Why the monopolist does not deviate from the symmetric equilibrium in a Model with Good-Specific Subsistence Points

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August 8, 2007

In this note, we investigate whether the monopolist has an incentive to deviate from the symmetric equilibrium in a model with good-specific subsistence points (Ravn, Schmitt-Grohé, and Uribe, forthcoming). This question is of interest because under good-specific subsistence points the demand function faced by each individual monopolist features an additive, perfectly price inelastic term. Thus, all other things equal, the monopolist has an incentive to set an infinite price. In this note, we argue that an infinite price is indeed suboptimal. Moreover, we show that any finite deviation from the symmetric equilibrium price is suboptimal. We note that a perfectly price inelastic term also arises in the demand functions associated with preferences displaying deep habits as in Ravn, Schmitt-Grohé, and Uribe (2006). For analytical convenience, we focus on a discrete variety space.

Why Is the Monopolist's Price Not Infinity?

Consider an economy with a fixed number of goods N. Households have preferences given by

$$C = \left[\sum_{i=1}^{n}(c_i - c^*)^{1-1/\eta}\right]^{\frac{1}{1-1/\eta}}, \tag{1}$$

defined over consumption of the different varieties, c_i, for $i = 1, 2, \ldots, n \leq N$. The variable $c^* > 0$ denotes a good-specific subsistence point assumed to be common across varieties for simplicity. When $c^* = 0$, the above preference specification collapses to the standard Dixit-Stiglitz utility function exhibiting a taste for diversity. The household's expenditure is given by

$$\sum_{i=1}^{n} P_i c_i, \tag{2}$$

where P_i denotes the price of good i.

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Without loss of generality, we assume that prices are in increasing order. That is, $P_1 \leq P_2 \leq \cdots \leq P_N$. The household chooses n and c_i, for $i = 1, \ldots, n$, to minimize (2) subject to (1) for a given C. The resulting demand functions are of the form

$$c_i = \left(\frac{P_i}{P(n)}\right)^{-\eta} C + c^*,$$

where $P(n)$ is defined as

$$P(n) = \left[\sum_{i=1}^{n} P_i^{1-\eta}\right]^{1/(1-\eta)}.$$

The implied expenditure function is given by

$$E(n) = P(n)C + c^* \sum_{i=1}^{n} P_i.$$

We say that preferences exhibit taste for diversity when given equal prices across varieties, $P_i = \bar{P}$ for all i, the expenditure function is decreasing in the number of varieties consumed, n. That is, preferences display taste for diversity when the condition $E(n-1) > E(n)$ is satisfied.

Suppose that prices P_i for $i \leq n-1$ are finite. We wish to show that if $P_n \to \infty$, then the household will drop variety n from the consumption basket. Because $\eta > 1$, we have that

$$\lim_{P_n \to \infty} P(n) = P(n-1).$$

The right-hand side of this expression is finite. Also, we have that

$$\lim_{P_n \to \infty} c^* \sum_{i=1}^{n} P_i = \infty.$$

It follows that

$$\lim_{P_n \to \infty} E(n) = \infty.$$

Note that because P_i is finite for $i \leq n-1$, we have that $E(n-1)$ remains finite as $P_n \to \infty$. It follows that

$$\lim_{P_n \to \infty} [E(n) - E(n-1)] = \infty,$$

which establishes that if P_n goes to infinity, the household drops good n from its consumption basket.

Does the Monopolist Have an Incentive to Deviate from the Symmetric Equilibrium?

It is clear from the previous analysis that profit maximization by the monopolist must satisfy a household participation constraint of the form

$$\frac{E(n)}{E(n-1)} \leq 1.$$

We now ask whether it is appropriate to ignore this constraint when considering a symmetric monopolistically competitive equilibrium. We wish to show that at the profit maximizing price the household participation constraint does not bind.

Suppose that $n-1$ firms price at P and the n-th firm prices at ρP, with $\rho \geq 1$. Then the ratio $E(n)/E(n-1)$ can be written as

$$\frac{E(n)}{E(n-1)} = \frac{\left((n-1)+\rho^{1-\eta}\right)^{1/(1-\eta)} + z(n-1) + z\rho}{(n-1)^{1/(1-\eta)} + z(n-1)},$$

where $z \equiv c^*/C$.

The Markup in the Symmetric Equilibrium

The monopolist maximizes

$$(p - mc)(p^{-\eta}C + c^*)$$

where $p = P_i/P(n)$ and $mc = MC/P(n)$. We assume that the monopolist takes $P(n)$ as given. The first-order condition is

$$p^{-\eta}C + c^* - \eta p^{-\eta-1}C(p - mc) = 0$$

Assuming that every monopolist charges the same price P, defining the markup of price over marginal cost as $\mu \equiv P/MC$, and rearranging, we obtain

