Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment*

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Abstract

This paper analyzes the inefficiencies that arise from the combination of a fixed exchange rate, nominal rigidities, and free capital mobility. We document that nominal wages are downwardly rigid in emerging countries. Motivated by this evidence, we develop a dynamic stochastic model of a small open economy that incorporates this nominal friction. The model predicts that currency pegs cause average rates of unemployment that are increasing in the variance of the underlying shocks. Under plausible calibrations, average unemployment exceeds 8 percent. Free capital mobility creates a negative externality that contributes to the unemployment problem. Optimal capital controls are shown to be prudential and to reduce unemployment to around 3 percent. The optimal exchange rate policy eliminates unemployment and calls for large devaluations during crises.

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1 Introduction

Fixed-exchange rate arrangements are often part of broader economic reform programs that include liberalization of international capital flows. For small emerging economies, such a policy combination has been a mixed blessing. A case in point is the European currency union, which imposes capital account liberalization as a prerequisite for admission. Figure 1 displays the average current-account-to-GDP ratio, an index of nominal hourly wages in Euros, and the rate of unemployment for a group of peripheral European countries that are either on or pegging to the Euro over the period 2000 to 2011. In the early 2000s, these countries enjoyed large capital inflows, which, through their expansionary effect on domestic absorption, led to sizable appreciations in hourly wages.

![Figure 1: Boom-Bust Cycle in Peripheral Europe: 2000-2011](image)


With the onset of the global recession in 2008, capital inflows dried up and aggregate demand collapsed. At the same time, nominal wages remained at the level they had achieved at the peak of the boom. The combination of depressed levels of aggregate demand and high nominal wages was associated with a massive increase in involuntary unemployment. In turn, local monetary authorities were unable to reduce real wages via a devaluation because of their commitment to the currency union.

This narrative evokes several interrelated questions. What is the optimal exchange-rate policy in an open economy with downward nominal wage rigidity. What are the welfare costs of currency pegs vis-à-vis the optimal exchange-rate policy in the presence of down-
ward nominal wage rigidity. Can fixed-exchange-rate regimes benefit from imposing capital controls. If so, are optimal capital controls prudential in nature, that is, is it optimal to tax capital inflows during booms and subsidize them during contractions. How large are the welfare gains of optimal capital controls?

In this paper, we address these questions both analytically and quantitatively. To this end, we develop a model of an open economy with downward nominal wage rigidity, a tradable sector, and a nontradable sector. The model economy is driven by stochastic disturbances to the country interest rate and the terms of trade. The motivation for focusing on downward nominal wage rigidity is empirical. There exists a large empirical literature suggesting that downward nominal wage rigidity is pervasive in developed countries. In this paper, we provide new evidence suggesting that this is also the case among emerging market economies.

An important prediction of our model is an endogenous connection between macroeconomic volatility and the mean level of unemployment. This connection is due to the nature of the labor contract implicit in our model, according to which employment is demand determined during contractions but supply and demand determined during booms. As a result, involuntary unemployment emerges during downturns and full employment during booms. Consequently, aggregate fluctuations cause unemployment on average. Importantly, the average level of unemployment is increasing in the amplitude of the business cycle, opening the door to large welfare gains from macroeconomic stabilization policy.

In our model, the combination of downward nominal wage rigidity, a fixed exchange rate, and free capital mobility creates a negative externality. The nature of this externality is that expansions in aggregate demand drive up wages, putting the economy in a vulnerable situation. For in the contractionary phase of the cycle, downward nominal wage rigidity and a fixed exchange rate prevent real wages from falling to the level consistent with full employment. Agents understand this mechanism, but are too small to internalize the fact that their individual expenditure decisions collectively cause inefficiently large increases in wages during expansions, which exacerbate unemployment during contractions.

The existence of the externality creates a rationale for government intervention. We consider three policy regimes. The baseline policy arrangement is the one that gives rise to the externality, namely, a currency peg coupled with free capital mobility. The second regime is the optimal exchange-rate policy. And the third regime is one in which the policymaker sticks to a currency peg but imposes optimal capital controls.

We show that the optimal exchange-rate policy eliminates involuntary unemployment and with it the aforementioned negative externality. The resulting equilibrium achieves the first-best allocation. The optimal exchange-rate policy consists in engineering large devaluations of the domestic currency during contractions driven by negative external shocks. The main
purpose of these devaluations is to reduce the real value of wages. Importantly, these optimal
devaluations are not of the beggar-thy-neighbor type. For they do not aim to foster exports
by altering the terms of trade. Rather, they are geared toward correcting the distortion in
real wages created by downward nominal wage rigidity.

In the context of our model, optimal exchange rate policy is not the only way for the
policymaker to achieve the first-best allocation. We show that the Pareto optimal allocation
can also be achieved through fiscal policy consisting in labor subsidies, or subsidies to the
absorption of nontradables.

Contrary to the policy instruments discussed thus far, capital controls represent a second-
best stabilization instrument in the sense that they cannot bring about the first-best allo-
cation. A natural question then is why bother characterizing optimal capital controls. The
reason is that policymakers may find that capital controls is the only instrument that they
can implement in practice. For instance, for many eurozone countries, and for reasons that
may exceed economic considerations, devaluing is not an option. In addition, the use of labor
subsidies to achieve the first best may be difficult from a political point of view. For instance,
we show that the labor subsidy scheme that implements the first best inherits the stochastic
properties of the underlying shocks, which in emerging countries are typically highly volatile.
Thus the optimal labor subsidy scheme would require large variations in wage subsidies at
a quarterly frequency. This may be problematic in light of the fact that the institutional
arrangements (especially the legislative process) that govern the determination of income
taxes is inertial, making large swings in labor subsidies on a quarter-to-quarter basis unreal-
realistic. By contrast, if capital controls are interpreted broadly as regulations of cross-border
financial flows, they appear as a more realistic policy prospect. For instance, the set of fi-
nancial reform measures developed by the Basel Committee on Banking Supervision, known
as Basel III, contemplates the use of procyclical capital requirements for banks. This type
of regulation is of interest because it tends to act like capital controls but without violat-
ing existing statutes governing the flow of financial capital across borders in the European
Union.

We model capital controls as a proportional tax (subsidy) on net external debt holdings.
The tax is equivalent to an interest rate markup on net foreign liabilities. We characterize
analytically and numerically optimal capital control policy under commitment. We show
that the Ramsey-optimal tax on external debt is positive on average and highly procyclical.
Thus, the optimal capital control policy is prudential in nature, as it restricts capital inflows
in good times and subsidizes external borrowing in bad times. The benevolent government
of a fixed-exchange-rate economy has an incentive to levy taxes on external debt during
expansions as a way to limit nominal wage growth. Moderating wage growth during booms
helps ameliorate the unemployment problem caused by downward wage rigidity during contractions. In turn, wage growth is affected by the aggregate desired absorption of tradable goods, which acts as a shifter of the demand for nontradables. As a result, the government can indirectly affect employment in the nontraded sector by manipulating the intertemporal price of tradables (the interest rate) via capital controls. Thus, the government in a fixed-exchange-rate economy determines the optimal capital control policy as the solution to a tradeoff between intertemporal distortions (caused by the capital controls themselves) and static distortions (caused by the combination of downward nominal wage rigidity and a fixed exchange rate).

In our model currency pegs coupled with free capital mobility lead to high average levels of unemployment. Versions of the model calibrated to emerging country data (Argentina and Greece) predict average rates of unemployment of more than 8 percent. In turn, large levels of unemployment translate into large welfare losses. Although capital controls represent a second-best instrument for dealing with the externality created by the combination of downward nominal wage rigidity, a currency peg, and free capital mobility, our quantitative analysis suggests that they can be highly effective. In our baseline calibration, optimal capital controls reduce the average rate of unemployment from over 8 percent to below 3 percent. This means that the tradeoff faced by the policymaker between alleviating the static distortions in the labor markets and interfering with the efficient intertemporal allocation of tradable absorption through capital controls is resolved largely in favor of the former. The predicted reduction in unemployment under optimal capital controls is reflected in lower welfare losses.

Our work is related to the Mundellian literature on the trilemma of international finance, according to which a country cannot have at the same time a fixed exchange rate, free capital mobility, and an independent interest rate policy. (For a recent treatment, see Obstfeld et al., 2010.) A number of studies have analyzed the welfare consequences of currency pegs in the context of models with nominal rigidities (e.g., Kollmann, 2002; and Galí and Monacelli, 2005). These papers are cast in the Calvo-Yun tradition, which assumes that nominal rigidities are symmetric, that is, product or factor price increases are as costly as decreases. The data however, suggests that nominal wage rigidity is one sided. A second key difference between this literature and our approach is that our assumed labor contract implies that employment is demand determined during contractions, but supply and demand determined during booms. In models of the Calvo-Yun style, by contrast, employment is always demand determined. As a result of these differences, our model predicts that aggregate instability has a first-order effect on mean unemployment.

There is also a body of work on the role of capital controls as a stabilization instrument.
A strand of this literature stresses financial distortions, such as collateral constraints on external borrowing as a rationale for capital controls (Auernheimer and García-Saltos, 2000; Uribe, 2006, 2007; Lorenzoni, 2008; Caballero and Lorenzoni, 2009; Korinek, 2010; Jeanne and Korinek, 2012; Benigno et al., 2011; Bianchi, 2011; and Bianchi and Mendoza, 2012). Another line of work is based on the classical trade theoretic argument that governments of large countries have incentives to apply capital controls as a means to induce households to internalize the country’s market power in financial markets (e.g., Obstfeld and Rogoff, 1996 section 1.4, and Costinot et al., 2011). Our theory of capital controls is distinct from the above two in that it does not assume the existence of collateral constraints or market power in financial markets. In a recent related paper, Farhi and Werning (2012) study capital controls in the context of a perfect-foresight, linearized version of the Galí and Monacelli (2005) sticky-price model.

The remainder of the paper is organized as follows. Section 2 develops a small open economy model with downward nominal wage rigidity and non-Walrasian labor markets. Section 3 identifies an externality arising from the combination of downward nominal wage rigidity, fixed exchange rates and free capital mobility. It also characterizes equilibrium dynamics under alternative policy arrangements. Section 4 presents empirical evidence on downward nominal wage rigidity and uses it to calibrate the model’s structural parameter governing this source of nominal friction. Section 5 uses data from Argentina to estimate the driving process and to calibrate the model. It analyzes quantitatively the adjustment of the economy to a boom-bust cycle under the three policy arrangements described above. It also contains the main quantitative results on the effects of the three policies on average unemployment and welfare. Section 6 conducts sensitivity analysis. It shows that the main findings are robust to allowing for production in the traded sector, estimating the driving process using data from Greece, considering a higher intertemporal elasticity of substitution, allowing for more wage flexibility, and introducing endogenous labor supply. Section 7 concludes.

2 An Open Economy With Downward Nominal Wage Rigidity

We develop a model of a small open economy in which nominal wages are downwardly rigid. The model features two types of good, tradables and nontradables. The economy is driven by two exogenous shocks, a country-interest-rate shock and a terms-of-trade shock.
2.1 Households

The economy is populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where $c_t$ denotes consumption. The period utility function $U$ is assumed to be strictly increasing and strictly concave and the parameter $\beta$, denoting the subjective discount factor resides in the interval $(0, 1)$. The symbol $E_t$ denotes the mathematical expectations operator conditional upon information available in period $t$. The consumption good is a composite of tradable consumption, $c_T^t$, and nontradable consumption, $c_N^t$. The aggregation technology is of the form

$$c_t = A(c_T^t, c_N^t),$$

where $A$ is an increasing, concave, and linearly homogeneous function.

We assume full liability dollarization. Specifically, households have access to a one-period, internationally traded, state non-contingent bond denominated in tradables. We let $d_t$ denote the level of debt assumed in period $t-1$ and due in period $t$ and $r_t$ the interest rate on debt held between periods $t$ and $t+1$. The sequential budget constraint of the household is given by

$$P_T^t c_T^t + P_N^t c_N^t + E_t d_t = P_T^t y_T^t + W_t h_t + \Phi_t + \frac{E_t d_{t+1}}{1 + r_t},$$

where $P_T^t$ denotes the nominal price of tradable goods, $P_N^t$ the nominal price of nontradable goods, $E_t$ the nominal exchange rate defined as the domestic-currency price of one unit of foreign currency, $y_T^t$ the endowment of traded goods, $W_t$ the nominal wage rate, $h_t$ hours worked, and $\Phi_t$ nominal profits from the ownership of firms. The variables $r_t$ and $y_T^t$ are assumed to be exogenous and stochastic. Movements in $y_T^t$ can be interpreted either as shocks to the physical availability of tradable goods or as shocks to the country's terms of trade.

Households supply inelastically $\bar{h}$ hours to the labor market each period. The assumption of an inelastic labor supply is motivated in part by microeconometric evidence (e.g., Blundell and MaCurdy, 1999) and macroeconometric evidence from models with nominal rigidities (e.g., Justiniano, Primiceri, and Tambalotti, 2010; and Smets and Wouters, 2007) suggesting that the labor supply elasticity is near zero. A second reason for assuming an inelastic labor supply is that it makes the workings of our two-sector model more transparent. In section 6.5, we relax this assumption by endogenizing the labor-supply decision. Because of the presence of downward nominal wage rigidity, households may not be able to sell all of the hours they
supply. As a result, households take employment, \( h_t \leq \bar{h} \), as exogenously given.

Households are assumed to be subject to the following debt limit, which prevents them from engaging in Ponzi schemes

\[
d_{t+1} \leq \bar{d},
\]

(4)

where \( \bar{d} \) denotes the natural debt limit.

We assume that the law of one price holds for tradables. Specifically, letting \( P_t^T \) denote the foreign currency price of tradables, the law of one price implies that

\[
P_T^T = P_T^T \cdot E_t.
\]

We further assume that the foreign-currency price of tradables is constant and normalized to unity, \( P_T^T = 1 \). (In our quantitative analysis, we relax this assumption.) Thus, we have that the nominal price of tradables equals the nominal exchange rate,

\[
P_T^T = E_t.
\]

Households choose contingent plans \( \{ c_t, c_T^T, c_N^T, d_{t+1} \} \) to maximize (1) subject to (2)-(4) taking as given \( P_T^T, P_N^T, E_t, W_t, h_t, \Phi_t, r_t, \) and \( y_T^T \). Letting \( p_t \equiv P_N^T / P_T^T \) denote the relative price of nontradables in terms of tradables and using the fact that \( P_T^T = E_t \), the optimality conditions associated with this problem are (2)-(4) and

\[
\frac{A_2(c_T^T, c_N^T)}{A_1(c_T^T, c_N^T)} = p_t,
\]

(5)

\[
\lambda_t = U'(c_t)\frac{A_1(c_T^T, c_N^T)}{A_1(c_T^T, c_N^T)},
\]

\[
\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t,
\]

\[
\mu_t \geq 0,
\]

\[
\mu_t(d_{t+1} - \bar{d}) = 0,
\]

where \( \lambda_t / P_T^T \) and \( \mu_t \) denote the Lagrange multipliers associated with (3) and (4), respectively.

Equation (5) describes the demand for nontradables as a function of the relative price of nontradables, \( p_t \), and the level of tradable absorption, \( c_T^T \). Given \( c_T^T \), the demand for nontradables is strictly decreasing in \( p_t \). This is a consequence of the assumptions made about the aggregator function \( A \). It reflects the fact that as the relative price of nontradables increases, households tend to consume relatively less nontradables. The demand function for nontradables is depicted in figure 2 with a downward sloping solid line. An increase in the
absorption of tradables shifts the demand schedule up and to the right, reflecting normality. Such a shift is shown with a dashed downward sloping line in figure 2 for an increase in traded consumption from $c^T_0$ to $c^T_1 > c^T_0$. It follows that absorption of tradables can be viewed as a shifter of the demand for nontradables. Of course, $c^T_t$ is itself an endogenous variable, which is determined simultaneously with all other endogenous variables of the model.

2.2 Firms

Nontraded output, denoted $y^N_t$, is produced by perfectly competitive firms. Each firm operates a production technology given by

$$y^N_t = F(h_t),$$

which uses labor services as the sole input. The function $F$ is assumed to be strictly increasing and strictly concave. Firms choose the amount of labor input to maximize profits, given by

$$\Phi_t \equiv P^N_t F(h_t) - W_t h_t.$$
The optimality condition associated with this problem is $P_t^N F'(h_t) = W_t$. Dividing both sides by $P_t^T$ and using the facts that $P_t^T = E_t$ and $h_t = F^{-1}(y^N_t)$ yields a supply schedule of nontradable goods of the form

$$p_t = \frac{W_t/E_t}{F'(F^{-1}(y^N_t))}.$$ 

This supply schedule is depicted with a solid upward sloping line in figure 3. Ceteris paribus, the higher is the relative price of the nontraded good, the larger is the supply of nontradable goods.

