Relative Deep Habits

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Abstract

This note presents a detailed formal derivation of the equilibrium conditions of a variation of the deep habit model developed by Ravn, Schmitt-Grohé, and Uribe (2003). In the present note, the single-period utility function depends on the quasi ratio of current consumption to the stock of habit of each variety, whereas in the original deep habit model it depends on the quasi difference of these two variables. We refer to the case discussed in this note as 'relative deep habits'. *JEL Classification: D10, D12, D42, E30.*

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1 Relative Deep Habits in an Economy Without Capital

1.1 Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0,1]$. Each household j has preferences defined over consumption of a continuum of differentiated consumption goods, c_{it}^j indexed by $i \in [0,1]$ and labor effort, h_t^j. Following Abel (1990), preferences feature 'catching up with the Joneses.' However, unlike in the work of Abel, we assume that consumption externalities operate at the level of each individual good rather than at the level of some composite final good. We refer to this variant as 'catching up with the Joneses good by good.' Specifically, we assume that household j derives utility from an object x_t^j defined by

$$x_t^j = \left[\int_0^1 \left(\frac{c_{it}^j}{s_{it-1}^{\theta}} \right)^{1-1/\eta} di \right]^{1/(1-1/\eta)}, \tag{1}$$

where s_{it-1} denotes the stock of external habit in consuming good i in period $t-1$, which is assumed to evolve over time according to the following law of motion

$$s_{it} = \rho s_{it-1} + (1-\rho) c_{it}, \tag{2}$$

where $c_{it} \equiv \int_0^1 c_{it}^j dj$ denotes the cross-section average level of consumption of variety i, which the household takes as exogenously given. The parameter θ measures the degree of time nonseparability in consumption of each variety. When $\theta = 0$, we have the benchmark case of time separable preferences. At the same time, the parameter $\eta > 0$ denotes the intratemporal elasticity of substitution of habit-adjusted consumption of different varieties. It will become clear shortly that the introduction of catching up with the Joneses at the level of individual goods introduces a supply-side channel of transmission inducing more persistence in aggregate variables. The utility function of the household is assumed to be of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j - v_t, h_t^j), \tag{3}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, twice continuously differentiable, and strictly concave. The variable v_t denotes an exogenous and stochastic preference shock that follows a univariate autoregressive process of the form

$$v_t = \rho_v v_{t-1} + \epsilon_t^v,$$

where $\rho_v \in [0,1)$ and $\epsilon_t^v \sim$ i.i.d. with mean zero and standard deviation σ_v. This shock is meant to capture innovations in private nonbusiness absorption.

Households have two sources of wealth: wages and pure profits from the ownership of firms. At the same time, households allocate their income to purchases of consumption goods, c_{it}^j. Households face the following sequential budget constraint:

$$\int_0^1 p_{it} c_{it}^j di = w_t h_t^j + \Phi_t^j, \tag{4}$$

where p_{it} denotes the relative price of good i, w_t denotes the real wage rate and Φ_t^j denotes lump-sum profits originated in household j's ownership of shares in firms. In addition, households are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games. Clearly, for any given levels of x_t^j, purchases of each variety $i \in [0,1]$ in period t must solve the dual problems of minimizing total expenditure, $\int_0^1 p_{it} c_{it}^j di$, subject to the aggregation constraints (1). The optimal level of c_{it}^j for $i \in [0,1]$ is given by

$$c_{it}^j = \left(\frac{p_{it}}{\tilde{p}_t}\right)^{-\eta} s_{it-1}^{\theta(1-\eta)} x_t^j, \tag{5}$$

where $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$, is a price index, $p_{it} \equiv \frac{P_{it}}{P_t}$, is the relative price of variety i in terms of the composite good, and

$$\tilde{p}_t \equiv \left[\int_0^1 \left(p_{it} s_{it-1}^{\theta}\right)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

is the relative price of x_t in terms of the composite good. Note that consumption of each variety depends negatively on its externality-adjusted relative price $p_{it}/\tilde{p}_t$. At the optimum, we have that $\tilde{p}_t x_t^j = \int_0^1 p_{it} c_{it}^j di$. Then, the representative household's problem can be stated as consisting in choosing processes x_t^j and h_t^j so as to maximize the lifetime utility function (3) subject to

$$\tilde{p}_t x_t^j = w_t h_t^j + \phi_t^j, \tag{6}$$

given processes $\tilde{p}_t$, w_t, and Φ_t^j. The first-order condition associated with the household's problem is

$$-\frac{U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)} = \frac{w_t}{\tilde{p}_t} \tag{7}$$

1.2 Firms

Goods are produced by monopolistic firms. Each good $i \in [0, 1]$ is produced using labor as the sole input via the following homogenous-of-degree-one production technology:

$$y_{it} = A_t h_{it} - \phi, \tag{8}$$

where y_{it} denotes output of good i, h_{it} denotes services of labor, ϕ denotes a fixed cost of production, and A_t denotes an aggregate technology shock. We assume that the logarithm of A_t follows a univariate autoregressive process of the form

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a, \tag{9}$$

where ϵ_t^a is a white noise with standard deviation σ^a. On aggregate, households demand $c_{it} \equiv \int_0^1 c_{it}^j dj$ units of good i for consumption purposes. Equation (30) implies that

$$c_{it} = \left(\frac{p_{it}}{\tilde{p}_t}\right)^{-\eta} s_{it-1}^{\theta(1-\eta)} x_t \tag{10}$$

where $x_t \equiv \int_0^1 x_t^j dj$. Firms are price setters. In exchange, they must stand ready to satisfy demand at the announced prices. Formally, firm i must satisfy

$$A_t h_{it} - \phi \geq c_{it}. \tag{11}$$

Firm i's problem consists in choosing processes p_{it}, c_{it}, s_{it}, and h_{it} so as to maximize the present discounted value of profits,

$$E_0 \sum_{t=0}^{\infty} r_{0,t} \left[p_{it} c_{it} - w_t h_{it}\right], \tag{12}$$

subject to (2), (39), and (41), given processes $r_{0,t}$, $\tilde{p}_t$, w_t, A_t, and x_t, and initial condition s_{-1}. The variable $r_{0,t}$ is a pricing kernel determining the period-zero utility value of one unit of the composite good delivered in a particular state of period t. It follows from the household's problem that $r_{0,t} \equiv \beta^t U_x(x_t, h_t)/\tilde{p}_t$. The Lagrangian associated with firm i's optimization problem can be written as

