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# **Is Optimal Capital-Control Policy Countercyclical In Open-Economy Models With Collateral Constraints?**

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## Starting Point

- Open-economy models with collateral constraints typically display a pecuniary externality.
- The externality originates in the fact that the price of pledgable objects is endogenous to the model but exogenous to individual agents.
- The existing literature has stressed that in those models positive capital controls are desirable.
- However, it has not given much attention to the cyclical properties of optimal capital control taxes.

## This Paper

shows that in open economy models with collateral constraints the optimal capital control policy is **procyclical**. The Ramsey planner lowers capital controls in good times and raises them in bad times. In that sense the pecuniary externality does not call for macroprudential policy.

**Intuition:** Optimal capital control taxes are determined as the trade-off between the desire to frontload consumption (i.e., to borrow as much as possible) and the desire to avoid a binding collateral constraint. Avoiding a binding collateral constraint requires taxing borrowing when the economy is about to hit the borrowing constraint, which occurs in recessions. As a result capital control taxes are tightened in recessions and eased in expansions.

## The Model

### Household problem

$$\max_{\{c_t, c_t^T, c_t^N, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

subject to

$$c_t = A(c_t^T, c_t^N) \equiv \left[ a c_t^T{}^{1-1/\xi} + (1-a) c_t^N{}^{1-1/\xi} \right]^{1/(1-1/\xi)}$$

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + \frac{d_{t+1}}{1+r_t}$$

$$d_{t+1} \leq \kappa (y_t^T + p_t y_t^N)$$

The endowment processes,  $y_t^T$  and  $y_t^N$ , and the interest rate,  $r_t$ , are assumed to be exogenous.

**Equilibrium**  $\{c_t, c_t^T, c_t^N, d_{t+1}, \lambda_t, \mu_t, p_t\}$  satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \quad (1)$$

$$c_t = \left[ a c_t^T{}^{1-\frac{1}{\xi}} + (1-a) c_t^N{}^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}} \quad (2)$$

$$\lambda_t = a c_t^{-\sigma} \left( \frac{c_t^T}{c_t} \right)^{-1/\xi} \quad (3)$$

$$\lambda_t \left[ \frac{1}{1 + r_t} - \mu_t \right] = \beta \mathbb{E}_t \lambda_{t+1} \quad (4)$$

$$p_t = \frac{1-a}{a} \left( \frac{c_t^T}{c_t^N} \right)^{1/\xi} \quad (5)$$

$$c_t^N = y_t^N \quad (6)$$

$$d_{t+1} \leq \kappa \left[ y_t^T + p_t y_t^N \right], \quad \mu_t \left[ \kappa \left( y_t^T + p_t y_t^N \right) - d_{t+1} \right] = 0, \quad \mu_t \geq 0 \quad (7)$$

given exogenous  $\{y_t^T, y_t^N, r_t\}$  and  $d_0$ .

## The Pecuniary Externality

As stressed in the related literature (Korinek 2011, Bianchi 2011 for example), this model has a pecuniary externality: Combining (1), (5), and (7) yields

$$d_{t+1} \leq \kappa \left[ y_t^T + \frac{1-a}{a} \left( \frac{y_t^T + \frac{d_{t+1}}{1+r_t} - d_t}{y_t^N} \right)^{1/\xi} y_t^N \right]$$

In equilibrium the value of collateral depends on the level of borrowing. Individual agents understand this mechanism but also understand that individually they are too small to affect the equilibrium price of nontradables,  $p_t$ . Hence they take  $p_t$  as given.

The existing literature has shown that the pecuniary externality makes positive capital controls taxes desirable (Bianchi, 2011, for example).

The present paper asks whether optimal capital controls are countercyclical.

## Point of comparison

An economy in which agents internalize the pecuniary externality. That is, an economy in which consumption and savings decisions take into account market clearing in the nontraded sector and the fact that the relative price of nontradables depends on the desired consumptions of tradables and nontradables. In this economy, the utility maximization problem is

$$\max_{\{c_t^T, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{A(c_t^T, y_t^N)^{1-\sigma} - 1}{1-\sigma}$$

subject to

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}$$

$$d_{t+1} \leq \kappa \left[ y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right]$$

This is the best allocation given the collateral constraint.

## Can The Best Allocation Be Supported As A Competitive Equilibrium?

Yes, by an appropriate use of **capital controls**.

$\tau_t$  = proportional tax on debt assumed in period  $t$ .

