
STEPHANIE SCHMITT-GROHÉ
MARTÍN URIBE

Foreign Demand for Domestic Currency and the Optimal Rate of Inflation

We characterize the Ramsey optimal rate of inflation in a model with a foreign demand for domestic currency. In the absence of such demand, the model implies that the Friedman rule—deflation at the real rate of interest—is optimal. We show analytically that in the presence of a foreign demand for domestic currency, this result breaks down. Calibrated versions of the model deliver optimal annual rates of inflation between 2% and 10%. The domestically benevolent government imposes an inflation tax to extract resources from the rest of the world in the form of seignorage revenue.

JEL codes: E5, E41

Keywords: foreign demand for money, optimal rate of inflation, Ramsey equilibrium, Friedman rule.

MORE THAN HALF OF U.S. currency circulates abroad. Porter and Judson (1996), for instance, estimate that at the end of 1995, \$200–\$250 billion of the \$375 billion of U.S. currency in circulation outside of banks was held abroad. The foreign demand for U.S. currency has remained strong across time. The 2006 Treasury, Federal Reserve, and Secret Service report on the use of U.S. currency abroad estimates that as of December 2005, about \$450 billion of the \$760 billion of circulated U.S. banknotes are held in other countries. Goldberg (2010) updates this figure and concludes that foreign holdings of U.S. currency had reached \$580 billion in March 2009. The estimated size of the foreign demand for U.S. currency suggests that the majority of the seignorage income of the United States is generated outside of its borders.

A second currency enjoying a strong foreign demand is the euro. A report by the European Central Bank (ECB, 2011) estimates that in 2010 about 25% of euro

We would like to thank an anonymous referee for comments.

STEPHANIE SCHMITT-GROHÉ is a Professor of Economics, Department of Economics, Columbia University, CEPR, and NBER (E-mail: ss3501@columbia.edu). MARTÍN URIBE is a Professor of Economics, Department of Economics, Columbia University and NBER (E-mail: martin.uribe@columbia.edu).

Received November 3, 2009; and accepted in revised form December 7, 2011.

Journal of Money, Credit and Banking, Vol. 44, No. 6 (September 2012)

© 2012 The Ohio State University

currency (or 205 billion euro) was held outside of the euro zone. One piece of circumstantial evidence pointing at the relevance that the ECB assigns to seigniorage income generated by foreign holdings of the euro is its decision to issue large-denomination bills (200 and 500 euros). This decision has been interpreted by academic researchers as an indication of the ECB's desire to extract seigniorage revenue from the underground economy as well as from foreign holders of euros (especially in the former Soviet block and the Middle East) (see, e.g., Rogoff 1998).

A natural question is therefore whether a country's optimal rate of inflation is influenced by the presence of a foreign demand for its currency. In this paper, we address this issue within the context of a dynamic Ramsey problem. We show that the mere existence of a foreign demand for domestic money can, under plausible parameterizations, justify sizable deviations from the rate of inflation associated with the Friedman rule. The basic intuition behind this finding is that adherence to the negative rate of inflation associated with the Friedman rule would represent a welfare-decreasing transfer of real resources by the domestic economy to the rest of the world, as nominal money balances held abroad increase in real terms at the rate of deflation. A benevolent government weighs this cost against the benefit of keeping the opportunity cost of holding money low to reduce transactions costs for domestic agents. Our analytical results show that this trade-off is resolved in favor of deviating from the Friedman rule. Our quantitative analysis suggests that for plausible calibrations that capture the range of estimates of the size of the foreign demand for U.S. currency, the optimal rate of inflation lies between 2% and 10% per year.

The reason why the Ramsey government finds it optimal to collect seigniorage revenues from the rest of the world is not the fact that such revenues allow the fiscal authority to lower distortionary taxes. Rather, it is the fact that the imposition of an inflation tax allows the domestic government to engineer an indirect transfer of real resources from foreign consumers to domestic consumers. We highlight this incentive by establishing that in the presence of a foreign demand for domestic currency, the Friedman rule is suboptimal even when the domestic government has access to lump-sum taxation.

The rest of the paper is organized as follows. Section 1 presents a dynamic monetary model with a foreign demand for domestic currency. Section 2 establishes that the Friedman rule is optimal in the absence of a foreign demand for domestic currency and that it fails to be optimal in the presence of such demand. Section 3 provides the estimates of the optimal rate of inflation in the context of a calibrated version of the model. Section 4 demonstrates that deviations from the Friedman rule are optimal even when the domestic government has access to lump-sum taxation. Section 5 concludes.

1. THE MODEL

Consider an economy populated by a large number of identical households. Each household has preferences defined over sequences of consumption and leisure and

described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \quad (1)$$

where c_t denotes the consumption, h_t denotes the labor effort, and $\beta \in (0, 1)$ denotes the subjective discount factor. The single-period utility function U is assumed to be increasing in consumption, decreasing in effort, and strictly concave.

A domestic demand for real balances is introduced into the model by assuming that nominal money holdings, denoted by M_t^d , facilitate consumption purchases. Specifically, consumption purchases are subject to a proportional transaction cost $s(v_t)$ that is decreasing in the household's money-to-consumption ratio, or consumption-based money velocity,

$$v_t = \frac{P_t c_t}{M_t^d}, \quad (2)$$

where P_t denotes the nominal price of the consumption good in period t . The transaction cost function, $s(v)$, satisfies the following assumptions: (i) $s(v)$ is nonnegative and twice continuously differentiable; (ii) there exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $s(\underline{v}) = s'(\underline{v}) = 0$; (iii) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$; and (iv) $2s'(v) + vs''(v) > 0$ for all $v \geq \underline{v}$. Assumption (ii) ensures that the Friedman rule, that is, a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption (iii) guarantees that in equilibrium money, velocity is always greater than or equal to the satiation level. Assumption (iv) ensures that the demand for money is a decreasing function of the nominal interest rate.

