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ABSTRACT

This paper generalizes the standard habit formation model to an environment in which agents form habits over individual varieties of goods as opposed to over a composite consumption good. We refer to this preference specification as `deep habit formation'. Under deep habits, the demand function faced by individual producers depends on past sales. This feature is typically assumed ad-hoc in customer market and brand switching cost models. A central result of the paper is that deep habits give rise to countercyclical markups, which is in line with the empirical evidence. This result is important because ad-hoc formulations of customer-market and switching-cost models have been criticized for implying procyclical and hence counterfactual markup movements. The paper also provides econometric estimates of the parameters pertaining to the deep habit model.

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1 Introduction

The standard habit-persistence model assumes that households form habits from consumption of a single aggregate good. An important consequence of this assumption is that the introduction of habit formation alters the propagation of macroeconomic shocks only insofar as it modifies the way in which aggregate demand and possibly the supply of labor respond to such shocks.

In this paper, we generalize the concept of habit formation by considering the possibility that private agents do not simply form habit from their overall consumption levels, but rather are capable of becoming addicted to the consumption of individual goods. We have in mind an environment in which consumers can become hooked separately to narrowly defined good categories, such as chocolate, alcohol, music, cars, etc., not just to consumption defined broadly. We believe that this description of preferences, to which we refer as 'deep habits', is more compelling than its standard, or, in our terminology, 'superficial,' counterpart.

The assumption that agents can form habits on a good-by-good basis has two important implications for aggregate dynamics. First, the demand side of the macroeconomy is indistinguishable from that pertaining to an environment in which agents have superficial habits. Second, and more significantly, the supply side of the economy is altered in fundamental ways. Specifically, when habits are formed at the level of individual goods, firms take into account that the demand they will face in the future depends on their current sales. This is because, higher consumption of a particular good in the current period makes consumers, all other things equal, more willing to buy such good in the future through the force of habit.

Thus, when habits are deeply rooted, the optimal pricing problem of the firm becomes dynamic. Furthermore, one can show that in this case the firm's problem is akin to those studied in partial equilibrium models of brand-switching costs (Klemperer, 1995) or customer-market pricing (Phelps and Winter, 1970). The deep habit formation model can therefore be viewed as providing further microfoundations to such models and as a natural vehicle for incorporating them into a dynamic general equilibrium framework.

We embed the deep habit formation assumption into an economy with imperfectly competitive product markets. This combination results in a model of endogenous, time-varying markups of prices over marginal cost. A central result of this paper is that in the deep habit model markups behave countercyclically in equilibrium. In particular, we show that expansions in output driven by preference shocks, government spending shocks, or productivity shocks are accompanied by declines in markups. This is in line with the empirical literature extant, which finds markups to be countercyclical (Rotemberg and Woodford, 1991). The countercyclicality of markups is a particularly interesting implication of the deep habit model. For existing general equilibrium versions of customer-market and switching cost models have been criticized on the grounds that they predict procyclical markups (Rotemberg and Woodford, 1991, 1995). This criticism, however, is based on customer-market models in which the demand function faced by individual firms is specified ad-hoc and not derived from the optimizing behavior of households. Our results thus show that once the demand for individual goods is derived from first principles, then a customer-market model is capable of predicting empirically relevant cyclical behavior of markups.

The intuition for why the deep habit model induces countercyclical movements in markups is relatively straightforward. In a simple version of the deep habit model, the demand faced by an individual firm, say firm i, in period t is of the form $q_{it} = p_{it}^{-\eta}(q_t - \theta q_{t-1}) + \theta q_{it-1}$, where q_{it} denotes the demand for good i, p_{it} denotes the relative price of good i, and q_t denotes the level of aggregate demand. Firm i takes the evolution of q_t as given. The parameter $\theta \in [0, 1)$ measures the strength of habit for good i. Because the term θq_{it-1} in the demand function originates exclusively from habitual consumption of good i it is completely insensitive to movements in the current relative price. In turn, this price inelastic term of the demand function causes the price elasticity of demand for good i, given by $\eta/[1 + \theta q_{it-1}(p_{it}^{-\eta}(q_t - \theta q_{t-1}))^{-1}]$, to be increasing in the aggregate shifter $q_t - \theta q_{t-1}$. Because markups tend to be inversely related to the elasticity of demand, it follows that periods in which the quasi difference in aggregate demand $q_t - \theta q_{t-1}$ is above average are likely to be associated with relatively low markups. Under either superficial habits or no habits the demand function faced by firm i collapses to $q_{it} = p_{it}^{-\eta} q_t$, which results when θ is equal to zero. In this case, the price elasticity of demand for good i is constant and equals η implying a time-invariant markup.

A further contribution of this paper is to estimate the structural parameters of the deep habit model. Existing econometric estimates of the degree of habit formation identify the parameter describing habits from the consumer's Euler equation. This restriction continues to be present in the deep habit model. Thus, available estimates of the degree of habit formation can be interpreted as estimates of the degree of deep habit formation. However, the deep habit model contains additional equilibrium conditions that can be used to identify the habit parameter, namely, supply-side restrictions stemming from the optimal pricing decision of firms. In our econometric work we exploit these additional identifying restrictions to obtain more efficient estimates of the habit parameter. Our results are consistent with previous studies, which rely solely on Euler equation estimations, in that our findings suggest a relatively high degree of habit persistence and an inertial evolution of the stock of habit over time.

Section 2 develops the deep habit model in the context of a simple endowment economy.

Section 3 compares the deep habit model to other models of endogenous markup determination. Section 4 studies the equilibrium dynamics of the deep habit model within a fully fledged real-business-cycle environment with endogenous labor supply and capital accumulation. It investigates the response of markups to a variety of shocks. It also compares the markup dynamics implied by the deep-habit model to those arising from an ad-hoc formulation of the customer-market model. Section 5 estimates econometrically the parameters of the deep habit model. Section 6 closes the paper.

2 An Economy with Deep Habits

2.1 Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0,1]$. Each household j has preferences defined over consumption of a continuum of differentiated goods, c_{it}^j. Good varieties are indexed by $i \in [0,1]$. Households also value leisure and thus derive disutility from labor effort, h_t^j. Following Abel (1990), preferences feature 'catching up with the Joneses.' An important difference between Abel's specification and ours is that we assume that consumption externalities operate at the level of each individual good rather than at the level of the composite final good. We refer to this variant as 'catching up with the Joneses good by good' or 'deep habits.' Specifically, we assume that household j derives utility from an object x_t^j defined by

$$x_t^j = \left[\int_0^1 \left(c_{it}^j - \theta c_{it-1}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}, \tag{1}$$

where $c_{it-1} \equiv \int_0^1 c_{it-1}^j dj$ denotes the lagged cross-section average level of consumption of variety i, which the household takes as exogenously given. The parameter θ measures the degree of time nonseparability in consumption of each variety. When $\theta = 0$, we have the benchmark case of time separable preferences. The parameter $\eta > 0$ denotes the intratemporal elasticity of substitution of habit-adjusted consumption across different varieties.

For any given level of x_t^j, purchases of each variety $i \in [0,1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it}^j di$, subject to the aggregation constraint (1), where P_{it} denotes the nominal price of a good of variety i at time t. The optimal level of c_{it}^j for $i \in [0,1]$ is then given by

$$c_{it}^j = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t^j + \theta c_{it-1}, \tag{2}$$

where $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$, is a nominal price index. Note that consumption of each variety is decreasing in its relative price, P_{it}/P_t, increasing in the level of habit-adjusted consumption, x_t^j, and, for $\theta > 0$, increasing in past aggregate consumption of the variety in question. At the optimum, we have that $P_t x_t^j = \int_0^1 P_{it}(c_{it}^j - \theta c_{it-1}) di$.

The utility function of the household is assumed to be of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j, h_t^j), \tag{3}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, twice continuously differentiable, and strictly concave.

In each period $t \geq 0$, households have access to complete contingent claims markets. Let $r_{t,t+j}$ denote the stochastic discount factor such that $E_t r_{t,t+j} z_{t+j}$ is the period-t price of a random payment z_{t+j} in period $t+j$. In addition, households are assumed to be entitled to the receipt of pure profits from the ownership of firms, Φ_t^j. Then, the representative household's period-by-period budget constraint can be written as

$$x_t^j + \omega_t + E_t r_{t,t+1} d_{t+1}^j = d_t^j + w_t h_t^j + \Phi_t^j, \tag{4}$$

where $\omega_t \equiv \theta \int_0^1 P_{it}/P_t c_{it-1} di$ and w_t denotes the wage rate. In addition, households are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games. The representative household's optimization problem consists in choosing processes x_t^j, h_t^j, and d_{t+1}^j so as to maximize the lifetime utility function (3) subject to (4) and a no-Ponzi-game constraint, taking as given the processes for w_t, ω_t, and Φ_t^j and initial asset holdings d_0^j.

The first-order conditions associated with the household's problem are (4),

$$-\frac{U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)} = w_t \tag{5}$$

and

$$U_x(x_t^j, h_t^j) r_{t,t+1} = \beta U_x(x_{t+1}^j, h_{t+1}^j). \tag{6}$$

2.2 Firms

Intermediate goods are produced by monopolistically competitive firms. Each good $i \in [0,1]$ is manufactured using labor as an input via the following production technology:

$$y_{it} = A_t h_{it},$$

where y_{it} denotes output of good i, h_{it} denotes labor input, and A_t denotes an aggregate technology shock. Equation (2) implies that aggregate demand for good i, $c_{it} \equiv \int_0^1 c_{it}^j dj$, is given by

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t + \theta c_{it-1},$$

where $x_t \equiv \int_0^1 x_t^j dj$.

It is of interest to compare the short- and long-run price elasticities of demand implied by the above demand function, which are given by

$$\epsilon^{short-run} \equiv \frac{\partial \ln \left[\left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t + \theta c_{it-1}\right]}{\partial \ln P_{it}} = -\eta \left(\frac{c_{it} - \theta c_{it-1}}{c_{it}}\right)$$

and

$$\epsilon^{long-run} \equiv \frac{\partial \ln \left[(1-\theta)^{-1} \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t\right]}{\partial \ln P_{it}} = -\eta.$$

These expressions state that in the presence of deep habit, that is, when $\theta > 0$, the short-run demand elasticity is smaller (in absolute terms) than the long-run elasticity. The reason why the short-run elasticity is smaller than its long-run counterpart is that habits take time to adjust. Perhaps more importantly, the short-run price elasticity is time dependent. This characteristic of the deep-habit model gives rise to a time varying markup. In an economy without deep habits ($\theta = 0$), the short- and long-run elasticities are identical. The price elasticity of demand is time invariant and equal to η in an economy with standard habit formation, that is, habit formation at the level of the composite good.

