A Model of the Twin Ds:
Optimal Default and Devaluation

by

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Motivation

There is a strong empirical link between sovereign default and large devaluations

- Reinhart (2002) examines data for 58 countries over the period 1970 to 1999 and finds that:

  - The unconditional probability of a large devaluation in any 24-month period is 17%.
  
  — The probability of a large devaluation conditional on the 24-month period containing a default is 84%.

- Reinhart refers to this phenomenon as the Twin Ds.
Median of cumulative devaluations conditional on default in year 0 minus unconditional median. Sample contains 116 default episodes between 1975 and 2013 in 70 countries. Source: Default dates, Uribe and Schmitt-Grohé (2014), chapter 11. Exchange rates, WDI.
Argentina 1996-2006


Vertical Line 2002, default and devaluation.
This Paper

- Develop a model that explains the Twin Ds phenomenon as an optimal policy outcome.

Main Elements

- Imperfect enforcement of debt contracts
- Downward nominal wage rigidity.

Intuition

- Under the optimal policy, default occurs during large recessions.
- A contracting demand for labor puts downward pressure on real wages.
- A large devaluation reduces the real value of wages, thereby preventing massive unemployment.
- The devaluation is necessary to bring real wages down because nominal wages are downwardly rigid.
Analytical Findings

Two Decentralizations of the Eaton-Gersovitz Model

• **Decentralization I:** Real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be decentralized via optimal capital controls.

• **Decentralization II:** Real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be interpreted as the centralized version of economies with default risk, downward nominal wage rigidity, optimal capital controls, and optimal devaluation policy.
Quantitative Findings

- The optimal policy features large devaluations of 40% around default events, which is in line with the data. That is, the Twin Ds emerge endogenously as an optimal policy outcome.

- Under currency pegs, default episodes are accompanied by massive involuntary unemployment of around 20%.

- Due to larger default incentives, currency-peg economies can support less external debt than optimally floating economies (20% versus 60% of traded GDP).
The Model: The Household’s Problem

\[
\begin{align*}
\max_{\{c_t^T, c_t^N, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \\
\text{subject to} \\
\quad c_t = A(c_t^T, c_t^N) \\
\quad c_t^T + p_t c_t^N + d_t = \tilde{y}_t^T + w_t h_t + (1 - \tau_t d) q_t d_{t+1} \\
\quad h_t \leq \bar{h}
\end{align*}
\]

Households take \( h_t \) as given. All debt is denominated in foreign currency (original sin).

The Demand For Nontradables

\[
\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t
\]
The Demand For Nontradables

\[
\frac{A_2(c_T^1, c^N)}{A_1(c_T^1, c^N)} > \frac{A_2(c_T^0, c^N)}{A_1(c_T^0, c^N)}
\]

A Contraction in Traded Absorption, \( c_T^1 \downarrow \), Shifts the Demand for Nontradables Down and to the Left

\[ c_T^1 < c_T^0 \]
Firms

$$\max \left\{ p_t F(h_t) - w_th_t \right\}$$

$$w_t \equiv \frac{W_t}{P_T^T}$$ real wage in terms of tradables.

$$W_t$$ nominal wage.

$$P_T^T$$ nominal price of tradable goods.

$$P_T^T = \mathcal{E}_t P_T^{T*}$$, law of one price.

$$\mathcal{E}_t$$ nominal exchange rate (pesos per dollar).

$$P_T^{T*}$$ foreign-currency (dollar) price of tradables, normalized to unity, $$P_T^{T*} = 1$$.

$$\Rightarrow P_T^T = \mathcal{E}_t$$.

Profit Maximization Condition

$$p_t = \frac{W_t/\mathcal{E}_t}{F'(h_t)}$$
The Supply of Nontraded Goods

\[ \frac{W_0}{\varepsilon_0} \frac{1}{F'(h)} \]

\[ \frac{W_0}{\varepsilon_1} \frac{1}{F'(h)} \]

\((\varepsilon_1 > \varepsilon_0)\)
Downward Nominal Wage Rigidity

\[ W_t \geq \gamma W_{t-1} \]

\( \gamma = \) degree of downward wage rigidity.