Households are assumed to have access to one-period nominal bonds, denoted as B_t , which carry a gross nominal interest rate of R_t when held from period t to period $t + 1$. Households supply labor services to competitive labor markets at the real wage rate w_t . In addition, households receive profit income in the amount Π_t from the ownership of firms, and pay income taxes at the flat rate τ_t . The flow budget constraint of the household in period t is then given by

$$P_t c_t [1 + s(v_t)] + M_t^d + B_t = M_{t-1}^d + R_{t-1} B_{t-1} + P_t (1 - \tau_t) (w_t h_t + \Pi_t). \quad (3)$$

In addition, it is assumed that the household is subject to the following borrowing limit that prevents it from engaging in Ponzi-type schemes:

$$\lim_{j \rightarrow \infty} \frac{M_{t+j}^d + R_{t+j} B_{t+j}}{\prod_{s=0}^j R_{t+s}} \geq 0. \quad (4)$$

This restriction states that in the long run, the household's net nominal liabilities must grow at a rate smaller than the nominal interest rate. It rules out, for example, schemes in which households roll over their net debts forever.

The household chooses sequences $\{c_t, h_t, v_t, M_t^d, B_t\}_{t=0}^{\infty}$ to maximize (1) subject to (2)–(4), taking as given the sequences $\{P_t, \tau_t, R_t, w_t, \Pi_t\}_{t=0}^{\infty}$ and the initial condition $M_{-1}^d + R_{-1}B_{-1}$. The first-order conditions associated with the household's maximization problem are (2), (3), and (4) holding with equality, and

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t}, \quad (5)$$

$$\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1 - \tau_t)w_t}{1 + s(v_t) + v_t s'(v_t)}, \quad (6)$$

$$\frac{U_c(c_t, h_t)}{1 + s(v_t) + v_t s'(v_t)} = \beta \frac{R_t P_t}{P_{t+1}} \frac{U_c(c_{t+1}, h_{t+1})}{[1 + s(v_{t+1}) + v_{t+1} s'(v_{t+1})]}. \quad (7)$$

Optimality condition (5) can be interpreted as a domestic demand for money or a domestic liquidity preference function. Given our maintained assumptions about the transactions technology $s(v_t)$, the implied domestic money demand function is decreasing in the gross nominal interest rate R_t . Further, our assumptions imply that as the interest rate vanishes, or R_t approaches unity, the domestic demand for money reaches a finite maximum level given by c_t/\underline{v} . At this level of money demand, households are able to perform transactions costlessly, as the transactions cost, $s(v_t)$, becomes nil. Optimality condition (6) shows that a level of money velocity above the satiation level \underline{v} , or, equivalently, an interest rate greater than zero, introduces a wedge, given by $1 + s(v_t) + v_t s'(v_t)$, between the marginal rate of substitution of consumption for leisure and the real wage rate. In addition, the labor supply distortion has a tax component given by $1 - \tau_t$, making the total wedge between the marginal rate of substitution of leisure for consumption and the real wage rate equal to $(1 - \tau_t)/[1 + s(v_t) + v_t s'(v_t)]$. This wedge induces households to move to an inefficient allocation featuring too much leisure and too little consumption. The wedge is increasing in the nominal interest rate and in the income tax rate, implying that the larger is the nominal interest rate or the income tax rate, the more distorted is the consumption–leisure choice. Optimality condition (7) is a Fisher equation, stating that the nominal interest rate must be equal to the sum of the expected rate of inflation and the real rate of interest. It is clear from the Fisher equation that intertemporal movements in the nominal interest rate create a distortion in the real interest rate perceived by households.

Final goods are produced by competitive firms using the technology $F(h_t)$ that takes labor as the only factor input. The production function F is assumed to be increasing and concave. Firms choose labor input to maximize profits, which are

given by

$$\Pi_t = F(h_t) - w_t h_t.$$

The first-order condition associated with the firm's profit maximization problem gives rise to the following demand for labor

$$F'(h_t) = w_t. \quad (8)$$

The rest of the world demands domestic nominal money balances in the amount M_t^f . We assume that the demand by foreigners for real domestic currency is a function of the level of foreign aggregate absorption, denoted as c_t^f , and the domestic nominal interest rate:

$$\frac{M_t^f}{P_t} = c_t^f L(R_t, x_t), \quad (9)$$

where the function L is assumed to be decreasing in its first argument. The variable x_t is a vector of shifters assumed to be exogenous.¹ Let the velocity of foreign-held domestic currency, denoted by v_t^f , be given by

$$v_t^f = \frac{P_t c_t^f}{M_t^f}. \quad (10)$$

The government prints money, issues nominal one-period bonds, and levies taxes to finance an exogenous stream of public consumption, denoted as g_t , and interest obligations on the outstanding public debt. Accordingly, the government's sequential budget constraint is given by

$$M_t^d + M_t^f + B_t = M_{t-1}^d + M_{t-1}^f + R_{t-1} B_{t-1} + P_t g_t - P_t \tau_t F(h_t). \quad (11)$$

Implicit in the sequential budget constraint of the government is the assumption that the government's consumption transactions are not subject to a monetary friction like the one imposed on private purchases of goods.

Combining the household's and the government's sequential budget constraints yields the following aggregate resource constraint:

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{M_t^f - M_{t-1}^f}{P_t}. \quad (12)$$

It is clear from this resource constraint that the domestic economy collects seignorage revenue from foreigners whenever nominal money balances held by foreigners increase.