$\tau_t > 0 \Rightarrow$  capital control tax

$\tau_t < 0 \Rightarrow$  borrowing subsidy

Capital controls are rebated via transfers  $\ell_t$ , which can be lump sum or proportional to any source of household income.

The Household budget constraint now is:

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + (1 - \tau_t) \frac{d_{t+1}}{1 + r_t} + \ell_t$$

The interest rate perceived by households is

$(1 + r_t)$  without capital controls and

$\frac{1+r_t}{1-\tau_t} > 1 + r_t$ , if  $\tau_t > 0$  with capital controls.



**The Competitive Equilibrium in the Economy with Capital Control Taxes** is a set of processes  $c_t^T$ ,  $d_{t+1}$ ,  $\lambda_t$ ,  $\mu_t$ , and  $p_t$  satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \quad (8)$$

$$\lambda_t = U'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N) \quad (9)$$

$$\left( \frac{1 - \tau_t}{1 + r_t} - \mu_t \right) \lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \quad (10)$$

$$p_t = \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} \quad (11)$$

$$d_{t+1} \leq \kappa \left[ y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right] \quad (12)$$

$$\mu_t [\kappa (y_t^T + p_t y_t^N) - d_{t+1}] = 0 \quad (13)$$

$$\mu_t \geq 0 \quad (14)$$

given  $\{\tau_t\}$ ,  $\{y_t^T\}$ ,  $\{y_t^N\}$ , and  $\{r_t\}$ , and  $d_0$ .

**The Optimal Capital Control Tax** solves the problem

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y_t^N))$$

subject to (8)-(14).

- This looks like a formidable task, but it turns out to be quite simple, for the following reason:
- As we show next, constraints (8)-(14) are satisfied if and only if constraints (8) and (12) are satisfied.
- The ‘only if’ part of this statement is trivial, since (8)-(14) include (8) and (12). The following two slides walk you through the proof of the ‘if’ part.

## Equations (8)-(14) Repeated

$$\boxed{c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}} \quad (8)$$

$$\lambda_t = U'(A(c_t^T, y_t^N)) A_1(c_t^T, y_t^N) \quad (9)$$

$$\left(\frac{1-\tau_t}{1+r_t} - \mu_t\right) \lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \quad (10)$$

$$p_t = \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} \quad (11)$$

$$\boxed{d_{t+1} \leq \kappa \left[ y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right]} \quad (12)$$

$$\mu_t [\kappa (y_t^T + p_t y_t^N) - d_{t+1}] = 0 \quad (13)$$

$$\mu_t \geq 0 \quad (14)$$

given  $\{\tau_t\}$ ,  $\{y_t^T\}$ ,  $\{y_t^N\}$ , and  $\{r_t\}$ , and  $d_0$ .

**Claim:** If conditions (8) and (12) for any pair of processes  $\{c_t^T, d_{t+1}\}$ , then conditions (8)-(14) are also satisfied for those processes.

**Proof:** Suppose  $\{c_t^T\}$  and  $\{d_{t+1}\}$  satisfy (8) and (12).

Then pick  $\{p_t\}$  to satisfy (11).

Pick  $\{\mu_t\} = 0 \forall t$ , then (13) and (14) hold.

Pick  $\lambda_t$  to satisfy (9).

Pick  $\tau_t$  to satisfy (10) ■

**Note** that when the collateral constraint (12) is binding,  $\tau_t$  is not unique, i.e., when the collateral constraint binds,  $\exists$  other picks for  $\mu_t$  and  $\tau_t$  that are consistent with the same allocation.

Thus, we have shown that the Ramsey Optimal Capital Control Tax Problem can be stated as:

$$\max_{\{c_t^T, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y_t^N))$$

subject to

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \quad (8)$$

$$d_{t+1} \leq \kappa \left[ y_t^T + \frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} y_t^N \right] \quad (12)$$

which says that with a capital control tax instrument the Ramsey planner induces households to fully internalize the pecuniary externality, thereby supporting the best allocation as a competitive equilibrium.

## **Is Optimal Capital Control Policy Countercyclical?**

## Calibration and Shocks

Exactly as in Bianchi (2011):

- Endowment shocks ( $y_t^T$  and  $y_t^N$  stochastic)
- Constant interest rate
- Model calibrated to Argentina

(Later, we will consider an alternative stochastic structure with interest-rate shocks.)