\( \gamma = 0 \Rightarrow \) fully flexible wages.

Think of \( \gamma \) as being around 1. Schmitt-Grohé and Uribe (2013) estimate \( \gamma = 0.99 \) at quarterly frequency.
Optimal Exchange-Rate Policy

\[
\frac{A_2(c^T_0, F(h))}{A_1(c^T_0, F(h))} \quad \frac{A_2(c^T_1, F(h))}{A_1(c^T_1, F(h))}
\]

\[
\frac{W_0}{W_0/\varepsilon_0} \quad \frac{W_0/\varepsilon_1}{F'(h)}
\]

\(c^T_1 < c^T_0\)

\(\varepsilon_1 > \varepsilon_0\) (optimal devaluation)
The Government

• Each period $t$, the government can be either in good financial standing or in bad financial standing.

• If the government is in good financial standing, it can choose to either honor its debt (indicated by $I_t = 1$) or default. If it defaults, it immediately acquires bad financial standing (indicated by $I_t = 0$).

• If the government is in bad financial standing in $t$, then it regains good financial standing in $t+1$ with exogenous probability $\theta$, and maintains bad standing with probability $1 - \theta$. 
Two Exogenous Cost of Default II

(1) Financial Exclusion: While the country is in bad financial standing \((I_t = 0)\), it cannot participate in international credit markets,

\[
(1 - I_t)d_{t+1} = 0.
\]

(2) Output Loss: The endowment received by households, is given by

\[
\begin{cases} 
  y^T_t & \text{if } I_t = 1 \text{ (good standing)} \\
  y^T_t - L(y^T_t) & \text{if } I_t = 0 \text{ (bad standing)}
\end{cases}
\]

where \(L(\cdot)\) is a nondecreasing (loss) function and \(y^T_t\) is an exogenous stochastic process.
A Typical Default Episode Under Optimal Policy
A Typical Default Episode Under A Currency Peg

- Tradable Endowment, $y_t^T$
- Consumption of Tradables, $c_t^T$
- Debt, $d_t$
- Unemployment Rate, $1-h_t$
- Real Wage, $w_t$
- Relative Price of Nontradables, $p_t$
- Risk premium, $r-r^*$
- Capital Control Tax, $\tau_t^d$

---

peg  ---  optimal devaluation
Peggers can support less external debt

Note. Debt distributions are conditional on being in good financial standing.
Conclusions

• This paper shows that real economies with default risk à la Eaton-Gersovitz can be interpreted as the centralized version of economies with default risk, downward nominal wage rigidity, optimal capital controls, and optimal devaluation policy.

• The model developed in this paper predicts that under the optimal policy defaults are accompanied by large devaluations. Hence the Twin Ds phenomenon emerges as an optimal outcome.

• In the model, the central role of devaluations around default episodes is to fend off unemployment.

• Under currency pegs, defaults are predicted to be accompanied by massive unemployment.

• Fixed-exchange-rate economies are shown to support much less debt in equilibrium than optimal float economies.
Extras
Functional Forms and Calibration

- Time unit: one quarter
- Probability of reentry: $\theta = 0.0385$
- $U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$; $\sigma = 2$
- $A(c^T, c^N) = \left[ a(c^T)^{1-\frac{1}{\xi}} + (1 - a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\xi}}$; $\xi = \frac{1}{2}$; $a = 0.26$
- $y^N_t = h^\alpha_t$; $\alpha = 0.75$
- Output loss function: $L(y^T_t) = \max\{0, \delta_1 y^T_t + \delta_2 (y^T_t)^2\}$
- Set $\beta = 0.85$, $\delta_1 = -0.35$, and $\delta_2 = 0.44$ to ensure:
  (a) $E(d_t/y^T_t) = 60\%$,
  (b) Prob of default equal to 2.6 per century, and
  (c) Average output loss in autarky of 7%.
- Set $\gamma = 0.99 \Rightarrow$ wages can fall by up to 4% per year.
- $y^T_t = 0.93 y^T_{t-1} + 0.037 \mu_t$; $\mu \sim N(0, 1)$ (Argentina, 1983:1-2001:4)
The Twin Ds: Six Examples