1. In an online appendix, available on the website of the *Journal of Money, Credit and Banking* and on that of the authors, we provide some microfoundations for this specification of the foreign demand for domestic currency.

A competitive equilibrium is a set of sequences $\{v_t, w_t, v_t^f, c_t, h_t, M_t^d, M_t^f, B_t, P_t\}_{t=0}^\infty$ satisfying (2) and (4) holding with equality, (5)–(12), and

$$R_t \geq 1, \quad (13)$$

given policies $\{R_t, \tau_t\}_{t=0}^\infty$, the exogenous sequences $\{g_t, c_t^f\}_{t=0}^\infty$, and the initial conditions $M_{-1}^d + R_{-1}B_{-1} > 0$ and M_{-1}^f . Equilibrium condition (13) imposes a zero lower bound on the nominal interest rate. Such a bound is required to prevent the possibility of unbounded arbitrage profits created by taking short positions in nominal bonds and long positions in nominal fiat money, which would result in ill-defined demands for consumption goods by households.

Our primary goal is to characterize the Ramsey optimal rate of inflation. To this end, we begin by deriving the primal form of the competitive equilibrium. Then, we state the Ramsey problem. And finally, we characterize optimal fiscal and monetary policy.

1.1 The Primal Form of the Competitive Equilibrium

Following a long-standing tradition in public finance, we study optimal policy using the primal-form representation of the competitive equilibrium. Finding the primal form involves the elimination of all prices and tax rates from the equilibrium conditions so that the resulting reduced form involves only real variables. In our economy, the real variables that appear in the primal form are consumption, hours, and domestic and foreign money velocity. The primal form of the equilibrium conditions consists of two equations. One equation is a feasibility constraint, given by the resource constraint (12), which must hold at every date. The other equation is a single, present-value constraint known as the implementability constraint. The implementability constraint guarantees that at the prices and quantities associated with every possible competitive equilibrium, the present discounted value of consolidated government surpluses equals the government's total initial liabilities.

Combining equations (5), (9), and (10), one can express foreign money velocity as a function of domestic money velocity alone:

$$v_t^f = \chi(v_t), \quad (14)$$

where we have omitted the exogenous argument x_t , which will be regarded as constant throughout the analysis.

The following proposition provides the primal form of the competitive equilibrium.

PROPOSITION 1. (*Primal Form of the Competitive Equilibrium*). *Given the initial conditions $(R_{-1}B_{-1} + M_{-1}^d)$ and M_{-1}^f and the initial price level P_0 , sequences*

$\{c_t, h_t, v_t\}_{t=0}^{\infty}$ satisfy the feasibility condition

$$[1 + s(v_0)]c_0 + g_0 = F(h_0) + \frac{c_0^f}{\chi(v_0)} - \frac{M_{-1}^f}{P_0} \quad (15)$$

in period 0 and

$$\begin{aligned} [1 + s(v_t)]c_t + g_t &= F(h_t) + \frac{c_t^f}{\chi(v_t)} - \frac{c_{t-1}^f}{\chi(v_{t-1})} (1 - v_{t-1}^2 s'(v_{t-1})) \\ &\quad \times \frac{U_c(c_{t-1}, h_{t-1})}{\gamma(v_{t-1})} \frac{\gamma(v_t)}{\beta U_c(c_t, h_t)}, \end{aligned} \quad (16)$$

for all $t > 0$, the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \{U_c(c_t, h_t)c_t + U_h(c_t, h_t)h_t\} = \frac{U_c(c_0, h_0)}{1 + s(v_0) + v_0 s'(v_0)} \frac{R_{-1}B_{-1} + M_{-1}^d}{P_0}, \quad (17)$$

and

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

if and only if they also satisfy the set of equilibrium conditions (2) and (4) holding with equality, and (5)–(13).

PROOF: See the Appendix.

2. THE RAMSEY EQUILIBRIUM

The government is assumed to be benevolent toward domestic residents. This means that the welfare function of the government coincides with the lifetime utility of the domestic representative agent, and that it is independent of the level of utility of foreign residents. The Ramsey problem consists in choosing a set of strictly positive sequences $\{c_t, h_t, v_t\}_{t=0}^{\infty}$ to maximize the utility function (1) subject to (15), (16), and (17), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$, given $R_{-1}B_{-1} + M_{-1}$, M_{-1}^f , and P_0 . We fix the initial price level arbitrarily to keep the Ramsey planner from engineering a large unexpected initial inflation aimed at reducing the real value of predetermined nominal government liabilities. This assumption is regularly maintained in the literature on optimal monetary and fiscal policy.

Write the feasibility constraint (16) as $H(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) = 0$ and the implementability constraint (17) as $\sum_{t=0}^{\infty} \beta^t K(c_t, h_t) = A(c_0, h_0, v_0)$. Let the Lagrange multiplier on the feasibility constraint (16) be denoted by ψ_t , the Lagrange multiplier on the implementability constraint (17) be denoted by λ , and the Lagrange multiplier on the constraint $v_t \geq \underline{v}$ be denoted by μ_t . Then, for any $t > 0$, the

first-order conditions of the Ramsey problem are

$$U_c(c_t, h_t) + \lambda K_c(c_t, h_t) + \psi_t H_1(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) + \beta \psi_{t+1} H_2(c_{t+1}, c_t, h_{t+1}, h_t, v_{t+1}, v_t) = 0, \quad (18)$$

$$U_h(c_t, h_t) + \lambda K_h(c_t, h_t) + \psi_t H_3(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) + \beta \psi_{t+1} H_4(c_{t+1}, c_t, h_{t+1}, h_t, v_{t+1}, v_t) = 0, \quad (19)$$

$$\psi_t H_5(c_t, c_{t-1}, h_t, h_{t-1}, v_t, v_{t-1}) + \beta \psi_{t+1} H_6(c_{t+1}, c_t, h_{t+1}, h_t, v_{t+1}, v_t) + \mu_t = 0, \quad (20)$$

$$(v_t - \underline{v})\mu_t = 0; \quad \mu_t \geq 0; \quad v_t \geq \underline{v}. \quad (21)$$

We do not include the constraint $v_t^2 s'(v_t) < 1$ in the Lagrangian. Therefore, we must check that the solution to the above system satisfies this constraint.