Also, all other things equal, the higher is the labor cost $W_t/E_t$, the smaller is the supply of nontradables at each level of the relative price $p_t$. That is, an increase in the nominal wage rate, holding constant the nominal exchange rate, causes the supply schedule to shift up and to the left. Figure 3 displays with a broken upward sloping line the shift in the supply schedule that results from an increase in the nominal wage rate from $W_0$ to $W_1 > W_0$, holding the nominal exchange rate constant at $E_0$. Similarly, a currency devaluation, holding the nominal wage constant, shifts the supply schedule down and to the right. Intuitively, a devaluation that is not accompanied by a change in nominal wages reduces the real labor cost thereby inducing firms to increase the supply of nontradable goods for any given relative price. Suppose, to keep the graph simple, that the government devalues the currency to a level $E_1 > E_0$ such that $W_1/E_1 = W_0/E_0$. Such a devaluation shifts the supply schedule indicated with a broken line back to its original position given by the solid line.
2.3 Downward Nominal Wage Rigidity

The central friction in the model is downward nominal wage rigidity. Specifically, we impose that

\[ W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \tag{6} \]

The parameter \( \gamma \) governs the degree of downward nominal wage rigidity. The higher is \( \gamma \), the more downwardly rigid are nominal wages. This setup nests the cases of absolute downward rigidity, when \( \gamma \geq 1 \), and full wage flexibility, when \( \gamma = 0 \). In section 4, we present empirical evidence suggesting that \( \gamma \) is close to unity.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear. Instead, involuntary unemployment, given by \( \bar{h} - h_t \), will be a regular feature of this economy. Actual employment must satisfy

\[ h_t \leq \bar{h} \tag{7} \]

at all times. At any point in time, wages and employment must satisfy the slackness condition

\[ (\bar{h} - h_t) (W_t - \gamma W_{t-1}) = 0. \tag{8} \]

This condition states that periods of unemployment \( (h_t < \bar{h}) \) must be accompanied by a binding wage constraint. It also states that when the wage constraint is not binding \( (W_t > \gamma W_{t-1}) \), the economy must be in full employment \( (h_t = \bar{h}) \).

We note that the assumed structure of the labor market is perfectly competitive. Both workers and employers are wage takers. Alternatively, one could assume market power on either side. In the related new Keynesian literature, it is customary to assume that workers have market power and set wages to maximize their lifetime utility. As emphasized by Elsby (2009), in the presence of a lower bound on nominal wages, this market structure might give rise to an endogenous compression of wage increases in anticipation of future adverse shocks. The empirical evidence, however, suggests that strategic wage compression may have played a relatively small role in recent boom-bust episodes. For instance, as documented in figure 1, nominal hourly wages in the periphery of the Euro zone increased over 60 percent during the boom of 2000-2008 in spite of virtually no growth in total factor productivity.\(^1\)

\(^1\)Barkbu et al. (2012) show that for the euro area as a whole, total factor productivity grew by less than 0.2 percent per year between 2000 and 2010. Productivity growth in the periphery of Europe was even weaker. According to data from the EU Klems Growth and Productivity Account Project, between 2000 and 2007 value added TFP fell by 4 percent in Spain and by 1 percent in Ireland.
2.4 Non-Walrasian Equilibrium

In equilibrium, the market for nontraded goods must clear at all times. That is, the condition
\[ c_t^N = y_t^N \]
must hold for all \( t \). Combining this condition, the production technology for nontradables, the household’s budget constraint, and the definition of firms’ profits, we obtain the following market-clearing condition for traded goods:
\[ c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}. \]

Letting
\[ w_t \equiv \frac{W_t}{E_t} \]
denote the real wage in terms of tradables and
\[ \epsilon_t \equiv \frac{E_t}{E_{t-1}} \]
the gross rate of devaluation of the domestic currency, we define a non-Walrasian equilibrium as follows.

**Definition 1 (Non-Walrasian Equilibrium)** A non-Walrasian equilibrium is a set of stochastic processes \( \{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, \mu_t\}_{t=0}^{\infty} \) satisfying
\[ c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \quad (9) \]
\[ d_{t+1} \leq \bar{d}, \quad (10) \]
\[ \mu_t \geq 0, \quad (11) \]
\[ \mu_t(d_{t+1} - \bar{d}) = 0, \quad (12) \]
\[ \lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \quad (13) \]
\[ \frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t, \quad (14) \]
\[ \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}. \quad (15) \]
\[ w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \quad (16) \]
\begin{align*}
    h_t &\leq \bar{h}, \\
    (h_t - \bar{h}) \left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) &= 0,
\end{align*}
\text{given an exchange rate policy, } \{\epsilon_t\}_{t=0}^\infty, \text{ initial conditions } w_{-1} \text{ and } d_0, \text{ and exogenous stochastic processes } \{r_t, y_t^T\}_{t=0}^\infty.

3 \hspace{1em} \textbf{Theoretical Analysis}

In this section we characterize analytically equilibrium under four alternative policy regimes, a currency peg with free capital mobility, the optimal exchange-rate policy, optimal fiscal policy, and a currency peg with optimal capital controls.

3.1 \hspace{1em} \textbf{Currency Pegs}

A currency peg is an exchange rate policy in which the nominal exchange rate is fixed. The gross devaluation rate therefore satisfies
\[
    \epsilon_t = 1,
\]
for \( t \geq 0 \). Under a currency peg, our model economy is subject to two nominal rigidities. One is policy induced: The nominal exchange rate, \( E_t \), is kept fixed by the monetary authority. The second is structural and is given by the downward rigidity of the nominal wage \( W_t \). The combination of these two nominal rigidities results in a real rigidity. Specifically, under a currency peg, the real wage expressed in terms of tradables, \( w_t \), is downwardly rigid, and adjusts only sluggishly, at the rate \( (1 - \gamma) \). The labor market is, therefore, in general, in disequilibrium and features involuntary unemployment. The magnitude of the labor market disequilibrium is a function of the amount by which the past real wage exceeds the current full-employment real wage. It follows that under a currency peg, the past real wage, \( w_{t-1} \), becomes a relevant state variable for the economy.

3.2 \hspace{1em} \textbf{A Peg-Induced Externality}

Figure 4 illustrates the adjustment of the economy to a boom-bust episode. Because in equilibrium \( c^N_t = y^N_t = F(h_t) \), the figure plots the demand and supply schedules for nontraded goods in terms of employment in the nontraded sector, so that the horizontal axis measures \( h_t \). The intersection of the demand and supply schedules, therefore, indicate the equilibrium demand for labor, given \( c^T_t \) and \( W_t/E_t \). The figure also shows with a dotted vertical line the
labor supply, $\bar{h}$. Suppose that the initial position of the economy is at point $A$, where the labor market is operating at full employment, $h_t = \bar{h}$. Suppose that in response to a positive external shock, such as a decline in the country interest rate, traded absorption increases from $c^T_0$ to $c^T_1 > c^T_0$ causing the demand function to shift up and to the right. If nominal wages stayed unchanged, the new intersection of the demand and supply schedules would occur at point $B$. However, at point $B$ the demand for labor would exceed the available supply of labor $\bar{h}$. The excess demand for labor drives up the nominal wage from $W_0$ to $W_1 > W_0$ causing the supply schedule to shift up and to the left. The new intersection of the demand and supply schedules occurs at point $C$, where full employment is restored and the excess demand for labor has disappeared.

Suppose now that the external shock fades away, and that, therefore, absorption of tradables goes back to its original level $c^T_0$. The decline in $c^T_1$ shifts the demand schedule back to its original position, indicated by the downward sloping solid line. However, the economy does not immediately return to point $A$. Due to downward nominal wage rigidity, the nominal wage stays at $W_1$, and because of the currency peg, the nominal exchange rate remains at $E_0$.\(^2\) As a result, the supply schedule does not move. The new intersection is at point $D$. There, the economy suffers involuntary unemployment equal to $\bar{h} - h_{\text{bust}}$.

\(^2\)For expositional convenience, in figures 4 and 5 $\gamma$ is set to unity.
The combination of downward nominal wage rigidity and a currency peg creates a negative externality. The nature of this externality is that in periods of economic expansion, elevated demand for nontradables drives nominal (and real) wages up placing the economy in a vulnerable situation. For in the contractionary phase of the cycle, downward nominal wage rigidity and the currency peg hinder the downward adjustment of real wages, causing unemployment. Individual agents understand this mechanism, but are too small to internalize the fact that their own expenditure choices collectively exacerbate disruptions in the labor market.

3.3 Volatility And Mean Unemployment

The present model implies an endogenous connection between the amplitude of the cycle and the average levels of involuntary unemployment and output. This connection opens the door to large welfare gains from optimal stabilization policy and is rooted in the fact that under a currency peg the economy adjusts asymmetrically to positive and negative external shocks. The adjustment to positive external shocks is efficient, as nominal wages adjust upward to ensure that firms are on their labor demand schedule and households on their labor supply schedule. In sharp contrast, the adjustment to negative external shocks is inefficient, as nominal wages fail to fall forcing households off their labor supply schedule and generating involuntary unemployment. It follows that over the business cycle, the model economy fluctuates between periods of full employment and an efficient level of production and periods of involuntary unemployment and inefficiently low levels of production. Therefore, the average levels of involuntary unemployment and nontraded output depend on the amplitude of the business cycle. That is, in this model, mean unemployment is increasing in the variance of the underlying shocks. This implication of our model is an important difference with existing sticky-wage models in the Erceg, Henderson, and Levin (2000) and Galí (2011) tradition. In that class of models, employment is always demand determined. As a result increases in involuntary unemployment during recessions are roughly offset by reductions in unemployment during booms. It follows that in the Erceg-Henderson-Levin-Galí sticky-wage framework the average levels of unemployment and output, and therefore welfare, do not depend in a quantitatively relevant way on the amplitude of the business cycle.
3.4 Optimal Exchange Rate Policy

Consider an exchange rate policy in which the central bank always sets the devaluation rate to ensure full employment in the labor market, that is, to ensure that

$$h_t = \bar{h},$$

for all $t \geq 0$. We refer to this exchange-rate arrangement as the full-employment exchange rate policy and will show that it supports the Pareto optimal allocation. The equilibrium dynamics associated with the full-employment exchange-rate policy are illustrated in figure 4. Suppose that, after being hit by a negative external shock, the economy is stuck at point $D$ with involuntary unemployment equal to $\bar{h} - h_{bust}$. At point $D$, the desired demand for tradables is $c_T^0$, the nominal wage is $W_1$, and the nominal exchange rate is $E_0$. Suppose that the central bank were to devalue the domestic currency so as to deflate the purchasing power of nominal wages to a point consistent with full employment. That is, suppose that the central bank sets the exchange rate at the level $E_1 > E_0$ satisfying $\left(W_1/E_1\right)/F'(\bar{h}) = A_2(c_T^0, F(\bar{h}))/A_1(c_T^0, F(\bar{h}))$. In this case the supply schedule would shift down and to the right intersecting the demand schedule at point $A$, where unemployment is nil ($h = \bar{h}$). Under the full-employment exchange-rate policy, the relative price of non-tradables falls from $p_{boom}$ at the peak of the cycle, to $p_0$ after the negative external shock. By contrast, if the central bank had kept the exchange rate fixed, then the relative price of nontradables would have fallen by less, namely from $p_{boom}$ to $p_{bust}$. The reason why in the currency peg economy firms are reluctant to implement sufficiently large price cuts is that real wages, and therefore labor costs, remain as high as they were during the boom. By contrast, the devaluation lowers the real cost of labor, making it viable for firms to slash prices. In turn, because under the peg prices remain high, households do not receive a strong enough signal to switch expenditures away from tradables and toward nontradables in a magnitude compatible with full employment.

The full-employment policy amounts to setting the devaluation rate to ensure that the real wage equals the full-employment real wage rate at all times. Formally, the full-employment exchange-rate policy ensures that

$$w_t = \omega(c_T^t),$$

where $\omega(c_T^t)$ denotes the full-employment real wage rate and is given by

$$\omega(c_T^t) \equiv \frac{A_2(c_T^t, F(\bar{h}))}{A_1(c_T^t, F(\bar{h}))} F'(\bar{h}). \quad (20)$$
The assumed properties of the aggregator function $A$ ensure that the function $\omega(\cdot)$ is strictly increasing in the domestic absorption of tradables, $c^T_t$,

$$\omega'(c^T_t) > 0.$$  

The full-employment exchange rate policy stipulates that should the nominal value of the full-employment real wage evaluated at last period’s nominal exchange rate, $\omega(c^T_t)E_{t-1}$, fall below the lower bound $\gamma W_{t-1}$, then the central bank devalues the domestic currency to ensure that $\omega(c^T_t)E_t \geq \gamma W_{t-1}$. That is, the devaluation rate makes the nominal wage, $W_t$, greater than or equal to its lower bound, $\gamma W_{t-1}$, and at the same time guarantees that the real wage, $w_t$, equal the full-employment real wage $\omega(c^T_t)$.

In general, any exchange rate policy satisfying

$$\epsilon_t \geq \frac{w_{t-1}}{\omega(c^T_t)} \quad (21)$$

ensures full employment at all times. All exchange rate policies pertaining to this family deliver the same real allocation and are therefore equivalent from a welfare point of view. Indeed, the real allocation induced by policies belonging to this family is Pareto optimal. The following proposition establishes these results:

**Proposition 1** Any policy satisfying condition (21) is consistent with a real allocation that exhibits full employment ($h_t = \bar{h}$) at all dates and states and, furthermore, is Pareto optimal.

**Proof:** See appendix A.

The Pareto optimal allocation, denoted $\{c^{OPT}_t, d^{OPT}_{t+1}, h^{OPT}_t\}_{t=0}^\infty$, is the solution to the following value function problem:

$$v^{OPT}(y^T_t, r_t, d_t) = \max_{\{c^T_t, d_{t+1}, h_t\}} \left\{ U(A(c^T_t, F(h_t))) + \beta E_t v^{OPT}(y^T_{t+1}, r_{t+1}, d_{t+1}) \right\} \quad (22)$$

subject to (7), (9), and (10), where the function $v^{OPT}(y^T_t, r_t, d_t)$ represents the welfare level of the representative agent in state $(y^T_t, r_t, d_t)$. It is straightforward to show that $h^{OPT}_t = \bar{h}$, that is, the Pareto optimal allocation guarantees full employment at all times. The equilibrium processes of all other endogenous variables of the model can be readily obtained from equations (21) and the conditions listed in Definition 1. The facts that the aggregate dynamics under optimal exchange rate policy can be described as the solution to a Bellman equation and that the past real wage, $w_{t-1}$, is not a relevant state variable in period $t$, greatly facilitate the quantitative characterization of the model’s predictions.
3.5 Fiscal Alternatives To Devaluations

Many observers have suggested the use of fiscal policy to ease the pains of currency pegs. However, advocates of active fiscal policy do not speak with a single voice. Some argue that the right medicine is fiscal restraint via tax increases and cuts in public expenditures. Others hold diametrically opposed views and argue that only widespread increases in government spending and tax cuts can offer pain relief. Our model suggests that both of these extreme views are misguided. Instead, the model suggests that the way to ease the pain of a currency peg by means of fiscal policy is more sophisticated in nature. Specifically, optimal fiscal policy in the context of a currency peg consists in a time-varying wage subsidy at the firm level that targets industries with high degrees of downward wage rigidity.

In our model, the Pareto optimal equilibrium can be supported under a currency peg by implementing a proportional wage subsidy of the form

$$\tau_t^h = \max \left\{ 0, 1 - \frac{\omega(c_t^T)}{\gamma w_{t-1}} \right\}.$$  \hspace{1cm} (23)

The following proposition establishes this result.

**Proposition 2 (Optimal Wage Subsidy)** Suppose the exchange-rate policy is characterized by a currency peg. Then, the labor subsidy given in equation (23) supports the Pareto optimal allocation.

**Proof:** See appendix B.

In states in which the combination of the currency peg and the downward nominal wage rigidity prevents the real wage from falling to the full-employment real wage, the subsidy is positive. Otherwise, the subsidy is zero. It is straightforward to show that the Pareto optimal allocation can also be brought about via consumption or sale subsidies in the nontraded sector at the same rate $\tau_t^h$ characterized in the above proposition.

The subsidy scheme given in equation (23) can be financed in a nondistorting fashion by an appropriate proportional tax on any source of income (labor income, $w_t h_t$, tradable income, $y_t^T$, profits, $\Phi_t / E_t$, or any combination thereof). We note that these financing schemes are nondistorting even when the labor supply is elastic. The reason is that the subsidy is positive only in states of the world in which, in the absence of the subsidy, households are off their labor supply schedule, or involuntarily unemployed.\(^3\) It is clear from equation (23) that the optimal subsidy inherits the stochastic properties of the optimal devaluation rate studied in previous sections (see equation (21)). Because, as we will see shortly, under plausible calibrations the optimal devaluation rate is found to be highly volatile.

\(^3\)In a more recent contribution, Farhi et al. (2011) expand this idea to other economic environments.
at business-cycle frequency, it follows that the fiscal alternative presented here may indeed introduce an impractically high level of volatility in labor subsidy or labor tax rates.

