Online Appendix To
Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment

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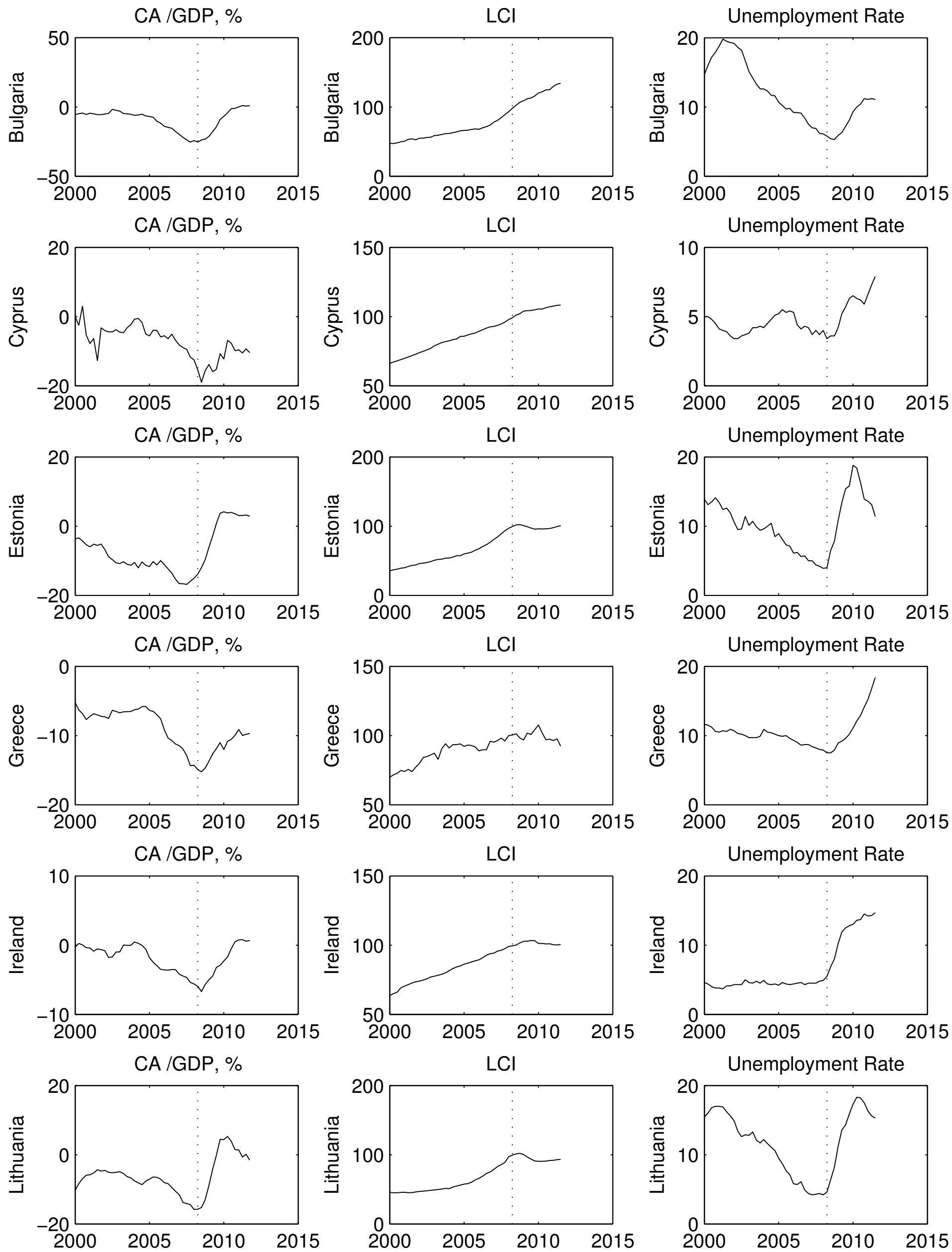
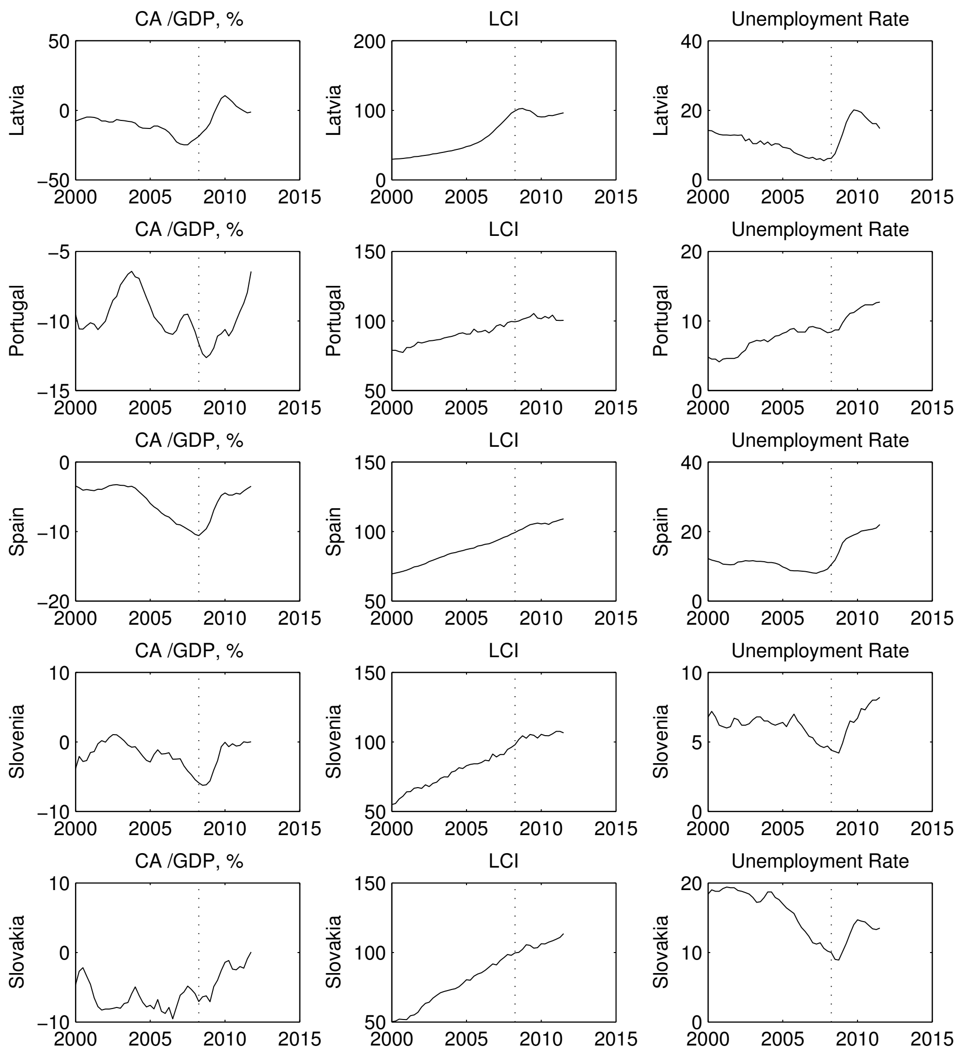
A Figure 1 Boom-Bust Cycle in Peripheral Europe: 2000-2011, Country-by-Country and Data Sources