$$\mu = \frac{\eta}{\eta - 1 - zn^{\eta/(\eta-1)}}$$

Deviations from the Symmetric Equilibrium

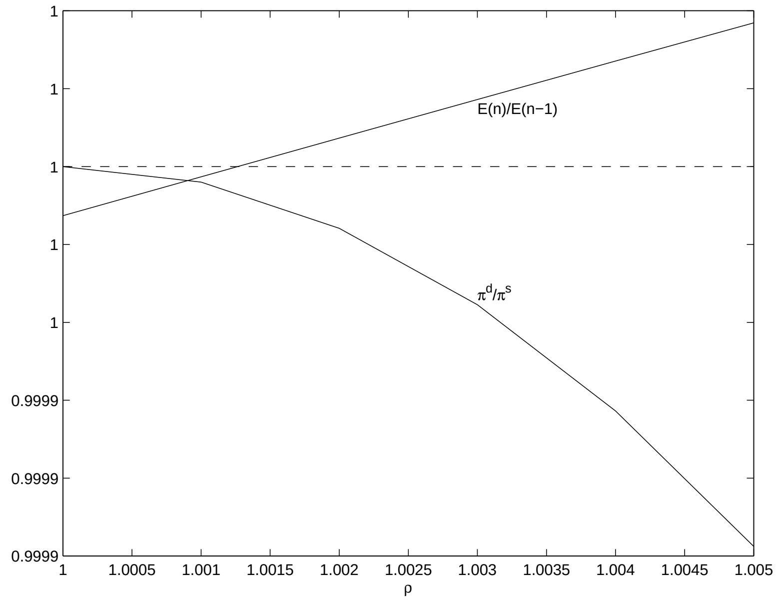
Let π^d denote the profits of a deviating firm that charges the price ρP, where P is the price in the symmetric equilibrium, and π^s profits if the firms charges the symmetric equilibrium price P. Then

$$\begin{aligned}
\frac{\pi^d}{\pi^s} &= \frac{(P_n - MC)\left(\left(\frac{P_n}{P(n)}\right)^{-\eta} C + c^*\right)}{(P - MC)\left(\left(\frac{P}{P(n)}\right)^{-\eta} C + c^*\right)} \\
&= \frac{(\rho - \mu^{-1})\left(\left(\rho\frac{P}{P(n)}\right)^{-\eta} + z\right)}{(1 - \mu^{-1})\left(\left(\frac{P}{P(n)}\right)^{-\eta} + z\right)} \\
&= \frac{(\rho - \mu^{-1})\left[\rho^{-\eta} n^{\eta/(1-\eta)} + z\right]}{(1 - \mu^{-1})\left(n^{\eta/(1-\eta)} + z\right)}
\end{aligned}$$

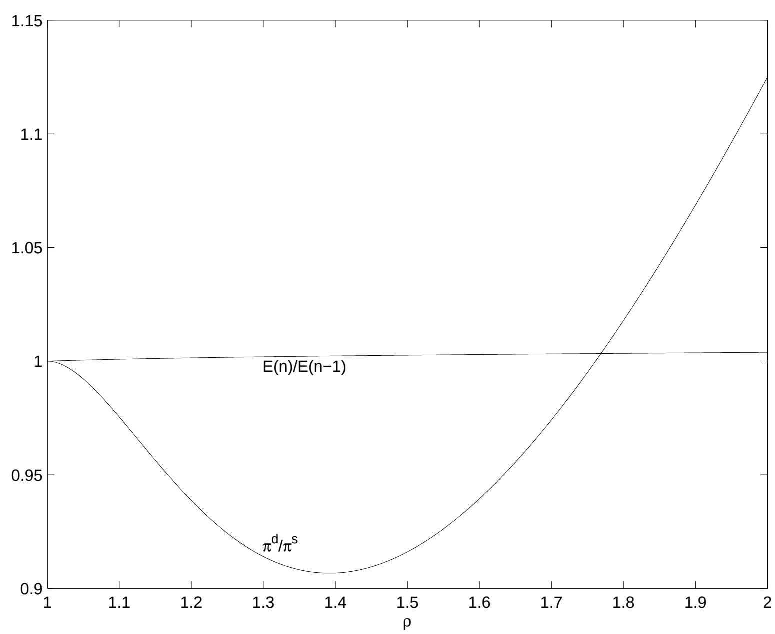
We now proceed to plot the ratios $E(n)/E(n-1)$ and π^d/π^s as a function of ρ. To this end, we set $n = 100$, $\eta = 5$, and z so that in the symmetric equilibrium the markup equals 33

Figure 1: π^d/π^s and $E(n)/E(n-1)$ as a function of ρ

(A) Zoom In View



(B) Zoom Out View



percent, or $\mu = 1.33$. Figure 1 displays the ratios π^d/π^s and $E(n)/E(n-1)$ as functions of the price deviation factor ρ. Notice that at the symmetric equilibrium, there is a taste for diversity. That is, when $\rho = 1$, we see that $E(n) < E(n-1)$. As ρ increases, the ratio $E(n)/E(n-1)$ increases. Eventually, this ratio crosses unity, indicating the critical value of ρ at which households drop variety n. The figure also shows that at any value of ρ less than or equal to this critical value, profits of the deviating monopolist do not exceed the level of profits associated with the symmetric equilibrium. This means that the monopolist will choose not to deviate from the symmetric equilibrium. This result is robust to setting n to 50, 200, 1000, and 2000, keeping μ constant at 1.33.

References

Ravn, Morten O., Stephanie Schmitt-Grohé, and Martín Uribe, "Deep Habits," *Review of Economic Studies 73,* January 2006, 195-218.

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