### 3.6 Optimal Capital Controls

In section 3.2, we established that the combination of a currency peg and downward nominal wage rigidity creates an externality. During episodes of large capital inflows, nominal wages rise making the economy vulnerable to unemployment once capital inflows dry up, as nominal wages cannot adjust downwardly to equilibrate the labor market. Individuals understand this source of fragility, but are too small to do anything about it. Consequently, the government has an incentive to intervene. In this section, we study the efficacy of capital controls in remedying the peg-induced externality.

Specifically, we explore the possibility that the government acts prudentially by imposing capital controls during booms. Such policy would tend to curb capital inflows, and in that way contain the rise in nominal wages and limit the size of involuntary unemployment once the boom is over. Our approach is not to assume that capital controls are prudential, but rather to investigate whether a Ramsey planner would indeed use capital controls, and, if so, whether he will apply them in a prudential fashion.

The intuition for why the government may wish to use capital controls in a prudential manner is illustrated in figure 5. Suppose the economy starts at point $A$. At that point traded consumption is equal to $c_0^T$ and the economy enjoys full employment. Assume now that the economy experiences a decrease in the country interest rate followed by an increase, that is, the country interest rate first declines and then rises. Assume that in the absence of capital controls, consumption of tradables rises from $c_0^T$ to $c_1^T > c_0^T$ when the country interest rate falls, and then declines back to $c_0^T$ once the country interest rate rises. As discussed earlier, in this case the economy moves from point $A$ to point $C$ during the boom and then from point $C$ to point $D$ during the bust. During the boom nominal wages rise from $W_0$ to $W_1 > W_0$. During the bust, the economy experiences involuntary unemployment in the amount of $h - h_{\text{bust}}$ because real wages are stuck at $W_1/E_0$ and downward nominal wage rigidity in combination with the currency peg prevents real wages from falling to a level consistent with full employment.\footnote{Recall that for graphical clarity, figure 5 is drawn under the assumption that $\gamma = 1$.}

Consider now the case that the government implements capital control taxes in response to the initial interest rate decline and that as a result of these taxes, the increase in traded consumption is smaller. Specifically, assume that traded consumption now increases from $c_0^T$ to $c_2^T < c_1^T$. The demand for nontradables, shown with a dashed downward-sloping
line in figure 5, shifts up and to the right. Were nominal wages to remain unchanged the new intersection of demand and supply would be at point \( B' \). However, at that point labor demand exceeds labor supply. As a result nominal wages increase to \( W_2 \) and the nontradables supply schedule, shown with a dashed upward sloping line, shifts up and to the left. The new intersection of the demand and supply schedules is at point \( C' \), where the economy enjoys full employment. Because the shift in the demand schedule in the presence of capital controls is smaller than in the absence of the imposition of capital control taxes, nominal wages rise by less, that is, \( W_2 < W_1 \). Assume again that when the positive external shock fades away consumption of tradables falls back to \( c^T_0 \). The resulting demand schedule is thus the same as the initial one, given by the solid downward sloping line in figure 5. The supply schedule does not shift because due to downward nominal wage rigidity nominal wages cannot decline, that is, wages remain at \( W_2 \), and due to the currency peg the exchange rate cannot change, that is, it remains at \( E_0 \). The new intersection of the demand and supply schedules is at point \( D' \), where \( \bar{h} - h_{bust}^{occ} \) workers are involuntarily unemployed. However, the level of unemployment at \( D' \) is lower than at \( D \). It follows that by imposing capital controls in response to a positive external shock, the government is able to reduce the amount of unemployment that occurs once the positive external shock is over.
But this government intervention comes at the cost of bringing about an inefficient allocation of traded consumption over time. For capital controls distort the interest rate perceived by domestic households. The imposition of capital controls induces households not to take full advantage of the cheaper cost of borrowing during the boom and to not reduce their absorption of tradables sufficiently once the interest rate rises. The figure illustrates the benefits in terms of lower unemployment that capital controls can bring. But the figure does not capture the costs in terms of a suboptimal time path of tradable consumption. To analyze the trade-off between less unemployment and an inefficient allocation of traded consumption over time, we next characterize Ramsey optimal capital control policies more formally.

Assume that the government taxes external debt at the proportional rate $\tau_d^t$, and rebates this source of revenue via a proportional income subsidy denoted $\tau_y^t$. The budget constraint of the household is then given by

$$c_t^T + p_t c_t^N + d_t = (1 + \tau_y^t)(y_t^T + w_t h_t + \phi_t) + \frac{(1 - \tau_d^t)d_{t+1}}{1 + r_t},$$

where $\phi_t \equiv \Phi_t / E_t$ are profits in terms of tradables. We note again that because all sources of income—$y_t^T$, $w_t h_t$, and $\phi_t$—are taken as exogenous by the household, the income subsidy used to rebate the revenues from capital controls is nondistorting. The introduction of a capital control tax changes the household’s first-order condition for holdings of foreign assets to:

$$\lambda_t \frac{1 - \tau_d^t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t. \quad (24)$$

According to this expression, the effective gross interest rate perceived by households is $(1 + r_t)/(1 - \tau_d^t)$, which is greater than the gross country interest rate, $1 + r_t$, when the government imposes capital controls, i.e., when $\tau_d^t > 0$.

The government is assumed to be benevolent and to be endowed with full commitment. We therefore refer to the fiscal authority as the Ramsey planner. We assume that the central bank pegs the currency and that the government cannot use labor subsidies of the type studied in the previous section. The budget constraint of the government is therefore given by

$$\tau_y^t(y_t^T + w_t h_t + \phi_t) = \frac{\tau_d^t d_{t+1}}{1 + r_t}. \quad (25)$$

The policy variable $\tau_d^t$ can take positive or negative values. In the former case it represents a tax on capital inflows and in the latter a subsidy.

Because the monetary authority pegs the currency at all times, equilibrium conditions (16)
and (18) become, respectively,

\[ w_t \geq \gamma w_{t-1}, \quad (26) \]

and

\[ (h_t - \bar{h})(w_t - \gamma w_{t-1}) = 0. \quad (27) \]

We then have the following definition of a non-Walrasian equilibrium in the economy with capital controls:

**Definition 2 (Equilibrium With Capital Controls)** A non-Walrasian equilibrium is a set of stochastic processes \( \{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, \mu_t, \tau_t^d\}_{t=0}^{\infty} \) satisfying (9)-(13), (15), (17), and (24)-(27), given a capital-control policy \( \{\tau_t^d\}_{t=0}^{\infty} \), initial conditions \( w_{-1} \) and \( d_0 \), and exogenous stochastic processes \( \{r_t, y_t^T\}_{t=0}^{\infty} \).

The Ramsey planner’s optimization problem consists in choosing a tax scheme \( \{\tau_t^d\} \) to maximize the household’s lifetime utility function (1) subject to the equilibrium conditions listed in definition 2. The strategy we follow to characterize the Ramsey allocation is to drop from the Ramsey planner’s problem all constraints except for (9), (10), (15), (17), and (26), and then show that the solution to this less constrained problem satisfies the omitted constraints.

Accordingly, the less constrained Ramsey problem (LCP) is given by

\[
\max_{\{c_t^T, d_{t+1}, h_t, w_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t)))
\]

subject to (9), (10), (15), (17), and (26), given \( d_0 \), \( w_{-1} \), and the stochastic processes \( \{r_t, y_t^T\}_{t=0}^{\infty} \).

We now show that the allocation \( \{c_t^T, d_{t+1}, h_t, w_t\}_{t=0}^{\infty} \) that solves the LCP satisfies the constraints of the original Ramsey problem listed in definition 2. To see this, use the LCP allocation to construct \( \lambda_t \) to satisfy (13). Next, set \( \mu_t = 0 \) for all \( t \). It follows that (11) and (12) are satisfied. Now construct \( \tau_t^d \) to satisfy (24) and \( \tau_t^y \) to satisfy (25). It remains to be shown that the allocation that solves the LCP satisfies the slackness condition (27). To see that this is the case, consider the following proof by contradiction. Suppose, contrary to what we wish to show, that the solution to the LCP implies \( h_t < \bar{h} \) and \( w_t > \gamma w_{t-1} \) at some date \( t' \geq 0 \). Consider now a perturbation to the allocation that solves the LCP consisting

\[ \text{Note that in states in which the allocation calls for setting } d_{t+1} < \bar{d}, \text{ } \mu_t \text{ must be chosen to be zero. However, in states in which the Ramsey allocation yields } d_{t+1} = \bar{d}, \text{ } \mu_t \text{ need not be chosen to be zero. In these states, any positive value of } \mu_t \text{ could be supported in the decentralization of the Ramsey equilibrium. Of course, in this case, } \tau_t^d \text{ will depend on the chosen value of } \mu_t. \text{ In particular, } \tau_t^d \text{ will be strictly decreasing in the arbitrarily chosen value of } \mu_t. \]
in a small increase in hours at time $t'$ from $h_{t'}$ to $\tilde{h}_{t'} \leq \bar{h}$. Clearly, this perturbation does not violate the resource constraint (9), since hours do not enter in this equation. From (15) we have that the real wage falls to $\tilde{w}_{t'} \equiv \frac{A_2(c^T_{t'}, F(h_{t'}))}{A_1(c^T_{t'}, F(h_{t'}))} F'(h_{t'}) < w_{t'}$ (recall that the expression in the middle is decreasing in hours). Because $A_1$, $A_2$, and $F'$ are continuous functions, expression (26) is satisfied provided the increase in hours is sufficiently small. In period $t' + 1$, restriction (26) is satisfied because $\tilde{w}_{t'} < w_{t'}$. We have therefore established that the perturbed allocation satisfies the restrictions of the LCP. Finally, the perturbation is clearly welfare increasing because it raises the consumption of nontradables in period $t'$ without affecting the consumption of tradables in any period or the consumption of nontradables in any period other than $t'$. It follows that an allocation that does not satisfy the slackness condition (27) cannot be a solution to the LCP. This completes the proof that the allocation that solves the LCP is indeed the Ramsey optimal allocation. We summarize this result in the following proposition:

**Proposition 3 (Optimal Capital Controls)** Let \( \{c^T_{t}, d_{t+1}^{OCC}, h_{t}^{OCC}, w_{t}^{OCC}\}_{t=0}^{\infty} \) be the allocation associated with the Ramsey optimal capital control policy. Then \( \{c^T_{t}, d_{t+1}^{OCC}, h_{t}^{OCC}, w_{t}^{OCC}\}_{t=0}^{\infty} \) is the solution to the problem of maximizing (1) subject to (9), (10), (15), (17), and (26), given \( d_0, w_{-1} \), and the stochastic processes \( \{r_t, y^T_{t}\}_{t=0}^{\infty} \).

A corollary of this proposition is that one can characterize the Ramsey allocation as the solution to the following Bellman equation problem:

\[
v^{OCC}(y^T_{t}, r^T_{t}, d_{t}, w_{t-1}) = \max \left[ U(c^T_{t}, F(h^T_{t})) + \beta \mathbb{E}_t v^{OCC}(y^T_{t+1}, r^T_{t+1}, d_{t+1}, w_{t}) \right] (28)
\]

subject to (9), (10), (15), (17), and (26), where \( v^{OCC}(y^T_{t}, r^T_{t}, d_{t}, w_{t-1}) \) denotes the value function of the representative household. We exploit this formulation of the Ramsey problem in our quantitative analysis.

We note that the allocation induced by the Ramsey optimal capital control policy can also be supported through consumption taxes. Specifically, assume that instead of taxing external debt, the government taxes total consumption expenditures, \( c^T_{t} + p_t c^N_{t} \) at the rate $\tau^c_{t-1}$, so that the after-tax cost of consumption in period $t$ is \( (c^T_{t} + p_t c^N_{t})(1 + \tau^c_{t-1}) \). The consumption tax rate is determined one period in advance. That is, in period $t$ the government announces the tax rate on consumption expenditures that will be in effect in period $t + 1$. One can show that the Ramsey allocation can be supported by a consumption-tax-rate process of the form \( 1 + \tau^c_{t} = (1 - \tau^d_{t})(1 + \tau^c_{t-1}) \), for any initial condition $\tau^c_{-1} > -1$, where $\tau^d_{t}$ represents the Ramsey optimal tax rate on external debt.

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We also note that the model with Ramsey optimal capital controls is equivalent to one in which a benevolent government chooses the level of external debt and households cannot participate in financial markets but are hand-to-mouth agents. In this formulation, households receive a transfer from the government each period and their choice is limited to the allocation of expenditure between tradable and nontradable goods. The government then chooses the aggregate level of external debt taking into account the externality created by the combination of downward nominal wage rigidity and a currency peg.

3.7 The Optimality of Prudential Capital Controls: An Analytical Example

In this section, we present an analytical example showing the prudential nature of optimal capital controls. Specifically, in the economy analyzed in this example, the Ramsey policy completely smoothes consumption in response to a temporary decline in the interest rate in order to attenuate the impact of this shock on unemployment once the interest rate goes back up to its long-run level.

Consider an economy like the one studied thus far in which the government pegs the nominal exchange rate. Assume that preferences are given by \( U(c_t) = \ln(c_t) \) and \( A(c^T_t, c^N_t) = c^T_t c^N_t \). The technology for producing nontradable goods is \( F(h_t) = h^\alpha_t \), with \( \alpha \in (0, 1) \). Assume that the economy starts period zero with no outstanding debt, \( d_0 = 0 \), that the endowment of tradables, \( y^T \), is constant over time, and that the real wage in period \(-1\) equals \( \alpha y^T \). Consider a situation in which the economy is subject to a temporary interest rate decline in period zero. Specifically, suppose that

\[
 r_t = \begin{cases} 
 r & t = 0 \\
 r > r & t \neq 0 
\end{cases}
\]

This interest-rate shock is assumed to be unanticipated. Finally, assume that \( \beta(1 + r) = 1 \), that \( \gamma = 1 \), and that \( \bar{h} = 1 \). The economy is assumed to have been at a full-employment equilibrium in periods \( t < 0 \), with \( d_t = 0 \), \( c^T_t = y^T \), \( h_t = \bar{h} \), and \( c^N_t = \bar{h}^\alpha \).

The following proposition presents the aggregate dynamics of this economy under free capital mobility, under optimal capital controls, and under the first-best equilibrium.

**Proposition 4 (The Prudential Nature of Optimal Capital Controls)** In the economy described above, aggregate dynamics under free capital mobility are given by

\[
c^T_0 = y^T \left[ \frac{1}{1 + r} + \frac{r}{1 + r} \right] > y^T
\]

\[ c_t^T = y^T \left[ \frac{1}{1+r} + \frac{r}{1+r} \right] < y^T; \quad t \geq 1 \]
\[ d_t = y^T \left[ 1 - \frac{1+r}{1+r} \right] > 0; \quad t \geq 1 \]
\[ h_0 = 1, \]
\[ h_t = \frac{1+r}{1+r} < 1; \quad t \geq 1. \]

And the Ramsey optimal allocation when the planner uses capital controls as the policy instrument is given by
\[ c_t^T = y^T; \quad t \geq 0 \]
\[ h_t = 1; \quad t \geq 0 \]
\[ d_t = 0; \quad t \geq 0 \]
and
\[ \tau_t^d = \begin{cases} 
1 - \frac{1+r}{1+r} & \text{for } t = 0 \\
0 & \text{for } t \geq 1 
\end{cases} \]

The first-best allocation is given by
\[ c_0^T = y^T \left[ \frac{1}{1+r} + \frac{r}{1+r} \right] > y^T \]
\[ c_t^T = y^T \left[ \frac{1}{1+r} + \frac{r}{1+r} \right] < y^T; \quad t \geq 1 \]
\[ d_t = y^T \left[ 1 - \frac{1+r}{1+r} \right] > 0; \quad t \geq 1 \]
\[ h_t = 1; \quad t \geq 0. \]

**Proof:** See appendix C.

Under free capital mobility, agents borrow internationally to take advantage of the temporarily lower interest rate. The resulting capital inflow drives up consumption of tradables and real wages. When the interest rate returns to its long-run level, aggregate demand falls and unemployment emerges as real wages are too high to be consistent with full employment. As stressed throughout the paper, the rigidity of real wages is caused by the combination of downward nominal wage rigidity and a fixed exchange rate. The optimal capital control policy taxes capital inflows in period 0 to curb the boom in aggregate demand and in this way also limit the appreciation of real wages in period 0. Indeed, the Ramsey planner finds it optimal to fully undue the temporary decline in the world interest rate. The effective interest
rate faced by domestic households is given by \( r \) even in period 0. In this way, consumption is fully smoothed over time and as a result the labor market is unaffected by the temporary decline in interest rates.

This example clearly illustrates the tradeoff between the efficient intertemporal allocation of expenditures in tradable goods and full employment. Under free capital mobility, the intertemporal allocation of tradable consumption is the one associated with the first-best equilibrium. However, output in the nontraded sector is inefficiently low in all periods following the initial one, as labor resources remain unemployed. By contrast, under optimal capital controls, the intertemporal allocation of tradable expenditure is inefficient. For it is not optimal to smooth consumption in response to changes in the interest rate. At the same time, the labor market operates under full employment at all times, which is consistent with the first-best allocation.