Note. CA/GDP= Current account to GDP ratio in percent, LCI= Nominal Labor Cost Index, 2008=100. The vertical dotted line indicates 2008:Q2, the onset of the Great Contraction in Europe. The sample period is 2000Q4 to 2011Q3. All data is from Eurostat `http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database`. Current account and gross domestic product are four-quarter averages. Nominal hourly wage data is EuroStat's Labour cost index, nominal value - Quarterly data (Nace R2), series name, `[lc_lci_r2_q]`. For all countries but Spain it covers the sectors industry, services and construction. For Spain the wage measure in Industry, Services, and Construction begins only in 2006Q1, therefore we use the corresponding wage measure for Business Economy. Unemployment data is Eurostat's series `une_rt_q]`, unemployment rate, quarterly average. The data is available in the spreadsheet fig1.xlsx.

B Proof of Proposition 1

Consider an equilibrium, that is, a set of stochastic processes $\{c_t^T,\ h_t,\ w_t,\ d_{t+1},\ \lambda_t,\ \mu_t\}_{t=0}^{\infty}$ satisfying (10)-(19) and the exchange rate policy (22).

We first show that under this exchange rate policy h_t must equal $\bar{h}$ at all times. This part of the proof is by contradiction. Suppose $h_t < \bar{h}$ for some $t \geq 0$. Then, by (19) we have that

$$w_t = \frac{\gamma w_{t-1}}{\epsilon_t}. \tag{B.1}$$

Solve this expression for ϵ_t. Then use the resulting expression to eliminate ϵ_t from (22) to obtain $w_t \leq \omega(c_t^T)$. Using (16) to replace w_t and (21) to replace $\omega(c_t^T)$, we can rewrite this inequality as

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \leq \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}).$$

Because the left-hand side of this expression is strictly decreasing in h_t, we have that h_t must equal $\bar{h}$, which is a contradiction. We have therefore shown that under the exchange rate policy given in (22), unemployment is nil at all dates and states.

It remains to be shown that the real allocation associated with the exchange rate policy (22) is Pareto optimal. Evaluate the equilibrium conditions (10)-(15) at $h_t = \bar{h}$ to obtain

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t},$$

$$d_{t+1} \leq \bar{d}$$

$$\mu_t \geq 0,$$

$$\mu_t(d_{t+1} - \bar{d}) = 0,$$

$$\lambda_t = U'(A(c_t^T, \bar{h}))A_1(c_t^T, \bar{h})$$

$$\frac{\lambda_t}{1 + r_t} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t,$$

which are precisely the first-order necessary and sufficient conditions associated with the social planner's problem consisting in maximizing (23) subject to (8), (10) and (11). The fact that the first-order conditions of the social planner's problem are necessary and sufficient follows directly from the strict concavity of the planner's objective and the convexity of the planner's constraint set.

C Proof of Proposition 2

The household's problem consists in maximizing the lifetime utility function (1) subject to the aggregator function (2), the no-Ponzi-game constraint (4) and the sequential budget constraint (24). The first-order conditions associated with this problem are (5), (6), and (11)-(13).

Using (5) to eliminate p_t from (25) yields

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} F'(h_t) = (1 - \tau_t^h) w_t. \tag{C.1}$$

In equilibrium, (26) implies that

$$\tau_t^y = \frac{\tau_t^h w_t h_t}{y_t^T + \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F(h_t)}. \tag{C.2}$$

Equilibrium under a currency peg ($\epsilon_t = 1$) and a wage subsidy is then defined as a set of stochastic processes $\{c_t^T,\ h_t,\ w_t,\ d_{t+1},\ \lambda_t,\ \mu_t,\ \tau_t^y\}_{t=0}^{\infty}$ satisfying (10)-(15), (18), and (C.1), (C.2), and

$$w_t \geq \gamma w_{t-1} \tag{C.3}$$

$$(h_t - \bar{h})\,(w_t - \gamma w_{t-1}) = 0. \tag{C.4}$$

Consider now the Pareto optimal allocation $\{c^{T^o}_t,\ d^o_{t+1},\ h^o_t\}_{t=0}^{\infty}$ defined in the body of the paper. To show that this allocation satisfies the equilibrium conditions of the economy with a currency peg and a wage subsidy, it suffices to establish that it satisfies conditions (C.1)-(C.4), since all other equilibrium conditions are common to those associated with the social planner's problem. Because $h_t^o = \bar{h}$ for all t, equation (C.4) is always satisfied. For the same reason, the left-hand side of (C.1) becomes $\omega(c_t^T)$. Now if $\omega(c_t^T) \geq \gamma w_{t-1}$, set $\tau_t^h = 0$. Then, by equation (C.1), $w_t = \omega(c_t^T)$, which, by assumption, is greater than or equal to γw_{t-1}. Therefore equilibrium condition (C.3) is satisfied in this case. If, on the other hand, $\omega(c_t^T) < \gamma w_{t-1}$, then set $1 - \tau_t^h = \frac{\omega(c_t^T)}{\gamma w_{t-1}}$. In this case, equation equation (C.1) implies that $w_t = \gamma w_{t-1}$, which satisfies condition (C.3). It follows that the proposed wage-subsidy process (27) supports the Pareto optimal allocation as a competitive equilibrium.