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} r_{0,t} \Big\{ p_{it} c_{it} - w_t h_{it} + \kappa_t \left[A_t h_{it} - \phi - c_{it} \right] \\ &+ \nu_t \left[\left(\frac{p_{it}}{\tilde{p}_t} \right)^{-\eta} x_t s_{it-1}^{\theta(1-\eta)} - c_{it} \right] + \lambda_t \left[\rho s_{it-1} + (1-\rho) c_{it} - s_{it} \right] \Big\},\end{aligned}$$

The first-order conditions associated with the firm's problem are equations (2), (30), (41), and (taking derivatives of the Lagrangian with respect to c_{it}, s_{it}, h_{it}, and p_{it} in this order)

$$p_{it} - \nu_t - \kappa_t + \lambda_t (1-\rho) = 0,$$

$$\theta(1-\eta) E_t r_{t,t+1} \nu_{t+1} \frac{c_{it+1}}{s_{it}} + \rho E_t r_{t,t+1} \lambda_{t+1} = \lambda_t$$

$$\kappa_t = \frac{w_t}{A_t h_{it}}$$

$$c_{it} - \eta \nu_t \frac{c_{it}}{p_{it}} = 0$$

1.3 Equilibrium

In any symmetric equilibrium $p_{it} = 1$. It then follows from the last equation of the previous subsection that the shadow value of a marginal sale in period t, ν_t, is constant and equal to $1/\eta$.

We keep the notation regarding habit formation as flexible as possible. Specifically, we allow for two different parameters, θ and θ^d. This distinction allows us to capture the following special cases: (1) Superficial habits, $\theta^d = 0$. (2) Deep habits $\theta = \theta^d > 0$. In most cases, we consider the special case of deep habits, θ^d. The equilibrium conditions are then given by

$$x_t = \frac{c_t}{s_{t-1}^{\theta}} \tag{13}$$

$$-\frac{U_h(x_t - v_t, h_t)}{U_x(x_t - v_t, h_t)} = \frac{w_t}{s_{t-1}^{\theta}} \tag{14}$$

$$A_t h_t - \phi = c_t \tag{15}$$

$$\frac{1 - \frac{1}{\eta} - \frac{w_t}{A_t}}{\rho - 1} = \beta E_t \frac{U_x(x_{t+1} - v_{t+1}, h_{t+1})}{U_x(x_t - v_t, h_t)} \frac{s_{t-1}^{\theta}}{s_t^{\theta}} \left[\theta^d \frac{(1-\eta)}{\eta} \frac{c_{t+1}}{s_t} + \rho \frac{1 - \frac{1}{\eta} - \frac{w_{t+1}}{A_{t+1}}}{\rho - 1} \right] \tag{16}$$

$$s_t = \rho s_{t-1} + (1-\rho) c_t \tag{17}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a \tag{18}$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{19}$$

This is a system of 7 nonlinear, stochastic, difference equations in 7 unknowns. We look for a stationary solution to this system.

1.4 Calibration and Functional Forms

As in Ravn, Schmitt-Grohé, and Uribe (2003), we assume that the period utility index is separable in consumption and leisure. Specifically, preferences are of the form

$$U(x,h) = \frac{x^{1-\sigma}-1}{1-\sigma} + \gamma\frac{(1-h)^{1-\chi}-1}{1-\chi},$$

where $0 < \sigma \neq 1$, $0 < \chi \neq 1$, and $\gamma > 0$. In the special case in which σ and χ approach unity, this utility function converges to the log-linear specification adopted in countless business-cycle studies (e.g., King, Plosser, and Rebelo, 1988).

We calibrate the model to the U.S. economy following Ravn, Schmitt-Grohé, and Uribe (2003). The time unit is meant to be one quarter. Table 2 summarizes the calibration. We

Table 1: Calibration

Symbol	Value	Description
β	0.9902	Subjective discount factor
σ	2	Inverse of intertemporal elasticity of substitution
θ, θ^d	-0.1	Degree of habit formation
ρ	0.85	Persistence of habit stock
δ	0.0253	Quarterly depreciation rate
η	5.3	Elasticity of substitution across varieties
ϵ_{hw}	1.3	Frisch elasticity of labor supply
h	0.2	Steady-State fraction of time devoted to work
$\bar{g}$	0.0318	Steady-state level of government purchases
ϕ	0.0853	Fixed cost
ρ_v, ρ_a	0.9	Persistence of exogenous shocks

set $\theta = -0.1$, $\rho = 0.85$, $\eta = 5.3$, and $\sigma = 2$. We fix the preference parameter γ to ensure that in the deterministic steady state households devote 20 percent of their time to market activities following Prescott (1986). The calibration restrictions that identify the remaining structural parameters of the model are taken from Rotemberg and Woodford (1992). We follow their calibration strategy to facilitate comparison of our model of endogenous markups due to deep habits to their ad-hoc version of the customer-market model. In particular, we set the annual real interest rate to 4 percent and the Frisch labor supply elasticity equal to

1.3. These restrictions imply that the subjective discount factor, β, is 0.99, and that the preference parameter χ is 3.08.

As shown later in this note, in our model the steady-state markup of prices over marginal costs, μ, is given by

$$\mu = \frac{\eta m}{\eta m - 1},$$

where

$$m \equiv \left(\frac{1-\beta\rho}{\rho-1}\right)\left[\beta\theta(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]^{-1}$$

Our calibration implies an average value-added markup of 1.13. Note that in the case of perfect competition, that is, when $\eta \to \infty$, the markup converges to unity. In the case of no relative deep habit, i.e., when $\theta = \theta^d = 0$, we have that m equals one, and the markup equals $\eta/(\eta-1) = 1.23$, which relates the markup to the intratemporal elasticity of substitution across varieties in the usual way. This expression for the steady-state markup is the one that emerges from models with imperfect competition and superficial habit (e.g., Giannoni and Woodford, 2003; and Christiano, Eichenbaum, and Evans, 2003). Because under deep relative habit, the parameter m is greater than unity, firms have less market power under deep relatie habits than under superficial habits. This finding is akin to the result of Phelps and Winter (1970), who show that in a customer-market model, the average markup is below the markup that arises under a standard imperfect competition environment.

We set the serial correlation of all two shocks to 0.9 (i.e., $\rho_v = \rho_a = 0.9.$). These values are in the ball park of available estimates.

The next section describes in detail how the calibration restrictions are used to identified the structural parameters of the model and how to solve for the steady-state values of the endogenous variables.