Firms are assumed to be price setters, to take the actions of all other firms as given, and to stand ready to satisfy demand at the announced prices. Formally, firm i must satisfy

$$y_{it} \geq c_{it}.$$

Firm i's profits in period t are given by

$$\Phi_t^i \equiv \frac{P_{it}}{P_t} c_{it} - w_t h_{it}.$$

Note that real marginal costs are equal to w_t/A_t or that nominal marginal costs are

$$MC_t = P_t w_t / A_t.$$

It follows that marginal costs are independent of scale and that all firms face the same marginal costs. Let μ_t^i denote the markup of prices over marginal costs charged by firm i, and μ_t the average markup charged in the economy, that is,

$$\mu_t^i = \frac{P_{it}}{MC_t}$$

and

$$\mu_t = \frac{P_t}{MC_t}$$

We then can express period profits as

$$\Phi_t^i = \frac{\mu_{it} - 1}{\mu_t} c_{it} \tag{7}$$

and the aggregate demand faced by firm i as

$$c_{it} = \left(\frac{\mu_{it}}{\mu_t}\right)^{-\eta} x_t + \theta c_{it-1}, \tag{8}$$

The firm's problem consists in choosing processes μ_{it} and c_{it} so as to maximize the present discounted value of profits,

$$E_t \sum_{j=0}^{\infty} r_{t,t+j} \Phi_{t+j}^i, \tag{9}$$

subject to (7) and (8), given processes $r_{t,t+j}$, μ_t, and x_t.

The Lagrangian of firm i's problem can be written as

$$\mathcal{L} \;=\; E_0 \sum_{t=0}^{\infty} r_{0,t} \left\{ \frac{\mu_{it} - 1}{\mu_t} c_{it} + \nu_t \left[\left(\frac{\mu_{it}}{\mu_t}\right)^{-\eta} x_t + \theta c_{it-1} - c_{it} \right] \right\},$$

where ν_t is a Lagrange multiplier. The associated first-order conditions are (8) and

$$\begin{aligned}
\nu_t &= \frac{\mu_{it}-1}{\mu_t} + \theta E_t r_{t,t+1}\nu_{t+1} \\
c_{it} &= \eta\nu_t \left(\frac{\mu_{it}}{\mu_t}\right)^{-\eta-1} x_t.
\end{aligned}$$

The multiplier ν_t represents the shadow value of selling a unit of goods in period t. The first of the above optimality conditions states that the value of selling an extra unit of good in period t, ν_t, has two components. One is the short-run profit of a sale, given by $(\mu_{it}-1)/\mu_t$. A unit increase in sale in the current period triggers additional sales in the amount of θ in the next period. The present discounted value of these additional sales is $\theta E_t r_{t,t+1}\nu_{t+1}$, which is the second component. The second optimality condition equates the cost and benefit of increasing the markup. The benefit is given by increase in revenue in the amount of c_{it}. The cost is the decline in the demand for goods that the markup increase induces, $\eta\left(\frac{\mu_{it}}{\mu_t}\right)^{-\eta-1} x_t$ evaluated at the shadow value of sales, ν_t.

2.3 Equilibrium

Because all households are identical, consumption and labor supplies are invariant across them. It follows that we can drop the superscript j from all variables. We assume that the initial conditions c_{it} for $t=-1$ are the same for all intermediate goods i. Further, we restrict attention to symmetric equilibria in which all firms charge the same price. Therefore, we can also drop the subscript i from all variables. A competitive equilibrium can then be defined as a set of contingent plans $\{x_t, c_t, h_t, r_{t,t+1}, w_t, \nu_t, \mu_t\}$ satisfying

$$\begin{aligned}
x_t &= c_t - \theta c_{t-1}, && (10)\\
U_x(x_t,h_t)r_{t,t+1} &= \beta U_x(x_{t+1},h_{t+1}), && (11)\\
w_t &= -\frac{U_h(x_t,h_t)}{U_x(x_t,h_t)}, && (12)\\
\mu_t &= \frac{A_t}{w_t}, && (13)\\
c_t &= A_t h_t, && (14)\\
\nu_t &= \theta E_t r_{t,t+1}\nu_{t+1} + 1 - \frac{1}{\mu_t}, && (15)\\
c_t &= \eta(c_t - \theta c_{t-1})\nu_t. && (16)
\end{aligned}$$

It is of interest to compare these equilibrium conditions to those arising from the standard habit formation model. That is, from a model where the single-period utility function depends on current and past consumption of the composite good as opposed to current and past consumption of each particular variety as in our formulation. It is straight forward to show that this more usual model of habit persistence, to which we refer as 'superficial habits', shares with our deep habit model equilibrium conditions (10)-(14). The only difference to our model is that equilibrium conditions (15) and (16) are replaced by the requirement that the markup must be constant and equal to $\mu_t = \eta/(\eta - 1)$. This is a significant difference. For the introduction of deep habit renders the markup time varying. That is, the presence of deep habits introduces a dynamic wedge between marginal products of factor inputs and factor prices.

Additionally, we note that because the Euler equation (11) is common to the deep habit model as well as to the standard habit formation models, existing Euler-equation-based empirical estimates of the degree of habit formation can interpreted as uncovering the degree of deep habit. Because in our model the parameter θ that measures the strength of deep habits appears in equations other than the Euler equation (11), our model provides additional identification restrictions. In section 5, we exploit these additional restrictions in conjunction with the Euler equation to obtain a more efficient estimate of θ.

To facilitate comparison with alternative models of time-varying markups, it is convenient to express the markup as a function of measures of current demand and expected future profits. The reason is that these variables have been stressed by the related literature as important determinants of markup dynamics.

Iterating equation (15) forward and assuming that $\lim_{j\to\infty} \theta^j E_t r_{t,t+j}\nu_{t+j} = 0$, one can express ν_t as the present discounted value of expected future per unit profits induced by a unit increase in current sales

$$\nu_t = E_t \sum_{j=0}^{\infty} \theta^j r_{t,t+j} \left(\frac{\mu_{t+j} - 1}{\mu_{t+j}} \right). \tag{17}$$

Define $\gamma_t \equiv c_t/c_{t-1}$. Then (16) implies that

$$\nu_t = \frac{\gamma_t}{\eta(\gamma_t - \theta)}.$$

Using this expression to eliminate ν_t from (15) yields

$$\left[1 - \frac{1}{\mu_t}\right] = \frac{1}{\eta}\frac{\gamma_t}{\gamma_t - \theta} - \theta E_t r_{t,t+1}\nu_{t+1}. \tag{18}$$

This expression implicitly defines the markup as a function of consumption growth, γ_t, and the present discounted value of future profits induced by a unit increase in current sales,

$\theta E_t r_{t,t+1}\nu_{t+1}$. Clearly, if $\theta = 0$, then the markup is constant and equal to $1/\eta$. If $\theta \in (0,1)$, then the markup is time varying. The markup is decreasing in γ_t, that is, ceteris paribus, if current demand is high, the markup is low. The reason is that each firm has an incentive to lower their current period price in order to increase their market share, thereby paving the way for higher future profits due to formation of habits. The second argument of the markup function has two components, the discount factor $r_{t,t+1}$ and the present value of future total per unit profits, ν_{t+1}. The equilibrium markup is decreasing in the discount factor, which implies that, all else constant, a rise in the real interest rate should be associated with an increase in the current markup. This is because if the real interest rate is higher, then the firm discounts future profits more and thus has less incentives to invest in market share today. To the extend that the exogenous shocks driving economic fluctuations have different effects on the real interest rate, under deep habit they would also have different implications for the behavior of the markup. Finally, the markup is decreasing in ν_{t+1}, the value of future per unit profits discounted to period $t+1$. The intuition for why the current markup is decreasing in ν_{t+1} is that if total future per unit profits are expected to be high, then the value of having market share in the future is also high, and thus there is an incentive to build market share today. A higher market share in the future can be achieved by charging lower markups today.

3 Comparison To Alternative Theories of Time-Varying Markups

In this section, we compare the equilibrium markup implied by the deep habit model to other theories of time-varying markups. Our comparison does not aspire to be exhaustive. For instance, we omit discussion of models where markups vary endogenously over time due to sluggish product-price adjustment, as in Yun (1996) and the vast neo-Keynesian literature that ensued. Neither do we consider model in which the markup varies due to changes in the composition of aggregate demand, as in Bils (1989) or Galí (1994). To facilitate comparison, it will prove useful to solve equation (18) for the markup. This yields the following representation of the equilibrium markup

$$\mu_t = \mu\left(\frac{c_t}{c_{t-1}}, E_t\sum_{j=1}^{\infty}\theta^j r_{t,t+j}\frac{\Phi_{t+j}}{c_{t+j}}\right), \quad \mu_1, \mu_2 < 0, \tag{19}$$

where

$$\Phi_t \equiv \frac{\mu_t - 1}{\mu_t}\, c_t \tag{20}$$

denotes current profits.

3.1 Relation To Switching Costs Models

Switching cost models of time-varying markups (e.g., Klemperer, 1995 and the references therein) are based on the assumption that customers incur an ad-hoc fixed cost of changing brands. In our model this cost is endogenous and takes the form of habit, embodied in the fact that current utility depends on past consumptions of each variety of goods.

In this subsection, we demonstrate the formal similarity between the deep habit model and the switching cost model. Specifically, we relate our model to the switching costs model presented in Klemperer (1995). To this end, it is instructive to present the firm's optimization problem in a slightly different form. For firm i the amount it sold last period c_{it-1} is a state variable because it determines the level of demand in the current period. For simplicity consider the case of perfect foresight. Let $V(c_{it-1})$ be the value of the maximized objective function (9) for any given c_{it-1}. Then we can represent the firm's profit maximization problem in period t by the functional equation

$$V(c_{it-1}) = \max_{\mu_{it}, c_{it}} \left\{ \frac{\mu_{it} - 1}{\mu_t} c_{it} + r_{t,t+1} V(c_{it}) \right\}$$

$$\text{s.t.} \quad c_{it} = \left(\frac{\mu_{it}}{\mu_t} \right)^{-\eta} x_t + \theta c_{it-1}.$$

Using the constraint to eliminate c_{it}, the first-order condition for this problem is

$$\frac{d \frac{\mu_{it}-1}{\mu_t} c_{it}}{d\mu_{it}} + r_{t,t+1} \frac{dV(c_{it})}{dc_{it}} \frac{dc_{it}}{d\mu_{it}} = 0 \tag{21}$$

This equation says that at the optimum the marginal increase in profits associated with an increase in the current markup $\left(\frac{\partial \frac{\mu_{it}-1}{\mu_t} c_{it}}{\partial \mu_{it}}\right)$ must equal the present value of the loss in the firm's future total discounted profits associated with the fall in the current customer base, $\left(r_{t,t+1} \frac{\partial V(c_{it})}{\partial c_{it}} \frac{\partial c_{it}}{\partial \mu_{it}}\right)$.