D The Analytical Example: Preliminary Results

Equilibrium for $t \geq 1$

We wish to show that in the economy analyzed in section 7, in which the nominal exchange rate is constant and capital controls are set in a Ramsey optimal fashion, the equilibrium allocation features constant values for consumption, debt, hours, and wages for $t \geq 1$.

The optimal capital control problem for $t \geq 1$ can be written as follows

$$\max_{\{c_t^T, h_t, w_t, d_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [\ln c_t^T + \alpha \ln h_t] \tag{D.1}$$

subject to

$$c_t^T + d_t = y^T + \frac{d_{t+1}}{1+r}, \tag{D.2}$$

$$\alpha \frac{c_t^T}{h_t} = w_t, \tag{D.3}$$

$$h_t \leq 1, \tag{D.4}$$

$$w_t \geq w_{t-1}, \tag{D.5}$$

and

$$d_{t+1} \leq \bar{d}, \tag{D.6}$$

given d_1 and w_0.

Consider next the less restrictive problem of maximizing (D.1) subject to (D.2), (D.4), and (D.6), given d_1. It is straightforward to see that the solution of this problem is $c_t^T = c^{T*} \equiv y^T - \frac{r}{1+r} d_1$, $d_{t+1} = d_1$, and $h_t = 1$ for all $t \geq 1$. For this solution to comply with equilibrium condition (D.3), the wage rate must satisfy $w_t = \alpha c^{T*}$ for all $t \geq 1$. In turn, for equilibrium condition (D.5) to hold, we need that

$$\alpha c^{T*} \geq w_0. \tag{D.7}$$

Therefore, if this condition holds, the solution to the less constrained problem is also the solution to the original Ramsey problem, and the Ramsey optimal allocation implies constant paths for consumption, debt, hours, and wages, which is what we set out to show.

Now assume that condition (D.7) is not satisfied, that is, assume that

$$\alpha c^{T*} < w_0. \tag{D.8}$$

Use equation (D.3) to eliminate h_t from the utility function and from (D.4). Then, we can rewrite the Ramsey problem as

$$\max_{\{c_t^T, w_t, d_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [(1+\alpha)\ln c_t^T + \alpha \ln \alpha - \alpha \ln w_t] \tag{D.9}$$

subject to (D.2), (D.5), (D.6), and

$$w_t \geq \alpha c_t^T, \tag{D.10}$$

given d_1 and w_0.

Consider the less restrictive problem consisting in dropping (D.10) from the above maximization problem. Since the indirect utility function (D.9) is separable in consumption of tradables and wages and since the only constraint in the less restrictive problem that features wages, namely, equation (D.5), contains neither consumption of tradables nor debt, we can separate the less restricted problem into two independent problems. One is

$$\max_{\{c_t^T, d_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [(1+\alpha)\ln c_t^T + \alpha \ln \alpha] \tag{D.11}$$

subject to (D.2) and (D.6). The solution of this problem is $c_t^T = c^{T*}$ and $d_{t+1} = d_1$ for all $t \geq 1$.

The second problem is

$$\max_{\{w_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [-\alpha \ln w_t]$$

subject to (D.5), given w_0. The solution to this problem is $w_t = w_0$ for all $t \geq 1$.

It remains to show that the solution to these two problems satisfy the omitted constraint (D.10). To see that this is indeed the case, note that $w_t = w_0 > \alpha c^{T*} = \alpha c_t^T$, where the inequality follows from (D.8).

We have therefore shown that in the Ramsey equilibrium all variables are constant for $t \geq 1$. In particular, $c_t^T = c^{T*} \equiv y^T - \frac{r}{1+r} d_1$, $d_{t+1} = d_1$, $h_t = \min\{1, \frac{\alpha c^{T*}}{w_0}\}$, and $w_t = \max\{w_0, \alpha c^{T*}\}$, for all $t \geq 1$.

Proof That $d_1 \geq 0$

We wish to prove that in equilibrium d_1 is nonnegative. To see this, assume that the planner is constrained to choose $d_1 \leq 0$. We will show that in this case, the optimal level of debt is 0. If $d_1 \leq 0$, we have that $c_0^T \leq y^T \leq c_1^T$. The full employment wage in period 0 is $\alpha c_0^T \leq \alpha y^T = w_{-1}$. We therefore have that $w_0 = w_{-1}$ and that $h_0 = \alpha c_0^T / w_{-1} = c_0^T / y^T \leq 1$. In period 1, the full employment wage is $\alpha c_1^T \geq \alpha y^T = w_0$, which does not violate the wage lower bound. So we have that $h_1 = 1$. The optimization problem of the Ramsey planner conditional on $d_1 \le 0$ is then given by

$$\max_{\{c_0^T, c_1^T, d_1\}} \left\{ \ln c_0^T + \alpha \ln(c_0^T/y^T) + \frac{\beta}{1-\beta} \ln c_1^T \right\}$$

subject to $d_1 \le 0$, $c_0^T = y^T + \frac{d_1}{1+\underline{r}}$, and $c_1^T = y^T - \frac{r d_1}{1+r}$. The optimality conditions associated with this problem are the above three constraints,

$$\frac{1+\alpha}{1+\underline{r}} \frac{1}{c_0^T} - \frac{1}{1+r} \frac{1}{c_1^T} \ge 0, \tag{D.12}$$

and the slackness condition

$$\left[\frac{1+\alpha}{1+\underline{r}} \frac{1}{c_0^T} - \frac{1}{1+r} \frac{1}{c_1^T} \right] d_1 = 0. \tag{D.13}$$

The left-hand side of (D.12) is positive for all $d_1 \le 0$. Then, by the slackness condition (D.13), we have that $d_1 = 0$, which is what we set out to establish.