Deterministic Steady State

Consider shutting off all sources of uncertainty and letting the system settle on a stationary point where any variable x_t satisfies $x_t = x_{t+1}$ for all t. In this state, the equilibrium conditions (43)-(19) collapse to:

$$x = c^{1-\theta} \tag{20}$$

$$\frac{\gamma c^{\sigma(1-\theta)+\theta}}{(1-h)^{\chi}} = w \tag{21}$$

$$h = c + \phi \tag{22}$$

$$[1-w] = \left(\frac{\rho-1}{1-\beta\rho}\right)\left[\beta\theta^d(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]\frac{1}{\eta} \tag{23}$$

$$s = c \tag{24}$$

$$A = 1$$

$$v = 0$$

This is a system of 7 equations in the following 16 unknowns: the 7 endogenous variables x, c, s, h, w, v, A; and 9 structural parameters σ, θ, β, η, ϕ, χ, γ, ρ, θ^d. To identify all 16 unknowns we impose 13 calibration restrictions:

$$\begin{aligned}
h - \phi - uk - wh &= 0 \qquad (25)\\
R &= 1.04^{1/4}\\
\theta = \theta^d &= 0.86\\
\rho &= 0.85\\
\eta &= 5.3\\
h &= 0.2\\
\sigma &= 2\\
\epsilon_{hw} \equiv \left.\frac{\partial \ln h}{\partial \ln w}\right|_{\lambda \text{ constant}} &= 1.3
\end{aligned}$$

Given the assumed preference specification, the Frisch labor supply elasticity is given by

$$\epsilon_{hw} = \frac{1-h}{h\chi}.$$

This expression can be solved for χ. The parameter β can be backed out from the assumed value for the interest rate

$$\beta = \frac{1}{R}.$$

Now using the fact that in the steady state the markup is given by $\mu = 1/w$, use equation (67) to write

$$1 = \eta\left[1 - \frac{1}{\mu}\right]\left(\frac{1-\beta\rho}{\rho-1}\right)\left[\beta\theta(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]^{-1}$$

Rearranging, we obtain the following expression for the steady-state markup

$$\mu = \frac{\eta m}{\eta m - 1},$$

where

$$m \equiv \left(\frac{1-\beta\rho}{\rho-1}\right)\left[\beta\theta(1-\eta)+\frac{1-\beta\rho}{\rho-1}\right]^{-1}$$

We note that if $\theta(\eta-1)>0(<0)$, then $m<1(>1)$.

Using the definition of the markup, we can write the zero-profit condition (77) as

$$\phi = \left(1-\frac{1}{\mu}\right)h$$

The production technology can be used to uncover the steady-state value of output:

$$y = h - \phi.$$

Consumption, in turn, satisfies,

$$c = y$$

The steady-state wage rate satisfies

$$w = \frac{1}{\mu}$$

Equations (71) determines directly the steady-state value of s. Finally, we can solve (62) for γ to obtain

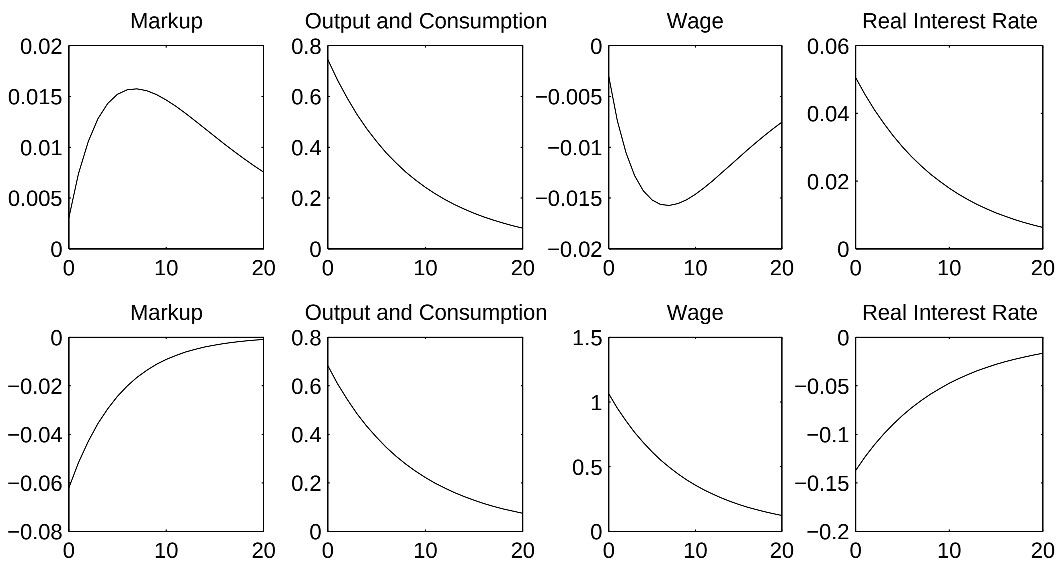
$$\gamma = \frac{w(1-h)^{\chi}}{c^{\sigma(1-\theta)+\theta}}$$

Figure 1 displays impulse responses to a positive preference shock (first row) and to a positive productivity shock (second row). The deep habit case is shown with a solid line, the superficial habit case is shown with a broken line, and the economy without habits is shown with a dotted line.

2 An Economy With Capital and Deep Habits Affecting All Components of Aggregate Demand

In this section we modify the simple model studied thus far in two dimensions. First, we introduce capital accumulation. Second, we assume that not only private consumption but also government spending and private investment are subject to deep habit formation. The model's parameterization is flexible enough to allow for the case in which any combination of aggregate demand components is subject to deep habits.

Figure 1: Impulse Responses to Positive Preference and Productivity Shocks Under Deep Habits, Superficial Habits, and No Habit in an Economy Without Capital



Row 1: Preference Shock. Row 2: Technology shock. Impulse responses are measured in percent deviations from steady state. Horizontal axes display the number of quarters after the shock.

2.1 Households

The economy is populated by a continuum of identical households of measure one indexed by $j \in [0,1]$. Each household j has preferences defined over consumption of a continuum of differentiated consumption goods, c^j_{it} indexed by $i \in [0,1]$ and labor effort, h^j_t. Following Abel (1990), preferences feature 'catching up with the Joneses.' However, unlike in the work of Abel, we assume that consumption externalities operate at the level of each individual good rather than at the level of some composite final good. We refer to this variant as 'catching up with the Joneses good by good.' Specifically, we assume that household j derives utility from an object x^j_t defined by equation (1). The utility function of the household is assumed to be of the form given in equation (3).