But this is just like equation (2') of Klemperer (1995). Klemperer shows that in many-period switching costs models firm F's first-order condition is

$$\frac{\partial \pi_t^F}{\partial p_t^F} + \delta \frac{\partial V_{t+1}^F}{\partial \sigma_t^F} \frac{\partial \sigma_t^F}{\partial p_t^F} = 0,$$

where he denotes firm $F's$ period-t price by p_t^F, profit by π_t^F, the value function by V_t^F, market share by σ_t^F, and the per-period discount factor by δ. Noting that p_t^F and σ_t^F are both linearly related to μ_{it} and c_{it}, respectively, the optimality condition shown by Klemperer is identical to (21). Klemperer (1995, section 5.1) discusses that a rise in the real interest rate (or a decline in the discount factor) should lead to an increase in firm F's prices and hence markups because a higher interest rate reduces the marginal present value of an investment in market share. In addition he points out that an increase in the marginal value of market share in the future will lower current prices and hence markups.

Thus deep habit gives rise to the same first-order optimality condition at the individual firm level as do switching costs models. However, an important difference is that in the deep habit model there is gradual substitution between differentiated goods rather than discrete switches among suppliers. One advantage of this, from the point of view of analytical tractability, is that under the deep habit formulation one does not face an aggregation problem. In equilibrium buyers can distribute their purchases identically and still suppliers face a gradual loss of customers if they raise their relative prices.

3.2 Relation To Customer-Market Models

Deep habits imply that the markup of prices over marginal costs varies endogenously over the business cycle. Rotemberg and Woodford (1991, 1992, 1995) have embedded models of endogenous markup determination into stochastic dynamic general equilibrium models of the business cycle. They focus on two particular models of markup determination, the customer market model and the implicit collusion model. The customer market model is based on Phelps and Winter (1970). Rotemberg and Woodford assume ad hoc that the demand faced by an individual firm is given by

$$c_{it} = \eta\left(\frac{\mu_{it}}{\mu_t}, c_t\right) m_{it} \quad \eta_1 < 0, \quad \eta(1, c) = c$$

where m_{it} is the market share of firm i, which is assumed to evolve over time according to the following law of motion

$$m_{it+1} = g\left(\frac{\mu_{it}}{\mu_t}\right) m_{it}; \quad g' < 0, \quad g(1) = 1.$$

Rotemberg and Woodford motivate these two expressions as intended to capture the idea that customers have switching costs as in the models of Klemperer (1995) and others. Rotemberg and Woodford (1995, p. 281) point out that "[i]t would be attractive to obtain such a specification of demand from underlying aggregator functions for consumers [...] which would depend on previous purchases. Unfortunately we are unable to do so...." Our paper can be interpreted as providing microfoundations for the Rotemberg and Woodford specification of the customer market model.

Under the specification of demand for consumption of each variety of the Rotemberg-Woodford customer market model, in equilibrium the markup, which we denote by μ_t^{CM}, can be expressed as a function of current output and the present discounted value of total future profits, that is,

$$\mu_t^{CM} = \mu\left(c_t, E_t \sum_{j=1}^{\infty} r_{t,t+j}\Phi_{t+j}\right),$$

where Φ_t denotes period-t profits and is defined in equation (20). Rotemberg and Woodford show that $\mu_2 < 0$, that is, when the present value of total future profits is high, the markup falls. However, the sign of μ_1 is ambiguous, that is, the markup may increase or decrease with the level of current output. Rotemberg and Woodford further show that in the special case in which $\eta(.,.)$ and $\eta_1(.,.)$ are proportional to c, the markup is increasing in c_t and depends only on the ratio of current aggregate demand to the present value of total future profits. So in this case we have

$$\mu_t^{CM} = \mu\left(\frac{E_t \sum_{j=1}^{\infty} r_{t,t+j}\Phi_{t+j}}{c_t}\right); \quad \mu' < 0.$$

In all of their numerical work they restrict attention to this special case.

There are several noteworthy similarities and differences between the Rotemberg and Woodford version of the customer market model and our deep habit model. The main similarity is that both imply that the firm faces a demand function that depends on past sales. In addition, both imply that the current markup can be expressed as a function of just two variables, some measure of current demand and some measure of expected future profits. However, in the deep habit model the profit measure is the present discounted value of *per unit* profits whereas in the Rotemberg-Woodford version of the costumer market model, the relevant profit measure is the present discounted value of total future profits. In both models, the markup is decreasing in the respective measure of future profits. Second, in the deep habit model, the markup depends on the current growth rate of aggregate demand, whereas in the customer market model of Rotemberg and Woodford it depends simply on the current level of aggregate demand. Third, in the deep habit model the markup falls when current demand is strong whereas in the customer market model specifications the response of the markup is ambiguous. Indeed, in the case that Rotemberg and Woodford focus on, the markup is an increasing function of the current level of aggregate demand. Thus deep habit and this particular specification of the customer market models may imply quite different markup dynamics. We will return to this issue in section 4 when we compare the quantitative business cycle implications of the deep habit model and the customer market model. In Ravn, Schmitt-Grohé, and Uribe (2002), we explore a slightly different deep habit formulation. Specifically, we assume that household j derives utility from an object x_t^j defined by $x_t^j = \left[\int_0^1 \left(\frac{c_{it}^j}{c_{it-1}^{\theta}}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}$, that is, habit is relative rather than additive. Such a specification implies that the demand function faced by firms is $c_{it} = \left(\frac{P_{it}}{\tilde{P}_t}\right)^{-\eta} c_{it-1}^{\theta(1-\eta)} x_t$. This specification appears to be more similar to the ad hoc specification of the demand function in the customer market model that Rotemberg and Woodford study because previous sales enter in a multiplicative rather than additive fashion and the price elasticity of the demand faced by individual firms is independent of the state of current aggregate demand.

3.3 Relation to the Implicit Collusion Model

Rotemberg and Saloner (1986) and Rotemberg and Woodford (1992) develop a model of time-varying markups due to implicit collusion in imperfectly competitive product markets. In their model, collusion among firms is sustained by the credible threat of reverting to a perfect competition, marginal-cost-pricing regime in the event that any firm fails to abide to the terms of the implicit agreement. Thus, the maximum sustainable markup in the collusive equilibrium is decreasing in the short-run benefit from deviating from the implicit collusion and increasing in the long-run benefit of staying in the collusive relationship. The benefit of cheating is an increasing function of current output, while the benefit of sticking to the implicit pricing arrangement is an increasing function of the present discounted value of future expected profits. More formally, the markup in the implicit collusion model, denoted μ_t^{IC}, can be written as

$$\mu_t^{IC} = \mu\left(\frac{E_t \sum_{j=1}^{\infty} r_{t,t+j} \alpha^j \Phi_{t+j}}{c_t}\right); \quad \mu' > 0,$$

where, as before, Φ_t denotes profits in period t and is defined in equation (20). The parameter $\alpha \in (0,1)$ denotes the probability that the collusive agreement will not be renegotiated and is assumed to be exogenous. This expression is similar to its counterpart in the deep habit model, given by equation (19). As in the deep habit model under implicit collusion, the markup is decreasing in current output. However, in the implicit collusion model the markup is increasing in the present discounted value of a measure of future expected profits, whereas in the deep habit model it is decreasing. The important difference is that the profit measure in the implicit collusion model is the present discounted value of *total* future profits, while in the deep habit model the profit measure is a *per-unit* value.

We conclude from the comparison performed in this section that: (a) Our deep habit model can be interpreted as providing microfoundations to Klemperer-style switching cost or Phelps-Winter-style customer market models. (b) The deep habit model delivers equilibrium formulations of the markup that are similar to existing ad-hoc version of the switching cost or the customer market model. However, important differences separate the micro-founded from the ad-hoc versions. Importantly, as we will show explicitly later in this paper these differences result in quite dissimilar equilibrium dynamics of markups; and (c) Both the implicit collusion and the deep habit models predict that current markups are decreasing in current output. The implicit collusion and deep habits models differ in that markups increase with expected future profits under implicit collusion but decrease under deep habit. However, the measure of future profits upon the markup depends in each of these models is not the same.

4 Deep Habits And Markup Dynamics

In this section we embed deep habits into a fully fledged dynamic general equilibrium model of the business cycle. The goal of this section is to numerically characterize the equilibrium behavior of markups in response to a variety of aggregate demand and supply shocks. We contrast the response of markups in the model with deep habits to that arising in a number of related models: (a) models featuring no habit formation; (b) the standard habit formation model incorporating dynamic complementarities at the level of aggregate consumption, or superficial habit; and (c) the Rotemberg-Woodford (1995) ad-hoc version of the Phelps-Winter customer market model.

4.1 The Fully Fledged Model

The theoretical framework considered here is richer than the one studied in section 2 in that it features capital accumulation, a more general formulation of deep habits, and three sources of aggregate fluctuations: government purchases, preference shocks, and productivity shocks.

4.1.1 Households

Household $j \in [0,1]$'s preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j - v_t, h_t^j), \tag{22}$$

where v_t is an exogenous and stochastic preference shock that follows a univariate autoregressive process of the form

$$v_t = \rho_v v_{t-1} + \epsilon_t^v,$$

with $\rho_v \in [0,1)$ and ϵ_t^v distributed i.i.d. with mean zero and standard deviation σ_v. This shock is meant to capture innovations to the level of private non-business absorption. The parameter $\sigma > 0$ is the reciprocal of the intertemporal elasticity of substitution.

Unlike in the simple model of section 2, we now consider a preference specification in which the stock of external habit depends not only upon consumption in the previous period but also on consumption in all past periods. Formally, the level of habit-adjusted consumption, x_t^j, is now given by

$$x_t^j = \left[\int_0^1 \left(c_{it}^j - \theta s_{it-1}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}, \tag{23}$$

where s_{it-1} denotes the stock of external habit in consuming good i in period $t-1$, which is assumed to evolve over time according to the following law of motion

$$s_{it} = \rho s_{it-1} + (1-\rho) c_{it}. \tag{24}$$

The parameter $\rho \in [0,1)$ measures the speed of adjustment of the stock of external habit to variations in the cross-sectional average level of consumption of variety i. When ρ takes the value zero, preferences reduce to the simple case studied in section 2. These preferences imply an aggregate demand for variety i of the form

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t + \theta s_{it-1}. \tag{25}$$

Households are assumed to own and invest in physical capital. At the beginning of a given period t, household j owns capital in the amount of k_t^j that it can rent out at the rate u_t in period t. The capital stock is assumed to evolve over time according to the following law of motion

$$k_{t+1}^j = (1-\delta) k_t^j + i_t^j, \tag{26}$$

where i_t^j denotes investment by household j in period t. Investment is a composite good produced using intermediate goods via the technology[1]

$$i_t^j = \left[\int_0^1 \left(i_{it}^j\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}. \tag{27}$$

For any given levels of i_t^j, purchases of each variety $i \in [0,1]$ in period t must solve the dual problems of minimizing total investment expenditure, $\int_0^1 P_{it}(i_{it}^j)di$, subject to the aggregation constraint (27). The optimal level of i_{it}^j for $i \in [0,1]$ is then given by

$$i_{it}^j = \left(\frac{P_{it}}{P_t}\right)^{-\eta} i_t^j. \tag{28}$$

At the optimum, we have that $P_t i_t^j = \int_0^1 P_{it} i_{it}^j di$. On aggregate, households demand $i_{it} \equiv \int_0^1 i_{it}^j dj$ units of good i for investment purposes. Equation (28) implies that

$$i_{it} = p_{it}^{-\eta} i_t, \tag{29}$$

where $i_t \equiv \int_0^1 i_t^j dj$.