Proof that $h_1 = c_1^T/c_0^T$

We have established that $c_0^T \ge y^T$. This implies that the full-employment real wage in period 0, αc_0^T, is greater than or equal to $w_{-1} \equiv \alpha y^T$. Therefore, we have that $h_0 = 1$ and $w_0 = \alpha c_0^T$. Also, the fact that $c_0^T \ge y^T$ implies that $c_1^T \le y^T$. In turn, this means that the full employment wage in period 1, αc_1^T, is less than or equal to $w_0 = \alpha c_0^T$. This implies, from the result $w_1 = \max\{w_0, \alpha c_1^T\}$, that $w_1 = w_0$. This result and the fact that $w_1 = \alpha c_1^T/h_1$ and $w_0 = \alpha c_0^T$ imply that $h_1 = c_1^T/c_0^T$. Intuitively, a contraction in tradable absorption in period 1 generates persistent unemployment.

For completeness, the following proposition presents the equilibrium allocation under a currency peg with free capital mobility, under a currency peg with optimal capital controls in the case $\alpha > r$, and under optimal exchange-rate (or optimal labor subsidy) policy.

Proposition D.1 (The Prudential Nature of Optimal Capital Controls) *In the economy of section 7, aggregate dynamics under a currency peg with free capital mobility are given by*

$$c_0^T = y^T \left[\frac{1}{1+\underline{r}} + \frac{r}{1+r} \right] > y^T$$

$$c_t^T = y^T \left[\frac{1}{1+r} + \frac{r}{1+r} \frac{1+\underline{r}}{1+r} \right] < y^T; \quad t \ge 1$$

$$d_t = y^T \left[1 - \frac{1+\underline{r}}{1+r}\right] > 0; \quad t \geq 1$$

$$h_0 = 1,$$

$$h_t = \frac{1+\underline{r}}{1+r} < 1; \quad t \geq 1.$$

And the Ramsey optimal allocation under a currency peg when the planner uses capital controls as the policy instrument and $\alpha > r$ is given by

$$c_t^T = y^T; \quad t \geq 0$$

$$h_t = 1; \quad t \geq 0$$

$$d_t = 0; \quad t \geq 0$$

and

$$\tau_t^d = \begin{cases} 1 - \frac{1+\underline{r}}{1+r} & \textit{for } t = 0 \\ 0 & \textit{for } t \geq 1 \end{cases}$$

The first-best allocation (achieved either by optimal exchange-rate policy or optimal labor-subsidy policy) is given by

$$c_0^T = y^T \left[\frac{1}{1+\underline{r}} + \frac{r}{1+r}\right] > y^T$$

$$c_t^T = y^T \left[\frac{1}{1+r} + \frac{r}{1+r}\frac{1+\underline{r}}{1+r}\right] < y^T; \quad t \geq 1$$

$$d_t = y^T \left[1 - \frac{1+\underline{r}}{1+r}\right] > 0; \quad t \geq 1$$

$$h_t = 1; \quad t \geq 0.$$

E Description of Argentine Data

The estimated process (34) is obtained using quarterly Argentine data over the period 1983:Q1 to 2001:Q4. The empirical measure of y_t^T is the cyclical component of Argentine GDP in agriculture, forestry, fishing, mining, and manufacturing at 1993 prices. The data were downloaded from www.indec.mecon.ar. We obtain the cyclical component by removing a log-quadratic time trend. We measure the country-specific real interest rate as the sum of the EMBI+ spread for Argentina and the 90-day Treasury-Bill rate, deflated using a measure of expected dollar inflation.[3]

[3]The country-specific interest rate reflects the fact that, in general, each country borrows at a different interest rate. The country interest rate captures factors such as country-specific repayment risk. These

Specifically, we construct the time series for the quarterly real Argentine interest rate, r_t, as $1 + r_t = (1 + i_t)E_t\frac{1}{1+\pi_{t+1}}$, where i_t denotes the dollar interest rate charged to Argentina in international financial markets and π_t is U.S. CPI inflation. For the period 1983:Q1 to 1997:Q4, we take i_t to be the Argentine interest rate series constructed by Neumeyer and Perri (2005) and posted at www.fperri.net/data/neuperri.xls. For the period 1998:Q1 to 2001:Q4, we measure i_t as the sum of the EMBI+ spread and the 90-day Treasury bill rate, which is in line with the definition used in Neumeyer and Perri. We measure $E_t\frac{1}{1+\pi_{t+1}}$ by the fitted component of a regression of $\frac{1}{1+\pi_{t+1}}$ onto a constant and two lags. This regression uses quarterly data on the growth rate of the U.S. CPI index from 1947:Q1 to 2010:Q2. The discretized process is contained in the .mat file tpm.mat available online.

F Numerical Algorithm for Approximating the Aggregate Dynamics Under a Currency Peg

Define the discretized state as follows:

$$Y^T = \{y_1^T, y_2^T, \ldots, y_{ny}^T\}$$

$$R = \{r_1, r_2, \ldots, r_{nr}\}$$

$$D = \{d_1, d_2, \ldots, d_{nd}\}$$

$$W = \{w_1, w_2, \ldots, w_{nw}\}.$$

In iteration n, suppose the guess for the solution for the marginal utility of tradable goods is given by the function Λ^n, mapping $Y^T \times R \times D \times W$ into $\mathbb{R}$. To obtain the next guess Λ^{n+1}, proceed as follows:

(1) For a given state $(y_i^T, r_j, d_k, w_\ell)$ with $i \in \{1, \ldots, ny\}$, $j \in \{1, \ldots, nr\}$, $k \in \{1, \ldots, nd\}$, $\ell \in \{1, \ldots, nw\}$, denote the level of debt due next period by d_s for $s \in \{1, \ldots, nd\}$.

(2) Use condition (10) to find the corresponding level of c^T as

$$c^T(d_s) = y_i^T + \frac{d_s}{1 + r_j} - d_k.$$

and use condition (21) to determine $\omega(c^T(d_s))$

$$\omega(c^T(d_s)) = \frac{A_2(c^T(d_s), F(\bar{h}))}{A_1(c^T(d_s), F(\bar{h}))}F'(\bar{h}).$$

idiosyncratic interest-rate differentials are present even for countries that are part of a monetary union, such as the members of the eurozone.