In this example, the optimal capital-control policy resolves the tradeoff between intertemporal distortions and static distortions in favor of eliminating all static distortions. As we will see shortly, the thrust of this result carries over to richer economic environments.

4 Evidence On Downward Nominal Wage Rigidity And Estimates Of \( \gamma \)

A central assumption in our theoretical framework is that nominal wages are downwardly rigid. This assumption is embodied in the parameter \( \gamma \), defining an upper bound on the rate of nominal wage decreases. In this section, we review existing and present new empirical evidence in support of this assumption. To stress the ubiquitousness of downward wage frictions, we consider evidence based on data from developed, emerging, and poor regions of the world, as well as from formal and informal labor markets.

One way in which the real effects of wage rigidity can be identified is by exploiting the seasonal dimension of wage adjustments. Olivei and Tenreyro (2007) document that in the United States nominal wage adjustments tend to be decided in the second half of the year and take effect at the very beginning of each year. This means that nominal wages are more rigid in the first than in the second half of the year. Olivei and Tenreyro exploit this seasonal pattern in wage adjustment and show, using VAR techniques, that monetary shocks that take place in the first half of the year have larger effects on aggregate activity than monetary shocks that occur in the second half of the year. Olivei and Tenreyro (2010) extend this result by observing that the seasonal component of nominal wage adjustments varies widely from country to country. They find that in countries in which wage setting is seasonal
(e.g., the United States and Japan), the real effects of monetary policy also depend on the season in which they occur. By contrast, in countries in which wage setting does not display a pronounced seasonal pattern (e.g., France, Germany, and the United Kingdom), the real effect of monetary policy innovations do not significantly depend on the season in which they are implemented. The findings of Olivei and Tenreyro (2007, 2010) provide evidence in favor of the hypothesis that nominal wage rigidity has significant effects on economic activity.

A further question is whether nominal wage rigidity is asymmetric as assumed in our theoretical model. The existing empirical evidence strongly supports this assumption. Gottschalk (2005), for example, uses SIPP panel data to estimate the probability of wage declines, increases, and no changes for male and female hourly workers working for the same employer over the period 1986-1993 in the United States. He finds that for males the probabilities of wage increases, wage constancy, and wage declines are, respectively, 41.2, 53.7, and 5.1 percent per year. The corresponding probabilities for females are 46.5, 49.2, and 4.3 percent. These findings suggest that over the course of one year a very small fraction of workers experiences a decline in nominal wages, while about half of workers experience no change. For the purpose of our argument, it is important to note that the sample period used by Gottschalk comprises the 1991 U.S. recession, for it implies that the observed scarcity of nominal wage cuts took place in the context of elevated unemployment. Barattieri, Basu, and Gottschalk (2010) report similar findings using data from the 1996-2000 SIPP panel. Interestingly, this study is not restricted to workers working for the same employer. A similar pattern of downward nominal wage rigidity is documented by Holden and Wulfsberg (2008) using industry-level wage data in 19 OECD countries over the period 1973-1999. Further evidence of downward nominal wage rigidity using microeconomic data is given in Fortin (1996) for Canada, Kuroda and Yamamoto (2003) for Japan, Fehr and Goette (2005) for Switzerland, and Daly et al. (2012) for the United States.

The evidence referenced above is based on data from formal labor markets in developed economies. However, a similar pattern of asymmetry in nominal wage adjustments emerges in informal labor markets located in poor areas of the world. Kaur (2012), for example, studies the behavior of nominal wages in casual daily agricultural labor markets in rural India. Specifically, she examines market-level wage and employment responses to local rainfall shocks in 500 Indian districts from 1956 to 2008. She finds that nominal wage adjustment is asymmetric. In particular, nominal wages rise in response to positive rain shocks but fail to fall during droughts. In addition, negative rain shocks cause labor rationing and unemployment. More importantly, inflation, which is uncorrelated with local rainfall shocks, moderates these effects. During periods of relatively high inflation, local droughts are more likely to result in lower real wages and less labor rationing.
The empirical literature surveyed thus far establishes that nominal wage rigidity is asymmetric and has significant economic effects. However, because it does not provide information on the speed of nominal downward wage adjustments, it does not lend itself to calibrating the wage-rigidity parameter $\gamma$. For this reason, we now propose an empirical strategy for identifying this parameter. It consists in observing the behavior of nominal wages during periods of rising unemployment and low inflation. We focus on episodes in which an economy undergoing a severe recession keeps the nominal exchange rate fixed. Two prominent examples are Argentina during the second half of the Convertibility Plan (1998-2001) and the periphery of Europe during the great recession of 2008.

Figure 6 displays the nominal exchange rate, subemployment (defined as the sum of unemployment and underemployment), nominal (peso) wages, and real (dollar) wages for Argentina during the period 1996-2006. The subperiod 1998-2001 is of particular interest because during that time the Argentine central bank was holding on to the currency peg in spite of the fact that the economy was undergoing a severe contraction and both unemployment and underemployment were in a steep ascent. In the context of a flexible-wage model, one would expect that the rise in unemployment would be associated with falling real wages. With the nominal exchange rate pegged, the fall in real wages must materialize through nominal wage deflation. However, during this period, the nominal hourly wage never fell. Indeed, it increased from 7.87 pesos in 1998 to 8.14 pesos in 2001. Our model predicts that with rising unemployment, the lower bound on nominal wages should be binding and therefore $\gamma$ should equal the gross growth rate of nominal wages. An estimate of the parameter $\gamma$ can then be constructed as the average quarterly growth rate of nominal wages over the three-year period considered, that is, $\gamma = \left(\frac{W_{2001}}{W_{1998}}\right)^{1/12}$. This yields a value of $\gamma$ of 1.0028.

In order for this estimate of $\gamma$ to represent an appropriate measure of wage rigidity in the context of our model, it must be adjusted to account for the fact that our model abstracts from foreign inflation and long-run productivity growth. To carry out this adjustment, we use the growth rate of the U.S. GDP deflator as a proxy for foreign inflation. Between 1998 and 2001, the U.S. GDP deflator grew by 1.77 percent per year on average. We set the long-run growth rate in Argentina at 1.07 percent per year, to match the average growth rate of Argentine per capita real GDP over the period 1900-2005 reported in García-Cicco et al. (2010). The adjusted value of $\gamma$ is then given by $1.0028/(1.0107 \times 1.0177)^{1/4} = 0.9958$. This value means that real wages can fall frictionlessly by 1.7 percent per year.

We note additionally that the fact that Argentine real wages fell significantly and persistently after the devaluation of 2002 (bottom right panel of figure 6) suggests that the 1998-2001 period was one of censored wage deflation, which further strengthens the view
Figure 6: Nominal Wages and Unemployment in Argentina, 1996-2006

Source. Nominal exchange rate and nominal wage, BLS. Subemployment, INDEC.
that nominal wages suffer from downward inflexibility.

Finally, we note that during the 1998-2001 Argentine contraction, consumer prices, unlike nominal wages, did fall significantly. The CPI rate of inflation was on average -0.86 percent per year over the period 1998-2001. It follows that real wages rose not only in dollar terms but also in terms of CPI units. Incidentally, this evidence provides some support for our assumption that downward nominal rigidities are less stringent for product prices than for factor prices.

The second episode from the emerging-market world that we use to infer the value of \( \gamma \) is the great recession of 2008 in the periphery of Europe. Table 1 presents an estimate of \( \gamma \) for eleven European economies that are either on the euro or pegging to the euro. The table shows the unemployment rate in 2008:Q1 and 2011:Q2. The starting point of this period corresponds to the beginning of the great recession in Europe according to the CEPR Euro Area Business Cycle Dating Committee. The 2008 crisis caused unemployment rates to rise sharply across all eleven countries. The table also displays the total growth of nominal hourly labor cost in manufacturing, construction and services (including the public sector) over the thirteen-quarter period 2008:Q1-2011:Q2.\(^6\) Despite the large surge in unemployment, nominal wages grew in most countries and in those in which they fell, the decline was modest. The implied value of \( \gamma \), shown in the last column of table 1, is given by the average growth rate of nominal wages over the period considered (that is, \( \gamma = (W_{2011:Q2}/W_{2008:Q1})^{1/13} \)). The estimated values of \( \gamma \) range from 0.996 for Lithuania to 1.028 for Bulgaria.

To adjust \( \gamma \) for foreign inflation, we use the fact that over the thirteen-quarter sample period considered in table 1 inflation in Germany was 3.6 percent, or about 0.3 percent per quarter. To adjust for long-run growth, we use the average growth rate of per capita output in the southern periphery of Europe of 1.2 percent per year or 0.3 percent per quarter.\(^7\) Allowing for these effects suggest an adjusted estimate of \( \gamma \) in the interval \([0.990, 1.022]\).

5 Quantitative Analysis

In this section we analyze quantitatively the behavior of the economy under under the three alternative policy arrangements analyzed theoretically in section 3. To this end, we begin by estimating the stochastic process of the exogenous driving forces and calibrating the remaining structural parameters of the model.

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\(^6\) The public sector is not included for Spain due to data limitations.

\(^7\) This figure corresponds to the average growth rate of per capita real GDP in Greece, Spain, Portugal, and Italy over the period 1990-2011 according to the World Development Indicators.
Table 1: Unemployment, Nominal Wages, and $\gamma$: Evidence from the Eurozone

<table>
<thead>
<tr>
<th>Country</th>
<th>Unemployment Rate 2008Q1 (in percent)</th>
<th>Unemployment Rate 2011Q2 (in percent)</th>
<th>Wage Growth $\frac{W_{2011Q2}}{W_{2008Q1}}$ (in percent)</th>
<th>Implied Value of $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulgaria</td>
<td>6.1</td>
<td>11.3</td>
<td>43.3</td>
<td>1.028</td>
</tr>
<tr>
<td>Cyprus</td>
<td>3.8</td>
<td>6.9</td>
<td>10.7</td>
<td>1.008</td>
</tr>
<tr>
<td>Estonia</td>
<td>4.1</td>
<td>12.8</td>
<td>2.5</td>
<td>1.002</td>
</tr>
<tr>
<td>Greece</td>
<td>7.8</td>
<td>16.7</td>
<td>-2.3</td>
<td>0.9982</td>
</tr>
<tr>
<td>Ireland</td>
<td>4.9</td>
<td>14.3</td>
<td>0.5</td>
<td>1.0004</td>
</tr>
<tr>
<td>Lithuania</td>
<td>4.1</td>
<td>15.6</td>
<td>-5.1</td>
<td>0.996</td>
</tr>
<tr>
<td>Latvia</td>
<td>6.1</td>
<td>16.2</td>
<td>-0.6</td>
<td>0.9995</td>
</tr>
<tr>
<td>Portugal</td>
<td>8.3</td>
<td>12.5</td>
<td>1.91</td>
<td>1.001</td>
</tr>
<tr>
<td>Spain</td>
<td>9.2</td>
<td>20.8</td>
<td>8.0</td>
<td>1.006</td>
</tr>
<tr>
<td>Slovenia</td>
<td>4.7</td>
<td>7.9</td>
<td>12.5</td>
<td>1.009</td>
</tr>
<tr>
<td>Slovakia</td>
<td>10.2</td>
<td>13.3</td>
<td>13.4</td>
<td>1.010</td>
</tr>
</tbody>
</table>

Note. $W$ is an index of nominal average hourly labor cost in manufacturing, construction, and services. Unemployment is the economy-wide unemployment rate. Source: EuroStat.

5.1 Calibration

The law of motion of tradable output and the country-specific interest rate is assumed to be given by the following autoregressive process:

\[
\begin{bmatrix}
\ln y^T_t \\
\ln \frac{1+r_t}{1+r}
\end{bmatrix} = A \begin{bmatrix}
\ln y^T_{t-1} \\
\ln \frac{1+r_{t-1}}{1+r}
\end{bmatrix} + \epsilon_t,
\]

where $\epsilon_t$ is a white noise of order 2 by 1 distributed $N(\theta, \Sigma_{\epsilon})$. The parameter $r$ denotes the deterministic steady-state value of $r_t$. We estimate this system using Argentine data over the period 1983:Q1 to 2001:Q4. (In section 6.2, we use Greek data.) We exclude the period post 2001 because Argentina was in default between 2002 and 2005 and excluded from international capital markets. The default was reflected in excessively high country premia. Excluding this period is in order because interest rates were not allocative, which is at odds with our maintained assumption that the country never loses access to international financial markets. This is a conservative choice, for inclusion of the default period would imply a more volatile driving force accentuating the real effects of currency pegs on unemployment.

Our empirical measure of $y^T_t$ is the cyclical component of Argentine GDP in agriculture,
forestry, fishing, mining, and manufacturing. We obtain the cyclical component by removing a log-quadratic time trend. We measure the country-specific real interest rate as the sum of the EMBI+ spread for Argentina and the 90-day Treasury-Bill rate, deflated using a measure of expected dollar inflation.

Our OLS estimates of the matrices $A$ and $\Sigma_\epsilon$ and of the scalar $r$ are

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \quad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \quad r = 0.0316.$$

According to these estimates, both $\ln y^T_t$ and $r_t$ are highly volatile, with unconditional standard deviations of 12.2 percent and 1.7 percent per quarter (6.8 percent per year), respectively. Also, the unconditional contemporaneous correlation between $\ln y^T_t$ and $r_t$ is high and negative at -0.86. This means that periods of relatively low traded output are associated with high interest rates and vice versa. The estimated joint autoregressive process implies that both traded output and the real interest rate are persistent, with first-order autocorrelations of 0.95 and 0.93, respectively. Finally, we estimate a steady-state real interest rate of 3.16 percent per quarter, or 12.6 percent per year. This high average value reflects the fact that our sample covers a period in which Argentina underwent a great deal of economic turbulence.

We set the parameter $\gamma$ governing the degree of downward nominal wage rigidity at 0.99. This is a conservative value. The estimates of $\gamma$ based on data from Argentina and the periphery of Europe presented in section 4 suggest that $\gamma$ lies in the interval [0.99, 1.022]. We set $\gamma$ to the lower bound of this interval, which represents the greatest degree of downward wage flexibility consistent with the data used in the estimation. In section 6.4, we set $\gamma$ to 0.98.

We adopt standard functional forms for preferences and technologies. Specifically, we assume a CRRA form for the period utility function, a CES form for the aggregator function, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

---

8The data were downloaded from www.indec.mecon.ar.

9Specifically, we construct the time series for the quarterly real Argentine interest rate, $r_t$, as $1 + r_t = (1 + i_t)E_{t+1}^{-1}1 + \pi_t + 1$, where $i_t$ denotes the dollar interest rate charged to Argentina in international financial markets and $\pi_t$ is U.S. CPI inflation. For the period 1983:Q1 to 1997:Q4, we take $i_t$ to be the Argentine interest rate series constructed by Neumeyer and Perri (2005) and posted at www.fperri.net/data/neuperri.xls. For the period 1998:Q1 to 2001:Q4, we measure $i_t$ as the sum of the EMBI+ spread and the 90-day Treasury bill rate, which is in line with the definition used in Neumeyer and Perri. We measure $E_{t+1}^{-1}1 + \pi_t + 1$ by the fitted component of a regression of $1 + \pi_t + 1$ onto a constant and two lags. This regression uses quarterly data on the growth rate of the U.S. CPI index from 1947:Q1 to 2010:Q2.
Table 2: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.99</td>
<td>Degree of downward nominal wage rigidity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>5</td>
<td>Inverse of intertemporal elasticity of consumption</td>
</tr>
<tr>
<td>$y^T$</td>
<td>1</td>
<td>Steady-state tradable output</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>1</td>
<td>Labor endowment</td>
</tr>
<tr>
<td>$a$</td>
<td>0.26</td>
<td>Share of tradables</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.44</td>
<td>Elasticity of substitution between tradables and nontradables</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Labor share in nontraded sector</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9375</td>
<td>Quarterly subjective discount factor</td>
</tr>
</tbody>
</table>

\[
A(c^T, c^N) = \left[ a(c^T)^{1-\frac{1}{\xi}} + (1 - a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi - 1}},
\]

and

\[
F(h) = h^\alpha.
\]

We calibrate the model at a quarterly frequency using data from Argentina as shown in table 2. Reinhart and Végh (1995) estimate the intertemporal elasticity of substitution to be 0.21 using Argentine quarterly data. We therefore set $\sigma$ equal to 5. We normalize the steady-state levels of output of tradables and hours at unity. Then, if the steady-state trade-balance-to-output ratio is small, the parameter $a$ is approximately equal to the share of traded output in total output. We set this parameter at 0.26, which is the share of traded output (as defined above) observed in Argentine data over the period 1980:Q1-2010:Q1. Using time series data for Argentina over the period 1993Q1-2001Q3, González Rozada et al. (2004) estimate the elasticity of substitution between traded and nontraded consumption, $\xi$, to be 0.44. This estimate is consistent with the cross-country estimates of Stockman and Tesar (1995). These authors include in their estimation both developed and developing countries. Restricting the sample to include only developing countries yields a value of $\xi$ of 0.43 (see Akinci, 2011). Following Uribe’s (1997) evidence on the size of the labor share in the nontraded sector in Argentina, we set $\alpha$ equal to 0.75.