Households have three sources of wealth: wages, rents from capital, and pure profits from the ownership of firms. At the same time, households allocate their income to purchases of consumption goods, c^j_{it}, purchases of investment goods, i^j_{it}, and to payment of taxes, τ_t. Households face the following sequential budget constraint:

$$\int_0^1 p_{it}(c^j_{it} + i^j_{it})di + \tau_t = w_t h^j_t + u_t k_t + \Phi^j_t \tag{26}$$

where p_{it} denotes the price of good i, w_t denotes the real wage rate, u_t denotes the rental rate of capital, and k^j_t denotes the stock of capital held by household j, Φ^j_t denotes lump-sum profits originated in household j's ownership of shares in firms, and τ_t denotes real lump-sum taxes levied in period t. The capital stock evolves according to the following law of motion

$$k^j_{t+1} = (1-\delta)k^j_t + {x^I}^j_t, \tag{27}$$

where ${x^I}^j_t$ denotes investment by household j in period t. Investment is a composite good produced using intermediate goods via the technology

$${x^I}^j_t = \left[\int_0^1 \left(\frac{i^j_{it}}{{s^I_{it-1}}^{\theta^I}}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}. \tag{28}$$

The variable s^I_{it-1} summarizes how productive a particular input will. The idea is if a firm has a lot of experience in purchasing intermediate input i, then current purchases of that intermediate good i will be more effective. For each intermediate good i the 'prior-business-capital-stock,' s^I_{it-1}, is assumed to evolve over time according to

$$s^I_{it} = \rho s^I_{it-1} + (1-\rho)i_{it} \tag{29}$$

In addition, households are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games. Clearly, for any given levels of x_t^j and ${x^{Ij}_t}$, purchases of each variety $i \in [0,1]$ in period t must solve the dual problems of minimizing total expenditure, $\int_0^1 p_{it}(c_{it}^j + i_{it}^j)di$, subject to the aggregation constraints (1) and (28). The optimal levels of c_{it}^j and i_{it}^j for $i \in [0,1]$ are given by

$$c_{it}^j = \left(\frac{p_{it}}{\tilde{p}_t}\right)^{-\eta} s_{it-1}^{\theta(1-\eta)} x_t^j \tag{30}$$

and

$$i_{it}^j = \left(\frac{p_{it}}{\tilde{p}_t^I}\right)^{-\eta} [s_{it-1}^I]^{\theta^I(1-\eta)} {x^{Ij}_t} \tag{31}$$

where

$$\tilde{p}_t \equiv \left[\int_0^1 \left(p_{it} s_{it-1}^{\theta}\right)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

is the price of x_t and

$$\tilde{p}_t^I \equiv \left[\int_0^1 \left(p_{it}[s_{it-1}^I]^{\theta}\right)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

is the price of a unit of investment, x_t^I. Note that consumption of each variety depends negatively on its externality-adjusted relative price $p_{it}/\tilde{p}_t$. At the optimum, we have that $\tilde{p}_t x_t^j = \int_0^1 p_{it} c_{it}^j di$ and $\tilde{p}_t^I x_t^{Ij} = \int_0^1 p_{it} i_{it}^j di$. Then, the representative household's problem can be stated as consisting in choosing processes x_t^j, x_t^{Ij}, h_t^j, and k_t^j, so as to maximize the lifetime utility function (3) subject to (27) and

$$\tilde{p}_t x_t^j + \tilde{p}_t^I x_t^{Ij} + \tau_t = w_t h_t^j + u_t k_t^j + \phi_t^j, \tag{32}$$

given processes $\tilde{p}_t$, $\tilde{p}_t^I$, w_t, u_t, and Φ_t^j. The first-order conditions associated with the household's problem are

$$-\frac{U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)} = \frac{w_t}{\tilde{p}_t} \tag{33}$$

$$\tilde{p}_t^I \frac{U_x(x_t^j, h_t^j)}{\tilde{p}_t} = \beta E_t \frac{U_x(x_{t+1}^j, h_{t+1}^j)}{\tilde{p}_{t+1}} [(1-\delta)\tilde{p}_{t+1}^I + u_{t+1}] \tag{34}$$

2.2 The Government

We assume that government expenditures,g_t, are exogenous, stochastic, and follow a univariate first-order autoregressive process of the form

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g,$$

where the innovation ϵ_t^g distributes i.i.d. with mean zero and standard deviation σ^g. The government allocates spending over intermediate goods g_{it} so as to maximize the quantity of a composite good produced with intermediate goods according to the relationship

$$x_t^g = \left[\int_0^1 \left(\frac{g_{it}}{[s_{it-1}^g]^{\theta^g}}\right)^{1-1/\eta} di\right]^{1/(1-1/\eta)}.$$

The variable s_{it}^g denotes the stock of habit in good i, and evolves over time as

$$s_{it}^g = \rho s_{it-1}^g + (1-\rho)g_{it}. \tag{35}$$

We justify our specification of the aggregator function for government consumption by assuming that private households value government spending in goods in a way that is separable from private consumption and leisure and that households derive habits on consumption of government provided goods. The government's problem consists in choosing g_{it}, $i \in [0,1]$, so as to maximize x_t^g subject to the budget constraint $\int_0^1 p_{it}g_{it} \leq g_t$ and taking as given the initial conditions s_{i-1}^g. In solving this maximization problem, the government takes as given the effect of current public consumption on the level of next period's composite good—i.e., habits in government consumption are external. Conceivably, government habits could be treated as internal to the government even if they are external to their beneficiaries, namely, households. This, alternative, however, is analytically less tractable. The case of no habits in government consumption results from setting $\theta^g = 0$ in the above aggregator function for public goods. We believe that this is not the case of greatest interest under our maintained assumption that government spending on goods is valued by habit-forming private agents.

The resulting demand for each good $i \in [0,1]$ by the public sector is

$$g_{it} = \left(\frac{p_{it}}{\tilde{p}_t^g}\right)^{-\eta} x_t^g s_{it-1}^{g\theta(1-\eta)}, \tag{36}$$

where

$$g_t = \tilde{p}_t^g x_t^g$$

$$\tilde{p}_t^g \equiv \left[\int_0^1 \left(p_{it}s_{it-1}^{g\theta}\right)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

Public spending is assumed to be fully financed by lump-sum taxation.

$$g_t = \tau_t,$$

for all t.