The representative household's problem can then be stated as consisting in choosing processes x_t^j, h_t^j, i_t^j, and k_t^j, so as to maximize the lifetime utility function (22) subject to (26) and

$$x_t^j + i_t^j + \omega_t + E_t r_{t,t+1} d_{t+1}^j = d_t^j + w_t h_t^j + \Phi_t^j + u_t k_t^j, \tag{30}$$

given processes v_t, ω_t, w_t, $r_{t,t+1}$, u_t, and Φ_t^j.

4.1.2 The Government

Each period $t \geq 0$, nominal government spending is given by $P_t g_t$. We assume that real government expenditures, denoted by g_t, are exogenous, stochastic, and follow a univariate first-order autoregressive process of the form

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g,$$

[1] Note that we do not assume any habit in the production of investment goods. However, if we were to reinterpret our catching-up-with-the-Joneses habit model as a switching costs model, then one may plausibly argue that in fact, the aggregate investment good should depend not only on the current level of purchases of intermediate investment goods but also on their respective past levels. Alternatively, one could assume that there is habit formation in investment as well. For example, Giannoni and Woodford (2003) introduce superficial habit formation into an otherwise standard Neo-Keynesian aggregate-supply aggregate-demand model and interpret the object that is subject to habit formation not simply as private consumption but as total (private) aggregate demand. They justify the assumption of habit in the investment demand by saying that it should be understood as a proxy for adjustment costs in investment expenditure that imply an inertial response in the rate of investment spending.

where the innovation ϵ_t^g distributes i.i.d. with mean zero and standard deviation σ^g. The government allocates spending over intermediate goods g_{it} so as to maximize the quantity of a composite good produced with intermediate goods according to the relationship

$$x_t^g = \left[\int_0^1 (g_{it} - \theta s_{it-1}^g)^{1-1/\eta} di\right]^{1/(1-1/\eta)}.$$

The variable s_{it}^g denotes the stock of habit in good i, and evolves over time as

$$s_{it}^g = \rho s_{it-1}^g + (1-\rho) g_{it}. \tag{31}$$

We justify our specification of the aggregator function for government consumption by assuming that private households value government spending in goods in a way that is separable from private consumption and leisure and that households derive habits on consumption of government provided goods. The government's problem consists in choosing g_{it}, $i \in [0,1]$, so as to maximize x_t^g subject to the budget constraint $\int_0^1 P_{it} g_{it} \leq P_t g_t$ and taking as given the initial condition $g_{it} = g_t$ for $t = -1$ and all i. In solving this maximization problem, the government takes as given the effect of current public consumption on the level of next period's composite good—i.e., habits in government consumption are external. Conceivably, government habits could be treated as internal to the government even if they are external to their beneficiaries, namely, households. This, alternative, however, is analytically less tractable. The case of no habits in government consumption results from setting $\theta = 0$ in the above aggregator function for public goods. We believe that this is not the case of greatest interest under our maintained assumption that government spending on goods is valued by habit-forming private agents.

The resulting demand for each intermediate good $i \in [0,1]$ by the public sector is

$$g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t^g + \theta s_{it-1}^g, \tag{32}$$

where

$$x_t^g = g_t - \theta \int_0^1 \frac{P_{it}}{P_t} s_{it-1}^g di.$$

Public spending is assumed to be fully financed by lump-sum taxation.

4.1.3 Firms

Each good $i \in [0, 1]$ is manufactured using labor and capital as inputs via the following homogenous-of-degree-one production technology:

$$y_{it} = A_t F(k_{it}, h_{it}) - \phi, \tag{33}$$

where y_{it} denotes output of good i, k_{it} and h_{it} denote services of capital and labor, and ϕ denotes fixed costs of production. The presence of fixed costs introduce increasing returns to scale in the production technology. We model fixed costs to ensure that profits are relatively small on average as is the case for the U.S. economy in spite of equilibrium markups of price over marginal cost significantly above zero. The variable A_t denotes an aggregate technology shock. We assume that the logarithm of A_t follows a first-order autoregressive process

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a, \tag{34}$$

where ϵ_t^a is a white noise disturbance with standard deviation σ^a.

Firms are price setters. In exchange, they must stand ready to satisfy demand at the announced prices. Formally, firm i must satisfy

$$A_t F(k_{it}, h_{it}) - \phi \geq c_{it} + i_{it} + g_{it}. \tag{35}$$

where c_{it} is given by equation (25), g_{it} by (32) and i_{it} by equation (29).

Firm i's problem consists in choosing processes p_{it}, c_{it}, g_{it}, i_{it}, h_{it}, and k_{it}, so as to maximize the present discounted value of profits, which is given by

$$E_0 \sum_{t=0}^{\infty} r_{0,t} \left[p_{it}(c_{it} + i_{it} + g_{it}) - w_t h_{it} - u_t k_{it} \right], \tag{36}$$

subject to (24), (25), (29), (31), (32), and (35), given processes $r_{0,t}$, w_t, u_t, A_t, x_t^g, and x_t and given c_{i-1} and g_{i-1}. Appendix A contains the first-order conditions associated with the firm's optimization problem. Appendix B lists the complete set of conditions defining a stationary equilibrium.

Of particular interest in our analysis is the equilibrium behavior of the markup of prices over marginal cost. Noting that the marginal cost is given by $w_t/[A_t F_h(k_t, h_t)]$, and that in equilibrium the relative price of any variety of intermediate good in terms of the composite habit-adjusted consumption good is unity, we have that the markup is given by is

$$\mu_t = \frac{A_t F_h(k_t, h_t)}{w_t}. \tag{37}$$

This expression shows that the markup acts as a time varying wedge between the wage rate and the marginal product of labor. Our focus will be in determining the cyclicality of this wedge in response to various aggregate disturbances.

4.2 Calibration and Functional Forms

Consistent with the estimation strategy we adopt later in section 5, we assume that the period utility index is separable in consumption and leisure. Specifically, preferences are of the form

$$U(x,h) = \frac{x^{1-\sigma}-1}{1-\sigma} + \gamma\frac{(1-h)^{1-\chi}-1}{1-\chi} \quad 0 < \sigma, \chi \neq 1,\ \gamma > 0.$$

In the special case in which σ and χ approach unity, this utility function converges to the log-linear specification used by King, Plosser, and Rebelo (1988) among many other business-cycle studies. The production function is assumed to be of the Cobb-Douglas type

$$F(k,h) = k^{\alpha}h^{1-\alpha}; \quad \alpha \in (0,1).$$

We calibrate the model to the U.S. economy. The time unit is meant to be one quarter. Table 1 summarizes the calibration. Later in section 5, we estimate the preference parameters

Table 1: Calibration

Symbol	Value	Description
β	0.9902	Subjective discount factor
σ	2	Inverse of intertemporal elasticity of substitution
θ	0.86	Degree of habit formation
ρ	0.85	Persistence of habit stock
α	0.25	capital elasticity of output
δ	0.0253	Quarterly depreciation rate
η	5.3	Elasticity of substitution across varieties
ϵ_{hw}	1.3	Frisch elasticity of labor supply
h	0.2	Steady-State fraction of time devoted to work
$\bar{g}$	0.0318	Steady-state level of government purchases
ϕ	0.0853	Fixed cost
ρ_v, ρ_g, ρ_a	0.9	Persistence of exogenous shocks

pertaining to the deep habit model. Based on that estimation we set $\theta = 0.86$, $\rho = 0.85$, $\eta = 5.3$, and $\sigma = 2$. We set the preference parameter γ to ensure that in the deterministic steady state households devote 20 percent of their time to market activities following Prescott (1986). The calibration restrictions that identify the remaining structural parameters of the model are taken from Rotemberg and Woodford (1992). We follow their calibration strategy to facilitate comparison of our model of endogenous markups due to deep habits to their ad-hoc version of the customer-market model. In particular, we set the labor share in GDP to 75 percent, the consumption share to 70 percent, the government consumption share to 12 percent, the annual real interest rate to 6 percent, and the Frisch labor supply elasticity equal to 1.3. These restrictions imply that the capital elasticity of output in production, α, is 0.25, the depreciation rate, δ, is 0.025 per quarter, the subjective discount factor, β, is 0.99, and that the preference parameter χ is 3.08.

In our model, the steady-state markup of price over marginal cost, μ, is endogenously given by

$$\mu = \frac{\eta m}{\eta m - 1},$$

where

$$m \equiv (s_c + s_g)\left[\frac{(1-\beta\rho)(1-\theta)}{\beta\theta(\rho - 1) + 1 - \beta\rho}\right] + s_i \leq 1.$$

Our calibration implies a somewhat high average markup of 1.32. Note that in the case of perfect competition, that is, when $\eta \to \infty$, the markup converges to unity. In the case of no deep habit, i.e., when $\theta = 0$, we have that m equals one, and the markup equals $\eta/(\eta - 1) = 1.23$, which relates the markup to the intratemporal elasticity of substitution across varieties in the usual way. This expression for the steady-state markup is the one that emerges from models with imperfect competition and superficial habit (e.g., Giannoni and Woodford, 2003; and Christiano, Eichenbaum, and Evans, 2003). Because under deep habit, the parameter m is less than unity, firms have more market power under deep habits than under superficial habits. This is because in the former formulation firms take advantage of the fact that when agents form habits on a variety-by-variety basis, the short-run price elasticity of demand for each variety is less than η.

Appendix C describes in detail how the calibration restrictions are used to identified the structural parameters of the model and how to solve for the steady-state values of the endogenous variables.