Note: Exchange rates are nominal dollar exchange rates, annual average, first observation normalized to unity. Data sources: Default dates from Moody’s (2011) and Standard & Poor’s (2006). Nominal exchange rates from World Development Indicators.
### Average Debt And Default Probability Across Devaluation Policies

<table>
<thead>
<tr>
<th></th>
<th>Optimal Devaluation Policy</th>
<th>Currency Peg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt-to-traded-output ratio (qtr)</td>
<td>60%</td>
<td>20%</td>
</tr>
<tr>
<td>Number of Defaults per century</td>
<td>2.6</td>
<td>2.0</td>
</tr>
</tbody>
</table>
### Data and Model Predictions: Optimal Devaluation Policy

<table>
<thead>
<tr>
<th></th>
<th>$E(r - r^*)$</th>
<th>$\sigma(r - r^*)$</th>
<th>$\text{corr}(r - r^*, y)$</th>
<th>$\text{corr}(r - r^*, tb/y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>7.4</td>
<td>2.9</td>
<td>-0.64</td>
<td>0.72</td>
</tr>
<tr>
<td>Model</td>
<td>3.5</td>
<td>3.2</td>
<td>-0.54</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2013. In the theoretical model, all moments are conditional on the country being in good financial standing.
### Business-Cycle Statistics:

**Data and Model Predictions Under Optimal Exchange Rate Policy**

<table>
<thead>
<tr>
<th></th>
<th>$\sigma(c)/\sigma(y)$</th>
<th>$\sigma(tb/y)/\sigma(y)$</th>
<th>corr($c, y$)</th>
<th>corr($tb/y, y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emerging Countries</td>
<td>1.23</td>
<td>0.69</td>
<td>0.72</td>
<td>-0.51</td>
</tr>
<tr>
<td>Argentina</td>
<td>1.11</td>
<td>0.48</td>
<td>0.75</td>
<td>-0.87</td>
</tr>
<tr>
<td>Model</td>
<td>1.22</td>
<td>0.57</td>
<td>0.88</td>
<td>-0.14</td>
</tr>
</tbody>
</table>
Do Countries Default in Bad Times?

Output Around Default Episodes

Consumption, Investment, The Trade Balance, and The Real Exchange Rate Around Default Episodes

Is there wage restraint during booms?

Example: Periphery of Europe during the 2000-2008 Boom
Boom-Bust Cycle in Peripherical Europe: 2000-2011

Data Source: Eurostat. Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia

⇒ Wages grew by 70 percent between 2000 and 2008!
Let’s take a closer look at Spain and Ireland ...
Nominal hourly wages in Spain increase by 44 percent during the 2000-2008 boom
Nominal hourly wages in Ireland increase by 57 percent during the 2000-2008 boom
... Despite No Growth in Total Factor Productivity
Total Factor Productivity: 2000-2007
(value added based), Index (1995=100)

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>96.2</td>
<td>95.6</td>
<td>94.8</td>
<td>94.1</td>
<td>93.4</td>
<td>92.4</td>
<td>91.9</td>
<td>92.1</td>
</tr>
<tr>
<td>Ireland</td>
<td>109.0</td>
<td>111.2</td>
<td>112.6</td>
<td>110.5</td>
<td>110.9</td>
<td>108.6</td>
<td>106.4</td>
<td>107.8</td>
</tr>
</tbody>
</table>