2.1 Optimality of the Friedman Rule in the Absence of a Foreign Demand for Money

When the foreign demand for domestic currency is nil, $M_t^f = 0$, any policy other than the Friedman rule fails to be Ramsey optimal. To see this, note that when $M_t^f = 0$ (or, equivalently, when $c^f / \chi(v_t) = 0$), the first-order condition of the Ramsey problem with respect to v_t , equation (20), becomes

$$-\psi_t s'(v_t)c_t + \mu_t = 0.$$

Consider any level of velocity v_t greater than \underline{v} , the level called for by the Friedman rule. Our assumptions regarding the transactions cost technology imply that $s'(v) > 0$ for any $v_t > \underline{v}$. Also, the fact that the period utility function U is strictly increasing implies that $\psi_t > 0$. It then follows from the above expression that μ_t is strictly positive when $v_t > \underline{v}$. This result and the fact that, by assumption, $v_t > \underline{v}$, imply that optimality condition (21) is violated. We conclude that in the case of no foreign demand for domestic currency, if a Ramsey equilibrium exists, then it must be characterized by a zero nominal interest rate for all $t > 0$. This is a standard result in the theory of optimal monetary and fiscal policy.

2.2 Failure of the Friedman Rule in the Presence of a Foreign Demand for Money

We now present the main result of this paper, namely, that the Friedman rule ceases to be Ramsey optimal in the presence of a foreign demand for domestic currency. To facilitate the exposition, we restrict attention to the steady state of the Ramsey equilibrium. That is, we restrict attention to solutions to (16) and (18)–(21) in which the endogenous variables c_t , h_t , v_t , ψ_t , and μ_t are constant given constant levels for

the exogenous variables g_t and c_t^f . To establish the failure of the Friedman rule when $M^f > 0$, we show that a Ramsey equilibrium in which v_t equals \underline{v} is impossible. In the steady state, the optimality condition (20) when evaluated at $v_t = \underline{v}$ becomes:

$$\frac{c^f}{\chi(\underline{v})} s''(\underline{v}) \underline{v} \left(1 - \frac{1}{\beta} + \underline{v} \right) + \mu = 0.$$

Under our maintained assumptions regarding the transactions cost technology, $s''(\underline{v})$ is positive. Also, under reasonable calibrations, the constant $1/\beta - 1$, which equals the steady-state real interest rate, is smaller than the velocity level \underline{v} . Then, the first term in the above expression is positive. This implies that the multiplier μ must be negative, which violates optimality condition (21).²

We conclude that in the presence of a foreign demand for domestic currency, if a Ramsey equilibrium exists, it involves a deviation from the Friedman rule. The intuition behind this result is that the presence of a foreign demand for domestic currency introduces an incentive for the fiscal authority to inflate in order to extract resources, in the form of seignorage, from the rest of the world (whose welfare does not enter the domestic planner's objective function). Indeed, at any negative inflation rate (and, most so at the level of inflation consistent with the Friedman rule), the domestic country actually derives negative seignorage income from the rest of the world, because foreign money holdings increase in real value as the price level falls. On the other hand, levying an inflation tax on foreign money holdings comes at the cost of taxing domestic money holdings as well. In turn, the domestic inflation tax entails a welfare loss, because domestic households must pay elevated transaction costs as they are forced to economize on real balances. Thus, the Ramsey planner faces a trade-off between taxing foreign money holdings and distorting the domestic real allocation. We have demonstrated analytically that the resolution of this trade-off leads to an inflation rate above the one called for by Friedman's rule. We now turn to the question of how large the optimal deviation from the Friedman rule is under a plausible calibration of our model.

3. QUANTIFYING DEVIATIONS FROM THE FRIEDMAN RULE

To gauge the quantitative implications of a foreign demand for money for the optimal rate of inflation, we parameterize the model and solve numerically for the steady state of the Ramsey equilibrium. We adopt the following functional forms for the period utility function, the transactions cost technology, and the foreign demand

2. One may argue that the assumption $2s'(v) + vs''(v) > 0$ for all $v \geq \underline{v}$, which implies that the nominal interest rate is a strictly increasing function of v for all $v \geq \underline{v}$ and, in particular, that the elasticity of the liquidity preference function at a zero nominal interest rate is finite is too restrictive. Suppose, instead, that the assumption in question is relaxed by assuming that it must hold only for $v > \underline{v}$ but not at $v = \underline{v}$. In this case, a potential solution to the first-order condition of the Ramsey problem with respect to v_t is $v = \underline{v}$, provided that $s''(\underline{v}) = 0$.

for domestic money:

$$U(c, h) = \ln(c) + \theta \ln(1 - h); \quad \theta > 0,$$

$$s(v) = Av + B/v - 2\sqrt{AB},$$

and

$$\chi(v) = v.$$

The assumed transactions cost function implies that the satiation level of velocity is $v = \sqrt{B/A}$ and a demand for money of the form

$$\frac{M_t^d}{P_t} = \frac{c_t}{\sqrt{\frac{B}{A} + \frac{1}{A} \frac{R_t - 1}{R_t}}}.$$

The assumed form for the function χ implies identical relationships between the nominal interest rate and domestic-money velocity in the domestic and the foreign economies.