We set $\bar{d}$ at the natural debt limit, which we define as the level of external debt that can be supported with zero tradable consumption when the household perpetually receives the lowest possible realization of tradable endowment, $y_{T \text{min}}$, and faces the highest possible realization of the interest rate, $r_{\text{max}}$. Formally, $\bar{d} \equiv y_{T \text{min}} (1 + r_{\text{max}})/r_{\text{max}}$. Using the minimum level of the cyclical component of traded output and the maximum level of the real interest rate in our sample to assign values to $y_{T \text{min}}$ and $r_{\text{max}}$, we obtain $\bar{d} = 8.34$.

The final parameter we calibrate is the subjective discount factor $\beta$. We set this parameter so as to match a foreign-debt-to-output ratio of 26 percent per year, a value in line with
that reported for Argentina over our calibration period by Lane and Milesi-Ferretti (2007).
In the context of our model, the task of calibrating $\beta$ is complicated by the fact that the
debt-to-output ratio is sensitive to the assumed monetary regime. In emerging countries
in general and in Argentina in particular, monetary regimes tend to change frequently and
widely. A compromise is therefore in order. In calibrating $\beta$, we assume that the underlying
monetary regime takes the form of a currency peg. This strategy results in a value of $\beta$ of
0.9375.

5.2 Approximating Equilibrium Dynamics

Here we sketch the numerical solution methods we employ to approximate the equilibrium
dynamics under the three policy arrangement we consider, namely, in ascending order of
computational complexity, the optimal exchange-rate policy, a currency peg with optimal
capital controls, and a currency peg with free capital mobility.

Under all three policy arrangements, the approximation involves discretizing the state
space. We discretize the exogenous driving processes $y^T_t$ and $r_t$ using 21 equally spaced
points for $\ln y^T_t$ and 11 equally spaced points for $\ln(1 + r_t)/(1 + r)$. We construct the
transition probability matrix of the state $(\ln y^T_t, \ln((1 + r_t)/(1 + r)))$ by simulating a time
series of length 1,000,000 drawn from the system (29). We associate each observation in
the time series with one of the 231 possible discrete states by distance minimization. The
resulting discrete-valued time series is used to compute the probability of transitioning from
a particular discrete state in one period to a particular discrete state in the next period. The
resulting transition probability matrix captures well the covariance matrices of order 0 and
1.

To discretize the endogenous state $d_t$, we use 501 equally spaced points. We fix the upper
bound of the debt grid at $\bar{d}$. We set the lower bound of the debt grid at -5.

When the exchange-rate policy takes the form of a currency peg (whether it is combined
with free capital mobility or with capital controls), a second endogenous state emerges,
namely, past real wages, $w_{t-1}$. We discretize this state using a grid of 500 equally spaced
points for the logarithm of $w_{t-1}$. We set the lowest grid value of $w_{t-1}$ at 0.5 and the highest
at 5.3.

The equilibrium dynamics under the optimal exchange rate policy are obtained from
the solution to the value function problem given by the functional equation (22) and the
constraints (7), (9), and (10). We numerically approximate this solution by applying the
method of value function iteration over a discretized state space. Under optimal exchange
rate policy, the state of the economy in period $t \geq 0$ is the triplet $\{y^T_t, r_t, d_t\}$. 

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The equilibrium dynamics under a currency peg with optimal capital control policy can be approximated by solving the functional equation (28) subject to subject to (9), (10), (15), (17), and (26). Approximating the equilibrium dynamics in this environment is more demanding than doing so under the optimal exchange-rate policy because the former problem includes an additional state variable, namely, \( w_{t-1} \). We numerically approximate the equilibrium dynamics by applying the method of value function iteration over a discretized state space. The state of the economy in period \( t \geq 0 \) consists of the quadruplet \( y_t^T, r_t, d_t, \) and \( w_{t-1} \).

The task of approximating the equilibrium dynamics becomes even more challenging under a currency peg with free capital mobility. As in the case of a currency peg with optimal capital controls, there are four state variables, \( y_t^T, r_t, d_t, \) and \( w_{t-1} \). However, under a currency peg with free capital mobility, aggregate dynamics cannot be cast in terms of a Bellman equation without introducing additional state variables, such as the individual level of debt, which households perceive as distinct from its aggregate counterpart. We therefore approximate the solution by Euler equation iteration over a discretized version of the state space \( (y_t^T, r_t, d_t, w_{t-1}) \). Appendix D describes our numerical algorithm in more detail.

### 5.3 Boom-Bust Cycles

We define a boom-bust episode as a situation in which tradable output, \( y_t^T \), is at or below trend in period 0, at least one standard deviation above trend in period 10, and at least one standard deviation below trend in period 20. To this end, we simulate the model economy for 20 million periods and select all subperiods that satisfy our definition of a boom-bust episode. We then average across these episodes.

The top two panels of figure 7 display the dynamics of the two exogenous driving forces, tradable output and the country interest rate. Because our estimate of the exogenous driving forces features a high negative correlation between traded output and the country interest rate, a boom-bust episode can also be interpreted as one in which the interest rate falls one standard deviation below mean in the following 10 quarters and then rises to one standard deviation above in the following 10 quarters.

The middle and bottom panels of the figure depict the model’s predictions during a boom-bust cycle. Solid lines correspond to the economy with a currency peg and free capital mobility, broken lines to the economy with a currency peg and optimal capital controls, and dotted lines to the economy with optimal exchange-rate policy.

The middle left panel of the figure shows that the optimal capital control policy is prudential. Capital controls increase significantly during the expansionary phase of the cycle,
Figure 7: Boom-Bust Dynamics

Traded Output, $y^T_t$

Annualized Interest Rate, $r_t$

Capital Control Rate, $\tau^d_t$

Annualized Devaluation Rate, $\epsilon_t$

Unemployment Rate, $1 - h_t$

Traded Consumption, $c^T_t$

No Capital Controls
Optimal Capital Controls
Optimal Exchange-Rate Policy
from about 2 percent at the beginning of the episode to 6 percent at the peak of the cycle. During the contractionary phase of the cycle, capital controls are drastically relaxed. Indeed at the bottom of the crisis, capital inflows are actually subsidized at a rate of about 2 percent. The sharp increase in capital controls during the expansionary phase of the cycle puts sand in the wheels of capital inflows, thereby restraining the boom in tradable consumption (see the bottom right panel). Under a peg with free capital mobility, during the boom, tradable consumption increases significantly more than under the optimal capital control policy. In the contractionary phase, the fiscal authority incentivates spending in tradables by subsidizing capital inflows. As a result consumption falls by much less in the regulated economy than it does in the unregulated one. During the recession, the optimal capital control policy, far from calling for austerity in the form of severe cuts in tradable consumption, supports this type of expenditure. That is, the capital control policy stabilizes the absorption of tradable goods over the cycle. It follows that the Ramsey-optimal capital control policy does not belong to the family of beggar-thy-neighbor policies, for it does not seek to foster trade surpluses during crises.

Because unemployment depends directly upon variations in the level of tradable absorption through the latter’s role as a shifter of the demand schedule for nontradedables, and because optimal capital controls stabilize the absorption of tradables, unemployment is also stable over the boom-bust cycle. Specifically, as can be seen from the bottom left panel of figure 7, in the peg economy without capital controls, unemployment increases sharply by over 20 percentage points during the recession. By contrast, under optimal capital controls the rate of unemployment rises relatively modestly by about 3 percentage points. It follows that the Ramsey planner’s tradeoff between distorting the intertemporal allocation of tradable consumption and reducing unemployment is overwhelmingly resolved in favor of the latter.

Indeed, the rate of unemployment in the peg economy with optimal capital controls is much closer to the unemployment rate under the optimal exchange rate policy (equal to zero at all times) than to the unemployment rate in the peg economy with free capital mobility. However, the means by which the policymaker achieves low unemployment in the peg economy with optimal capital controls and in the optimal exchange-rate-policy economy are quite different. In the optimal-capital control economy lower unemployment is the consequence of stabilizing traded absorption (i.e., stabilizing the demand schedule in figure 5). By contrast under the optimal exchange rate policy, low unemployment is achieved through a series of large currency devaluations (middle right panel) that lower the labor cost in the nontraded sector during crises (i.e., by shifts in the supply schedule in figure 5)).
### Table 3: Optimal Capital Controls: Level, Volatility and Welfare Effects

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Control  Rate</td>
<td>$\tau^d_t$</td>
<td>0</td>
<td>2.4</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>$\bar{h} - h_t$</td>
<td>0</td>
<td>3.1</td>
</tr>
<tr>
<td>Consumption</td>
<td>$c_t$</td>
<td>0.93</td>
<td>0.97</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>$y_t^T - c_t^T$</td>
<td>0.18</td>
<td>0.05</td>
</tr>
<tr>
<td>Real Wage</td>
<td>$W_t/E_t$</td>
<td>1.5</td>
<td>2.1</td>
</tr>
<tr>
<td>Traded Output</td>
<td>$y_t^T$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>$r_t$</td>
<td>13.2</td>
<td>13.2</td>
</tr>
<tr>
<td>External Debt</td>
<td>$d_t$</td>
<td>5.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Debt-to-Output Ratio</td>
<td>$d_t/4(y_t^T + p_t c_t^N)$</td>
<td>57.9</td>
<td>11.2</td>
</tr>
</tbody>
</table>

| Welfare Cost     | $\lambda^{FCM}, \lambda^{OCC}$ | 0 | 3.7 | 11.6 |

Note. $\tau^d_t$, $\bar{h} - h_t$, and $d_t/4(y_t^T + p_t c_t^N)$ are expressed in percent, $r_t$ is expressed in percent per year, and $c_t$, $y_t^T - c_t^T$, $W_t/E_t$, $y_t^T$, and $d_t$ are expressed in levels. Welfare costs are relative to the optimal exchange-rate policy (or first-best allocation) and are expressed in percent of consumption per period (see expressions (30) and (31)).

#### 5.4 Level and Volatility Effects of Optimal Capital Controls

Table 3 displays unconditional first and second moments of macroeconomic indicators of interest. On average, the Ramsey planner imposes a positive tax on external debt of 2.4 percent. This figure implies large average levels of capital controls, for the effective interest rate faced by domestic debtors, given by $(1 + r_t)/(1 - \tau^d_t)$, increases from an average of 13.2 percent per year under free capital mobility to 24.8 percent per year under optimal capital controls. The main reason why the Ramsey planner finds it optimal to impose capital controls on average is to lower the average level of external debt holdings. We postpone an explanation of why this is optimal until section 5.5.

Table 3 also shows that the tax on debt is highly volatile, with a standard deviation of 5.2 percentage points per quarter. The main payoff of imposing cyclical capital controls is a significant reduction in the average rate of unemployment from 13.5 percent under free capital mobility to 3.1 percent under the optimal capital control policy. This reduction in
unemployment is welfare increasing because it raises the average level of production, and hence also absorption, of nontradables, which provide utility to domestic households.

The reduction in unemployment is mediated by a significant reduction in the volatility of the growth rate of tradable absorption. The standard deviation of the growth rate of tradable consumption, \( c^T_t / c^T_{t-1} \), not shown in the table, falls from 5.3 percent under free capital mobility to 2.9 percent under optimal capital controls. The connection between the volatility of tradable consumption growth and the average level of unemployment follows from the fact that, as shown in section 2.1, consumption of tradables plays the role of a shifter of the demand for nontradables. In turn, the Ramsey planner succeeds in curbing the variance of tradable expenditure growth by raising the cost of external borrowing during booms and lowering it during recessions. The correlation between traded output \( y^T_t \) and the capital control rate \( \tau^d_t \) is 0.54 and the correlation between the interest rate \( r_t \) and \( \tau^d_t \) is -0.58. Indeed, the Ramsey planner engineers an effective interest rate that is positively correlated with traded output in spite of the fact that the interest rate itself is strongly negatively correlated with the latter.

Table 3 shows that the first and second moments of the real (and nominal) wage rates are not significantly affected by the presence of capital controls. This prediction of the model might appear as surprising because downward wage rigidity is the sole friction in the present model, and because unemployment behaves markedly differently in the peg economy with free capital mobility and the peg economy with optimal capital controls. A reason why the unconditional moments of real wages seem little affected by capital controls is that the lower bound on wages is binding most of the time in both economies (85 percent of the time under free capital mobility and 65 percent of the time under optimal capital controls), and, when this happens, the wage rate falls at the common gross rate \( \gamma \). A reason why the first and second moments of unemployment are so different under free capital mobility and under optimal capital controls in spite of the similarity in the corresponding moments of real wages is that when the wage constraint is binding the magnitude of the unemployment rate depends on the strength of the domestic absorption of tradables, which is significantly affected by capital controls.

An important distinction in wage dynamics across capital control regimes that is not captured by the unconditional moments shown in table 3 is the behavior of wages during booms. During economic expansions, the Ramsey fiscal authority, through capital controls, limits the appreciation of real wages. In this way, it also reduces the need for large decreases in the real wage once the boom is over. To visualize the role of optimal capital controls in limiting wage increases during booms, figure 8 displays the cumulative probability distribution of positive wage changes under free capital mobility and under optimal capital controls.
in the peg economies. Under optimal capital controls the vast majority of wage increases is small. Specifically, 90 percent of wage increases are less than 5 percent in magnitude. By contrast, only about half of all wage increases that occur under free capital mobility are smaller than 5 percent. This difference underlines the prudential nature of optimal capital controls.

### 5.5 Peg-Induced Overborrowing

Table 3 shows that the average level of external debt in the peg economy with free capital mobility is more than three times higher than it is in the peg economy with optimal capital controls. This prediction of the model is also evident from figure 9, which shows the unconditional distribution of external debt in the economy with a fixed exchange rate and free capital mobility (solid line) and in the economy with a fixed exchange rate and optimal capital controls (dashed line). The Ramsey planner induces a lower average level of external debt by taxing borrowing at a positive rate. Recall that the average tax rate on debt is 2.4 percent per quarter. It follows that the peg economy with free capital mobility accumulates inefficiently large amounts of external debt relative to the peg economy with optimal capital controls. In other words, conditional on being on a fixed-exchange-rate regime, economies with free capital mobility overborrow.
The reason why the average level of external debt is lower under optimal capital controls than under free capital mobility is that the Ramsey planner finds it optimal to induce an external debt position that is significantly more volatile than the one associated with free capital mobility. As shown in table 3, the standard deviation of external debt is 2.3 under optimal capital controls, but only 0.7 under free capital mobility. Similarly, figure 9 shows that the distribution of external debt is significantly more dispersed under optimal capital controls than under free capital mobility. A more volatile process for external debt requires centering the debt distribution further away from the natural debt limit for precautionary reasons. In turn, the reason why the Ramsey planner finds wide swings in the external debt position desirable is that such variations allow him to insulate the domestic absorption of tradable goods from exogenous disturbances buffeting the economy. Put differently, in the Ramsey economy, external debt plays the role of shock absorber to a much larger extent than it does in the economy with free capital mobility.

The purpose of optimal capital controls is not to close the current account. On the contrary, under optimal capital controls, the economy makes more heavy use of the current account to smooth consumption than it does under free capital mobility. To see this, note that the current account is given by the change in net external debt, and that, as is apparent from figure 9, the net external debt has a much more dispersed distribution under optimal controls.
capital controls than under free capital mobility.

Figure 9 shows that the distribution of net external debt under the optimal exchange rate policy has a higher mean and is much less dispersed than under a currency peg with optimal capital controls. This difference highlights the fact that the two policies achieve reductions in unemployment (relative to a currency peg with free capital mobility) through very different means. Under the optimal exchange rate policy, the government induces shifts in the supply of nontradables to offset variations in the demand for nontradables brought about by fluctuations in the desired consumption of tradables. In this way, the policy maker achieves two goals, full employment and an efficient allocation of tradable consumption over time. Under a currency peg with optimal capital controls, the policymaker cannot induce shifts in the supply of nontradables, since its hands are tied by the currency peg and the presence of downward nominal wage rigidity. Instead, the government reduces unemployment by minimizing shifts in the demand for nontradables. To this end, the government levies capital controls to stabilize the desired demand for tradable consumption, which is the key shifter of the demand schedule for nontradables. In turn, as explained above, a smooth path for tradable consumption can only be supported by large swings in the country’s external debt position, which by necessity must be centered further away from the natural debt limit than it would be if tradable consumption were allocated optimally.