The natural logarithms of the traded and nontraded endowments follow a bivariate AR(1), which is estimated on annual HP-filtered Argentine data spanning the period 1965 to 2007. Traded GDP: Manufacturing and primary products. Nontraded GDP: remaining components.

$$\begin{bmatrix} \ln y_t^T \\ \ln y_t^N \end{bmatrix} = \begin{bmatrix} 0.901 & -0.453 \\ 0.495 & 0.225 \end{bmatrix} \begin{bmatrix} \ln y_{t-1}^T \\ \ln y_{t-1}^N \end{bmatrix} + \epsilon_t, \quad (15)$$

where  $\epsilon_t \sim N(\emptyset, \Omega_\epsilon)$ , with  $\Omega_\epsilon = \begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}$ .



## Some Unconditional Summary Statistics of the Driving Process

Statistic	$\ln y^T$	$\ln y^N$
Std. Dev.	6%	6%
Serial Corr.	0.53	0.62
$\text{Corr}(\ln y_t^T, \ln y_t^N)$	0.83	

Comments:

- (1) High volatility of tradable and nontradable endowment;
- (2) Strong positive correlation between  $y_t^T$  and  $y_t^N$ ;

## Discretization of the State Space

There are 4 distinct grid points for  $\ln(y^T)$ ,

$$\begin{bmatrix} -0.1093 \\ -0.0347 \\ 0.0347 \\ 0.1093 \end{bmatrix}$$

and 16 distinct pairs  $(y^T, y^N)$ .

There are 800 grid points for  $d_t$ .

The total grid has  $16 \times 800 = 12,800$  points.

## Summary of the Calibration

Time unit is one year.

Parameter	Value	Description
$\kappa$	$0.32(1 + r)$	Parameter of collateral constraint
$\sigma$	2	Inverse of intertemporal elasticity of consumption
$\beta$	0.91	Subjective discount factor
$r$	0.04	Interest rate (annual)
$\xi$	0.83	Elasticity of substitution between tradables and nontradables
$a$	0.31	Weight on tradables in CES aggregator
$y^N$	1	Steady-state nontradable output
$y^T$	1	Steady-state tradable output
$n_y$	16	Number of grid points for $(\ln y_t^T, \ln y_t^N)$
$n_d$	800	Number of grid points for $d_t$ , equally spaced
$[\ln \underline{y}^T, \ln \bar{y}^T]$	$[-0.1093, 0.1093]$	Range for tradable output
$[\ln \underline{y}^N, \ln \bar{y}^N]$	$[-0.1328, 0.1328]$	Range for nontradable output
$[\underline{d}/(1 + r), \bar{d}/(1 + r)]$	$[0.4 \ 1.02]$	Range for debt

A comment:

Agents are quite impatient

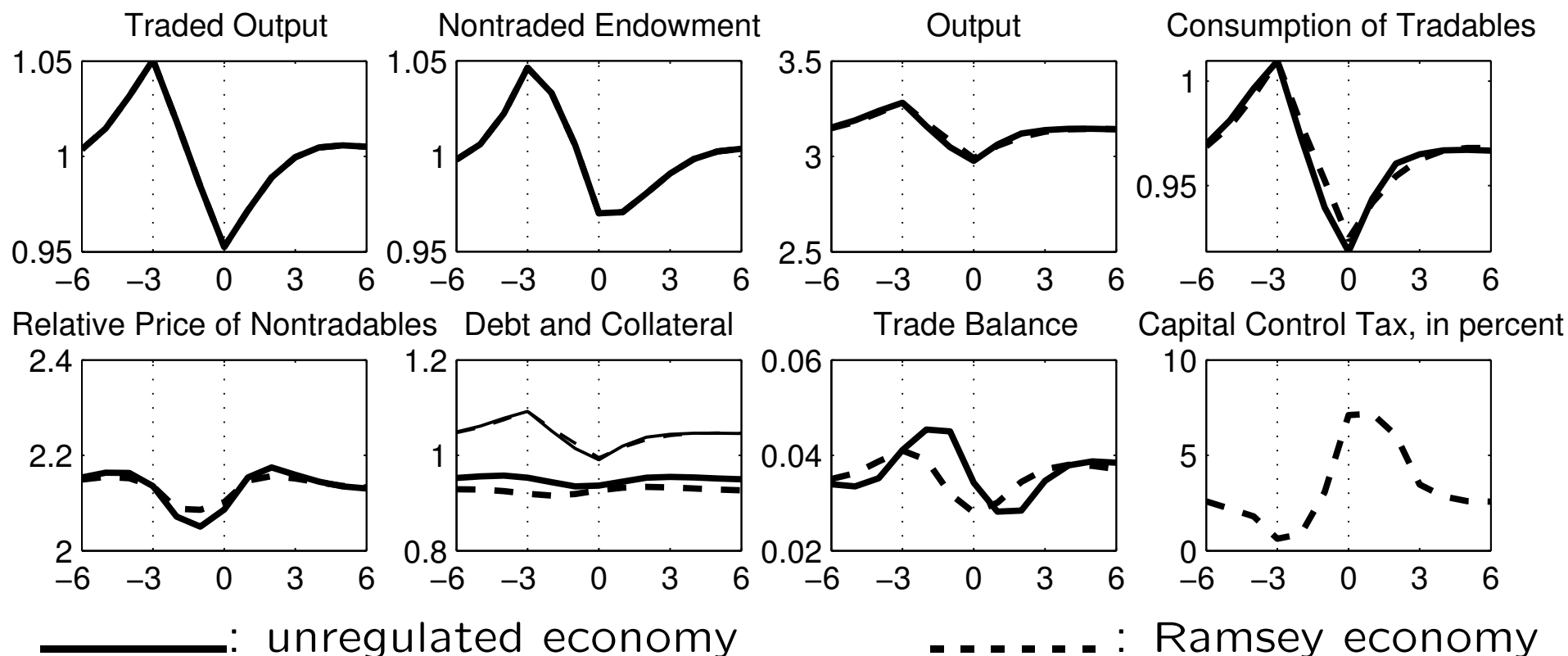
$$r = 0.04;$$

$$\beta = 0.91;$$

$$\beta(1 + r) = 0.95$$

The high degree of impatience influences how far apart the debt densities are in the economy without the collateral constraint and the economy with the collateral constraint. (see EXTRAS)

## The Typical Boom-Bust Cycle in the Endowment-Shock Economy



Definition of boom-bust episode:  $y_{-3}^T > 1$  and  $y_0^T < 1$ ; given grid this implies that during a typical boom bust episode output falls from 5% above mean to 5% below mean over 3 years. Frequency, 12.3%. Each line is the mean across all windows containing a boom-bust cycle in a time series of 1 million years. For the capital-control tax rate, the figure displays the median instead of the mean across windows because this variable is skewed, with an unconditional mean of 4.2 percent and an unconditional median of 2.5 percent. Because the capital control tax rate is indeterminate when the collateral constraint binds under the Ramsey policy, this variable is given a number only if the collateral constraint is slack under the Ramsey policy. Replication file `typical_boom_bust.m` in `sgu_endowment_shocks.zip`, on the authors' websites.

Observation:

Over the typical boom bust cycle the optimal capital control tax is not countercyclical. It is lowered during booms and raised during recessions.

What is the role of Ramsey optimal capital control taxes? — To avoid a binding constraint.

- Capital Control Taxes are positive on average  $\text{Median}(\tau_t) = 0.0258$