We follow Schmitt-Grohé and Uribe (2004) and set $\beta = 1/1.04$, $\theta = 2.90$, $B = 0.07524$, and $g_t = 0.04$ for all t , which implies a share of government spending of about 20%.³ We set $c^f = 0.06$ and $A = 0.0056$ to match the empirical regularities that about 50% of the U.S. currency (or about 26% of M1) is held outside of the United States and that the M1-to-consumption ratio is about 29%.⁴ Finally, we set the level of debt in the Ramsey steady state to 20% of GDP.⁵

We develop a numerical algorithm that delivers the exact solution to the steady state of the Ramsey equilibrium. The mechanics of the algorithm are as follows: (i) Pick a positive value of λ . (ii) Given this value of λ , solve the nonlinear system (16) and (18)–(21) for c , h , v , ψ , and μ . (iii) Calculate w from (8), τ from (6), R from (5), π from (7), v^f from (14), M_t^d/P_t from (2), and M_t^f/P_t from (10). (iv) Calculate the steady-state debt-to-output ratio, which we denote by $s_d \equiv B_t/(P_t y_t)$, from (11), taking into account that $y = h$. (v) If s_d is larger than the calibrated value of 0.2,

3. To identify the parameter B , in Schmitt-Grohé and Uribe (2004), we estimate the implied money demand equation $v_t^2 = B/A + 1/A(R_t - 1)/R_t$ on quarterly U.S. postwar data. We measure v_t as the ratio of nondurable consumption and services expenditures to M_1 , and R_t as the 3-month T-bill rate. We estimate the money demand equation using OLS and IV techniques.

4. For an estimate of the amount of U.S. currency circulating abroad, see the joint press release of the Board of Governors of the Federal Reserve System and the Department of the Treasury of October 25, 2006, available online at www.federalreserve.gov.

5. This debt level implies that the pre-Ramsey reform debt-to-output ratio in the economy without a foreign demand for domestic currency and with a prereform inflation rate of 4.2% is about 44%. The reason why the Ramsey steady-state level of debt is much lower than the pre-Ramsey-reform level is that the reform induces a drop in expected inflation of about 8%, which causes a large asset substitution away from government bonds and toward real money balances. The overall level of government liabilities (money plus bonds) is relatively unaffected by the Ramsey reform.

TABLE 1
 RAMSEY POLICY WITH FOREIGN DEMAND FOR DOMESTIC CURRENCY

	$\frac{M^f}{M^f+M^d}$	$\frac{M^f+M^d}{Pc}$	π	R	τ
No foreign demand: $c^f = 0$	0.00	0.27	-3.85	0.00	17.56
Baseline calibration: $c^f = 0.06$	0.22	0.26	2.10	6.18	16.15
Higher foreign demand: $c^f = 0.1$	0.32	0.24	10.52	14.94	14.64
Low domestic demand: $A = 0.0014$	0.22	0.13	2.11	6.19	16.33
High interest elasticity: $B = 0.0376$	0.22	0.37	-0.96	3.00	16.95
High debt-to-output ratio: $\frac{B}{P_y} = 0.50$	0.22	0.26	2.21	6.30	17.50
Lump-sum taxes	0.20	0.27	0.85	4.88	0.00
Lump-sum taxes and $g_t = 0$	0.19	0.27	0.59	4.62	-

NOTE: The baseline calibration is: $A = 0.0056$, $B = 0.07524$, $\frac{B}{P_y} = 0.2$, and $c^f = 0.06$. The interest rate, R , and the inflation rate, π , are expressed in percent per annum, and the income tax rate, τ , is expressed in percent.

lower λ . If, instead, s_d is smaller than the calibrated value of 0.2, then increase the value of λ . (vi) Repeat steps (i)–(v) until s_d has converged to its calibrated value.

Table 1 presents our numerical results. The first row of the table shows that when foreign demand for domestic currency is nil, which we capture by setting $c^f = 0$, then as we have shown analytically above the Friedman rule is Ramsey optimal; that is, the nominal interest rate is zero in the steady state of the Ramsey equilibrium. The inflation rate is -3.85% and the income tax rate is about 18%. In this case, because the foreign demand for domestic currency is nil, the domestic government has no incentives to levy an inflation tax, as it would generate no revenues from the rest of the world. The second row of the table considers the case that the foreign demand for domestic currency is positive. In particular, we set $c^f = 0.06$ and obtain that in the Ramsey steady state, the ratio of foreign currency to total money is 22% and that total money holdings are 26% of consumption. Both figures are broadly in line with observation in the U.S. economy. The table shows, again in line with the analytical results obtained above, that the Ramsey optimal rate of interest is positive; that is, the Friedman rule is no longer optimal. Of greater interest, however, is the size of the deviation from the Friedman rule. The table shows that the Ramsey optimal inflation rate is 2.10% per year about 6 percentage points higher than the value that obtains in the absence of a foreign demand for domestic currency. The optimal rate of interest now is 6.2%. When we increase foreign demand for domestic currency by assuming a larger value of foreign demand, $c^f = 0.1$, then the share of foreign holdings of domestic currency in total money increases by 10 percentage points to 0.32 and the Ramsey optimal inflation rate is more than 10% per year. In this calibration, the benefit from collecting an inflation tax from foreign holdings of currency appears to strongly dominate the costs that such a high inflation tax represents for domestic agents in terms of a more distorted consumption–leisure choice and elevated transaction costs. The larger inflation tax revenue of the government relaxes the budget constraint of the government allowing for a decline in the Ramsey optimal tax rate of about 1.5 percentage points.