Finally, it is of interest to point out that the optimal capital control policy characterized here is complementary to the one that emerges in models of overborrowing due to collateral constraints (e.g., Korinek, 2010; and Bianchi, 2011). In this class of models, households’ ability to borrow is increasing in the value of output in terms of tradables. In turn, the value of output in terms of tradables is increasing in the relative price of nontradables. This relative price is endogenous to the model but exogenous to the household, which creates a pecuniary externality. The government understands that in equilibrium the relative price of nontradables is increasing in the absorption of tradables. Consequently, to induce a more relaxed collateral constraint on average, the government implements a policy that raises the average consumption of traded goods. To support higher average consumption of tradables, the economy must hold a lower average stock of external debt. This is achieved through capital controls. Thus, this literature shares two characteristics with the present model, namely, an externality that leads to overborrowing, and positive average capital controls as a second-best remedy. The key difference, of course, resides in the nature of the externality, financial frictions or downward wage rigidity.
5.6 Welfare Costs of Free Capital Mobility for Peggers

We have established that in the peg economy free capital mobility entails excessive external debt and unemployment. Both of these factors tend to depress consumption and therefore reduce welfare. In this section, we quantify the welfare losses associated with free capital mobility in economies subject to a currency peg.

Define the welfare cost of a currency peg under free capital mobility (FCM) conditional on state \( s_t \equiv \{y_T^t, r_t, d_t, w_{t-1}\} \), denoted \( \lambda^{FCM}(s_t) \), as the percent increase in the lifetime consumption stream required by an individual living in the economy with a currency peg and free capital mobility in state \( s_t \) to be as well off as an individual living in an economy with optimal exchange rate policy. Formally, \( \lambda^{FCM}(s_t) \) is implicitly given by

\[
E \left\{ \sum_{j=0}^{\infty} \beta^j U \left( c_{t+j}^{FCM} \left( 1 + \frac{\lambda^{FCM}(s_t)}{100} \right) \right) \left| s_t \right. \right\} = v^{OPT}(y_T^t, r_t, d_t),
\]

where \( c_t^{FCM} \) denotes consumption in the economy with a peg and free capital mobility, and, as defined earlier, \( v^{OPT}(y_T^t, r_t, d_t) \) denotes the value function in the economy with the optimal exchange rate policy in state \( (y_T^t, r_t, d_t) \). Because the state vector \( s_t \) is stochastic, the conditional welfare cost measure, \( \lambda^{FCM}(s_t) \), is itself stochastic. We report the mean of \( \lambda^{FCM}(s_t) \) over the distribution of \( s_t \) in the peg economy with free capital mobility. Formally, let \( \pi^{FCM}(s_t) \) denote the unconditional probability of \( s_t \) under a peg with free capital mobility. Then,

\[
\lambda^{FCM} = \sum_{s_t} \pi^{FCM}(s_t) \lambda^{FCM}(s_t). \tag{30}
\]

Similarly, the welfare cost of a currency peg under the optimal capital control (OCC) policy conditional on state \( s_t \), denoted \( \lambda^{OCC}(s_t) \), is defined as the permanent percent increase in the lifetime consumption stream required by an individual living in the economy with a currency peg and optimal capital controls in state \( s_t \) to be as well off as an individual living in the economy with optimal exchange rate policy. That is, \( \lambda^{OCC}(s_t) \) is implicitly given by

\[
E \left\{ \sum_{j=0}^{\infty} \beta^j U \left( c_{t+j}^{OCC} \left( 1 + \frac{\lambda^{OCC}(s_t)}{100} \right) \right) \left| s_t \right. \right\} = v^{OPT}(y_T^t, r_t, d_t),
\]

where \( c_t^{OCC} \) denotes consumption in the economy with a peg and optimal capital controls. Letting \( \pi^{OCC}(s_t) \) denote the unconditional probability of \( s_t \) under a peg with optimal capital controls, we have that the expected value of the welfare cost of a peg under the optimal capital
The control policy is given by

$$\lambda^{OCC} = \sum_{s_t} \pi^{OCC}(s_t) \lambda^{OCC}(s_t).$$

(31)

Recalling that the optimal exchange-rate policy achieves the Pareto optimal allocation, one can interpret \(\lambda^{FCM}\) as the distance, in welfare terms, between the first best allocation and the allocation induced by a currency peg with free capital mobility. Similarly, \(\lambda^{OCC}\) can be interpreted as the distance between the first-best allocation and the one induced by a currency peg coupled with Ramsey optimal capital controls.

Table 3 shows that the average welfare costs of free capital mobility for a pegging economy are large. The representative household living in the economy with free capital mobility requires on average an increase of 11.6 percent in consumption every period to be indifferent between living under a peg with free capital mobility and living in an economy with optimal exchange-rate policy.

The optimal capital control policy greatly reduces the pains of currency pegs. Households living in an economy with a currency peg and optimal capital controls require a 3.7 percent increase in consumption each period to be as well off as living in the economy with optimal exchange-rate policy. The welfare gain of moving from a peg, with or without optimal capital controls, to the optimal exchange rate policy has three components: one is a reduction in unemployment which translates into higher production and consumption of nontradables. This benefit is larger for the peg economy with free capital mobility. The second component is related to transitional dynamics along which households liquidate precautionary savings through higher-than-average consumption of tradables. This effect can be seen from figure 9, showing that the average level of external debt is higher under the optimal exchange-rate policy than under either of the two peg arrangements. The reduction in savings is greater for the peg economy with optimal capital controls. The third component is a lower long-run level of tradable consumption under the optimal exchange rate policy. This component represents a cost and is higher under optimal capital controls than under free capital mobility. Overall, the welfare gains are dominated by the reduction in unemployment.

A relevant question from a policy perspective is what are the welfare gains for a pegging economy with free capital mobility to adopt optimal capital controls. One might be tempted to conclude that the answer is \(\lambda^{FCM} - \lambda^{OCC}\), or 7.9 percent of consumption per period. But this computation would fail to correctly take into account the transitional debt dynamics involved in moving from free capital mobility to optimal capital controls. As can be seen from figure 9, moving from a peg economy with free capital mobility to a peg with optimal capital controls requires reducing the average level of external debt. This deleveraging is costly, since it implies a temporarily lower-than-average level of consumption of tradables.
This sacrifice is painful by itself, but it also causes unemployment to be higher along the transition (recall that consumption of tradables is a shifter of the demand for nontradables). In the long-run the economy with optimal capital controls will enjoy a higher average level of tradable consumption than the economy with free capital mobility. In sum, moving from free capital mobility to optimal capital controls has the benefits of lower long-run unemployment (13.5 versus 3.1), higher long-run consumption of tradables (6.9 percent), but a transitional cost associated with debt deleveraging. Of course, by definition, there must be net gains from moving from free capital mobility to optimal capital controls. In our economy these gains turn out to be 2.2 percent of consumption per period.\(^{10}\) This is a large number as welfare costs go in business cycle analysis, but is not as high as the measure \(\lambda_{FCM}^{\text{FCM}} - \lambda_{OC}^{\text{OCC}}\).

Our finding of large welfare costs of currency pegs (with or without capital controls) stands in stark contrast to a large body of work, pioneered by Lucas (1987), suggesting that the costs of business cycles (not just of suboptimal monetary policy) are small. Lucas’ approach to computing the welfare costs of business cycles abstracts from two features that are central determinants of welfare costs in our model, namely the effect of volatility on mean unemployment and transitional dynamics. The lack of connection between volatility and means within Lucas’ approach can be seen by noticing that it consists in first removing a trend from a consumption time series and then evaluating a second-order approximation of welfare using observed deviations of consumption from trend. Under this approach, the welfare cost of business cycles depends only upon the volatility of the cyclical component of consumption. Implicit in this methodology is the assumption that the trend is unaffected by policy. In our model, however, suboptimal monetary policy, creates an endogenous connection between the amplitude of the business cycle and the average rate of unemployment. In turn, through its effect on the average level of unemployment, suboptimal exchange-rate policy has a significant effect on the average level of consumption of nontradables. It follows that applying Lucas’ methodology to data stemming from our model would overlook the effects of policy on trend consumption and therefore would result in spuriously low welfare costs.

The importance of transitional dynamics for welfare can be seen by comparing the mean and standard deviation of consumption in the economy with a peg and optimal capital controls and the economy with optimal exchange-rate policy. The welfare cost of pegs with optimal capital controls relative to the optimal exchange-rate policy is 3.7 percent of con-

---

\(^{10}\)Formally, the welfare gain of switching from a peg with free capital mobility to a peg with optimal capital controls, denoted \(\lambda_{FCM}^{\text{FCM}} - \lambda_{OC}^{\text{OCC}}(s_t)\) is given by

\[
\lambda_{FCM}^{\text{FCM}} - \lambda_{OC}^{\text{OCC}}(s_t) = \sum_{j=0}^{\infty} \beta^j U \left( c_{t+j}^{\text{FCM}} \left( 1 + \frac{\lambda_{FCM}^{\text{FCM}} - \lambda_{OC}^{\text{OCC}}(s_t)}{100} \right) \right) | s_t | = \mathbb{E} \left\{ \sum_{j=0}^{\infty} \beta^j U \left( c_{t+j}^{\text{OC}} \right) | s_t \right\}.
\]
sumption per period eventhough consumption under the former policy has a higher mean (0.97 versus 0.93) and the same volatility (0.08) as under the latter. The reason why the peg with optimal capital controls is welfare dominated by the economy with optimal exchange-rate policy is that the former is associated with an inefficiently low level of external debt. As a result, an economy that switches from a peg with optimal capital controls to the optimal exchange rate policy enjoys a transition with high absorption of tradables as it accumulates external debt.

Because our model is stylized, we interpret the present welfare evaluation as suggestive. The sensitivity analysis conducted in the next section provides some more support for the size of the welfare losses reported here.

6 Sensitivity Analysis

In this section, we perform a number of variations in the structure of the model and parameter values.

6.1 Production in the Traded Sector

Our baseline model specification assumes that the supply of tradables, $y_T^t$, is exogenous. We now relax this assumption and assume instead that tradables are produced with labor. Specifically, we assume that

$$y_T^t = e^{z_t} (h_T^t)^{\alpha_T},$$

where $y_T^t$ denotes output of tradable goods, $h_T^t$ denotes labor employed in the traded sector, and $\alpha_T \in (0, 1)$ is a parameter. The variable $z_t$ is assumed to be exogenous and stochastic. We interpret $z_t$ either as a productivity shock in the traded sector or as a disturbance in the country’s terms of trade. We assume that, as in the nontraded sector, firms in the traded sector are perfectly competitive in product and labor markets. Further, we assume that labor is perfectly mobile across sectors. We make this assumption to create a sharp contrast with the baseline formulation in which labor is completely immobile across sectors. A more realistic formulation would be one in which, in the short run, labor does move across sectors, but sluggishly. The assumption of free labor mobility across sectors implies that wages are equalized across sectors.

Firms in the traded sector choose labor to maximize profits, which are given by

$$P_T e^{z_t} (h_T^t)^{\alpha_T} - W_t h_T^t.$$
Table 4: Sensitivity Analysis

<table>
<thead>
<tr>
<th>Economy</th>
<th>Welfare Cost Peg with No Capital Controls</th>
<th>Welfare Cost Peg with Optimal Capital Controls</th>
<th>Unemployment Rate Peg with No Capital Controls</th>
<th>Unemployment Rate Peg with Optimal Capital Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Baseline</td>
<td>11.6</td>
<td>3.7</td>
<td>13.5</td>
<td>3.1</td>
</tr>
<tr>
<td>2. Production in Traded Sector</td>
<td>10.1</td>
<td>5.0</td>
<td>7.8</td>
<td>1.9</td>
</tr>
<tr>
<td>3. Greece</td>
<td>17.6</td>
<td>6.0</td>
<td>15.3</td>
<td>3.7</td>
</tr>
<tr>
<td>4. Higher intertemp. Elast. of Subst. ((\sigma = 1/\xi = 2.27))</td>
<td>8.4</td>
<td>0.6</td>
<td>12.4</td>
<td>0.5</td>
</tr>
<tr>
<td>4.a. Less Downward Nominal Wage Rigidity ((\gamma = 0.98))</td>
<td>6.2</td>
<td>0.4</td>
<td>9.5</td>
<td>0.4</td>
</tr>
<tr>
<td>4.b. Endogenous Labor Supply ((\delta = 0.5))</td>
<td>19.0</td>
<td>0.8</td>
<td>33.5</td>
<td>1.3</td>
</tr>
<tr>
<td>4.c. Endogenous Labor Supply ((\delta = 0.75))</td>
<td>9.3</td>
<td>0.6</td>
<td>33.5</td>
<td>1.8</td>
</tr>
<tr>
<td>4.d. Endogenous Labor Supply ((\delta = 1))</td>
<td>2.1</td>
<td>0.3</td>
<td>33.5</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Note. Welfare costs are relative to the optimal exchange-rate policy (or first-best allocation) and are expressed in percent of consumption per period (see expressions (30) and (31)). Unemployment rates are expressed in percent.

The first-order condition associated with the firm’s profit maximization problem is

\[ \alpha_T P_t^T e^{z_t} (h_t^T)^{\alpha_T - 1} = W_t. \]  

(32)

Letting \(h_t^N\) denote hours employed in the nontraded sector, total hours worked, denoted by \(h_t\), are then given by

\[ h_t = h_t^T + h_t^N. \]

All other conditions of the model are as in the baseline formulation.

We assume that \(z_t\) and \(r_t\) follow the joint stochastic process given in equation (29), with \(z_t\) taking the place of \(\ln y_T^t\). This strategy for calibrating the law of motion of \(z_t\) results in a standard deviation of \(\ln y_T^t\) of 0.14 under a peg which is slightly above the value in the baseline model.\(^{11}\) Following Uribe (1997), we set \(\alpha_T = 0.5\). All other parameters take the values indicated in Table 2.

Table 4 shows that the welfare cost of currency pegs in the present economy is 10.1 percent of consumption per period on average and that optimal capital controls reduce these costs to 5.0 percent. The intuition for why the welfare costs of currency pegs continue to be

\(^{11}\)Ideally, one would like to use data on total factor productivity in the traded sector to calibrate the parameters defining the process \(z_t\). We do not pursue this avenue here for lack of reliable sectoral data.
large even when the supply of tradables is endogenous, can be illustrated by considering the 
adjustment of the economy to negative shocks when the lower bound on wages is binding. 
Consider first a negative interest-rate shock (i.e., an increase in $r_t$). If the wage rigidity is 
binding, then, as in the baseline economy, employment in the nontraded sector falls because 
of a weaker demand for this type of goods. At the same time, optimality condition (32) 
indicates that employment in the traded sector is unchanged, since wages are downwardly 
rigid and the exchange rate is pegged. This means that the unemployment that emerges in 
the nontraded sector will not be absorbed by the traded sector. Consider now the effect of 
a deterioration in the terms-of-trade or a negative productivity shock in the traded sector 
(i.e., a decline in $z_t$). Suppose again that the lower bound on nominal wages is binding. In 
this case, optimality condition (32) implies that employment in the traded sector will fall. 
This is because the product wage is unchanged but the marginal product of labor falls at any 
given level of employment. In the nontraded sector, demand declines because the negative 
productivity shocks produces a negative income effect. It follows that the nontraded sector 
does not absorb the hours lost in the traded sector. On the contrary, employment in the 
nontraded sector will also decline due to a weaker demand. This explains why the economy 
with production in the traded sector continues to exhibit large levels of unemployment under 
a currency peg.

Nonetheless, the unemployment rate under a currency peg is lower in the economy with 
production in the traded sector than in the baseline economy (7.8 versus 13.5 percent). 
The reason is that employment in the traded sector acts as a stabilizer of the wage rate 
during booms, thereby attenuating the negative externality caused by the combination of 
downward nominal wage rigidity and a currency peg. To see this, consider a decline in the 
country interest rate that raises the desired absorption of tradable and nontradable goods. 
This shock causes the demand for labor to increase in the nontraded sector driving wages 
up. This increase in wages induces firms in the traded sector to reduce employment. In 
turn, these freed up hours dampen the increase in wages required to clear the labor market. 
This dampening effect is beneficial because it means that once the boom is over the economy 
enters its way down to trend with lower real wages making the downward wage rigidity less 
stringent.

It is worth pointing out that the magnitude of the reduction in unemployment explained 
by this effect depends on our assumption that labor is perfectly mobile across sectors. To the 
extent that in the short run labor is sector specific, the reduction in the cyclical component 
of unemployment induced by the introduction of production in the traded sector should be 
expected to be smaller.
6.2 Greece

In this section we calibrate the model to Greece. This case is of interest because, relative to Argentina (from which we derive our baseline calibration) Greece displays less volatility in percent deviations of traded output from trend (6.5 versus 12.3 percent), less volatility in the country interest rate (5.2 versus 7.4 percentage points per annum), and a lower average country interest rate (4.5 versus 13.2 percent per annum). Another difference between Greece and Argentina is that the former exhibits a larger net foreign liability position as a fraction of GDP (117 percent versus 26 percent around the end of the respective calibration period).