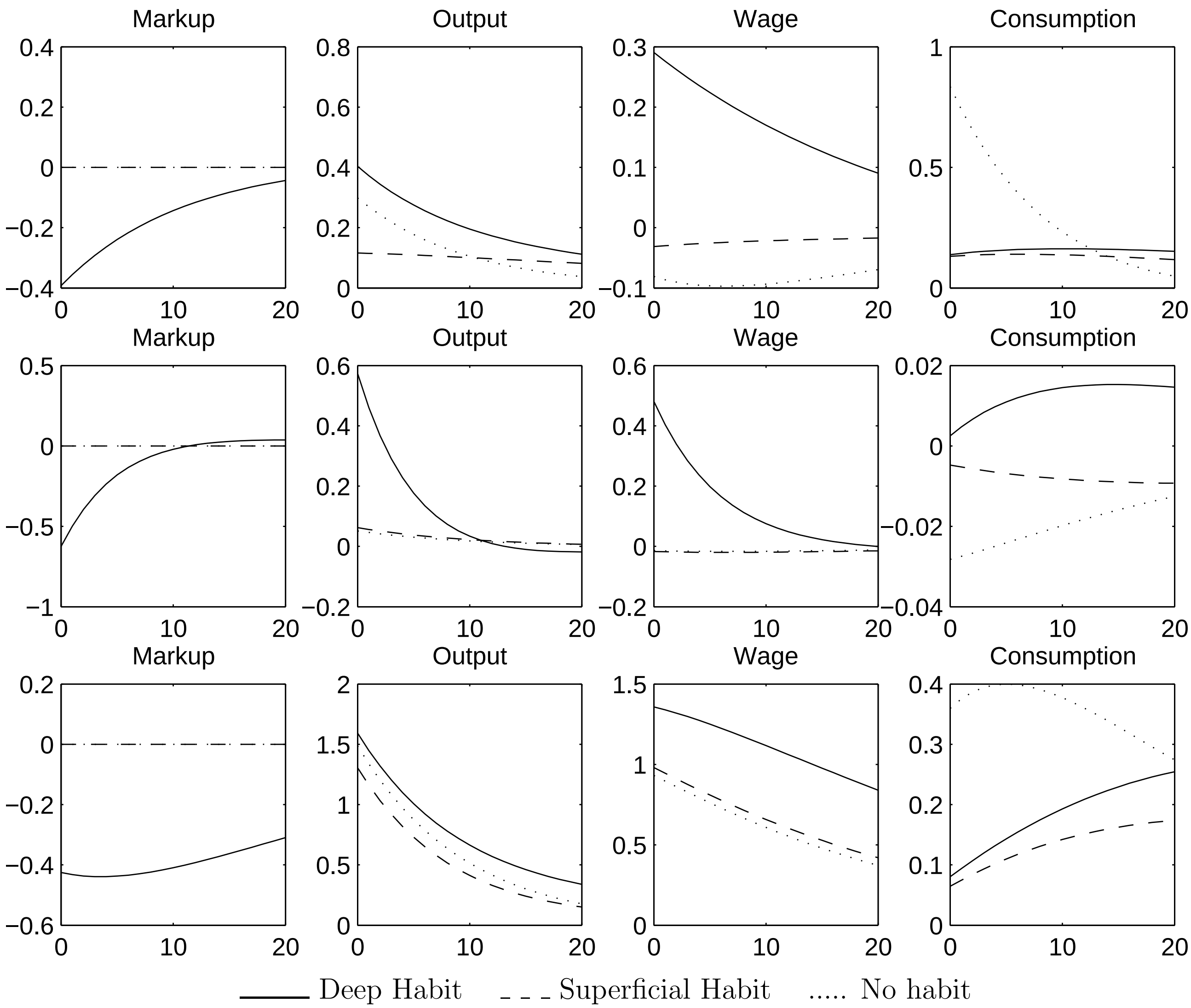
We set the serial correlation of all three shocks to 0.9 (i.e., $\rho_v = \rho_g = \rho_a = 0.9.$). These values are in the ball park of available estimates.

4.3 Aggregate Dynamics

We now characterize quantitatively the response of the deep habit model to a variety of exogenous shocks. We consider three sources of aggregate fluctuations: preference shocks, v_t, government spending shocks, g_t, and productivity shocks, A_t. The first row of figure 1 displays impulse responses of the markup, output, wages, and consumption to an increase in the preference shock v_t in the amount of 1 percent of steady-state consumption. The response of the deep habit model is shown with a solid line. For comparison, the figure also depicts the response of economies with superficial habit, shown with a dashed line, and no habit, shown with a dotted line. Under all three model specifications, consumption and output increase as a result of the preference shock. Consumption increases because the shock raises the marginal utility of consumption. At the same time, the shock induces an expansion in labor supply because it reduces the value of leisure in terms of consumption. This explains why output increases in all three cases. The expansion in the supply of labor puts downward pressure on wages. In the economies with superficial habit or no habit, the labor demand schedule is unaffected by the preference shock on impact. In these two economies, the combination of an unchanged labor demand schedule and an increase in the labor supply, causes the equilibrium wage to fall. By contrast, in the deep habit model the markup falls significantly on impact by about 3 percent. This decline in the markup leads to an increase in the demand for labor at any given wage rate. This expansion in labor demand more than compensates the increase in labor supply, resulting in an equilibrium increase in wages of about 2 percent. The reason why markups fall in the deep habit model is that in response to the increase in the demand for goods, firms try to widen their customer base by offering lower prices. This is a deliberate strategy aimed at inducing agents to form habits which firms can later on exploit by charging higher markups. Summarizing, the difference between the deep habit model and the models with either superficial or no habit is that under deep habits the markup behaves countercyclically and real wages are procyclical in response to an expansionary preference shock.

Row two of figure 1 shows the response of the three economies under analysis to a 1 percent increase in government consumption. Government spending shocks are similar to preference shocks of the type described above in that they increase aggregate absorption and labor supply. In the case of government spending shocks, the labor supply increases because the expansion in unproductive public spending leaves households poorer. The increase in labor supply induces a fall in the equilibrium wage rate in the economies with superficial or no habits. In the economy with deep habits firms reduce markups by about 1 percent. The resulting expansion in labor demand produces a rise in wages. As in the case of innovations in private spending, government purchases shocks trigger countercyclical markup movements

Figure 1: Impulse Responses to Positive Preference, Government Spending, and Productivity Shocks Under Deep Habit, Superficial Habit, and No Habit



Row 1: Preference Shock. Row 2: Government Spending Shock. Row 3: Technology shock. Impulse responses are measured in percent deviations from steady state. Horizontal axes display the number of quarters after the shock.

and procyclical wage movements.

Finally, the third row of figure 1 displays the consequences of a 1 percent increase in total factor productivity. Here under deep habits firms also choose to cut markups to gain customer base. As a result, the wage rate increases by more under deep habits than under either superficial habits or no habits.

We conclude that in response to all three shocks considered here, the markup displays a strong countercyclical behavior. This result stands in stark contrast to ad-hoc versions of switching cost or customer market models such as the one developed in Rotemberg and Woodford (1991, 1995) and discussed in section 3 above. To stress this point, figure 2 displays the response of the deep habit model and the Rotemberg and Woodford (1991, 1995) version of the customer market model to preference, government spending, and productivity shocks. In the Rotemberg-Woodford customer-market economy, the markup is given by

$$\hat{\mu}_t = \epsilon_\mu(\hat{X}_t - \hat{y}_t); \quad \epsilon_\mu < 0,$$

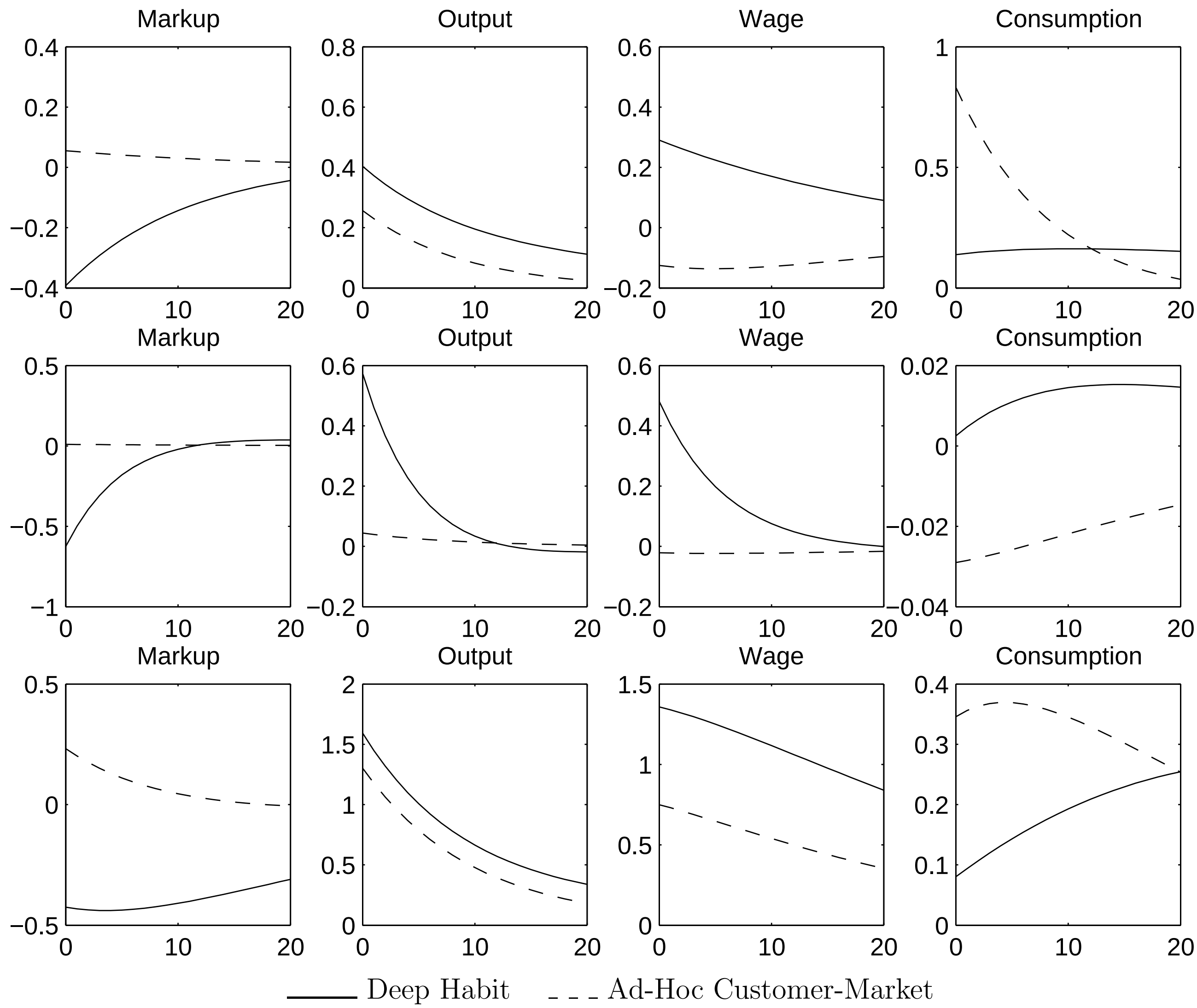
where a hat over a variable denotes percent deviation from its steady-state value. The variable $y_t \equiv c_t + g_t + i_t$ denotes aggregate demand and X_t is a measure of the present discounted value of profits and is defined as

$$X_t \equiv E_t \sum_{j=1}^{\infty} \alpha_{RW}^j r_{t,t+j} \frac{\mu_{t+j} - 1}{\mu_{t+j}} y_{t+j}.$$

The Rotemberg-Woodford version of the customer-market model can easily be embedded into the basic structure of our fully-fledged deep-habit model by substituting the above two expressions for equilibrium conditions (48)-(50) shown in appendix B and shutting off habit by setting $\rho = \theta = 0$. In addition, following Rotemberg and Woodford (1995) we set the parameter α_{RW}, reflecting the probability that any given firm will continue to exist (and thus continue to exploit the market share built up over time) to 0.9, and the elasticity of the markup with respect to the ratio of future expected profits to current demand, ϵ_μ, to -1.