(3) If the full-employment wage violates constraint (17) for $\epsilon_t = 1$, then the current wage must be equal to γw_ℓ. Therefore, we have that the current wage is given by

$$w' = \max \left\{ \gamma w_\ell, \omega(c^T(d_s)) \right\}.$$

Pick the current wage, which will be a state variable for the next period, so that the wage rate takes on one of the values in the set W. Formally, we have that

$$qq = \text{argmin}_{q \in \{1,\ldots,nw\}}. |w_q - w'|$$

and then denote the current wage choice given d_s as w_{qq}.

(4) To find the level of employment associated with d_s, note that if $\omega(c^T(d_s)) \geq \gamma w_\ell$, then $h(d_s) = \bar{h}$, else $h(d_s)$ solves

$$\gamma w_\ell = \frac{A_2(c^T(d_s), F(h(d_s))}{A_1(c^T(d_s), F(h(d_s)))} F'(h(d_s)).$$

(5) Find the level of nontraded consumption as

$$c^N(d_s) = F(h(d_s)),$$

the level of consumption of the aggregate good from equation (2) as

$$c(d_s) = A(c^T(d_s), c^N(d_s)),$$

and the current value of the marginal utility of consumption of tradables from (14) as

$$\lambda(d_s) = U'(c(d_s))A_1(c^T(d_s), c^N(d_s))$$

(6) Use equation (15) to construct μ as

$$\mu(d_s) = \frac{\lambda(d_s)}{1+r_j} - \beta \sum_{ii=1}^{ii=ny} \sum_{jj=1}^{nr} Prob(y^T_{ii}, r_{jj}|y^T_i, r_j)\Lambda^n(y^T_{ii}, r_{jj}, d_s, w_{qq}).$$

If $\mu \geq 0$ and $s = nd$, then d_s is the optimal choice of debt in the current period and $s^* = nd$. In this case:

$$\Lambda^{n+1} = U'(c(d_{nd}))A_1(c^T(d_{nd}), c^N(d_{nd})).$$

Else construct μ for all $s \in \{1, \ldots, nd-1\}$. Find the optimal s as $s^* = \text{argmin}_{s \in \{1,\ldots,nd-1\}} |\mu(d_s)|$.

Construct

$$\Lambda^{n+1}(y_i^T, r_j, d_k, w_\ell) = U'(c(d_{s^*}))A_1(c^T(d_{s*}), c^N(d_{s^*})).$$

(7) Keep iterating in this way, until the maximum distance (taken over the $ny \times nr \times nd \times nw$ states between Λ^{n+1} and Λ^n is less than $1e - 8$.

G Sensitivity Analysis

In this section, we perform a number of variations in the structure of the model and parameter values.

G.1 Production in the Traded Sector

Our baseline model specification assumes that the supply of tradables, y_t^T, is exogenous. We now relax this assumption and assume instead that tradables are produced with labor. Specifically, we assume that

$$y_t^T = e^{z_t} \left(h_t^T\right)^{\alpha_T},$$

where y_t^T denotes output of tradable goods, h_t^T denotes labor employed in the traded sector, and $\alpha_T \in (0, 1)$ is a parameter. The variable z_t is assumed to be exogenous and stochastic. We interpret z_t either as a productivity shock in the traded sector or as a disturbance in the country's terms of trade. We assume that, as in the nontraded sector, firms in the traded sector are perfectly competitive in product and labor markets. Further, we assume that labor is perfectly mobile across sectors. We make this assumption to create a sharp contrast with the baseline formulation in which labor is completely immobile across sectors. A more realistic formulation would be one in which, in the short run, labor does move across sectors, but sluggishly. The assumption of free labor mobility across sectors implies that wages are equalized across sectors.

Firms in the traded sector choose labor to maximize profits, which are given by

$$P_t^T e^{z_t} \left(h_t^T\right)^{\alpha_T} - W_t h_t^T.$$

The first-order condition associated with the firm's profit maximization problem is

$$\alpha_T P_t^T e^{z_t} \left(h_t^T\right)^{\alpha_T - 1} = W_t. \tag{G.1}$$

Letting h_t^N denote hours employed in the nontraded sector, total hours worked, denoted by h_t, are then given by

$$h_t = h_t^T + h_t^N.$$

Table G.1: Sensitivity Analysis

	Welfare Cost		Unemployment Rate	
Economy	Peg with No Capital Controls	Peg with Optimal Capital Controls	Peg with No Capital Controls	Peg with Optimal Capital Controls
1. Baseline	11.6	3.7	13.5	3.1
2. Production in Traded Sector	10.1	5.0	7.8	1.9
3. Greece	17.6	6.0	15.3	3.7
4. $\sigma = 1/\xi = 2.27$ and $\beta = 0.962$	8.4	0.6	12.4	0.5
4.a. Less Downward Nominal Wage Rigidity ($\gamma = 0.98$)	6.2	0.4	9.5	0.4
4.b. Endogenous Labor Supply ($\delta = 0.5$)	19.0	0.8	33.5	1.3
4.c. Endogenous Labor Supply ($\delta = 0.75$)	9.3	0.6	33.5	1.8
4.d. Endogenous Labor Supply ($\delta = 1$)	2.1	0.3	33.5	8.4

Note. Welfare costs are relative to the optimal exchange-rate policy (or first-best allocation) and are expressed in percent of consumption per period (see expressions (35) and (36)). Unemployment rates are expressed in percent.

All other conditions of the model are as in the baseline formulation.

We assume that z_t and r_t follow the joint stochastic process given in equation (34), with z_t taking the place of $\ln y_t^T$. This strategy for calibrating the law of motion of z_t results in a standard deviation of $\ln y_t^T$ of 0.14 under a peg which is slightly above the value in the baseline model.[4] Following Uribe (1997), we set $\alpha_T = 0.5$. All other parameters take the values indicated in table 2.