We estimate the law of motion of traded output and the country interest rate using quarterly data from Greece over the period 1981-2011. See Appendix E for details. We calibrate all other structural parameters of the model as shown in table 2.

The reduced level of external uncertainty estimated for Greece relative to that estimated for Argentina and the lower real interest rate observed in Greece makes external borrowing more attractive in the model economy. As a result, the model delivers a mean debt-to-output ratio of 113 percent under a peg with free capital mobility.

The welfare costs of currency pegs in the model calibrated to Greece are large, 17.6 percent of consumption per period under free capital mobility and 6.0 percent under optimal capital controls. Both of these figures are higher than the corresponding ones for the model calibrated to Argentina, in spite of the fact that Greece is hit by less volatile external shocks. This result is explained by the fact that Greece has a larger external debt. Lower uncertainty and higher levels of external debt have opposite effects on the welfare costs of currency pegs. On the one hand, all other things equal, less uncertainty makes currency pegs less costly. Recall that in the present model, the average level of unemployment is increasing in the amplitude of the cycle. On the other hand, a higher level of external debt increases the welfare cost of pegs. The reason is that the higher is the level of external debt, the larger is the interest obligation created by a given increase in the interest rate. As a result, the higher is the country’s external debt, the larger are the income effects caused by stochastic variations in the country interest rate. Recalling that positive and negative income shocks have asymmetric effects on unemployment, it follows that higher levels of debt induce higher average levels of unemployment. As it turns out, the cost induced by the larger level of external indebtedness in Greece more than offsets the benefits stemming from less volatile external shocks and lower average country interest rates.

Optimal capital controls continue have a large beneficial effects. The welfare cost of a currency peg relative to the first-best allocation is 11 percentage points higher under free capital mobility than under optimal capital controls.

To gain further insight into the relationship between indebtedness and the welfare costs of

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pegs, we recalibrate the subjective discount factor to induce a lower average debt-to-output ratio. Specifically, we raise $\beta$ from 0.9375 to 0.975. This parameterization results in an average debt-to-GDP ratio of 81.3 percent. This value is in line with the average debt-to-GDP observed in Greece since its adoption of the Euro in 2001. In this case, the welfare cost of a currency peg with free capital mobility relative to the first-best allocation is 6.3 percent and the average unemployment rate is 7.5 percent. This result suggests that the welfare costs of currency pegs are larger the more impatient (or debt hungry) are households. At the same time, capital controls continue to be highly effective in easing the pains of pegs, as they lower welfare costs and unemployment to 1.7 and 1.3 percent, respectively.

### 6.3 Higher Intertemporal Elasticity of Substitution

Here, we investigate the sensitivity of our findings to increasing the intertemporal elasticity of substitution, $1/\sigma$. Specifically, we lower $\sigma$ from its baseline value of 5 to 2.27. We pick this value for two reasons. First, it is close to 2, which is a value widely used in the business cycle literature (see Uribe, 2011, and the references cited therein). Second, it is equal to the inverse of our assumed value for the intratemporal elasticity of substitution $1/\xi$. This is a convenient parameterization because it implies that the behavior of external debt and consumption of tradables is identical under a currency peg with free capital mobility and under the optimal exchange-rate policy (or the first-best allocation). To see this, note that when $\sigma = 1/\xi$, the period utility index, $U(A(c^T, c^N))$, becomes additively separable in consumption of tradables and nontradables and equal to $\left( (c^T)^{1-\sigma} + (c^N)^{1-\sigma} - 1 \right) / (1 - \sigma)$. This means that the marginal utility of consumption of tradables becomes independent of consumption of nontradables. It is then straightforward to show that under a currency peg or the optimal exchange rate policy the variables $c^T_t$ and $d_t$ are determined by the solution of the system composed by expressions (9)-(12) and

$$(c^T_t)^{-\sigma} = \beta(1 + r_t)\mathbb{E}_t (c^T_{t+1})^{-\sigma} + \mu_t.$$  

Under this parameterization, the welfare costs of currency pegs relative to the optimal exchange-rate policy are attributable exclusively to the unemployment consequences of pegs.

Raising the intertemporal elasticity of substitution makes households less risk averse and as a result more willing to assume external debt. Holding all parameters other than $\sigma$ constant at their baseline values, the lowering of $\sigma$ results in debt distributions (under both the currency peg regime and the optimal exchange rate regime) that pile up to the left of the natural debt limit. The implied debt-to-output ratios are many times larger than those observed over our calibration period. For this reason, we adjust the value of $\beta$ from
its baseline value of 0.9375 to 0.962 to ensure that together with a value of $\sigma = 2.27$, the currency-peg economy delivers an external debt share in line with that observed over the calibration period (26 percent of annual output).

Table 4 shows that under this alternative calibration the welfare costs of currency pegs with free capital mobility relative to the optimal exchange-rate policy continue to be high with a mean of 8.4 percent of consumption per period. This figure is smaller than its baseline counterpart. This is expected because less risk averse agents are more tolerant to economic fluctuations.

As in the baseline calibration, optimal capital controls bring the economy quite close to the first-best allocation in terms of welfare. Specifically, the welfare cost of a currency peg coupled with optimal capital controls relative to the optimal exchange-rate policy is only 0.6 percent of consumption.

### 6.4 Less Downward Nominal Wage Rigidity

Because in the present model involuntary unemployment is the main source of welfare losses associated with currency pegs the key parameter determining the magnitude of these welfare losses is $\gamma$, which governs the degree of downward nominal wage rigidity. Our baseline calibration ($\gamma = 0.99$) implies that nominal wages can fall frictionlessly up to four percent per year. As argued in section 4, this is a conservative value in the sense that it allows for falls in nominal wages during crises that are larger than those observed either in the 2001 Argentine crisis or the ongoing crisis in peripheral Europe even after correcting for foreign inflation and long-run growth. We now set $\gamma$ to 0.98, which allows for frictionless nominal wage declines of 8 percent per year. Taking into account that the largest wage decline observed in Argentina in 2001 or in the periphery of Europe since the onset of the great recession was 1.6 percent per year (Lithuania, see table 1), it follows that we are considering a degree of wage rigidity substantially lower than those implied by observed wage movements during recent large contractions. To avoid large changes in the distribution of external debt caused by parameter changes, we continue to assume here that $\sigma = 1/\xi = 2.27$. As explained in the previous subsection, this restriction ensures that the equilibrium distribution of external debt is the same under a peg with free capital mobility and in the first-best allocation (or under the optimal exchange-rate policy).

Table 4 shows that the mean welfare cost of a currency peg falls from 8.4 to 6.2 as we lower $\gamma$ from 0.99 to 0.98. This welfare cost is still a large figure compared to existing results in monetary economics. The intuition why currency pegs are less painful when wages are more downwardly flexible is straightforward. A negative aggregate demand shock reduces the
demand for nontradables which requires a fall in the real wage rate to avoid unemployment. Under a currency peg this downward adjustment must be brought about exclusively by a fall in nominal wages. The less downwardly rigid are nominal wages, the faster is the downward adjustment in both the nominal and the real wage and therefore the smaller is the resulting level of unemployment.

As under the baseline value of $\gamma$, the peg economy with optimal capital controls results in welfare levels very close to those achieved by the optimal exchange-rate policy. In this regard, the result that capital controls can go a long way toward alleviating the pains of pegs is robust to allowing for more wage flexibility.

### 6.5 Endogenous Labor Supply

We now relax the assumption of an inelastic labor supply schedule. Specifically, we consider a period-utility specification of the form

$$U(c_t, \ell_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \varphi \frac{\ell_t^{1-\theta} - 1}{1 - \theta},$$

where $\ell_t$ denotes leisure in period $t$, and $\varphi$ and $\theta$ are positive parameters. As in the previous two subsections, we impose the restriction $\sigma = 1/\xi$.

Under this specification, the household’s optimization problem features a new first-order condition of the form

$$\varphi (\ell^v_t)^{-\theta} = w_t \lambda_t,$$

where $\ell^v_t$ denotes the desired (or voluntary) amount of leisure. The above expression is a notional labor supply, in the sense that the household may not be able to work the desired number of hours and therefore may be forced to have more leisure than desired. We assume that households are endowed with $\bar{h}$ hours per period. Let $h^s_t$ denote the number of hours households supply to the market. As before, households may not be able to sell all of the hours they supply to the labor market. Let $h_t$ denote the actual number of hours worked. Then, we impose

$$h^s_t \geq h_t,$$

and

$$(h^s_t - h_t) \left( w_t - \gamma \frac{w_t-1}{\ell_t} \right) = 0.$$

These two expression are the counterparts to conditions (7) and (18) of the baseline economy. All other conditions describing aggregate dynamics are as before.

In the present economy, leisure has two component, voluntary leisure and involuntary
leisure. Voluntary leisure, denoted $\ell_t^v$, is given by the difference between the time endowment and the number of hours the household wishes to work at the market wage, that is,

$$\ell_t^v = \bar{h} - h_t^s.$$  

Involuntary leisure equals involuntary unemployment, which we denote by $u_t$. It is the difference between the number of hours the household would like to work at the going wage and the number of hours the household is actually employed, that is,

$$u_t = h_t^s - h_t.$$  

An important issue is how voluntary and involuntary leisure enter in the utility function. One possibility is to assume that voluntary and involuntary leisure are perfect substitutes. In this case, we have that $\ell_t = \ell_t^v + u_t$. However, there exists an extensive empirical literature suggesting that voluntary and involuntary leisure are far from perfect substitutes. For instance, Krueger and Mueller (2012), using longitudinal data from a survey of unemployed workers in New Jersey find that despite the fact that the unemployed spend relatively more time in leisure-related activities they enjoy these activities to a lesser degree than their employed counterparts and thus, on an average day, report higher levels of sadness than the employed. Similarly, Winkelmann and Winkelmann (1998), using longitudinal data of working-age men in Germany find that, after controlling for individual fixed effects and income, unemployment has a large non-pecuniary detrimental effect on life satisfaction. Another source of non-substitutability between voluntary and involuntary leisure stems from the fact that the unemployed spend more time than the employed looking for work, an activity that they perceive as highly unsatisfying. Krueger and Mueller (2012), for example, report that the unemployed work 391 minutes less per day than the employed but spend 101 minutes more per day on job search. In addition, these authors find that job search generates the highest feeling of sadness after personal care out of 13 time-use categories.

Based on this evidence, it is important to consider specifications in which voluntary and involuntary leisure are imperfect substitutes in utility. Specifically, we model leisure as

$$\ell_t = \ell_t^v + \delta u_t.$$  

The existing literature strongly suggests that $\delta$ is less than unity. However, estimates of this parameter are not available. For this reason, we consider three values of $\delta$, 0.5, 0.75, and 1.

We calibrate the remaining new parameters of the model as follows: we assume that under full employment households spend a third of their time working. We adopt a Frisch
wage elasticity of labor supply of 2, which is on the high end of available empirical estimates from micro and aggregate data (see, for example, Blundell and MaCurdy, 1999; Justiniano, Primiceri, and Tambalotti, 2010; and Smets and Wouters, 2007). Finally, we normalize the number of hours worked under full employment at unity so as to preserve the size of the nontraded sector relative to the traded sector. This calibration strategy yields $\varphi = 1.11$, $\bar{h} = 3$, and $\theta = 1$.

Lines 4.b to 4.d of table 4 show the welfare cost of currency pegs with and without capital controls implied by the present model specification. The welfare cost of a currency peg with free capital mobility depend significantly on the degree of substitutability between voluntary and involuntary leisure. The more substitutable voluntary and involuntary leisure are, the lower are the welfare cost of currency pegs. This result should be expected. Consider the case in which voluntary and involuntary unemployment are perfect substitutes ($\delta = 1$). In this case, pegs reduce welfare because involuntary unemployment reduces the production and hence consumption of nontradable goods. However, unemployment increases leisure one for one and in this way increases utility, greatly offsetting the negative welfare effect of lower nontradable consumption. As $\delta$ falls, the marginal contribution of unemployment to leisure also falls, reducing the offsetting effect of leisure. For a value of $\delta$ of 0.75, for instance, the welfare cost of currency pegs with free capital mobility is 9.3 percent, which is higher than in the case with inelastic labor supply (see line 4 of table 4). If the marginal contribution of involuntary leisure to total leisure is half as large as that of voluntary leisure ($\delta = 0.5$), the welfare cost of currency pegs with free capital mobility increases to 19.0 percent of consumption per year.

Regardless of the precise value assumed for $\delta$, capital controls are highly effective in reducing the welfare costs of pegs. In all cases, the welfare costs of pegs are cut by at least a factor of 7 when the peg is coupled with optimal capital controls.

7 Conclusion

In this paper we present a model-based assessment of the costs of currency pegs in economies with downward nominal wage rigidity. Our theoretical approach is based on a familiar narrative that goes back to Keynes’ (1925) and Friedman’ (1953) reservations against fixed exchange-rate arrangements. We incorporate this narrative into a dynamic stochastic model of the open economy amenable to welfare evaluations.

We focus attention on downward nominal wage rigidity as the central source of monetary nonneutrality. This assumption represents a departure from models in the new Keynesian tradition, which assume that nominal rigidities are symmetric. Our modeling choice is guided
by evidence presented in this paper suggesting that downward nominal wage rigidity is pervasive in emerging countries. We further assume that employment is demand determined during recessions, but that the labor market clears during booms. This assumption represents a second departure of our model from formulations in the new-Keynesian tradition in which employment is always demand determined. As a consequence of this assumption, our model generates a quantitatively important endogenous connection between the amplitude of the business cycle and the average rate of unemployment. This connection opens the door to large welfare gains from optimal stabilization policy.

We show that the combination of downward nominal wage rigidity and a fixed exchange rate gives rise to a negative externality. The nature of the externality is that private absorption expands too much in response to favorable shocks, causing inefficiently large increases in real wages. No problems are manifested in this phase of the cycle. However, as the economy returns to its trend path, wages fail to fall quickly enough because they are downwardly rigid. In addition, the central bank, having its hands tied by the commitment to a fixed exchange rate, cannot deflate the real value of wages via a devaluation. In turn, high real wages and a contracting level of aggregate absorption cause involuntary unemployment. Individual agents are conscious of this mechanism, but are too small to internalize it. The government, on the other hand, does internalize the distortion and therefore has an incentive to intervene.

Using a calibrated version of our model, we find that external crises can be highly contractionary when the exchange rate is fixed and capital flows are unfettered. Specifically, the model predicts that a large external shock, defined as a two-standard-deviation collapse in the terms of trade and a two-standard-deviation increase in the country interest rate, causes an increase in unemployment of more than 20 percent of the labor force. This figure is consistent with the unemployment rates observed in the aftermath of recent large contractions in emerging market economies that followed a fixed exchange rate regime, including Argentina 1998-2001 and the periphery of the European Union post 2008.

Our model predicts that currency pegs with free capital mobility are costly not only during crises but also over regular business-cycle fluctuations. Under plausible calibrations, this policy arrangement leads to average rates of unemployment in excess of 8 percent. We show that the optimal exchange rate policy eliminates unemployment and calls for large devaluations during external crises. In response to large downturns like the one described above, the optimal rate of devaluation can be as high as 50 percent per year for several quarters.

For many economies, especially those that are members of a currency union, breaking away from a fixed exchange-rate arrangement is not an option. For this reason, we charac-
terize the optimal capital control policy, broadly interpreted as regulations of cross-border financial flows, as an alternative way to address the aforementioned negative externality. We show that the optimal capital control policy is prudential in nature. The benevolent government taxes capital inflows in good times and subsidizes external borrowing in bad times. The key role of capital controls is to insulate the domestic absorption of tradable goods from external shocks. In this way, the government avoids that external disturbances spill over into the nontraded sector where they would otherwise cause unemployment. Although they represent a second-best policy, capital controls can go a long way toward restoring full employment in fixed exchange-rate economies. Under our baseline calibration, the average rate of unemployment falls from 13.5 to 3.1 percent when the currency peg is coupled with optimal capital controls. These results suggest that, when labor markets suffer from downward nominal wage rigidity and the exchange rate is fixed, capital controls can be an effective instrument for macroeconomic stabilization. More importantly, the predictions of our model suggest that governments of fixed-exchange-rate economies should concentrate effort not on crisis management, but rather on crisis prevention.
Appendix A: Proof of Proposition 1

Consider an equilibrium, that is, a set of stochastic processes \( \{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, \mu_t\}_{t=0}^{\infty} \) satisfying (9)-(18) and the exchange rate policy (21).

We first show that under this exchange rate policy \( h_t \) must equal \( \bar{h} \) at all times. This part of the proof is by contradiction. Suppose \( h_t < \bar{h} \) for some \( t \geq 0 \). Then, by (18) we have that

\[
w_t = \frac{\gamma w_{t-1}}{\epsilon_t}.
\]

(33)

Solve this expression for \( \epsilon_t \). Then use the resulting expression to eliminate \( \epsilon_t \) from (21) to obtain \( w_t \leq \omega(c_t^T) \). Using (15) to replace \( w_t \) and (20) to replace \( \omega(c_t^T) \), we can rewrite this inequality as

\[
\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \leq \frac{A_2(c_T^T, F(\bar{h}))}{A_1(c_T^T, F(\bar{h}))} F'(\bar{h}).
\]

Because the left-hand side of this expression is strictly decreasing in \( h_t \), we have that \( h_t \) must equal \( \bar{h} \), which is a contradiction. We have therefore shown that under the exchange rate policy given in (21), unemployment is nil at all dates and states.

