Supplementary Material for "Foreign Demand for Domestic Currency and the Optimal Rate of Inflation," by Stephanie Schmitt-Grohé and Martín Uribe to be posted as an online appendix

Appendix: Some Microfoundations for the Foreign Demand for Domestic Currency

In this appendix, we derive the foreign demand for domestic (i.e., U.S.) currency. By the domestic economy, we mean the United States, and by the foreign economy we mean the collection of countries, other than the United States, that hold U.S. dollars. Suppose that there are $N \geq 1$ foreign countries demanding U.S. currency. We index these countries by $i = 1, \ldots, N$. Suppose that each country i faces transaction costs given by $s^i(v_t^i)$, where s^i is a positive, increasing function. The variable v_t^i denotes money velocity in country i in period t and is defined as

$$v_t^i = \frac{P_t^i c_t^i}{M_t^i},$$

where P_t^i denotes the price level in country i, c_t^i denotes consumption in country i, and M_t^i denotes money holdings in country i. We assume that there is currency substitution in country i. Specifically, money holdings, M_t^i, are assumed to be a composite of local currency, $M_{1,t}^i$, and domestic (U.S.) currency, $M_{2,t}^i$, given by

$$M_t^i = A^i(M_{1,t}^i, M_{2,t}^i E_t^i),$$

where the aggregator function A^i is assumed to be positive, increasing, and homogeneous of degree one, and E_t^i denotes the nominal exchange rate defined as the price of one unit of the domestic currency (i.e., the price of the U.S. dollar) in terms of units of currency of country i. We further assume that there is a single traded good and that purchasing power parity holds, so that $P_t^i = P_t E_t^i$, where P_t denotes the price level in the domestic economy (i.e., in the United States).

Letting

$$v_{1,t}^i \equiv \frac{P_t^i c_t^i}{M_{1t}^i}$$

and

$$v_{2,t}^i \equiv \frac{P_t^i c_t^i}{E_t^i M_{2t}^i} = \frac{P_t c_t^i}{M_{2t}^i},$$

velocity of country i, v_t^i, can be written as

$$v_t^i = \frac{1}{A^i\left(\frac{1}{v_{1,t}^i}, \frac{1}{v_{2,t}^i}\right)}.$$

We can then write the transaction cost faced by the representative household of country i as

$$\tilde{s}^i(v_{1,t}^i, v_{2,t}^i) \equiv s^i\left(\frac{1}{A^i\left(\frac{1}{v_{1,t}^i}, \frac{1}{v_{2,t}^i}\right)}\right)$$

The sequential budget constraint of the representative household in country i is given by

$$[1+\tilde{s}^i(v_{1,t}^i, v_{2,t}^i)]c_t^i + m_{1,t}^i + m_{2,t}^i + b_t^i \leq \frac{m_{1,t-1}^i}{\pi_t \epsilon_t^i} + \frac{m_{2,t-1}^i}{\pi_t} + \frac{R_{t-1}}{\pi_t} b_{t-1}^i + y_t^i,$$

where $\epsilon_t^i \equiv E_t^i/E_{t-1}^i$ denotes the depreciation rate of the currency of country i, $\pi_t \equiv P_t/P_{t-1}$ denotes the domestic (U.S.) rate of inflation, y_t^i denotes income of country i in period t, R_t denotes the U.S. nominal interest rate, and $b_t^i \equiv B_t^i/P_t$, where B_t^i denotes country i's holdings of dollar-denominated bonds, which pay the gross nominal interest rate R_t in dollars. We could assume additionally the existence of a domestic bond denominated in units of currency of country i that pays the gross nominal interest rate R_t^i when held between periods t and $t+1$. With free capital mobility R_t^i and R_t would be related by the no-arbitrage condition $R_t^i = R_t \epsilon_{t+1}^i$. This bond is redundant in our environment, however, because of the absence of uncertainty.

The representative household of country i maximizes its lifetime utility function subject to the above sequential budget constraint and to some no-Ponzi-game borrowing limit. For the purpose of deriving the demand for domestic currency (dollars) by country i, it suffices to consider the first-order conditions of the household's optimization problem with respect to $m^i_{1,t}$, $m^i_{2,t}$, and b^i_t, which are respectively given by

$$\xi^i_t[1-(v^i_{1,t})^2\tilde{s}_1(v^i_{1,t},v^i_{2,t})]=\beta\frac{\xi^i_{t+1}}{\pi_{t+1}\epsilon^i_{t+1}}$$

$$\xi^i_t[1-(v^i_{2,t})^2\tilde{s}_2(v^i_{1,t},v^i_{2,t})]=\beta\frac{\xi^i_{t+1}}{\pi_{t+1}}$$

$$\xi^i_t=\beta R_t\frac{\xi^i_{t+1}}{\pi_{t+1}},$$

where ξ^i_t denotes the Lagrange multiplier associated with the household's sequential budget constraint. Using the third first-order condition to eliminate ξ^i_t from the first two first-order conditions, we obtain

$$[1-(v^i_{1,t})^2\tilde{s}_1(v^i_{1,t},v^i_{2,t})]=\frac{1}{R_t\epsilon^i_{t+1}}$$

$$[1-(v^i_{2,t})^2\tilde{s}_2(v^i_{1,t},v^i_{2,t})]=\frac{1}{R_t}$$

Solving these two expressions for $v^i_{1,t}$ and $v^i_{2,t}$, yields a money demand function for domestic currency (i.e., U.S. dollars) by country i of the form

$$m^i_{2,t}=c^i_tL^i(R_t,\epsilon^i_{t+1}).$$

The total foreign demand for U.S. dollars, $m^f_t\equiv M^f_t/P_t\equiv\sum_{i=1}^N m^i_{2,t}$, is given by

$$m^f_t=\sum_{i=1}^N c^i_tL^i(R_t,\epsilon^i_{t+1}).$$

Let $c^f_t\equiv\sum_{i=1}^N c^i_t$ denote the total absorption of goods in foreign countries that hold U.S.

currency. We then define the share of country i in c_t^f as

$$\alpha_t^i \equiv \frac{c_t^i}{c_t^f}$$

Let

$$L(R_t, x_t) \equiv \sum_{i=1}^{N} \alpha_t^i L^i(R_t, \epsilon_{t+1}^i),$$

and

$$x_t = [\epsilon_{t+1}^1, \ldots, \epsilon_{t+1}^N, \alpha_t^1, \ldots, \alpha_t^N]$$

denote a vector of shifters of the foreign demand for domestic currency. Then, we can rewrite the foreign demand for domestic (U.S.) currency as

$$m_t^f = c_t^f L(R_t, x_t),$$

which is the expression used in the main body of the paper. We assume functional forms for the transactions cost s and the aggregator function A that ensure that L is decreasing in it first argument. We assume that the domestic economy takes x_t as exogenous. It would be of interest to consider extensions of the model in which elements of x_t are endogenous.