Under all three shocks, the Rotemberg and Woodford customer-market model predicts that the markup moves in the same direction as output. Also, in response to demand shocks, whether private or public, wages move countercyclically. To understand why the Rotemberg and Woodford version of the customer-market model produces so different markup behavior than the customer-market model based on deep habit formation, it is important to consider separately the response of markups to future expected profits and to current demand conditions. Under both formulations markups are predicted to decline in response to an increase in the present discounted value of expected future profits. However, the two models differ in

Figure 2: Impulse Responses to Positive Preference, Government Spending, and Productivity Shocks in the Deep-Habit and Ad-Hoc Customer-Market Models



Row 1: Preference Shock. Row 2: Government Spending Shock. Row 3: Technology shock. Impulse responses are measured in percent deviations from steady state. Horizontal axes display the number of quarters after the shock.

their prediction regarding the response of markups to the state of current aggregate demand. Under deep habits, periods of strong aggregate demand are associated with firms building a customer base by lowering profit margins. Thus, all other things equal, markups are low when current output is high. The contrary is true in the Rotemberg and Woodford version of the customer market model.

A key difference between the deep-habit model and the Rotemberg and Woodford customer-market model is that in the deep habit model the demand faced by an individual firm features a positive, price-insensitive term, θc_{it-1}, that depends only on past sales. Due to this term, the price elasticity of demand for an individual variety increases with current aggregate demand, providing an incentive for firms to lower markups. In the Rotemberg-Woodford customer-market model there is no such price-insensitive term. As a result, the elasticity of demand for an individual variety is independent of current aggregate demand conditions. As we discussed earlier in subsection 3.2, a version of our deep-habit model in which the single-period utility function depends not on the quasi-difference between current and past consumption of each variety but rather on the quasi-ratio of these two variables, delivers a specification for the demand function faced by an individual firm that is closer to the one adopted ad-hoc by Rotemberg and Woodford (see Ravn, Schmitt-Grohé, and Uribe, 2002). We therefore conjecture that the assumption that deep habits are additive rather than relative might play a significant role in determining the cyclicality of markups.

5 Estimating Deep Habits

In this section, we estimate the habit formation model described in section 2 using a non-linear GMM estimator. The approach that we take is to exploit that the deep habit parameter enters both the Euler equation (11) and the conditions that determine the dynamics of the markup. This aspect of the model is particularly useful because it allows for a more efficient estimation of the habit parameter than standard estimates of habit parameters that are derived solely from the Euler equation.

For the estimation we will assume that utility is separable in consumption and leisure. Specifically, we assume that

$$U(x, h) = \frac{x^{1-\sigma} - 1}{1 - \sigma} + f(1 - h); \quad \sigma > 0.$$

In this case we have

$$U_x(x_t, h_t) = x_t^{-\sigma} = (c_t - \theta c_{t-1})^{-\sigma}.$$

Consider equilibrium condition (11) and use it to price a nominally risk free one-period bond.

Let R_t denote the gross (quarterly) nominal interest rate on such a bond held from period t to period $t+1$ and π_{t+1} the gross inflation rate between periods t and $t+1$. Then it must be the case that

$$U_x(x_t, h_t) = \beta R_t E_t U_x(x_{t+1}, h_{t+1})/\pi_{t+1}.$$

For the particular functional form assumed here this pricing equation can be written as

$$(\gamma_t - \theta)^{-\sigma} = \beta R_t E_t (\gamma_t)^{-\sigma} (\gamma_{t+1} - \theta)^{-\sigma} / \pi_{t+1}. \tag{38}$$

Such Euler equations have formed the basis of the estimation of habits in previous papers. Abel (1990, 1996), Dunn and Singleton (1986), and Ferson and Constantinides (1991) assume like us that habits depend on one-period lagged consumption and Ferson and Constantinides (1991) estimate θ to be above 0.90 using quarterly data.[2] Others, including Constantinides (1990), Campbell and Cochrane (1999), and Heaton (1995) instead allow for habits that exhibit gradual response to consumption. Campbell and Cochrane (1999) also allow for non-linearities in the process for the surplus ratio (the ratio of the difference between current consumption less the "habit" to current consumption). These authors calibrate the parameters pertaining to habits so that their model is consistent with key features of asset prices. Tallarini and Zhang (2003) estimate this model using an EMM estimator and find very persistent habits. All of these analysis, however, assume "superficial habits" and therefore base the analysis on the implications for the Euler equation. As we have discussed, in our model "deep" habits affect also the firms' pricing decisions and we can exploit this to obtain a more efficient estimate of the habit parameter.

Combining (13)-(16) yields

$$\frac{\gamma_t}{\eta(\gamma_t - \theta)} = \theta \beta E_t \left(\frac{\gamma_{t+1} - \theta}{\gamma_t - \theta} \right)^{-\sigma} (\gamma_t)^{-\sigma} \frac{\gamma_{t+1}}{\eta (\gamma_{t+1} - \theta)} + 1 - \frac{w_t}{A_t}.$$

Finally, letting $s_t = w_t h_t / y_t$ denote labor's share of income, and multiplying by η gives us that:

$$\frac{\gamma_t}{\gamma_t - \theta} = \theta \beta E_t (\gamma_t)^{-\sigma} \left(\frac{\gamma_{t+1} - \theta}{\gamma_t - \theta} \right)^{-\sigma} \frac{\gamma_{t+1}}{\gamma_{t+1} - \theta} + \eta(1 - s_t). \tag{39}$$

We then use equations (38) and (39) to estimate the parameters σ, θ, β, and η. We rewrite the two equations as orthogonality conditions and apply a non-linear GMM system estimator constraining the parameters to be identical in the two equations. The orthogonality

[2] Ferson and Constantinides (1991) assume internal habits and their results are therefore not directly comparable to ours. Braun, Constantinides, and Ferson (1993) extend the Ferson and Constantinides (1991) analysis to other OECD countries and find less evidence of very persistent habits.

conditions are given by:

$$
\begin{aligned}
0 &= \gamma_t^{\sigma} - \beta R_t E_t \left(\frac{\gamma_t - \theta}{\gamma_{t+1} - \theta} \right)^{\sigma} / \pi_{t+1} && (40) \\
0 &= \left(\frac{\gamma_t}{\gamma_t - \theta} \right) && (41) \\
&\quad - \beta \theta E_t \left(\frac{\gamma_{t+1}}{\gamma_{t+1} - \theta} \right) \left(\frac{\gamma_t - \theta}{\gamma_{t+1} - \theta} \right)^{\sigma} (\gamma_t)^{-\sigma} - \eta \left(1 - s_t\right).
\end{aligned}
$$

We use U.S. quarterly data spanning the period 1967:Q1 to 2003:Q1. We experiment with measuring γ_t on the basis of either total private consumers expenditure (C^{tot}) or private consumers expenditure on durables (C^{ndur}). In both cases, consumption growth rates are divided by civilian population to convert the numbers into per capita terms. The 3-months nominal interest rate is measured either by the 3-months T-Bill rate or by the federal funds rate. Inflation is measured by the implicit deflator of the relevant consumption series. Finally, we measure the labor share as compensation of employees divided by GDP at factor costs. As instruments we use 4 lags of consumption growth, inflation rates, nominal interest rates, labor shares, GDP growth rates, the term spread, and nominal wage inflation (and a constant). We included the term spread because it may have predictive power for consumption growth rates, and we introduce wage inflation because it may have predictive power for the labor share. The GMM estimator is over-identified and we can thus test the overidentifying restrictions using a J-test which is distributed as a χ^2 (with the degrees of freedom equal to the number of orthogonality conditions times the number of instruments less the number of estimated coefficients).

The parameter estimates are reasonable and appear to be robust to changes in the measurement of consumption growth rates and interest rates. The only parameter that appears sensitive to measurement is σ which is lower when we specify consumption growth on the basis of non-durables consumption than when we base the measurement on total consumption growth. This is perhaps not surprising given the higher variance of total consumption growth.

We find an estimate of the key parameter θ in the neighborhood of 0.6. This value is significantly lower than alternative estimates in the literature based on Euler equations but still gives rise to considerable non-time separability. The estimate is closer to the values obtained by Fuhrer (2000) who derive parameter estimates on the basis of a FIML estimator.[3] Furthermore, our estimate appears to be associated with quite small sampling uncertainty. The estimates of the other structural parameters are very reasonable as well. We find an

[3]Fuhrer (2000) also applies an alternative GMM estimator which yields a higher estimate of the habit parameter.

Table 2: GMM Estimates of Structural Parameters, US 1967-2003 (Quarterly Data).

Measurement						
C	R	σ	θ	η	β	J-test
C^{tot}	R^{TB3}	1.155	0.622	3.629	0.9986	90.10
		(0.041)	(0.011)	(0.008)	(0.0011)	[0.011]
C^{tot}	R^{FFR}	1.090	0.618	3.639	0.9970	90.04
		(0.040)	(0.010)	(0.007)	(0.0011)	[0.012]
C^{ndur}	R^{TB3}	1.581	0.611	3.643	0.9955	91.73
		(0.069)	(0.011)	(0.009)	(0.0013)	[0.008]
C^{ndur}	R^{FFR}	1.500	0.618	3.650	0.9938	92.60
		(0.065)	(0.011)	(0.009)	(0.0013)	[0.007]

Notes: Numbers in parentheses are heteroscedasticity consistent standard errors. Numbers in square brackets are P-values.

estimate of σ close to 1.5 which is considered reasonable in models without habit formation. The subjective discount factor, β, is estimated to be very close to 1 and implies an annual real interest rate of around 3.6 percent.[4] The estimate of the long-run price elasticity of demand is just around 3.6. This estimate implies a steady state markup of 38 percent which is somewhat high but not out of line with the values implies by other estimates in the empirical literature. The J-test formally rejects the overidentifying restrictions a finding that is consistent with results in the asset pricing literature that has examined standard "superficial" habit models.

Altogether, the empirical estimates are satisfactory and for our purposes, it is especially interesting that we are able to estimate the habit parameter with great precision and that we find a relatively high value for this parameter.

5.1 Estimation of the Generalized Deep Habits Model

In this section, we estimate a variation of the deep habit model in which the stock of external habit for each variety of goods is no longer equal to the previous period's consumption of that variety but rather equals a weighted average of all past consumptions of the good in question. Specifically, as in section 4, we assume that the first argument of the single-period utility function is given by $x_t = \left[\int_0^1 c_{it} - \theta\Phi_{it-1})^{1-1/\eta} di\right]^{\eta/(\eta-1)}$, where Φ_{it} denotes the stock of external habit of consumption of good i and evolves over time as $\Phi_{it} = (1-\rho)c_{it} + \rho\Phi_{it-1}$. For estimation purposes, we assume that output is produced without capital and with a

[4]The steady state real interest rate is given as γ^σ/β. The average quarterly per capita consumption growth rate is around 0.4 percent.

technology that is linear in labor. The superficial version of this generalized model of habit formation is similar to the types of habit formation that have been applied in the empirical finance literature by, for instance, Constantinides (1990), Campbell and Cochrane (1999), and Heaton (1995). Campbell and Cochrane introduce non-linearities in the modeling of habits but we will neglect such non-linearities mainly because such features significantly complicate the implications for firms' optimal pricing behavior.

