption and leisure. Specifically, preferences are of the form  $l=\sigma - 1 - \sigma - 1$ 

$$U(x,h) = \frac{x^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{(1-h)^{1-\chi} - 1}{1-\chi},$$

where  $0 < \sigma \neq 1$ ,  $0 < \chi \neq 1$ , and  $\gamma > 0$ . In the special case in which  $\sigma$  and  $\chi$  approach unity, this utility function converges to the log-linear specification used by King, Plosser, and Rebelo (1988) among many other business-cycle studies. The production function is assumed to be of the Cobb-Douglas type

$$F(k,h) = k^{\alpha} h^{1-\alpha}; \quad \alpha \in (0,1).$$

## 4 Deterministic Steady State

Consider shutting off all sources of uncertainty and letting the system settle on a stationary point where for any variable  $z_t$  we have  $z_t = z_{t+1} = z$  for all t. In this state, the equilibrium conditions (17)-(29) collapse to:

$$\frac{\gamma[c(1-\theta)]^{\sigma}}{(1-h)^{\chi}} = w \tag{30}$$

$$1 = \beta [1 - \delta + u] \tag{31}$$

$$k^{\alpha}h^{1-\alpha} = c + i + g + \phi \tag{32}$$

$$i = \delta k \tag{33}$$

$$\frac{c+g+i}{\eta} = \nu c(1-\theta^d) + \nu^g g(1-\theta^g) + i \left[1 - \frac{w}{(1-\alpha)(k/h)^{\alpha}}\right]$$
(34)

$$\left[1 - \frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] = \nu \left[\frac{\beta\theta^d(\rho-1) + 1 - \beta\rho}{1-\beta\rho}\right]$$
(35)

$$\left[1 - \frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] = \nu^g \left[\frac{\beta\theta^g(\rho-1) + 1 - \beta\rho}{1-\beta\rho}\right]$$
(36)

$$\frac{1-\alpha}{\alpha}\frac{k}{h} = \frac{w}{u} \tag{37}$$

$$s = c \tag{38}$$

$$s^g = g \tag{39}$$

$$A = 1$$

$$v = 0$$

$$g = \bar{g}.$$
(40)

This is a system of 13 equations in the following 26 unknowns: the 13 steady-state values of the variables variables  $c_t$ ,  $s_t$ ,  $h_t$ ,  $w_t$ ,  $v_t$ ,  $u_t$ ,  $A_t$ ,  $i_t$ ,  $g_t$ ,  $k_t$ ,  $\nu_t$ ,  $\nu_t^g$ , and  $s_t^g$ ; and 13 structural parameters  $\sigma$ ,  $\theta$ ,  $\delta$ ,  $\beta$ ,  $\eta$ ,  $\alpha$ ,  $\phi$ ,  $\chi$ ,  $\gamma$ ,  $\rho$ ,  $\theta^d$ ,  $\theta^g$ ,  $\bar{g}$ , To identify all 26 unknowns, we impose 13 calibration restrictions, whose empirical justification is provided later in section 5:

$$s_{h} \equiv \frac{wh}{k^{\alpha}h^{1-\alpha} - \phi} = 0.75$$

$$s_{c} \equiv \frac{c}{k^{\alpha}h^{1-\alpha} - \phi} = 0.7$$

$$s_{g} \equiv \frac{\bar{g}}{k^{\alpha}h^{1-\alpha} - \phi} = 0.12$$

$$k^{\alpha}h^{1-\alpha} - \phi - uk - wh = 0$$

$$R \equiv 1 - \delta + u = 1.04^{1/4}$$

$$\theta = \theta^{d} = \theta^{g} = 0.86$$

$$\rho = 0.85$$

$$\eta = 5.3$$

$$h = 0.2$$

$$\sigma = 2$$

$$\epsilon_{hw} \equiv \frac{\partial \ln h}{\partial \ln w}\Big|_{\lambda \text{ constant}}$$

$$= \frac{1 - h}{h\chi}$$

$$= 1.3$$

The 9th and the last restrictions can be solved for  $\chi$ . Equation (31) can be solved for  $\beta$ 

$$\beta = \frac{1}{R}.$$

Using equation (16) defining the equilibrium markup  $\mu_t$ , we can write (34) as

$$\frac{1}{\eta} = \nu s_c (1 - \theta^d) + \nu^g s_g (1 - \theta^g) + s_i \left[ 1 - \frac{1}{\mu} \right]$$

Now use equations (35) and (36) to eliminate  $\nu$  and  $\nu^{g}$  from this expression.

$$1 = \left[1 - \frac{1}{\mu}\right] \eta \left\{ s_c \left[\frac{1 - \beta\rho}{\beta\theta^d(\rho - 1) + 1 - \beta\rho}\right] (1 - \theta^d) + \left[\frac{1 - \beta\rho}{\beta\theta^g(\rho - 1) + 1 - \beta\rho}\right] s_g(1 - \theta^g) + s_i \right\}$$

Rearranging, we obtain the following expression for the steady-state markup

$$\mu = \frac{\eta m}{\eta m - 1},$$

where

$$m \equiv s_c \left[ \frac{(1-\beta\rho)(1-\theta^d)}{\beta\theta^d(\rho-1)+1-\beta\rho} \right] + s_g \left[ \frac{(1-\beta\rho)(1-\theta^g)}{\beta\theta^g(\rho-1)+1-\beta\rho} \right] + s_i \le 1$$
(41)