Table G.1 shows that the welfare cost of currency pegs in the present economy is 10.1 percent of consumption per period on average and that optimal capital controls reduce these costs to 5.0 percent. The intuition for why the welfare costs of currency pegs continue to be large even when the supply of tradables is endogenous, can be illustrated by considering the adjustment of the economy to negative shocks when the lower bound on wages is binding. Consider first a negative interest-rate shock (i.e., an increase in r_t). If the wage rigidity is binding, then, as in the baseline economy, employment in the nontraded sector falls because of a weaker demand for this type of goods. At the same time, optimality condition (G.1) indicates that employment in the traded sector is unchanged, since wages are downwardly rigid and the exchange rate is pegged. This means that the unemployment that emerges in

[4] Ideally, one would like to use data on total factor productivity in the traded sector to calibrate the parameters defining the process z_t. We do not pursue this avenue here for lack of reliable sectoral data.

the nontraded sector will not be absorbed by the traded sector. Consider now the effect of a deterioration in the terms-of-trade or a negative productivity shock in the traded sector (i.e., a decline in z_t). Suppose again that the lower bound on nominal wages is binding. In this case, optimality condition (G.1) implies that employment in the traded sector will fall. This is because the product wage is unchanged but the marginal product of labor falls at any given level of employment. In the nontraded sector, demand declines because the negative productivity shocks produces a negative income effect. It follows that the nontraded sector does not absorb the hours lost in the traded sector. On the contrary, employment in the nontraded sector will also decline due to a weaker demand. This explains why the economy with production in the traded sector continues to exhibit large levels of unemployment under a currency peg.

Nonetheless, the unemployment rate under a currency peg is lower in the economy with production in the traded sector than in the baseline economy (7.8 versus 13.5 percent). The reason is that employment in the traded sector acts as a stabilizer of the wage rate during booms, thereby attenuating the negative externality caused by the combination of downward nominal wage rigidity and a currency peg. To see this, consider a decline in the country interest rate that raises the desired absorption of tradable and nontradable goods. This shock causes the demand for labor to increase in the nontraded sector driving wages up. This increase in wages induces firms in the traded sector to reduce employment. In turn, these freed up hours dampen the increase in wages required to clear the labor market. This dampening effect is beneficial because it means that once the boom is over the economy enters its way down to trend with lower real wages making the downward wage rigidity less stringent.

It is worth pointing out that the magnitude of the reduction in unemployment explained by this effect depends on our assumption that labor is perfectly mobile across sectors. To the extent that in the short run labor is sector specific, the reduction in the cyclical component of unemployment induced by the introduction of production in the traded sector should be expected to be smaller.

G.2 Greece

In this section we calibrate the model to Greece. This case is of interest because, relative to Argentina (from which we derive our baseline calibration) Greece displays less volatility in percent deviations of traded output from trend (6.5 versus 12.3 percent), less volatility in the country interest rate (5.2 versus 7.4 percentage points per annum), and a lower average country interest rate (4.5 versus 13.2 percent per annum). Another difference between Greece and Argentina is that the former exhibits a larger net foreign liability position as a fraction of GDP (117 percent versus 26 percent around the end of the respective calibration period).

We estimate the law of motion of traded output and the country interest rate using quarterly data from Greece over the period 1981-2011. See Appendix H for details. We calibrate all other structural parameters of the model as shown in table 2.

The reduced level of external uncertainty estimated for Greece relative to that estimated for Argentina and the lower real interest rate observed in Greece makes external borrowing more attractive in the model economy. As a result, the model delivers a mean debt-to-output ratio of 113 percent under a peg with free capital mobility.

The welfare costs of currency pegs in the model calibrated to Greece are large, 17.6 percent of consumption per period under free capital mobility and 6.0 percent under optimal capital controls. Both of these figures are higher than the corresponding ones for the model calibrated to Argentina, in spite of the fact that Greece is hit by less volatile external shocks. This result is explained by the fact that Greece has a larger external debt. Lower uncertainty and higher levels of external debt have opposite effects on the welfare costs of currency pegs. On the one hand, all other things equal, less uncertainty makes currency pegs less costly. Recall that in the present model, the average level of unemployment is increasing in the amplitude of the cycle. On the other hand, a higher level of external debt increases the welfare cost of pegs. The reason is that the higher is the level of external debt, the larger is the interest obligation created by a given increase in the interest rate. As a result, the higher is the country's external debt, the larger are the income effects caused by stochastic variations in the country interest rate. Recalling that positive and negative income shocks have asymmetric effects on unemployment, it follows that higher levels of debt induce higher average levels of unemployment. As it turns out, the cost induced by the larger level of external indebtedness in Greece more than offsets the benefits stemming from less volatile external shocks and lower average country interest rates.

Optimal capital controls continue have a large beneficial effects. The welfare cost of a currency peg relative to the first-best allocation is 11 percentage points higher under free capital mobility than under optimal capital controls.

To gain further insight into the relationship between indebtedness and the welfare costs of pegs, we recalibrate the subjective discount factor to induce a lower average debt-to-output ratio. Specifically, we raise β from 0.9375 to 0.975. This parameterization results in an average debt-to-GDP ratio of 81.3 percent. This value is in line with the average debt-to-GDP observed in Greece since its adoption of the Euro in 2001. In this case, the welfare cost of a currency peg with free capital mobility relative to the first-best allocation is 6.3 percent and the average unemployment rate is 7.5 percent. This result suggests that the welfare costs of currency pegs are larger the more impatient (or debt hungry) are households. At the same time, capital controls continue to be highly effective in easing the pains of pegs, as they lower welfare costs and unemployment to 1.7 and 1.3 percent, respectively.

G.3 Higher Intertemporal Elasticity of Substitution And Lower Discount Rate

Here, we investigate the sensitivity of our findings to increasing the intertemporal elasticity of substitution, $1/\sigma$. Specifically, we lower σ from its baseline value of 5 to 2.27. We pick this value for two reasons. First, it is close to 2, which is a value widely used in the business cycle literature (see Uribe and Schmitt-Grohé, 2014, and the references cited therein). Second, it is equal to the inverse of our assumed value for the intratemporal elasticity of substitution $1/\xi$. This is a convenient parameterization because it implies that the behavior of external debt and consumption of tradables is identical under a currency peg with free capital mobility and under the optimal exchange-rate policy (or the first-best allocation). To see this, note that when $\sigma = 1/\xi$, the period utility index , $U(A(c^T, c^N))$, becomes additively separable in consumption of tradables and nontradables and equal to $\left[\left(c^T\right)^{1-\sigma} + \left(c^N\right)^{1-\sigma} - 1\right]/(1-\sigma)$. This means that the marginal utility of consumption of tradables becomes independent of consumption of nontradables. It is then straightforward to show that under a currency peg or the optimal exchange rate policy the variables c_t^T and d_t are determined by the solution of the system composed by expressions (10)-(13) and

$$\left(c_t^T\right)^{-\sigma} = \beta(1+r_t)\mathbb{E}_t\left(c_{t+1}^T\right)^{-\sigma} + \mu_t.$$

Under this parameterization, the welfare costs of currency pegs relative to the optimal exchange-rate policy are attributable exclusively to the unemployment consequences of pegs.