The model that we want to estimate can be summarized by the following equations:

$$\begin{aligned}
U(x,h) &= \frac{x^{1-\sigma}-1}{1-\sigma}+f(1-h); \quad \sigma>0 \\
U_x(x_t,h) &= x_t^{-\sigma}=(c_t-\theta\Phi_{t-1})^{-\sigma} \\
\Phi_t &= \rho\Phi_{t-1}+(1-\rho)\,c_t \\
\upsilon_t &= \frac{1}{\eta}\frac{c_t}{c_t-\theta\Phi_{t-1}} \\
1-\upsilon_t-\frac{1}{\mu_t} &= E_t r_{t,t+1}\left[(\rho-1)\,\theta\upsilon_{t+1}+\rho\left[1-\upsilon_{t+1}-\frac{1}{\mu_{t+1}}\right]\right],
\end{aligned}$$

where we once again assume that utility is separable in leisure and consumption. This model differs from the model we estimate earlier in this section because the introduction of persistence in habits introduces the variable, Φ_t, which summarizes the effects of past consumption choices on current utility.

Tallarini and Zhang (2003) estimate a superficial habit model similar to this specification using an EMM estimator. The EMM estimator combines GMM with a simulation approach where the model is solved numerically and used to generate expectations of the scores which are then used as the moment conditions in the GMM estimation. The application of the EMM estimator is considerably more complicated in our set-up because the habits affect not only the Euler equation but also firms' pricing policies. For that reason the estimation procedure need not only incorporate the simulation of consumption choice in the estimation algorithm but also the equations describing the firms' pricing choices and the equilibrium input and output allocations. Estimating this system is numerically very challenging.

Given our purposes, we choose instead to apply a simpler framework in which we apply a GMM estimator combined with a grid search approach. In particular, we log linearize the system of equations which can be summarized by:

$$
\begin{aligned}
0 &= \ln\phi_t - \rho\ln\phi_{t-1} + \rho\ln\gamma_t \\
0 &= \gamma E_t\ln\gamma_{t+1} - \theta\ln\gamma_t - \theta\left(\ln\phi_t - \ln\phi_{t-1}\right) - \frac{\gamma-\theta}{\sigma}E_t\ln r_{t+1} && (42) \\
0 &= a_1E_t\ln\gamma_{t+1} + a_2\ln\gamma_t + a_3\ln\phi_t + a_4\ln\phi_{t-1} + a_5E_t\ln s_{t+1} + a_6\ln h_t && (43) \\
a_1 &= \frac{\left[(\rho-1)\theta-\rho\right]\gamma + \eta\rho\left[1-s\right]\gamma}{\left[\left[(\rho-1)\theta-\rho\right]\gamma + \eta\rho\left[1-s\right](\gamma-\theta)\right]} - \frac{\gamma(1+\sigma)}{\gamma-\theta} \\
a_2 &= \frac{\sigma\theta+\gamma}{\gamma-\theta} - \frac{\left(\eta(1-s)-1\right)\gamma}{\eta(1-s)(\gamma-\theta)-\gamma} \\
a_3 &= \frac{\theta(1+\sigma)}{\gamma-\theta} - \frac{\eta\rho\left[1-s\right]\theta}{\left[\left[(\rho-1)\theta-\rho\right]\gamma + \eta\rho\left[1-s\right](\gamma-\theta)\right]} \\
a_4 &= -\frac{\theta(1+\sigma)}{\gamma-\theta} + \frac{\eta(1-s)\theta}{\eta(1-s)(\gamma-\theta)-\gamma} \\
a_5 &= -\frac{\eta\rho(\gamma-\theta)s}{\left[\left[(\rho-1)\theta-\rho\right]\gamma + \eta\rho\left[1-s\right](\gamma-\theta)\right]} \\
a_6 &= \frac{\eta(\gamma-\theta)s}{\eta(1-s)(\gamma-\theta)-\gamma}
\end{aligned}
$$

where s_t is the labor share and $\phi_t = \Phi_t/c_t$.

We notice that, for a given value of ρ, ϕ_t is observable over the sample path subject to an initial condition for ϕ_{-1}. In order to estimate this system, we employ a grid search GMM estimator. In particular, we estimate the orthogonality conditions (42) and (43) over a grid of values for $\phi_{-1} \in \Phi$ which we specify as the unit interval. For each point in this grid we evaluate the quadratic form:

$$Q\left(\Psi|\phi_{-1}\right) = g_T'\left(\Psi|\phi_{-1}\right)W g_T\left(\Psi|\phi_{-1}\right)$$

where $\Psi = [\sigma, \theta, \eta, \rho]$, $g_T\left(\Psi|\phi_{-1}\right) = \frac{1}{T}\sum_t h_t\left(\Psi|\phi_{-1}\right)$, and $h_t\left(\Psi|\phi_{-1}\right)$ denotes the orthogonality conditions, and W is the weighting matrix.

The estimator was then implemented by sequentially refining the grid until we obtained no further improvement in Q. In practice, we found that this procedure performed well and produced, in most cases, uniform convergence towards the optimum. We estimate the model using the same data as in the previous section.

We report the estimates of the structural parameters both on the basis of the GMM estimates of the Euler equation (42) estimated by itself and on the basis of the Euler equation estimated jointly with equation (43). Regardless of the measurement of consumption growth, the measurement of the risk free rate, and of whether we estimate the parameters on the basis of the system of equations or only using the Euler equation, we find significantly

Table 3: Generalized Deep-Habit Model: GMM Estimates of Structural Parameters, US 1967-2003 (Quarterly Data).

Measurement		Euler Equation Based Estimates				Joint System Based Estimates				
C	R	σ	θ	ρ	J-test	σ	θ	η	ρ	J-test
C^{tot}	R^{TB3}	1.216	0.727	0.728	31.92	2.777	0.686	2.453	0.717	51.53
		(0.452)	(0.153)	(0.079)	[0.371]	(0.638)	(0.059)	(0.614)	(0.051)	[0.826]
C^{tot}	R^{FFR}	1.384	0.814	0.731	33.04	1.854	0.861	5.261	0.848	54.22
		(0.737)	(0.147)	(0.064)	[0.321]	(0.583)	(0.039)	(1.062)	(0.027)	[0.749]
C^{ndur}	R^{TB3}	3.638	0.537	0.723	28.29	1.487	0.929	2.663	0.811	44.80
		(1.318)	(0.170)	(0.101)	[0.555]	(0.497)	(0.010)	(0.737)	(0.027)	[0.951]
C^{ndur}	R^{FFR}	3.852	0.614	0.777	28.22	1.764	0.950	4.408	0.826	45.69
		(1.752)	(0.175)	(0.078)	[0.559]	(0.495)	(0.006)	(0.991)	(0.029)	[0.940]

Notes: Numbers in parentheses are heteroscedasticity consistent standard errors. Numbers in square brackets are P-values.

positive estimates of the habit persistent parameter, ρ, around 0.6-0.8. The estimate of θ is also fairly invariant across specifications, significantly positive, and in the range of 0.5-0.95. Furthermore, we find that under all of the alternative specifications of the empirical model, we fail to reject the orthogonality conditions. Thus, the estimates are supportive of the existence of persistent habits. The estimate of the persistence of the habit is lower than alternative estimates in the literature, e.g., Tallarini and Zhang (2003) but the estimates are not directly comparable because we have assumed away non-linearities in the habit and because we log-linearize the orthogonality conditions.[5] The estimates of σ and η, however, appear sensitive to measurement. In particular, the estimate of η varies considerably over the various specification of consumption growth and the nominal interest rate. The estimates of σ also vary but in each case the estimate is economically sensible.

6 Conclusion

In recent macroeconomic modeling, it is commonplace to assume that households have preferences over a large number of differentiated goods. This assumption is made, for instance, in models with imperfectly competitive product markets. At the same time, there appears

[5]We note that the Euler equation implied by our model of deep habits is observationally equivalent to the standard habit formation model. Thus, when the habit parameters are estimated on the basis of the Euler equation alone, our results should be comparable to estimates derived on the basis of standard habit formation models. This is not the case, however, for the joint system estimator because deep habits have different implications for price dynamics than standard habit formation models.

to be some consensus that the assumption of habit formation is of great use in accounting for key business-cycle regularities, in particular, consumption and asset-price dynamics. An obvious question that emerges in modeling economies with habit formation and a large variety of goods available for consumption is at what level habits are formed. That is, are habits created at the level of each individual consumption good or at the level of a consumption aggregate. The existing literature has focused exclusively on the latter modeling strategy. This paper is motivated by our belief that the former alternative is at least equally compelling.

A central finding of our investigation is that the level at which habit formation is assumed to occur is of great macroeconomic consequences. When habits are formed at the level of each individual variety of consumption goods, the demand function faced by a firm depends not only on the relative price of the good and aggregate income—as in the standard case—but also on past sales of the particular good in question. This characteristic of the demand function alters the optimal pricing behavior of the firm. For today's prices are set taking into account that they will affect not just today's sales but also future sales through their effect on future demand. In this way, the assumption of deep habits results in a theory of time-varying markups.

The deep habit model developed in this paper provides microfoundations for other models in which past sales affect current demand conditions at the level of each individual good, such as customer-market and switching-cost models. General equilibrium versions of these models have been criticized for having the counterfactual implication that markups are procyclical. To our knowledge, all existing general equilibrium treatments of customer-market/switching-cost models use ad-hoc specifications of the demand function faced by individual firms. In this paper we show that once the demand faced by firms is derived from the behavior of optimizing households, the resulting equilibrium comovement between markups and aggregate activity is in line with the empirical evidence.