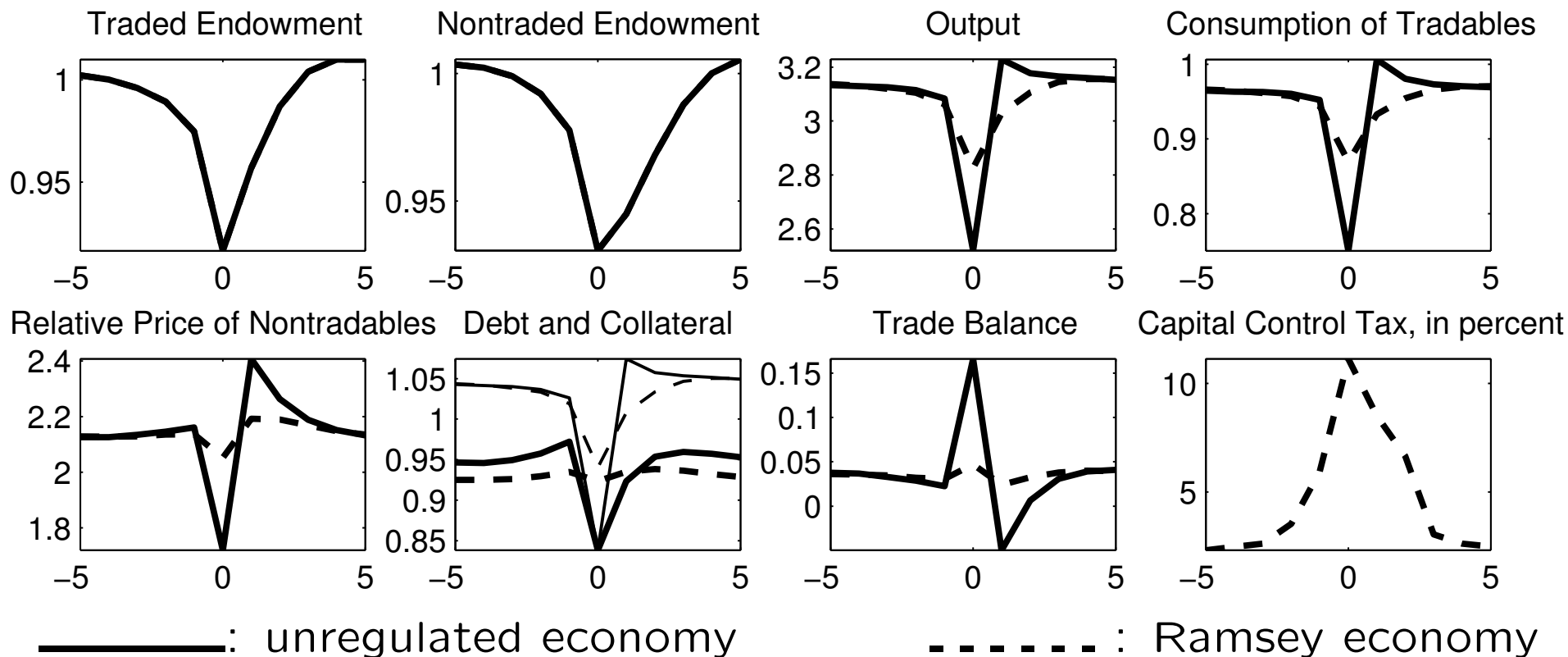
this should lower debt. Mean debt in the Ramsey economy is 0.926 (or 28.5% of output ) as opposed to 0.9483 (or 29.2% of output) in the unregulated economy.

Positive taxes help the economy stay clear of a binding constraint.

- In addition taxes are quite cyclical,  $\text{std}(\tau_t)$  is 4.2%. This also helps avoid the binding constraint
- Frequency of binding constraint in Ramsey 3.9% (or once every 26 years) and in unregulated economy 8.5% (or once every 12 years).

Why is it so important to avoid a binding constraint? Because it leads to a deep (albeit short) contraction:

## The Typical Financial Crisis in the Endowment-Shock Economy



Note. Each line is the mean across all 11-year windows containing a binding collateral constraint in the center in a one-million-year time series from the unregulated economy. For the capital-control tax rate, the figure displays the median instead of the mean across windows because this variable is skewed, with an unconditional mean of 4.2 percent and an unconditional median of 2.5 percent. Because the capital control tax rate is indeterminate when the collateral constraint binds under the Ramsey policy, this variable is given a number only if the collateral constraint is slack under the Ramsey policy. Replication file `typical_crisis.m` in `sgu_endowment_shocks.zip`, on the authors' websites.



Observation:

Ramsey planner **raises** capital control taxes in run up to crisis and lowers them once crisis is over.  $\Rightarrow$  optimal capital controls are **not** countercyclical

## What about unconditionally?

$$\text{corr}(\tau_t, \ln y_t) = -\mathbf{0.84},$$

$$\text{corr}(\tau_t, \ln c_t^T) = -\mathbf{0.88}$$

⇒ optimal capital controls are **not** countercyclical in that sense either.

## Alternative Calibration: Traded Endowment and Interest Rate Shocks

Why? Interest-rate shocks are important source of fluctuations for emerging markets. Further, interest rate shocks may exacerbate the pecuniary externality. Periods of low interest rates may induce agents to borrow too much making them vulnerable to a binding collateral constraint once rates rise again.

Parameters other than those related to the driving forces: keep as in baseline calibration, in particular, keep  $\beta(1 + r) = 0.95$

Annualize the quarterly process estimated in Schmitt-Grohé and Uribe (2016). There we use Argentine quarterly data over the period 1983:Q1 to 2001:Q4. The resulting annual process is

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t, \quad (16)$$

where  $\epsilon_t \sim N(\emptyset, \Sigma_\epsilon)$ , with

$$A = \begin{bmatrix} 0.48 & -0.77 \\ -0.08 & 0.68 \end{bmatrix}; \quad \Sigma_\epsilon = \begin{bmatrix} 0.0031 & -0.0015 \\ -0.0015 & 0.0014 \end{bmatrix}; \quad r = 0.1325.$$