2.3 Firms

Goods are produced by monopolistic firms. Each good $i \in [0,1]$ is produced using labor and capital as inputs via the following homogenous-of-degree-one production technology:

$$y_{it} = A_t F(k_{it}, h_{it}) - \phi, \tag{37}$$

where y_{it} denotes output of good i, k_{it} and h_{it} denote services of capital and labor, ϕ denotes a fixed cost of production, and A_t denotes an aggregate technology shock. We assume that the logarithm of A_t follows a univariate autoregressive process of the form

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a, \tag{38}$$

where ϵ_t^a is a white noise with standard deviation σ^a. On aggregate, households demand $c_{it} \equiv \int_0^1 c_{it}^j dj$ units of good i for consumption purposes and $i_{it} \equiv \int_0^1 i_{it}^j dj$ units for investment purposes. Equations (30) and (31) imply that

$$c_{it} = \left(\frac{p_{it}}{\tilde{p}_t}\right)^{-\eta} s_{it-1}^{\theta(1-\eta)} x_t \tag{39}$$

and

$$i_{it} = \left(\frac{p_{it}}{\tilde{p}_t^I}\right)^{-\eta} [s_{it-1}^I]^{\theta^I(1-\eta)} x_t^I \tag{40}$$

where $x_t \equiv \int_0^1 x_t^j dj$ and $x_t^I \equiv \int_0^1 x_t^{Ij} dj$. Firms are price setters. In exchange, they must stand ready to satisfy demand at the announced prices. Formally, firm i must satisfy

$$A_t F(k_{it}, h_{it}) - \phi \geq c_{it} + i_{it} + g_{it}. \tag{41}$$

Firm i's problem consists in choosing processes p_{it}, c_{it}, i_{it}, g_{it}, s_{it}, s_{it}^g, h_{it}, and k_{it} so as to maximize the present discounted value of profits,

$$E_0 \sum_{t=0}^{\infty} r_{0,t} \left[p_{it}(c_{it} + i_{it} + g_{it}) - w_t h_{it} - u_t k_{it}\right], \tag{42}$$

subject to (2), (29) (35), (36), (39), (40), and (41), given processes $r_{0,t}$, $\tilde{p}_t$, $\tilde{p}_t^g$, i_t, x_t^g, w_t, u_t, A_t, and x_t, and initial conditions s_{-1} and s_{-1}^g. The variable $r_{0,t}$ is a pricing kernel determining the period-zero utility value of one unit of the composite good delivered in a particular state of period t. It follows from the household's problem that $r_{0,t} \equiv \beta^t U_x(x_t, h_t)/\tilde{p}_t$. The Lagrangian associated with firm i's optimization problem can be written as

$$
\begin{aligned}
\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} r_{0,t} \{ p_{it}c_{it} + p_{it}i_{it} + p_{it}g_{it} - w_t h_{it} - u_t k_{it} \\
& + \kappa_t \left[A_t F(k_{it}, h_{it}) - \phi - c_{it} - i_{it} - g_{it} \right] \\
& + \nu_t \left[\left(\frac{p_{it}}{\tilde{p}_t} \right)^{-\eta} x_t s_{it-1}^{\theta(1-\eta)} - c_{it} \right] + \lambda_t \left[\rho s_{it-1} + (1-\rho) c_{it} - s_{it} \right] \\
& + \nu_t^I \left[\left(\frac{p_{it}}{\tilde{p}_t^I} \right)^{-\eta} x_t^I [s_{it-1}^I]^{\theta^I(1-\eta)} - i_{it} \right] + \lambda_t^I \left[\rho s_{it-1}^I + (1-\rho) i_{it} - s_{it}^I \right] \\
& + \nu_t^g \left[\left(\frac{p_{it}}{\tilde{p}_t^g} \right)^{-\eta} x_t^g [s_{it-1}^g]^{\theta^g(1-\eta)} - g_{it} \right] + \lambda_t^g \left[\rho s_{it-1}^g + (1-\rho) g_{it} - s_{it}^g \right] \Big\},
\end{aligned}
$$

The first-order conditions associated with the firm's problem are equations (2), (29), (35), (36), (39), (40), (41) and (taking derivatives of the Lagrangian with respect to c_{it}, s_{it}, i_{it}, s_{it}^I, g_{it}, s_{it}^g, h_{it}, k_{it}, and p_{it} in this order)

$$p_{it} - \nu_t - \kappa_t + \lambda_t(1-\rho) = 0,$$

$$\theta(1-\eta) E_t r_{t,t+1} \nu_{t+1} \frac{c_{it+1}}{s_{it}} + \rho E_t r_{t,t+1} \lambda_{t+1} = \lambda_t$$

$$p_{it} - \nu_t^I - \kappa_t + \lambda_t^I(1-\rho) = 0,$$

$$\theta^I(1-\eta) E_t r_{t,t+1} \nu_{t+1}^I \frac{i_{it+1}}{s_{it}^I} + \rho E_t r_{t,t+1} \lambda_{t+1}^I = \lambda_t^I$$

$$p_{it} - \nu_t^g - \kappa_t + \lambda_t^g(1-\rho) = 0,$$

$$\theta(1-\eta) E_t r_{t,t+1} \nu_{t+1}^g \frac{g_{it+1}}{s_{it}^g} + \rho E_t r_{t,t+1} \lambda_{t+1}^g = \lambda_t^g$$

$$\kappa_t = \frac{w_t}{A_t F_h(k_{it}, h_{it})}$$

$$\kappa_t = \frac{u_t}{A_t F_k(k_{it}, h_{it})}$$

$$c_{it} + i_{it} + g_{it} - \eta \nu_t \frac{c_{it}}{p_{it}} - \eta \nu_t^I \frac{i_{it}}{p_{it}} - \eta \nu_t^g \frac{g_{it}}{p_{it}} = 0$$

2.4 Equilibrium

We keep the notation regarding habit formation as flexible as possible. Specifically, we allow for three different habit parameters θ, θ^I and θ^g. This distinction allows us to capture the following special cases: (1) Deep habit on private consumption but not on government consumption and not on investment $\theta > 0$ and $\theta^g = \theta^I = 0$; (2) Deep habit on private and public consumption, $\theta^g = \theta^d > 0$ and $\theta^I > 0$; and (3) In the main text, we consider the special case of deep habit uniform across all private and public consumption, $\theta^d = \theta^g = \theta^I$.