Appendix A: The Firm's Problem in the Fully Fledged Model

The Lagrangian of firm i's problem can be written as

$$\begin{aligned}
\mathcal{L} &= E_0 \sum_{t=0}^{\infty} r_{0,t} \left\{ p_{it}c_{it} + p_{it}^{1-\eta} i_t + p_{it}g_{it} - w_t h_{it} - u_t k_{it} \right. \\
&\quad + \kappa_t \left[A_t F(k_{it}, h_{it}) - \phi - c_{it} - p_{it}^{-\eta} i_t - g_{it} \right] \\
&\quad + \nu_t \left[p_{it}^{-\eta} x_t + \theta s_{it-1} - c_{it} \right] + \lambda_t \left[\rho s_{it-1} + (1-\rho) c_{it} - s_{it} \right] \\
&\quad \left. + \nu_t^g \left[p_{it}^{-\eta} x_t^g + \theta s_{it-1}^g - g_{it} \right] + \lambda_t^g \left[\rho s_{it-1}^g + (1-\rho) g_{it} - s_{it}^g \right] \right\},
\end{aligned}$$

The first-order conditions associated with the firm's problem are equations (24), (25), (31), (32), (29), (35), and (taking derivatives of the Lagrangian with respect to c_{it}, s_{it}, g_{it}, s_{it}^g, h_{it}, k_{it}, and p_{it} in this order)

$$p_{it} - \nu_t - \kappa_t + \lambda_t(1-\rho) = 0,$$

$$\theta E_t r_{t,t+1} \nu_{t+1} + \rho E_t r_{t,t+1} \lambda_{t+1} = \lambda_t$$

$$p_{it} - \nu_t^g - \kappa_t + \lambda_t^g (1-\rho) = 0,$$

$$\theta E_t r_{t,t+1} \nu_{t+1}^g + \rho E_t r_{t,t+1} \lambda_{t+1}^g = \lambda_t^g$$

$$\kappa_t = \frac{w_t}{A_t F_h(k_{it}, h_{it})}$$

$$\kappa_t = \frac{u_t}{A_t F_k(k_{it}, h_{it})}$$

$$c_{it} + (1-\eta) p_{it}^{-\eta} i_t + g_{it} + \eta \kappa_t p_{it}^{-\eta-1} i_t - \eta \nu_t p_{it}^{-\eta-1} x_t - \eta \nu_t^g p_{it}^{-\eta-1} x_t^g = 0$$

Appendix B: Symmetric Equilibrium

In this appendix we keep the notation regarding habit formation as flexible as possible. Specifically, we allow for three different parameters θ, θ^d, and θ^g. This distinction allows us to capture the following special cases: (1) Superficial habit, $\theta^d = \theta^g = 0$. (2) Deep habit on private consumption but not on government consumption $\theta = \theta^d > 0$ and $\theta^g = 0$; (3) Deep habit on private and public consumption, $\theta = \theta^d > 0$ and $\theta^g > 0$. In the main text, we consider the special case of deep habit uniform across private and public consumption, $\theta^s = \theta^d = \theta^g$.

In any symmetric equilibrium $p_{it} = 1$. The equilibrium conditions are then given by

$$-\frac{U_h(c_t - \theta s_{t-1} - v_t, h_t)}{U_x(c_t - \theta s_{t-1} - v_t, h_t)} = w_t \tag{44}$$

$$U_x(c_t - \theta s_{t-1} - v_t, h_t) = \beta E_t U_x(c_{t+1} - \theta s_t - v_{t+1}, h_{t+1})[1 - \delta + u_{t+1}] \tag{45}$$

$$A_t F(k_t, h_t) - \phi = c_t + i_t + g_t \tag{46}$$

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{47}$$

$$\frac{c_t + g_t + i_t}{\eta} = \nu_t(c_t - \theta^d s_{t-1}) + \nu_t^g(g_t - \theta^g s_{t-1}^g) + i_t(1 - \frac{w_t}{A_t F_h(k_t, h_t)}) \tag{48}$$

$$\frac{1 - \nu_t - \frac{w_t}{A_t F_h(k_t,h_t)}}{\rho - 1} = \beta E_t \frac{U_x(c_{t+1} - \theta s_t - v_{t+1}, h_{t+1})}{U_x(c_t - \theta s_{t-1} - v_t, h_t)} \left\{ \theta^d \nu_{t+1} + \rho \frac{1 - \nu_{t+1} - \frac{w_{t+1}}{A_{t+1} F_h(k_{t+1},h_{t+1})}}{\rho - 1} \right\} \tag{49}$$

$$\frac{1 - \nu_t^g - \frac{w_t}{A_t F_h(k_t,h_t)}}{\rho - 1} = \beta E_t \frac{U_x(c_{t+1} - \theta s_t - v_{t+1}, h_{t+1})}{U_x(c_t - \theta s_{t-1} - v_t, h_t)} \left\{ \theta^g \nu_{t+1}^g + \rho \frac{1 - \nu_{t+1}^g - \frac{w_{t+1}}{A_{t+1} F_h(k_{t+1},h_{t+1})}}{\rho - 1} \right\} \tag{50}$$

$$\frac{F_h(k_t, h_t)}{F_k(k_t, h_t)} = \frac{w_t}{u_t} \tag{51}$$

$$s_t = \rho s_{t-1} + (1 - \rho)c_t \tag{52}$$

$$s_t^g = \rho s_{t-1}^g + (1 - \rho)g_t \tag{53}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a \tag{54}$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{55}$$

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g \tag{56}$$

This is a system of 13 nonlinear, stochastic, difference equations in 13 unknowns. We look for a stationary solution to this system.

Appendix C: Deterministic Steady State

Consider shutting off all sources of uncertainty and letting the system settle on a stationary point where any variable x_t satisfies $x_t = x_{t+1}$ for all t. In this state, the equilibrium conditions (44)-(56) collapse to:

$$\frac{\gamma[c(1-\theta)]^{\sigma}}{(1-h)^{\chi}} = w \tag{57}$$

$$1 = \beta[1-\delta+u] \tag{58}$$

$$k^{\alpha}h^{1-\alpha} = c+i+g+\phi \tag{59}$$

$$i = \delta k \tag{60}$$

$$\frac{c+g+i}{\eta} = \nu c(1-\theta^{d}) + \nu^{g} g(1-\theta^{g}) + i\left[1-\frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] \tag{61}$$

$$\left[1-\frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] = \nu\left[\frac{\beta\theta^{d}(\rho-1)+1-\beta\rho}{1-\beta\rho}\right] \tag{62}$$

$$\left[1-\frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] = \nu^{g}\left[\frac{\beta\theta^{g}(\rho-1)+1-\beta\rho}{1-\beta\rho}\right] \tag{63}$$

$$\frac{1-\alpha}{\alpha}\frac{k}{h} = \frac{w}{u} \tag{64}$$

$$s = c \tag{65}$$

$$s^{g} = g \tag{66}$$

$$A = 1$$

$$v = 0$$

$$g = \bar{g}. \tag{67}$$

This is a system of 13 equations in the following 26 unknowns: the 13 endogenous variables c, s, h, w, v, u, A, i, g, k, ν, ν^{g}, and s^{g}; and 13 structural parameters σ, θ, δ, β, η, α,

ϕ, χ, γ, ρ, θ^d, θ^g, $\bar{g}$, To identify all 26 unknowns we impose 13 calibration restrictions:

$$\begin{aligned}
s_h \equiv \frac{wh}{k^\alpha h^{1-\alpha} - \phi} &= 0.75 \\
s_c \equiv \frac{c}{k^\alpha h^{1-\alpha} - \phi} &= 0.7 \\
s_g \equiv \frac{\bar{g}}{k^\alpha h^{1-\alpha} - \phi} &= 0.12 \\
k^\alpha h^{1-\alpha} - \phi - uk - wh &= 0 \\
R \equiv 1 - \delta + u &= 1.06^{1/4} \\
\theta = \theta^d = \theta^g &= 0.86 \\
\rho &= 0.85 \\
\eta &= 5.3 \\
h &= 0.2 \\
\sigma &= 2 \\
\epsilon_{hw} \equiv \left.\frac{\partial \ln h}{\partial \ln w}\right|_{\lambda \text{ constant}} &= 1.3
\end{aligned} \tag{68}$$

Given the assumed preference specification, the Frisch labor supply elasticity is given by

$$\epsilon_{hw} = \frac{1-h}{h\chi}.$$

This expression can be solved for χ. Equation (58) can be solved for β

$$\beta = \frac{1}{R}.$$

Using equation (37) defining the equilibrium markup μ_t, we can write (61) as

$$\frac{1}{\eta} = \nu s_c(1-\theta^d) + \nu^g s_g(1-\theta^g) + s_i\left[1 - \frac{1}{\mu}\right]$$

Now use equations (62) and (63) to eliminate ν and ν^g from this expression.

$$1 = \left[1 - \frac{1}{\mu}\right]\eta\left\{s_c\left[\frac{1-\beta\rho}{\beta\theta^d(\rho-1)+1-\beta\rho}\right](1-\theta^d) + \left[\frac{1-\beta\rho}{\beta\theta^g(\rho-1)+1-\beta\rho}\right]s_g(1-\theta^g) + s_i\right\}$$

Rearranging, we obtain the following expression for the steady-state markup

$$\mu = \frac{\eta m}{\eta m - 1},$$

where

$$m \equiv s_c \left[\frac{(1-\beta\rho)(1-\theta^d)}{\beta\theta^d(\rho-1)+1-\beta\rho} \right] + s_g \left[\frac{(1-\beta\rho)(1-\theta^g)}{\beta\theta^g(\rho-1)+1-\beta\rho} \right] + s_i \leq 1$$

Using (37) and (64) we can write the zero-profit condition (68) as

$$\phi = \left(1 - \frac{1}{\mu}\right) k^\alpha h^{1-\alpha}$$

It follows that the labor share, $s_h \equiv wh/y$, is given by

$$s_h = 1 - \alpha.$$

Finally, to determine δ we use the following relation

$$\begin{aligned} \delta &= \frac{i}{k} \\ &= s_i \frac{y}{k} \\ &= \frac{s_i}{s_k} u \\ &= \frac{1 - s_c - s_g}{s_h} \frac{1-\alpha}{\alpha} [\beta^{-1} - 1 + \delta] \end{aligned}$$

The first equality uses (60), and the last one uses (59), (58), and (64). At this point, we have identified 10 of the 13 structural parameters, namely, σ, θ, δ, β, η, α, χ, ρ, θ^d, θ^g. It remains to determine values for the parameters γ, $\bar{g}$ and ϕ and steady-state values for the endogenous variables of the model. We accomplish this task next. The steady-state value of the rental rate of capital, u, is given by

$$u = \beta^{-1} - (1 - \delta)$$

To obtain the deterministic-steady-state level of the capital stock, solve (60) for k. This yields

$$\begin{aligned} k &= \frac{i}{\delta} \\ &= \frac{s_i}{\delta} [k^\alpha h^{1-\alpha} - \phi] \\ &= \frac{s_i}{\delta\mu} k^\alpha h^{1-\alpha} \\ &= \left[\frac{s_i}{\delta\mu} \right]^{\frac{1}{1-\alpha}} h \end{aligned}$$

Knowing k and h, the parameter ϕ was found above to be equal to $(1-1/\mu)k^{\alpha}h^{1-\alpha}$. The production technology delivers the steady-state value of output:

$$y = k^{\alpha}h^{1-\alpha} - \phi.$$

The steady-state values of the components of aggregate demand follow immediately

$$c = s_c y$$

$$i = s_i y$$

$$g = s_g y$$

Finally, equations (62), (63), (64), (65), (66), and (67) determine directly the steady-state values of ν, ν^g, w, s, s^g, and $\bar{g}$, respectively. Finally, we can solve (57) for γ to obtain

$$\gamma = \frac{w(1-h)^{\chi}}{[(1-\theta)c]^{\sigma}}$$

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