It remains to be shown that the real allocation associated with the exchange rate policy (21) is Pareto optimal. Evaluate the equilibrium conditions (9)-(14) at \( h_t = \bar{h} \) to obtain

\[
c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t},
\]

\[
d_{t+1} \leq \bar{d}
\]

\[
\mu_t \geq 0,
\]

\[
\mu_t(d_{t+1} - \bar{d}) = 0,
\]

\[
\lambda_t = U'(A(c_t^T, \bar{h}))A_1(c_t^T, \bar{h})
\]

\[
\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t,
\]

which are precisely the first-order necessary and sufficient conditions associated with the social planner’s problem consisting in maximizing (22) subject to (7), (9) and (10). The fact that the first-order conditions of the social planner’s problem are necessary and sufficient follows directly from the strict concavity of the planner’s objective and the convexity of the planner’s constraint set.
Appendix B: Proof of Proposition 2

Profits of firms expressed in terms of tradables are given by \( p_t F(h_t) - (1 - \tau^h_t)w_t h_t \). The optimality condition for profit maximization is \( p_t = \frac{(1 - \tau^h_t)w_t}{F(h_t)} \). Using (5) to eliminate \( p_t \) from this expression yields

\[
\frac{A_2(c^T_t, c^N_t)}{A_1(c^T_t, c^N_t)} F'(h_t) = (1 - \tau^h_t)w_t. \tag{34}
\]

We assume that the subsidy is financed by income taxes at the household level. Thus, the budget constraint of the government becomes \( \tau^h_t w_t h_t = \tau^y_t[y^T_t + w_t h_t + \Phi_t / E_t] \). In equilibrium, this expression implies that

\[
\tau^y_t = -\frac{\tau^h_t w_t h_t}{y^T_t + \frac{A_2(c^T_t, h_t)}{A_1(c^T_t, h_t)} F(h_t)} \tag{35}
\]

Equilibrium under a currency peg (\( \epsilon_t = 1 \)) is then defined as a set of stochastic processes \( \{c^T_t, h_t, w_t, d_{t+1}, \lambda_t, \mu_t, \tau^y_t \}_{t=0}^{\infty} \) satisfying (9)-(14), (17), and (34), (35), and

\[
w_t \geq \gamma w_{t-1} \tag{36}
\]

\[
(h_t - \bar{h}) (w_t - \gamma w_{t-1}) = 0. \tag{37}
\]

Consider now the Pareto optimal allocation \( \{c^{OPT}_t, d^{OPT}_{t+1}, h^{OPT}_t \}_{t=0}^{\infty} \) defined in the body of the paper. To show that this allocation satisfies the equilibrium conditions of the economy with a currency peg and a labor subsidy, it suffices to establish that it satisfies conditions (34)-(37), since all other equilibrium conditions are common to those associated with the social planner’s problem. Because \( h^{OPT}_t = \bar{h} \) for all \( t \), equation (37) is always satisfied. For the same reason, the left-hand side of (34) becomes \( \omega(c^T_t) \). Now if \( \omega(c^T_t) \geq \gamma w_{t-1} \), set \( \tau^h_t = 0 \). Then, by equation (34), \( w_t = \omega(c^T_t) \), which, by assumption, is greater than or equal to \( \gamma w_{t-1} \). Therefore equilibrium condition (36) is satisfied in this case. If on the other hand \( \omega(c^T_t) < \gamma w_{t-1} \), then set \( 1 - \tau^h_t = \frac{\omega(c^T_t)}{\gamma w_{t-1}} \). In this case, equation equation (34) implies that \( w_t = \gamma w_{t-1} \), which satisfies condition (36). It follows that the proposed tax process supports the Pareto optimal allocation as a competitive equilibrium.

Appendix C: Proof of Proposition 4

Allocation Under Free Capital Mobility

From period 1 onward, the economy faces a constant interest rate forever. Therefore, all endogenous variables are constant over time. The resource constraints in periods 0 and 1...
are then given by
\[ c_0^T = y^T + d_1/(1 + r) \]
and
\[ c_1^T + d_1 = y^T + d_1/(1 + r), \]
respectively. The second equation uses the fact that \( d_2 = d_1 \). The Euler equation in period 0 is
\[ c_1^T = \beta(1 + r)c_0^T. \]
This is a system of three equations in three unknowns, \( c_0^T, c_1^T, \) and \( d_1 \). Solving this system, we obtain
\[ c_0^T = y^T \left[ \frac{1}{1 + r} + \frac{r}{1 + r} \right] \]
\[ c_1^T = y^T \left[ \frac{1}{1 + r} + \frac{r}{1 + r} \frac{1 + r}{1 + r} \right], \]
and
\[ d_1 = y^T \left[ 1 - \frac{1 + r}{1 + r} \right]. \]
Notice that if \( r_\infty = r \), then \( c_1^T = y^T \) and \( d_1 = 0 \). However, because \( r_\infty < r \), the economy experiences a boom in consumption in period 0. This boom is financed with external debt, \( d_1 > 0 \). From period 1 onward, traded consumption must fall because the economy needs to generate resources to service the external debt. The contraction in the absorption of tradables that takes place after period 0 causes unemployment in the nontraded sector. By equation (15), the real wage in period 0 is given by \( \alpha c_0^T > w_{-1} \), indicating that capital inflows in period 0 cause an increase in the real wage. This elevation in real wages puts the economy in a vulnerable situation in period 1, when the interest rate goes back up to its permanent level \( r \). The full-employment real wage in period 1 is \( \alpha c_1^T \), which is lower than \( w_0 (= \alpha c_1^T) \), because \( c_1^T < c_0^T \). As a result, unemployment emerges and is equal to
\[ 1 - h_t = 1 - \frac{1 + r}{1 + r} > 0, \]
for \( t \geq 1 \). Notice that the larger the decline in the interest rate in period 0, the larger is the unemployment rate in periods \( t \geq 1 \). This level of involuntary unemployment will persist forever, unless the policy authority does something to lower it.

**Allocation Under Ramsey Optimal Capital Controls**

In line with the approach adopted in section 3.6, the optimal capital control policy is the solution to a social planner problem in which the government picks the level of traded
consumption in period 0, $c^T_0$, to maximize the welfare of the representative agent, taking into
account the effect of movements in traded consumption on unemployment in the nontraded
sector. Intuitively, the government wants to curb consumption of tradables in period 0 to
dampen the rise in real wages. Then, capital controls are chosen so as to be consistent with
the desired path of consumption.

The solution of the Ramsey problem is $c^*_t = y^T$ and $h_t = 1$ for all $t \geq 0$. The associated
level of lifetime welfare is
\[
\frac{1}{1 - \beta} \ln y^T.
\]

To see this, consider first a solution in which $c^T_0 > y^T$. In this case, $d_1 > 0$ and therefore
$c^T_t < y^T$ for all $t \geq 1$. In period 0, the full-employment wage is $\alpha c^T_0 > w_{-1}$. It follows that
$h_0 = 1$ and $w_0 = \alpha c^T_0$. In period 1, the full-employment wage rate is $\alpha c^T_1$, which is less than
$w_0$. As a result, we have that the lower bound on wages binds, $w_1 = w_0$. Equation (15) then
implies that $h_t = c^*_t / c^T_0 < 1$ for all $t \geq 1$. Lifetime utility is then given by
\[
\frac{1 - \beta(1 + \alpha)}{1 - \beta} \ln c^*_0 + \frac{\beta(1 + \alpha)}{1 - \beta} \ln c^*_1.
\]

Assuming that $\alpha > r$, we have that welfare at $c^*_0 = y^T$ is larger than welfare at any $c^T_0 > y^T$.

Now we wish to show that the proposed solution to the Ramsey problem dominates any
other one in which $c^T_0 < y^T$. If $c^T_0 < y^T$, then $d_1 < 0$, and therefore $c^*_t > y^T$, for all $t \geq 1$. By
combining the sequential budgets constraints in periods 0 and 1, given by $c^*_0 = y^T + d_1 / (1 + r)$
and $c^*_1 = y^T - \frac{r}{1 + r} d_1$, we obtain that $c^*_1 = \left[1 + r \frac{1 + \alpha}{1 + r}\right] y^T - \frac{r(1 + \alpha)}{(1 + r)} c^*_0$. The full-employment real
wage in period 0 is $\alpha c^*_0 < w_{-1}$, which implies the existence of involuntary unemployment in
period 0, $h_0 = c^*_0 / y^T < 1$. By a similar logic, there is full employment starting in period 1, $h_t = 1$ for $t \geq 1$. Lifetime welfare is then given by
\[
(1 + \alpha) \ln c^*_0 + \frac{\beta}{1 - \beta} \ln \left[\left(1 + r \frac{1 + \alpha}{1 + r}\right) y^T - \frac{r(1 + \alpha)}{(1 + r)} c^*_0\right] - \alpha \ln y^T
\]

Notice that this expression reduces to the lifetime utility level under the proposed solution
when $c^*_0 = y^T$. Moreover, the derivative of the above expression with respect to $c^*_0$ is positive
for any $c^*_0 \leq y^T$. This shows that the proposed solution dominates one in which $c^*_0 < y^T$.

Finally, the capital control policy that supports the Ramsey equilibrium can be read off
the household’s Euler equation for consumption of tradables evaluated at $c^*_0 = c^*_1 = y^T$, which yields
\[
\tau^d_0 = 1 - \frac{1 + r}{1 + r} > 0
\]
for \( t = 0 \) and \( \tau_t^d = 0 \)

for \( t \geq 1 \).

**Appendix D: Numerical Algorithm for Approximating the Aggregate Dynamics Under a Currency Peg**

Define the discretized state as follows:

\[ Y_T = \{y_{T1}, y_{T2}, \ldots, y_{Tny}\} \]

\[ R = \{r_1, r_2, \ldots, r_{nr}\} \]

\[ D = \{d_1, d_2, \ldots, d_{nd}\} \]

\[ W = \{w_1, w_2, \ldots, w_{nw}\} \]

In iteration \( n \), suppose the guess for the solution for the marginal utility of tradable goods is given by the function \( \Lambda^n \), mapping \( Y_T \times R \times D \times W \) into \( \mathbb{R} \). To obtain the next guess \( \Lambda^{n+1} \), proceed as follows:

1. For a given state \((y^T_i, r_j, d_k, w_\ell)\) with \( i \in \{1, \ldots, ny\} \), \( j \in \{1, \ldots, nr\} \), \( k \in \{1, \ldots, nd\} \), \( \ell \in \{1, \ldots, nw\} \), denote the level of debt due next period by \( d_s \) for \( s \in \{1, \ldots, nd\} \).

2. Use condition (9) to find the corresponding level of \( c_T \) as

\[ c_T^T(d_s) = y^T_i + \frac{d_s}{1 + r_j} - d_k. \]

and use condition (20) to determine \( \omega(c_T^T(d_s)) \)

\[ \omega(c_T^T(d_s)) = \frac{A_2(c_T^T(d_s), F(\bar{h}))}{A_1(c_T^T(d_s), F(\bar{h}))} F'(\bar{h}). \]

3. If the full-employment wage violates constraint (16) for \( \epsilon_t = 1 \), then the current wage must be equal to \( \gamma w_\ell \). Therefore, we have that the current wage is given by

\[ w' = \max \{\gamma w_\ell, \omega(c_T^T(d_s))\}. \]

Pick the current wage, which will be a state variable for the next period, so that the wage
rate takes on one of the values in the set $W$. Formally, we have that

$$qq = \arg\min_{q \in \{1, \ldots, nw\}} |w_q - w'|$$

and then denote the current wage choice given $d_s$ as $w_{qq}$.

(4) To find the level of employment associated with $d_s$, note that if $\omega(c^T(d_s)) \geq \gamma w_\ell$, then $h(d_s) = \bar{h}$, else $h(d_s)$ solves

$$\gamma w_\ell = \frac{A_2(c^T(d_s), F(h(d_s)))}{A_1(c^T(d_s), F(h(d_s))))} F'(h(d_s)).$$

(5) Find the level of nontraded consumption as

$$c^N(d_s) = F(h(d_s)),$$

the level of consumption of the aggregate good from equation (2) as

$$c(d_s) = A(c^T(d_s), c^N(d_s)),$$

and the current value of the marginal utility of consumption of tradables from (13) as

$$\lambda(d_s) = U'(c(d_s)) A_1(c^T(d_s), c^N(d_s))$$

(6) Use equation (14) to construct $\mu$ as

$$\mu(d_s) = \frac{\lambda(d_s)}{1 + r_j} - \beta \sum_{i=1}^{ny} \sum_{j=1}^{nr} \text{Prob}(y_{ii}^T, r_{jj} | y_i^T, r_j) \Lambda^n(y_{ii}^T, r_{jj}, d_s, w_{qq}).$$

If $\mu \geq 0$ and $s = nd$, then $d_s$ is the optimal choice of debt in the current period and $s^* = nd$. In this case:

$$\Lambda^{n+1} = U'(c(d_{nd})) A_1(c^T(d_{nd}), c^N(d_{nd})).$$

Else construct $\mu$ for all $s \in \{1, \ldots, nd-1\}$. Find the optimal $s$ as $s^* = \arg\min_{s \in \{1, \ldots, nd-1\}} |\mu(d_s)|$.

Construct

$$\Lambda^{n+1}(y_i^T, r_j, d_k, w_\ell) = U'(c(d_{s^*})) A_1(c^T(d_{s^*}), c^N(d_{s^*})).$$

(7) Keep iterating in this way, until the maximum distance (taken over the $ny \times nr \times nd \times nw$ states between $\Lambda^{n+1}$ and $\Lambda^n$ is less than $1e - 8$. 

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Appendix E: Data Description For Greece

In this appendix, we report the estimate of the exogenous driving process \( y_t^T r_t \)' for the case of Greece. We also describe how the empirical measures of \( y_t^T \) and \( r_t \) were constructed.

The estimation uses quarterly data from 1981:Q1 to 2011:Q3. Greece did not produce sectoral GDP data between 1991 and 1999. For this reason, we proxy traded output by an index of industrial production. Specifically, we use the index of total manufacturing production 2005=100 from the OECD seasonally adjusted at the source. The original series begins in 1955:Q1 and ends in 2011:Q3. We removed a cubic trend from the natural logarithm of the index over the period 1955:Q1 to 2011:Q3. We use observations of the detrended series for the period 1981:Q1 to 2011:Q3 to make the range compatible with the one corresponding to the country real interest rate.

We measure the real interest rate in terms of tradables using the formula

\[
1 + r_t = (1 + i_t) E_t \left[ \frac{E_t P_{t}^T}{E_{t+1} P_{t+1}^T} \right],
\]

where \( r_t \) denotes the real country interest rate in terms of tradables, \( i_t \) denotes the nominal interest rate in terms of national currency, \( E_t \) denotes the nominal exchange rate defined as units of domestic currency per unit of ECU or Euro as applicable (Greece’s legal tender changed to the Euro in 2001), \( P_{t}^T \) denotes the foreign-currency price of tradables, and \( E_t \) denotes the expectations operator conditional on information available in period \( t \). This formula assumes that the marginal rate of substitution is uncorrelated with the inverse of the domestic rate of inflation of tradable goods. The source for \( E_t \) is Eurostat (code ert_h_eur_q). We measure \( P_{t}^T \) by the German consumer price index published by the OECD. We measure \( i_t \) as follows. For the period 1981:Q1 to 1992:Q3 it is the overnight interest rate published by the Bank of Greece. For the period 2001:Q1 to 2011:Q3 we proxy \( i_t \) by the interest rate on 10-year Greek treasury bonds published by Eurostat (code irt_lt_mcby_q). For the period 1992:Q4 to 2000:Q4, we measure \( i_t \) as the average of the above two interest rates. We proxy \( E_t \left[ \frac{E_{t+1} P_{t+1}^T}{E_t P_{t}^T} \right] \) by the one-period ahead forecast of \( \frac{E_{t+1} P_{t}^T}{E_t P_{t+1}^T} \) implied by an estimated AR(2) process for this variable.

The estimates of \( A, \Sigma_\nu, \) and \( r \) defining the exogenous bivariate first-order autoregressive process given in equation (29) are

\[
A = \begin{bmatrix} 0.88 & -0.42 \\ -0.05 & 0.59 \end{bmatrix}; \quad \Sigma_\nu = \begin{bmatrix} 0.000536 & -0.000010 \\ -0.000010 & 0.000060 \end{bmatrix}; \quad r = 0.011.
\]

We discretize the driving process following the same procedure described in the body of
the paper for the case of Argentina.
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