Source: EU KLEMS Growth and Productivity Accounts. This database includes measures of output and input growth, and derived variables such as multifactor productivity at the industry level. The input measures include various categories of capital (K), labour (L), energy (E), material (M) and service inputs (S). The measures are developed for 25 individual EU member states, the US and Japan and cover the period from 1970 to 2007. The variables are organised around the growth accounting methodology, a major advantage of which is that it is rooted in neo-classical production theory. It provides a clear conceptual framework within which the interaction between variables can be analysed in an internally consistent way. The data series are publicly available on http://www.euklems.net. November 2009 release.
Evidence On Downward Nominal Wage Rigidity

- Downward nominal wage rigidity is the central friction in the present model ⇒ natural to ask if it is empirically relevant.

- Downward wage rigidity is a widespread phenomenon:
  - Evident in micro and macro data.
  - Rich, emerging, and poor countries.
  - Developed and underdeveloped regions of the world.

- Byproduct: Will obtain an estimate of the parameter $\gamma$ governing wage stickiness in the model (useful for quantitative analysis).
Downward Nominal Wage Rigidity: Evidence From Micro Data
Probability of Decline, Increase, or No Change in Wages

U.S. data, SIPP panel 1986-1993, between interviews one year apart.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decline</td>
<td>5.1%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Constant</td>
<td>53.7%</td>
<td>49.2%</td>
</tr>
<tr>
<td>Increase</td>
<td>41.2%</td>
<td>46.5%</td>
</tr>
</tbody>
</table>

Source: Gottschalk (2005)

- Large mass at ‘Constant’ suggests nominal wage rigidity.
- Small mass at ’Decline’ suggests downward nominal wage rigidity.
Distribution of Non-Zero Nominal Wage Changes
United States 1996-1999

Source: Barattieri, Basu, and Gottschalk (2012)
Evidence From The Great Contraction Of 2007
Distribution of Nominal Wage Changes, U.S. 2011

Figure 2
Distribution of observed nominal wage changes

Source: Daly, Hobijn, and Lucking (2012).
Micro Evidence On Downward Nominal Wage Rigidity From Other Developed Countries

- Switzerland: Fehr and Goette (2005).
Evidence From Informal Labor Markets

• Kaur (2012) examines the behavior of nominal wages, employment, and rainfall in casual daily agricultural labor markets in rural India (500 districts from 1956 to 2008).

• Finds asymmetric nominal wage adjustment:
  — $W_t$ increases in response to positive rainfall shocks
  — $W_t$ fails to fall, labor rationing, and unemployment are observed in response to negative rain shocks.

• Inflation (uncorrelated with local rain shocks) tends to moderate rationing and unemployment during negative rain shocks, suggesting downward rigidity in nominal rather than real wages.
Evidence From the Great Depression, 1929-1933

- Enormous contraction in employment: 31% between 1929 and 1931.

- Nonetheless, during this period nominal wages fell by 0.6% per year, while consumer prices fell by 6.6% per year. See the figure on the next slide.

- A similar pattern is observed during the second half of the Depression. By 1933, real wages were 26% higher than in 1929, in spite of a highly distressed labor market.
Nominal Wage Rate and Consumer Prices, United States
1923:1-1935:7

Evidence From the Great Depression In Europe

• Countries that left the gold standard earlier recovered faster than countries that remained on gold.

— Left Gold Early (sterlingbloc): United Kingdom, Sweden, Finland, Norway, and Denmark.

— Countries That Stuck To Gold (gold bloc): France, Belgium, the Netherlands, and Italy.

• Think of the gold standard as a currency peg (a peg not to a currency, but to gold).

• When sterling-bloc left gold, they effectively devalued, as their currencies lost value against gold.

• Look at the figure on the next slide. Between 1929 and 1935, sterling-bloc countries experienced less real wage growth and larger increases in industrial production than gold-bloc countries.
Changes In Real Wages and Industrial Production, 1929-1935

Evidence From Emerging Countries

• Argentina: pegged the peso at a 1-to-1 rate to the dollar between 1991 and 2001.