Row 4 of Table 1 considers a calibration that implies a weaker demand for money both domestically and abroad. Specifically, we lower the coefficient A in the transactions cost function by a factor of 4. Because the demand for money is proportional to the square root of A , this parameter change implies that the ratio of money to consumption falls by a factor of 2. In the Ramsey steady state, the money-to-consumption ratio falls from 26% to 13%. The relative importance of foreign demand for money is unchanged. It continues to account for 22% of total money demand. The optimal rate of inflation is virtually the same as in the baseline case. The reason why the inflation tax is virtually unchanged in this case is that the reduction in A induces proportional declines in both the domestic and the foreign demands for domestic currency. The decline in foreign money demand is equivalent to a decline in c^f and therefore induces the Ramsey planner to lower the rate of inflation. At the same time, the decline in the domestic demand for money reduces the cost of inflation for domestic agents, inducing the Ramsey planner to inflate more. In our parameterization, these two opposing effects happen to offset each other almost exactly.

Row 5 of Table 1 analyzes the sensitivity of our results to raising the interest elasticity of money demand. Under a higher interest elasticity, the Ramsey optimal rate of interest and inflation are lower than in the baseline case. The nominal interest rate falls from 6% to 3% and the inflation rate falls from about 2% to -1%. In this case, while the Ramsey policy deviates from the Friedman rule, the deviation is not large enough to render positive inflation Ramsey optimal. The last row of the table shows that our results are very little changed when we increase the steady-state debt level. We conclude from the results presented in Table 1 that the trade-off between collecting seignorage from foreign holders of domestic currency and keeping the opportunity cost of holding money low for domestic agents is overwhelmingly resolved in favor of collecting seignorage income from foreign holdings of domestic currency.

The numerical results of this section suggest that an inflation target of about 2% per annum may be rationalized on the basis of an incentive to tax foreign holdings of domestic currency. This argument could, in principle, be raised to explain average rates of inflation in countries such as the United States, which we used as a point of reference in our calibration, or in the euro area, whose currency is held widely in eastern Europe, Russia, and certain parts of Asia minor. However, the fact that a number of developed countries whose currency is not used outside of their geographic borders, such as Australia, Canada, and New Zealand, also maintain inflation targets of about 2% per year, indicate that the reason why inflation targets in the developed world are as high as observed may not exclusively originate from the desire to extract seignorage revenue from foreigners. Our investigation calls attention to the fact that, all other things equal, countries whose currencies are demanded outside of their borders should have greater incentives to deviate from the Friedman rule than countries whose currencies are locally circumscribed. We also view our finding as a challenge for future research to ascertain why countries whose monies are widely held abroad do not appear to exploit this margin fully.

4. LUMP-SUM TAXATION

The reason why the benevolent government finds it desirable to deviate from the Friedman rule in the presence of a foreign demand for currency is not entirely to finance its budget with seignorage revenue extracted from foreign residents. Rather, the government imposes an inflation tax on foreign residents to increase the total amount of resources available to domestic residents for consumption. To show that this is indeed the correct interpretation of our results, we now consider a variation of the model in which the government can levy lump-sum taxes on domestic residents. Specifically, we assume that the labor income tax rate τ_t is zero at all times, and that the government sets lump-sum taxes to ensure fiscal solvency.

A competitive equilibrium is then given by sequences $\{v_t, v_t^f, c_t, h_t, M_t^d, M_t^f, P_t\}_{t=0}^\infty$ satisfying (2), (5), (7), (9), (10), (12), (13), and

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{1}{1 + s(v_t) + v_t s'(v_t)}, \quad (22)$$

given an interest rate sequence $\{R_t\}_{t=0}^\infty$, and the exogenous sequences $\{c_t^f, g_t\}_{t=0}^\infty$.

One can show that, given the initial condition M_{-1}^f and the initial price level P_0 , sequences $\{c_t, h_t, v_t\}_{t=0}^\infty$ satisfy the feasibility conditions (15) and (16), the labor supply equation (22), and

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

if and only if they also satisfy the set of equilibrium conditions (2), (5), (7), (9), (10), (12), (13), and (22). This primal form is essentially the same as the one associated with the economy with distortionary taxes and government spending except that the implementability constraint is replaced by equation (22), which states that in equilibrium, labor demand must equal labor supply. Noting that equation (22) appears in both the standard and the primal forms of the competitive equilibrium, it follows that the proof of the above statement is a simplified version of the one presented in the Appendix. The Ramsey problem then consists in maximizing the utility function (1) subject to the feasibility constraints (15) and (16) and the restrictions $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$, given P_0 and M_{-1}^f .

Row 7 of Table 1 presents the steady state of the Ramsey equilibrium in the economy with lump-sum taxes. All parameters of the model are calibrated as in the economy with distortionary taxes. The table shows that the optimal rate of inflation equals 0.85%. This means that the presence of a foreign demand for money gives rise to an optimal inflation bias of about 5 percentage points above the level of inflation called for by the Friedman rule. This inflation bias emerges even though the government can resort to lump-sum taxes to finance its budget. The optimal inflation bias is smaller than in the case with distortionary taxes. This is because distortionary taxes, through their depressing effect on employment and output, make

the pre-foreign-seignorage level of consumption lower, raising the marginal utility of wealth, and as a result providing bigger incentives for the extraction of real resources from the rest of the world.