In any symmetric equilibrium $p_{it} = 1$. The equilibrium conditions are then given by

$$x_t = \frac{c_t}{s_{t-1}^\theta} \tag{43}$$

$$x_t^I = \frac{i_t}{[s_{t-1}^I]^{\theta^I}} \tag{44}$$

$$-\frac{U_h(x_t - v_t, h_t)}{U_x(x_t - v_t, h_t)} = \frac{w_t}{s_{t-1}^\theta} \tag{45}$$

$$[s_{t-1}^I]^{\theta^I} \frac{U_x(x_t - v_t, h_t)}{s_{t-1}^\theta} = \beta E_t \frac{U_x(x_{t+1} - v_{t+1}, h_{t+1})}{s_t^\theta}[(1-\delta)[s_t^I]^{\theta^I} + u_{t+1}] \tag{46}$$

$$A_t F(k_t, h_t) - \phi = c_t + i_t + g_t \tag{47}$$

$$k_{t+1} = (1-\delta)k_t + x_t^i \tag{48}$$

$$\frac{c_t + g_t + i_t}{\eta} = \nu_t c_t + \nu_t^g g_t + \nu_t^I i_t \tag{49}$$

$$\frac{1 - \nu_t - \frac{w_t}{A_t F_h(k_t,h_t)}}{\rho - 1} = \beta E_t \frac{U_x(x_{t+1} - v_{t+1}, h_{t+1})}{U_x(x_t - v_t, h_t)} \frac{s_{t-1}^\theta}{s_t^\theta} \left[\theta^d (1-\eta)\nu_{t+1} \frac{c_{t+1}}{s_t} + \rho \frac{1 - \nu_{t+1} - \frac{w_{t+1}}{A_{t+1}F_h(k_{t+1},h_{t+1})}}{\rho - 1}\right] \tag{50}$$

$$\frac{1 - \nu_t^I - \frac{w_t}{A_t F_h(k_t,h_t)}}{\rho - 1} = \beta E_t \frac{U_x(x_{t+1} - v_{t+1}, h_{t+1})}{U_x(x_t - v_t, h_t)} \frac{s_{t-1}^\theta}{s_t^\theta} \left[\theta^I (1-\eta)\nu_{t+1}^I \frac{i_{t+1}}{s_t^I} + \rho \frac{1 - \nu_{t+1}^I - \frac{w_{t+1}}{A_{t+1}F_h(k_{t+1},h_{t+1})}}{\rho - 1}\right] \tag{51}$$

$$\frac{1 - \nu_t^g - \frac{w_t}{A_t F_h(k_t,h_t)}}{\rho - 1} = \beta E_t \frac{U_x(x_{t+1} - v_{t+1}, h_{t+1})}{U_x(x_t - v_t, h_t)} \frac{s_{t-1}^\theta}{s_t^\theta} \left[\theta^g (1-\eta)\nu_{t+1}^g \frac{g_{t+1}}{s_t^g} + \rho \frac{1 - \nu_{t+1}^g - \frac{w_{t+1}}{A_{t+1}F_h(k_{t+1},h_{t+1})}}{\rho - 1}\right] \tag{52}$$

$$\frac{F_h(k_t, h_t)}{F_k(k_t, h_t)} = \frac{w_t}{u_t} \tag{53}$$

$$s_t = \rho s_{t-1} + (1-\rho)c_t \tag{54}$$

$$s_t^I = \rho s_{t-1}^I + (1-\rho)i_t \tag{55}$$

$$s_t^g = \rho s_{t-1}^g + (1-\rho)g_t \tag{56}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_t^a \tag{57}$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{58}$$

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g \tag{59}$$

This is a system of 17 nonlinear, stochastic, difference equations in 17 unknowns. We look for a stationary solution to this system.

2.5 Calibration and Functional Forms

As in Ravn, Schmitt-Grohé, and Uribe (2003), we assume that the period utility index is separable in consumption and leisure. Specifically, preferences are of the form

$$U(x,h) = \frac{x^{1-\sigma}-1}{1-\sigma} + \gamma\frac{(1-h)^{1-\chi}-1}{1-\chi},$$

where $0 < \sigma \neq 1$, $0 < \chi \neq 1$, and $\gamma > 0$. In the special case in which σ and χ approach unity, this utility function converges to the log-linear specification adopted in countless business-cycle studies (e.g., King, Plosser, and Rebelo, 1988). The production function is assumed to be of the Cobb-Douglas type

$$F(k,h) = k^{\alpha}h^{1-\alpha}; \quad \alpha \in (0,1).$$

We calibrate the model to the U.S. economy following Ravn, Schmitt-Grohé, and Uribe (2003). The time unit is meant to be one quarter. Table 2 summarizes the calibration.

Table 2: Calibration

Symbol	Value	Description
β	0.9902	Subjective discount factor
σ	2	Inverse of intertemporal elasticity of substitution
$\theta, \theta^d, \theta^s$	-0.1	Degree of habit formation
ρ	0.85	Persistence of habit stock
α	0.25	capital elasticity of output
δ	0.0253	Quarterly depreciation rate
η	5.3	Elasticity of substitution across varieties
ϵ_{hw}	1.3	Frisch elasticity of labor supply
h	0.2	Steady-State fraction of time devoted to work
$\bar{g}$	0.0318	Steady-state level of government purchases
ϕ	0.0853	Fixed cost
ρ_v, ρ_g, ρ_a	0.9	Persistence of exogenous shocks

We set $\theta = \theta^s = \theta^g = -0.1$, $\rho = 0.85$, $\eta = 5.3$, and $\sigma = 2$. We fix the preference parameter γ to ensure that in the deterministic steady state households devote 20 percent of their time to market activities following Prescott (1986). The calibration restrictions that identify the remaining structural parameters of the model are taken from Rotemberg and

Woodford (1992). We follow their calibration strategy to facilitate comparison of our model of endogenous markups due to deep habits to their ad-hoc version of the customer-market model. In particular, we set the labor share in GDP to 75 percent, the consumption share to 70 percent, the government consumption share to 12 percent, the annual real interest rate to 4 percent, and the Frisch labor supply elasticity equal to 1.3. These restrictions imply that the capital elasticity of output in production, α, is 0.25, the depreciation rate, δ, is 0.025 per quarter, the subjective discount factor, β, is 0.99, and that the preference parameter χ is 3.08.