Raising the intertemporal elasticity of substitution makes households less risk averse and as a result more willing to assume external debt. Holding all parameters other than σ constant at their baseline values, the lowering of σ results in debt distributions (under both the currency peg regime and the optimal exchange rate regime) that pile up to the left of the natural debt limit. The implied debt-to-output ratios are many times larger than those observed over our calibration period. For this reason, we adjust the value of β from its baseline value of 0.9375 to 0.962 to ensure that together with a value of $\sigma = 2.27$, the currency-peg economy delivers an external debt share in line with that observed over the calibration period (26 percent of annual output).

Table 4 shows that under this alternative calibration the welfare costs of currency pegs with free capital mobility relative to the optimal exchange-rate policy continue to be high with a mean of 8.4 percent of consumption per period. This figure is smaller than its baseline counterpart. This is expected because less risk averse agents are more tolerant to economic fluctuations.

As in the baseline calibration, optimal capital controls bring the economy quite close to the first-best allocation in terms of welfare. Specifically, the welfare cost of a currency peg coupled with optimal capital controls relative to the optimal exchange-rate policy is only 0.6 percent of consumption.

G.4 Less Downward Nominal Wage Rigidity

Because in the present model involuntary unemployment is the main source of welfare losses associated with currency pegs the key parameter determining the magnitude of these welfare losses is γ, which governs the degree of downward nominal wage rigidity. Our baseline calibration ($\gamma = 0.99$) implies that nominal wages can fall frictionlessly up to four percent per year. As argued in section 8, this is a conservative value in the sense that it allows for falls in nominal wages during crises that are larger than those observed either in the 2001 Argentine crisis or the ongoing crisis in peripheral Europe even after correcting for foreign inflation and long-run growth. We now set γ to 0.98, which allows for frictionless nominal wage declines of 8 percent per year. Taking into account that the largest wage decline observed in Argentina in 2001 or in the periphery of Europe since the onset of the great recession was 1.6 percent per year (Lithuania, see table 1), it follows that we are considering a degree of wage rigidity substantially lower than those implied by observed wage movements during recent large contractions. To avoid large changes in the distribution of external debt caused by parameter changes, we continue to assume here that $\sigma = 1/\xi = 2.27$. As explained in the previous subsection, this restriction ensures that the equilibrium distribution of external debt is the same under a peg with free capital mobility and in the first-best allocation (or under the optimal exchange-rate policy).

Table 4 shows that the mean welfare cost of a currency peg falls from 8.4 to 6.2 as we lower γ from 0.99 to 0.98. This welfare cost is still a large figure compared to existing results in monetary economics. The intuition why currency pegs are less painful when wages are more downwardly flexible is straightforward. A negative aggregate demand shock reduces the demand for nontradables which requires a fall in the real wage rate to avoid unemployment. Under a currency peg this downward adjustment must be brought about exclusively by a fall in nominal wages. The less downwardly rigid are nominal wages, the faster is the downward adjustment in both the nominal and the real wage and therefore the smaller is the resulting level of unemployment.

As under the baseline value of γ, the peg economy with optimal capital controls results in welfare levels very close to those achieved by the optimal exchange-rate policy. In this regard, the result that capital controls can go a long way toward alleviating the pains of pegs is robust to allowing for more wage flexibility.

G.5 Endogenous Labor Supply

We now relax the assumption of an inelastic labor supply schedule. Specifically, we consider a period-utility specification of the form

$$U(c_t, \ell_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{\ell_t^{1-\theta} - 1}{1-\theta},$$

where ℓ_t denotes leisure in period t, and φ and θ are positive parameters. As in the previous two subsections, we impose the restriction $\sigma = 1/\xi$.

Under this specification, the household's optimization problem features a new first-order condition of the form

$$\varphi(\ell_t^v)^{-\theta} = w_t \lambda_t,$$

where ℓ_t^v denotes the desired (or voluntary) amount of leisure. The above expression is a notional labor supply, in the sense that the household may not be able to work the desired number of hours and therefore may be forced to have more leisure than desired. We assume that households are endowed with $\bar{h}$ hours per period. Let h_t^s denote the number of hours households supply to the market. As before, households may not be able to sell all of the hours they supply to the labor market. Let h_t denote the actual number of hours worked. Then, we impose

$$h_t^s \geq h_t,$$

and

$$(h_t^s - h_t)\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) = 0.$$

These two expression are the counterparts to conditions (8) and (19) of the baseline economy. All other conditions describing aggregate dynamics are as before.

In the present economy, leisure has two component, voluntary leisure and involuntary leisure. Voluntary leisure, denoted ℓ_t^v, is given by the difference between the time endowment and the number of hours the household wishes to work at the market wage, that is,

$$\ell_t^v = \bar{h} - h_t^s.$$

Involuntary leisure equals involuntary unemployment, which we denote by u_t. It is the difference between the number of hours the household would like to work at the going wage and the number of hours the household is actually employed, that is,

$$u_t = h_t^s - h_t.$$

An important issue is how voluntary and involuntary leisure enter in the utility function. One possibility is to assume that voluntary and involuntary leisure are perfect substitutes. In this case, we have that $\ell_t = \ell_t^v + u_t$. However, there exists an extensive empirical literature suggesting that voluntary and involuntary leisure are far from perfect substitutes. For instance, Krueger and Mueller (2012), using longitudinal data from a survey of unemployed workers in New Jersey find that despite the fact that the unemployed spend relatively more time in leisure-related activities they enjoy these activities to a lesser degree than their employed counterparts and thus, on an average day, report higher levels of sadness than the employed. Similarly, Winkelmann and Winkelmann (1998), using longitudinal data of working-age men in Germany find that, after controlling for individual fixed effects and income, unemployment has a large non-pecuniary detrimental effect on life satisfaction. Another source of non-substitutability between voluntary and involuntary leisure stems from the fact that the unemployed spend more time than the employed looking for work, an activity that they perceive as highly unsatisfying. Krueger and Mueller (2012), for example, report that the unemployed work 391 minutes less per day than the employed but spend 101 minutes more per day on job search. In addition, these authors find that job search generates the highest feeling of sadness after personal care out of 13 time-use categories.