The last row of Table 1 displays the steady state of the Ramsey equilibrium in the case in which government consumption equals zero at all times ($g_t = 0$ for all t). All other things equal, the domestic economy has access to a larger amount of resources than in the economy with positive government consumption. As a result, the government has fewer incentives to collect seignorage income from the rest of the world. This is reflected in a smaller optimal rate of inflation of 0.59%. It is remarkable, however, that even in the absence of distortionary taxes and government expenditures, the government finds it optimal to deviate from the Friedman rule by about 4.5 percentage points. This result clearly shows that the ultimate purpose of positive interest rates in the presence of a foreign demand for money is the extraction of real resources from the rest of the world for private domestic consumption.

5. CONCLUSION

In this paper, we demonstrate that the presence of a foreign demand for domestic currency of the size observed for the U.S. dollar can introduce incentives for the monetary authority to generate positive rates of inflation. The inflation rate acts as a tax on foreign holdings of domestic currency that allows the domestic government to effectively extract real resources from the rest of the world. In our model, the Ramsey planner weights this incentive against the cost that inflation causes to domestic households. In the absence of a foreign demand for money, the optimal policy calls for adopting Friedman's rule, or deflating at the real rate of interest. We find that for plausible calibrations of our model, the trade-off between taxing the rest of the world and keeping domestic transactions costs low is resolved in favor of taxing foreign holdings of domestic currency at rates ranging from 2% to 10% per year.

APPENDIX: PROOF OF PROPOSITION 1

We first show that plans $\{c_t, h_t, v_t\}$ satisfying the equilibrium conditions (2) and (4) holding with equality, and (5)–(13) also satisfy (16) and (17), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$. Let $\gamma(v_t) \equiv 1 + s(v_t) + v_t s'(v_t)$. Note that (5), (13), and our maintained assumptions regarding $s(v)$ together imply that $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$.

Let $W_{t+1} = R_t B_t + M_t^d + M_t^f$. Use this expression to eliminate B_t from (11) and multiply by $q_t \equiv \prod_{s=0}^{t-1} R_s^{-1}$ to obtain

$$q_t (M_t^d + M_t^f) (1 - R_t^{-1}) + q_{t+1} W_{t+1} - q_t W_t = q_t [P_t g_t - \tau_t P_t F(h_t)].$$

Sum for $t = 0$ to $t = T$ to obtain

$$\sum_{t=0}^T [q_t(M_t^d + M_t^f)(1 - R_t^{-1}) - q_t(P_t g_t - \tau_t P_t F(h_t))] = -q_{T+1}W_{T+1} + W_0.$$

In writing this expression, we define $q_0 = 1$. Solve (6) for τ_t and (8) for w_t and use $F(h) = h$ to obtain $\tau_t F(h_t) = h_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t$. Use this expression to eliminate $\tau_t F(h_t)$ from the above equation, which yields

$$\begin{aligned} & \sum_{t=0}^T \left\{ q_t(M_t^d + M_t^f)(1 - R_t^{-1}) - q_t P_t \left[g_t - \left[h_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t \right] \right] \right\} \\ & = -q_{T+1}W_{T+1} + W_0. \end{aligned}$$

Use the feasibility constraint (12) to replace $h_t - g_t$ with $[1 + s(v_t)]c_t - \frac{M_t^f - M_{t-1}^f}{P_t}$.

$$\begin{aligned} & \sum_{t=0}^T q_t P_t \left\{ \frac{M_t^d + M_t^f}{P_t} (1 - R_t^{-1}) + [1 + s(v_t)]c_t \right. \\ & \quad \left. - \frac{M_t^f - M_{t-1}^f}{P_t} + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t \right\} = -q_{T+1}W_{T+1} + W_0. \end{aligned}$$

Use (2) and (5) to replace $\frac{M_t^d}{P_t}(1 - R_t^{-1})$ with $v_t s'(v_t)c_t$

$$\begin{aligned} & \sum_{t=0}^T q_t P_t \left\{ v_t s'(v_t)c_t - \frac{M_t^f}{P_t R_t} + [1 + s(v_t)]c_t + \frac{M_{t-1}^f}{P_t} + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t \right\} \\ & = -q_{T+1}W_{T+1} + W_0. \end{aligned}$$

Collect terms in c_t and replace $1 + s(v_t) + v_t s'(v_t)$ with $\gamma(v_t)$ and rearrange

$$\begin{aligned} & \sum_{t=0}^T q_t P_t \left\{ \gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t - \frac{M_t^f}{P_t R_t} + \frac{M_{t-1}^f}{P_t} \right\} \\ & = -q_{T+1}W_{T+1} + W_0. \end{aligned}$$

Noting that by definition $q_t/R_t = q_{t+1}$, write the above expression as

$$\begin{aligned} & \sum_{t=0}^T q_t P_t \left\{ \gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)}\gamma(v_t)h_t \right\} + \sum_{t=0}^T (M_{t-1}^f q_t - M_t^f q_{t+1}) \\ & = -q_{T+1}W_{T+1} + W_0. \end{aligned}$$

Evaluate the second sum on the left-hand side and recall that by definition $q_0 = 1$ to obtain

$$\sum_{t=0}^T q_t P_t \left\{ \gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} \gamma(v_t)h_t \right\} + M_{-1}^f - M_T^f q_{T+1} = -q_{T+1}W_{T+1} + W_0.$$

Using the definition of W_t , we can write the above expression as

$$\sum_{t=0}^T q_t P_t \left\{ \gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} \gamma(v_t)h_t \right\} = -q_{T+1} (R_T B_T + M_T^d) + R_{-1}B_{-1} + M_{-1}^d. \tag{A1}$$