As shown later in this note, in our model the steady-state markup of price over marginal cost, μ, is given by

$$\mu = \frac{\eta m}{\eta m - 1},$$

where

$$m \equiv \left(\frac{1-\beta\rho}{\rho-1}\right)\left[\beta\theta(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]^{-1} s_c + \left(\frac{1-\beta\rho}{\rho-1}\right)\left[\beta\theta^g(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]^{-1} s_g + s_i$$

Our calibration implies an average value-added markup of 1.13. Note that in the case of perfect competition, that is, when $\eta \to \infty$, the markup converges to unity. In the case of no relative deep habit, i.e., when $\theta = \theta^d = \theta^g = 0$, we have that m equals one, and the markup equals $\eta/(\eta-1) = 1.23$, which relates the markup to the intratemporal elasticity of substitution across varieties in the usual way. This expression for the steady-state markup is the one that emerges from models with imperfect competition and superficial habit (e.g., Giannoni and Woodford, 2003; and Christiano, Eichenbaum, and Evans, 2003). Because under deep relative habit, the parameter m is greater than unity, firms have less market power under deep relatie habits than under superficial habits. This finding is akin to the result of Phelps and Winter (1970), who show that in a customer-market model, the average markup is below the markup that arises under a standard imperfect competition environment.

We set the serial correlation of all three shocks to 0.9 (i.e., $\rho_v = \rho_g = \rho_a = 0.9.$). These values are in the ball park of available estimates.

The next section describes in detail how the calibration restrictions are used to identified the structural parameters of the model and how to solve for the steady-state values of the endogenous variables.

Deterministic Steady State

Consider shutting off all sources of uncertainty and letting the system settle on a stationary point where any variable x_t satisfies $x_t = x_{t+1}$ for all t. In this state, the equilibrium conditions (43)-(59) collapse to:

$$x = c^{1-\theta} \tag{60}$$

$$x^I = i^{1-\theta^I} \tag{61}$$

$$\frac{\gamma c^{\sigma(1-\theta)+\theta}}{(1-h)^{\chi}} = w \tag{62}$$

$$1 = \beta[1 - \delta + i^{-\theta^I} u] \tag{63}$$

$$k^{\alpha} h^{1-\alpha} = c + i + g + \phi \tag{64}$$

$$i^{1-\theta^I} = \delta k \tag{65}$$

$$\frac{c+g+i}{\eta} = \nu c + \nu^g g + \nu^I i \tag{66}$$

$$\left[1 - \frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] = \left(\frac{\rho-1}{1-\beta\rho}\right)\left[\beta\theta^d(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]\nu \tag{67}$$

$$\left[1 - \frac{w}{(1-\alpha)(k/h)^{\alpha}}\right] = \left(\frac{\rho-1}{1-\beta\rho}\right)\left[\beta\theta^I(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]\nu^I \tag{68}$$

$$\left[1 - \frac{w_t}{(1-\alpha)(k/h)^{\alpha}}\right] = \left(\frac{\rho-1}{1-\beta\rho}\right)\left[\beta\theta^g(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]\nu^g \tag{69}$$

$$\frac{1-\alpha}{\alpha}\frac{k}{h} = \frac{w}{u} \tag{70}$$

$$s = c \tag{71}$$

$$s^I = i \tag{72}$$

$$s^g = g \tag{73}$$

$$A = 1 \tag{74}$$

$$v = 0 \tag{75}$$

$$g = \bar{g}. \tag{76}$$

This is a system of 17 equations in the following 31 unknowns: the 17 endogenous variables x, x^I, c, s, s^I, h, w, v, ν^I, u, A, i, g, k, ν, ν^g, and s^g; and 14 structural parameters σ, θ, δ, β, η, α, ϕ, χ, γ, ρ, θ^d, θ^I, θ^g, $\bar{g}$, To identify all 31 unknowns we impose 14 calibration restrictions:

$$\begin{aligned}
s_h \equiv \frac{wh}{k^\alpha h^{1-\alpha} - \phi} &= 0.75 \\
s_c \equiv \frac{c}{k^\alpha h^{1-\alpha} - \phi} &= 0.7 \\
s_g \equiv \frac{\bar{g}}{k^\alpha h^{1-\alpha} - \phi} &= 0.12 \\
k^\alpha h^{1-\alpha} - \phi - uk - wh &= 0 \\
R \equiv 1 - \delta + u i^{-\theta^I} &= 1.04^{1/4} \\
\theta = \theta^d = \theta^g = \theta^I &= -0.1 \\
\rho &= 0.85 \\
\eta &= 5.3 \\
h &= 0.2 \\
\sigma &= 2 \\
\epsilon_{hw} \equiv \left.\frac{\partial \ln h}{\partial \ln w}\right|_{\lambda \text{ constant}} &= 1.3
\end{aligned} \tag{77}$$

Given the assumed preference specification, the Frisch labor supply elasticity is given by

$$\epsilon_{hw} = \frac{1-h}{h\chi}.$$

This expression can be solved for χ. Equation (63) can be solved for β

$$\beta = \frac{1}{R}.$$

Equation (67) and (69) imply that

$$\frac{\nu^g}{\nu} = \frac{\left[\beta\theta^d(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]}{\left[\beta\theta^g(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]}$$

And equations (67) and (68) imply that

$$\frac{\nu^I}{\nu} = \frac{\left[\beta\theta^d(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]}{\left[\beta\theta^I(1-\eta) + \frac{1-\beta\rho}{\rho-1}\right]}$$

Using those expression we can solve (66) for ν

$$\nu = \frac{1}{\eta}\frac{1}{\left[s_c + \frac{\nu^g}{\nu}s_g + \frac{\nu^I}{\nu}s_i\right]}$$

Note that in the special case that $\theta^I = \theta^g = \theta^d$, we have $\nu = \nu^g = \nu^I$ and thus $\nu = 1/\eta$.

With ν at hand we obtain:

$$\nu^g = \nu \times \frac{\nu^g}{\nu}$$

and

$$\nu^I = \nu \times \frac{\nu^I}{\nu}$$

Now use equations (67) to find the steady state markup. First, use $1/mu = w/F_h$:

$$\left[1 - \frac{1}{\mu}\right] = \left[1 + \beta\theta^d(1-\eta)\left(\frac{\rho-1}{1-\beta\rho}\right)\right]\nu$$

Rearranging, we obtain the following expression for the steady-state markup

$$\mu = \frac{1}{1 - \left[1 + \beta\theta^d(1-\eta)\left(\frac{\rho-1}{1-\beta\rho}\right)\right]\nu},$$

Using the definition of the markup and (70) we can write the zero-profit condition (77) as

$$\phi = \left(1 - \frac{1}{\mu}\right)k^\alpha h^{1-\alpha}$$

It follows that the labor share, $s_h \equiv wh/y$, is given by

$$s_h = 1 - \alpha.$$

Finally, to determine δ we use the following relation

$$\begin{aligned}
\delta &= \frac{i^{1-\theta^I}}{k} \\
&= s_i \frac{y i^{-\theta^I}}{k} \\
&= \frac{s_i}{s_k} u i^{-\theta^I} \\
&= \frac{1-s_c-s_g}{s_h} \frac{1-\alpha}{\alpha} [\beta^{-1}-1+\delta]
\end{aligned}$$

The first equality uses (65), and the last one uses (64), (63), and (70). At this point, we have identified 10 of the 13 structural parameters, namely, σ, θ, δ, β, η, α, χ, ρ, θ^d, θ^g. It remains to determine values for the parameters γ, $\bar{g}$ and ϕ and steady-state values for the endogenous variables of the model. We accomplish this task next.