• Starting in 1998, the economy was buffeted by a number of large negative shocks (weak commodity prices, large devaluation in Brazil, large increase in country premium, etc.).

• Not surprisingly, between 1998 and 2001, unemployment rose sharply.

• Nonetheless, nominal wages remained remarkably flat.

• This evidence suggests that nominal wages are downwardly rigid, and that $\gamma$ is about 1.

• Why $\gamma \approx 1$? The slackness condition $(\bar{h} - h_t)(W_t - \gamma W_{t-1})$ (recall $\epsilon_t = 1$ during this period), implies that if unemployment is growing, wages must grow at the gross rate $\gamma$. 
Evidence From Peripheral Europe (2008-2011)

- Look at the table on the next slide.

- Between 2008 and 2011, all countries in the periphery of Europe experienced increases in unemployment. Some very large increases.

- In spite of this context of extreme duress, nominal hourly wages experienced significant increases in most countries and modest declines in very few.

- The slide following the table explains how to use the information in the table to infer a range for $\gamma$. 
Unemployment, Nominal Wages, and $\gamma$
Evidence from the Eurozone

<table>
<thead>
<tr>
<th>Country</th>
<th>Unemployment Rate 2008Q1 (in percent)</th>
<th>Unemployment Rate 2011Q2 (in percent)</th>
<th>Wage Growth $W_{2011Q2}/W_{2008Q1}$ (in percent)</th>
<th>Implied Value of $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulgaria</td>
<td>6.1</td>
<td>11.3</td>
<td>43.3</td>
<td>1.028</td>
</tr>
<tr>
<td>Cyprus</td>
<td>3.8</td>
<td>6.9</td>
<td>10.7</td>
<td>1.008</td>
</tr>
<tr>
<td>Estonia</td>
<td>4.1</td>
<td>12.8</td>
<td>2.5</td>
<td>1.002</td>
</tr>
<tr>
<td>Greece</td>
<td>7.8</td>
<td>16.7</td>
<td>-2.3</td>
<td>0.9982</td>
</tr>
<tr>
<td>Ireland</td>
<td>4.9</td>
<td>14.3</td>
<td>0.5</td>
<td>1.0004</td>
</tr>
<tr>
<td>Italy</td>
<td>6.4</td>
<td>8.2</td>
<td>10.0</td>
<td>1.007</td>
</tr>
<tr>
<td>Lithuania</td>
<td>4.1</td>
<td>15.6</td>
<td>-5.1</td>
<td>0.996</td>
</tr>
<tr>
<td>Latvia</td>
<td>6.1</td>
<td>16.2</td>
<td>-0.6</td>
<td>0.9995</td>
</tr>
<tr>
<td>Portugal</td>
<td>8.3</td>
<td>12.5</td>
<td>1.91</td>
<td>1.001</td>
</tr>
<tr>
<td>Spain</td>
<td>9.2</td>
<td>20.8</td>
<td>8.0</td>
<td>1.006</td>
</tr>
<tr>
<td>Slovenia</td>
<td>4.7</td>
<td>7.9</td>
<td>12.5</td>
<td>1.009</td>
</tr>
<tr>
<td>Slovakia</td>
<td>10.2</td>
<td>13.3</td>
<td>13.4</td>
<td>1.010</td>
</tr>
</tbody>
</table>

How To Infer $\gamma$

The model implies that if unemployment increases from one period to the next, then nominal wages must be growing at the rate $\gamma$.

How to calculate $\gamma$:

$$\gamma = \left( \frac{W_{2011:Q2}}{W_{2008:Q1}} \right)^{\frac{1}{13}}$$

Subtract 0.6% per quarter to adjust for foreign inflation and long-run growth (because they are not explicitly incorporated in the model) to obtain the estimate:

$$\gamma \in [0.99, 1.022]$$