Based on this evidence, it is important to consider specifications in which voluntary and involuntary leisure are imperfect substitutes in utility. Specifically, we model leisure as

$$\ell_t = \ell_t^v + \delta u_t.$$

The existing literature strongly suggests that δ is less than unity. However, estimates of this parameter are not available. For this reason, we consider three values of δ, 0.5, 0.75, and 1.

We calibrate the remaining new parameters of the model as follows: we assume that under full employment households spend a third of their time working. We adopt a Frisch wage elasticity of labor supply of 2, which is on the high end of available empirical estimates from micro and aggregate data (see, for example, Blundell and MaCurdy, 1999; Justiniano, Primiceri, and Tambalotti, 2010; and Smets and Wouters, 2007). Finally, we normalize the number of hours worked under full employment at unity so as to preserve the size of the nontraded sector relative to the traded sector. This calibration strategy yields $\varphi = 1.11$, $\bar{h} = 3$, and $\theta = 1$.

Lines 4.b to 4.d of table 4 show the welfare cost of currency pegs with and without capital controls implied by the present model specification. The welfare cost of a currency peg with free capital mobility depend significantly on the degree of substitutability between voluntary and involuntary leisure. The more substitutable voluntary and involuntary leisure are, the lower are the welfare cost of currency pegs. This result should be expected. Consider the case in which voluntary and involuntary unemployment are perfect substitutes ($\delta = 1$). In this case, pegs reduce welfare because involuntary unemployment reduces the production and hence consumption of nontradable goods. However, unemployment increases leisure one for one and in this way increases utility, greatly offsetting the negative welfare effect of lower nontradable consumption. As δ falls, the marginal contribution of unemployment to leisure also falls, reducing the offsetting effect of leisure. For a value of δ of 0.75, for instance, the welfare cost of currency pegs with free capital mobility is 9.3 percent, which is higher than in the case with inelastic labor supply (see line 4 of table 4). If the marginal contribution of involuntary leisure to total leisure is half as large as that of voluntary leisure ($\delta = 0.5$), the welfare cost of currency pegs with free capital mobility increases to 19.0 percent of consumption per year.

Regardless of the precise value assumed for δ, capital controls are highly effective in reducing the welfare costs of pegs. In all cases, the welfare costs of pegs are cut by at least a factor of 7 when the peg is coupled with optimal capital controls.

Note that the equilibrium level of unemployment is independent of the parameter δ. This reflects the fact that the equilibrium allocation is unaffected by δ, because the household takes labor conditions as exogenously given. The parameter δ simply measures how differently households feel about voluntary and involuntary leisure.

H Description of Greek Data

In this appendix, we report the estimate of the exogenous driving process $[y_t^T \; r_t]'$ for the case of Greece. We also describe how the empirical measures of y_t^T and r_t were constructed.

The estimation uses quarterly data from 1981:Q1 to 2011:Q3. Greece did not produce sectoral GDP data between 1991 and 1999. For this reason, we proxy traded output by an index of industrial production. Specifically, we use the index of total manufacturing production 2005=100 from the OECD seasonally adjusted at the source. The original series begins in 1955:Q1 and ends in 2011:Q3. We removed a cubic trend from the natural logarithm of the index over the period 1955:Q1 to 2011:Q3. We use observations of the detrended series for the period 1981:Q1 to 2011:Q3 to make the range compatible with the one corresponding to the country real interest rate.

We measure the real interest rate in terms of tradables using the formula

$$1 + r_t = (1 + i_t)\mathbb{E}_t \left[\frac{E_t P_t^{T*}}{E_{t+1} P_{t+1}^{T*}} \right],$$

where r_t denotes the real country interest rate in terms of tradables, i_t denotes the nominal interest rate in terms of national currency, E_t denotes the nominal exchange rate defined as units of domestic currency per unit of ECU or Euro as applicable (Greece's legal tender changed to the Euro in 2001), P_t^{T*} denotes the foreign-currency price of tradables, and $\mathbb{E}_t$ denotes the expectations operator conditional on information available in period t. This formula assumes that the marginal rate of substitution is uncorrelated with the inverse of the domestic rate of inflation of tradable goods. The source for E_t is Eurostat (code `ert_h_eur_q`). We measure P_t^{T*} by the German consumer price index published by the OECD. We measure i_t as follows. For the period 1981:Q1 to 1992:Q3 it is the overnight interest rate published by the Bank of Greece. For the period 2001:Q1 to 2011:Q3 we proxy i_t by the interest rate on 10-year Greek treasury bonds published by Eurostat (code `irt_lt_mcby_q`). For the period 1992:Q4 to 2000:Q4, we measure i_t as the average of the above two interest rates. We proxy $\mathbb{E}_t \left[\frac{E_t P_t^{T*}}{E_{t+1} P_{t+1}^{T*}} \right]$ by the one-period ahead forecast of $\frac{E_t P_t^{T*}}{E_{t+1} P_{t+1}^{T*}}$ implied by an estimated AR(2) process for this variable.

The estimates of A, Σ_ν, and r defining the exogenous bivariate first-order autoregressive process given in equation (34) are

$$A = \begin{bmatrix} 0.88 & -0.42 \\ -0.05 & 0.59 \end{bmatrix}; \quad \Sigma_\nu = \begin{bmatrix} 0.000536 & -0.000010 \\ -0.000010 & 0.000060 \end{bmatrix}; \quad r = 0.011.$$

We discretize the driving process following the same procedure described in the body of the paper for the case of Argentina.

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