Using (16) and (37) we can write the zero-profit condition (41) as

$$\phi = \left(1 - \frac{1}{\mu}\right) k^{\alpha} h^{1 - \alpha}$$

It follows that the labor share,  $s_h \equiv wh/y$ , is given by

$$s_h = 1 - \alpha.$$

To determine  $\delta$  we use the following relation

$$\delta = \frac{i}{k}$$

$$= s_i \frac{y}{k}$$

$$= \frac{s_i}{s_k} u$$

$$= \frac{1 - s_c - s_g}{s_h} \frac{1 - \alpha}{\alpha} [\beta^{-1} - 1 + \delta]$$

The first equality uses (33), and the last one uses (32), (31), and (37). At this point, we have identified 10 of the 13 structural parameters, namely,  $\sigma$ ,  $\theta$ ,  $\delta$ ,  $\beta$ ,  $\eta$ ,  $\alpha$ ,  $\chi$ ,  $\rho$ ,  $\theta^d$ ,  $\theta^g$ . It remains to determine values for the parameters  $\gamma$ ,  $\bar{g}$  and  $\phi$  and steady-state values for the endogenous variables of the model. The steady-state value of the rental rate of capital, u, is

given by

$$u = \beta^{-1} - (1 - \delta)$$

To obtain the deterministic-steady-state level of the capital stock, solve (33) for k. This yields

$$k = \frac{i}{\delta}$$
$$= \frac{s_i}{\delta} [k^{\alpha} h^{1-\alpha} - \phi]$$
$$= \frac{s_i}{\delta \mu} k^{\alpha} h^{1-\alpha}$$
$$= \left[ \frac{s_i}{\delta \mu} \right]^{\frac{1}{1-\alpha}} h$$

Knowing k and h, the parameter  $\phi$  was found above to be equal to  $(1 - 1/\mu)k^{\alpha}h^{1-\alpha}$ . The production technology delivers the steady-state value of output:

$$y = k^{\alpha} h^{1-\alpha} - \phi.$$

The steady-state values of the components of aggregate demand follow immediately

$$c = s_c y$$
$$i = s_i y$$
$$g = s_g y$$

Equations (35), (36), (37), (38), (39), and (40) determine directly the steady-state values of  $\nu$ ,  $\nu^{g}$ , w, s,  $s^{g}$ , and  $\bar{g}$ , respectively. Finally, we can solve (30) for  $\gamma$  to obtain

$$\gamma = \frac{w(1-h)^{\chi}}{[(1-\theta)c]^{\sigma}}$$

## 5 Calibration

We calibrate the model to the U.S. economy. The time unit is meant to be one quarter. Table 1 summarizes the calibration. In "Deep Habits," we estimate the preference parameters pertaining to the deep habit model. Based on that estimation we set  $\theta = 0.86$ ,  $\rho = 0.85$ ,  $\eta = 5.3$ , and  $\sigma = 2$ . ollowing Prescott (1986), we set the preference parameter  $\gamma$  to ensure that in the deterministic steady state households devote 20 percent of their time to market

Symbol	Value	Description
$\beta$	0.9902	Subjective discount factor
$\sigma$	2	Inverse of intertemporal elasticity of substitution
heta	0.86	Degree of habit formation
$\rho$	0.85	Persistence of habit stock
$\alpha$	0.25	capital elasticity of output
$\delta$	0.0253	Quarterly depreciation rate
$\eta$	5.3	Elasticity of substitution across varieties
$\epsilon_{hw}$	1.3	Frisch elasticity of labor supply
h	0.2	Steady-State fraction of time devoted to work
$ar{g}$	0.0318	Steady-state level of government purchases
$\phi$	0.0853	Fixed cost
$ ho_v,  ho_g,  ho_a$	0.9	Persistence of exogenous shocks

Table 1: Calibration

activities. The calibration restrictions that identify the remaining structural parameters of the model are taken from Rotemberg and Woodford (1992). We follow their calibration strategy to facilitate comparison of our model of endogenous markups due to deep habits to their ad-hoc version of the customer-market model. In particular, we set the labor share in GDP to 75 percent, the consumption share to 70 percent, the government consumption share to 12 percent, the annual real interest rate to 4 percent, and the Frisch labor supply elasticity to 1.3. These restrictions imply that the capital elasticity of output in production,  $\alpha$ , is 0.25, the depreciation rate,  $\delta$ , is 0.025 per quarter, the subjective discount factor,  $\beta$ , is 0.99, and that the preference parameter  $\chi$  is 3.08.

As whown earlier, the steady-state markup of price over marginal cost,  $\mu$ , is given by  $\mu = \eta m/(\eta m - 1)$ , where  $m \leq 1$  is given in equation (41). Our calibration implies a somewhat high average markup of 1.32. Note that in the case of perfect competition, that is, when  $\eta \to \infty$ , the markup converges to unity. In the absence of deep habits, i.e., when  $\theta = 0$ , we have that m equals one, and the markup equals  $\eta/(\eta - 1) = 1.23$ , which relates the markup to the intratemporal elasticity of substitution across varieties in the usual way. This expression for the steady-state markup is the one that emerges from models with imperfect competition and superficial habit (e.g., Giannoni and Woodford, 2003; and Christiano, Eichenbaum, and Evans, 2003). Because under deep habits the parameter m is less than unity, firms have more market power under deep habits than under superficial habits. This is because firms take advantage of the fact that when agents form habits on a variety-by-variety basis, the short-run price elasticity of demand for each variety is less than  $\eta$ .

Finally, we set the serial correlation of all three shocks to 0.9 (i.e.,  $\rho_v = \rho_g = \rho_a = 0.9$ .).

These values are in the ball park of available estimates.