Take limits for $T \rightarrow \infty$. Then by (4), holding with equality, the limit of the right-hand side is well defined and equal to $R_{-1}B_{-1} + M_{-1}^d$. Thus, the limit of the left-hand side exists. This yields:

$$\sum_{t=0}^{\infty} q_t P_t \left\{ \gamma(v_t)c_t + \frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} \gamma(v_t)h_t \right\} = R_{-1}B_{-1} + M_{-1}^d.$$

By (7), we have that $P_t q_t = \beta^t U_c(c_t, h_t) / \gamma(v_t) P_0 / U_c(c_0, h_0) \gamma(v_0)$. Use this expression to eliminate $P_t q_t$ from the above equation to obtain

$$\sum_{t=0}^{\infty} \beta^t [U_c(c_t, h_t)c_t + U_h(c_t, h_t)h_t] = \left(\frac{U_c(c_0, h_0)}{\gamma(v_0)} \right) \left(\frac{R_{-1}B_{-1} + M_{-1}^d}{P_0} \right),$$

which is (17).

We next show that the competitive equilibrium conditions imply (16). For $t = 0$, equation (16) follows directly from (10) and (14). For $t > 0$, use (10) to eliminate M_t^f and M_{t-1}^f from (12) to obtain:

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{c_t^f}{v_t^f} - \frac{c_{t-1}^f}{v_{t-1}^f} \frac{1}{\pi_t}.$$

Now, use (7) to eliminate π_t . This yields:

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{c_t^f}{\chi(v_t)} - \frac{c_{t-1}^f}{\chi(v_{t-1})} \frac{U_c(c_{t-1}, h_{t-1})}{R_{t-1} \gamma(v_{t-1})} \frac{\gamma(v_t)}{\beta U_c(c_t, h_t)},$$

Using (5) to replace R_{t-1} yields (16). This completes the proof that the competitive equilibrium conditions imply the primal form conditions.

We now show that plans $\{c_t, h_t, v_t\}$ satisfying (16), (17), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$ also satisfy the equilibrium conditions (2) and (4) holding with equality, and (5)–(13).

Given a plan $\{c_t, h_t, v_t\}$ proceed as follows. Use (5) to construct R_t and (9) to construct v_t^f . Note that under the maintained assumptions on $s(v)$, the constraints $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$ ensure that $R_t \geq 1$. Let w_t be given by (8) and τ_t by (6).

To construct plans for M_t^d, M_t^f, P_{t+1} , and B_t , for $t \geq 0$, use the following iterative procedure: (i) set $t = 0$; (ii) use equation (2) to construct M_t^d and equation (10) to construct M_t^f (recall that P_0 is given); (iii) set B_t so as to satisfy equation (11); (iv) set P_{t+1} to satisfy (7); (v) increase t by 1 and repeat steps (i)–(v). To show that (12) holds use (16). Combining (10) and (14) with (15), it is obvious that (12) holds for $t = 0$. To show that it also holds for $t > 0$, combine (10), (14), and (16) to obtain:

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{M_t^f}{P_t} - \frac{M_{t-1}^f}{P_{t-1}} (1 - v_{t-1}^2 s'(v_{t-1})) \\ \times \frac{U_c(c_{t-1}, h_{t-1})}{\gamma(v_{t-1})} \frac{\gamma(v_t)}{\beta U_c(c_t, h_t)}.$$

Using (5), one can write this expression as

$$[1 + s(v_t)]c_t + g_t = F(h_t) + \frac{M_t^f}{P_t} - \frac{M_{t-1}^f}{P_{t-1}} (1/R_{t-1}) \frac{U_c(c_{t-1}, h_{t-1})}{\gamma(v_{t-1})} \\ \times \frac{\gamma(v_t)}{\beta U_c(c_t, h_t)}.$$

Finally, combining this expression with (7) yields (12).

It remains to be shown that (4) holds with equality. Follow the steps shown above to arrive at equation (A1). Notice that these steps make use only of equilibrium conditions that we have already shown are implied by the primal form. Now use (7) to replace $P_t q_t$ with $\beta^t U_c(c_t, h_t)/\gamma(v_t) P_0/U_c(c_0, h_0)\gamma(v_0)$ to obtain

$$\sum_{t=0}^T \beta^t [U_c(c_t, h_t)c_t + U_h(c_t, h_t)h_t] = -q_{T+1}(R_T B_T + M_T^d) \left(\frac{U_c(c_0, h_0)}{P_0 \gamma(v_0)} \right) \\ + \left(\frac{U_c(c_0, h_0)}{\gamma(v_0)} \right) \left(\frac{R_{-1} B_{-1} + M_{-1}^d}{P_0} \right).$$

Taking limit for $T \rightarrow \infty$, recalling the definition of q_t , and using (17) yield (4) holding with equality. This completes the proof.

LITERATURE CITED

European Central Bank. (2011) “The International Role of the Euro.”
 Goldberg, Linda S. (2010) “Is the International Role of the Dollar Changing?” *Current Issues in Economics and Finance*, Federal Reserve Bank of New York, 16 (1) 1–7.

Porter, Richard D., and Ruth A. Judson. (1996) "The Location of U.S. Currency: How Much Is Abroad?" *Federal Reserve Bulletin*, 82 (10), 883–903.

Rogoff, Kenneth. (1998) "Blessing or Curse? Foreign and Underground Demand for Euro Notes," *Economic Policy*, 13, 261–303.

Schmitt-Grohé, Stephanie, and Martín Uribe (2004) "Optimal Fiscal and Monetary Policy under Imperfect Competition," *Journal of Macroeconomics*, 26, 183–209.