To obtain the deterministic-steady-state level of the capital stock, solve (65) for k. This yields

$$\begin{aligned}
k &= \frac{i^{1-\theta^I}}{\delta} \\
&= \frac{(s_i y)^{1-\theta^I}}{\delta} \\
&= \frac{(s_i[k^\alpha h^{1-\alpha}-\phi])^{1-\theta^I}}{\delta} \\
&= \frac{(s_i[k^\alpha h^{1-\alpha}]/\mu)^{1-\theta^I}}{\delta} \\
&= \left(\frac{(s_i h^{1-\alpha}/\mu)^{1-\theta^I}}{\delta}\right)^{\frac{1}{1-\alpha(1-\theta^I)}}
\end{aligned}$$

Knowing k and h, the parameter ϕ was found above to be equal to $(1-1/\mu)k^\alpha h^{1-\alpha}$. The production technology delivers the steady-state value of output:

$$y = k^\alpha h^{1-\alpha} - \phi.$$

The steady-state values of the components of aggregate demand follow immediately

$$c = s_c y$$

$$i = s_i y$$

$$g = s_g y$$

The steady-state value of the rental rate of capital, u, is given by

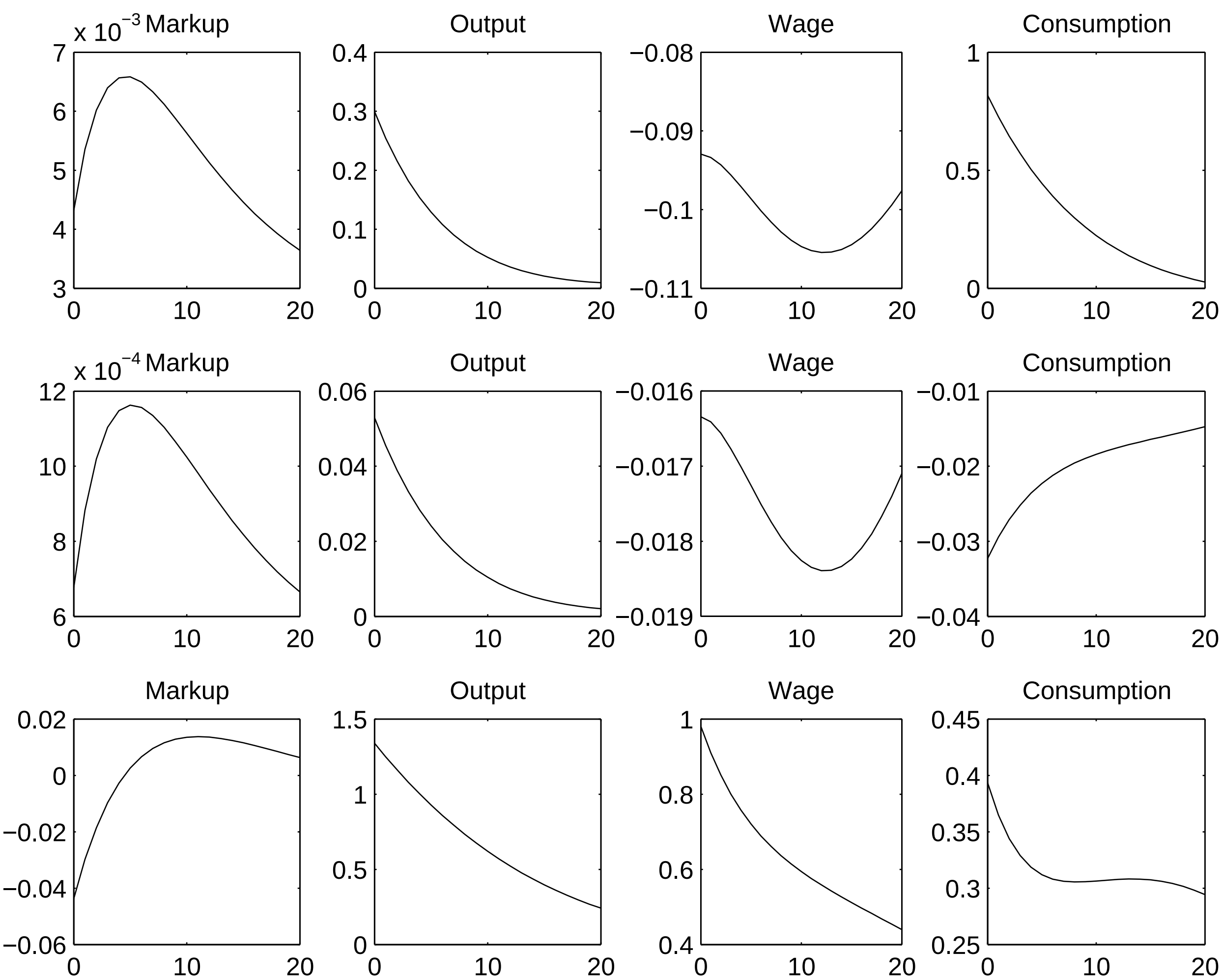
$$u = \frac{\beta^{-1} - (1-\delta)}{i^{-\theta^I}}$$

Equation (70) determines the steady-state wage rate. Equations (67), (69), (71), (73), and (76) determine directly the steady-state value of ν, ν^g, s, s^g, and $\bar{g}$, respectively. Finally, we can solve (62) for γ to obtain

$$\gamma = \frac{w(1-h)^\chi}{c^{\sigma(1-\theta)+\theta}}$$

Figure 2 displays impulse responses to a positive preference shock (first row), to a positive government spending shock (second row), and to a positive productivity shock (third row).

Figure 2: Impulse Responses to Positive Preference, Government Spending, and Productivity Shocks Under Deep Relative Habits in All Components of Aggregate Demand



Row 1: Preference Shock. Row 2: Gov't Spending Shock Row 3: Technology shock. Impulse responses are measured in percent deviations from steady state. Horizontal axes display the number of quarters after the shock.

References

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