## Some Unconditional Summary Statistics

Statistic	$y^T$	$r$
Mean	1	13.25%
Std. Dev.	11.7%	6.5%
Serial Corr.	0.85	0.78
Corr( $y_t^T, r_t$ )	-0.87	

Comments:

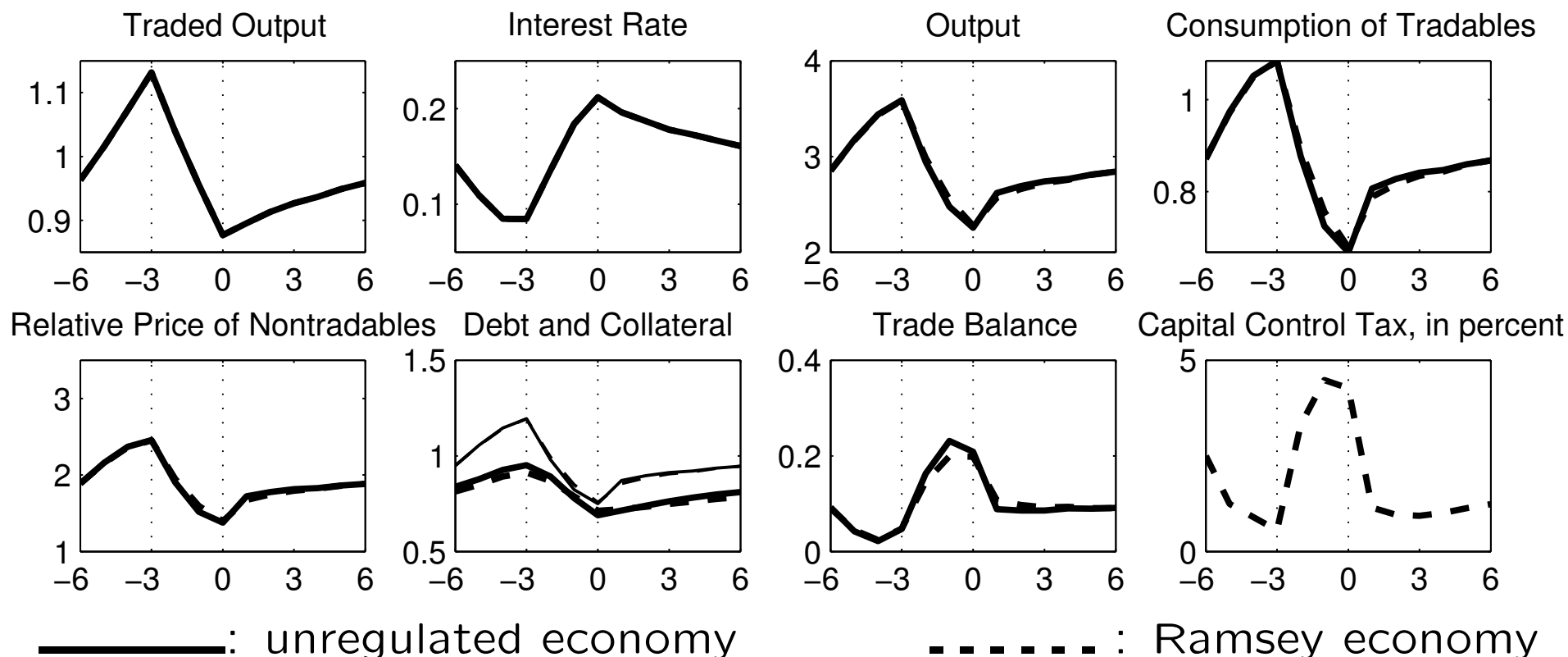
- (1) High volatility of tradable  $y_t^T$  and  $r_t$ .
- (2) Negative correlation between  $y_t^T$  and  $r_t$ , when it rains it pours.
- (3) High mean country interest rate, 13 percent per year.

## Calibration of the Economy with Interest-Rate Shocks

Parameter	Value	Description
$\kappa$	0.3328	Parameter of collateral constraint
$\sigma$	2	Inverse of intertemporal elast. of subst.
$\beta$	0.8357	Subjective discount factor
$r$	0.1325	Steady state country interest rate
$\xi$	0.83	Intratemporal elast. of subst.
$a$	0.31	Weight on tradables in CES aggregator
$y^N$	1	Nontradable output
$y^T$	1	Steady-state tradable output
$n_{y^T}$	21	Grid points for $\ln y_t^T$ , equally spaced
$n_r$	11	Grid points for $\ln \left( \frac{1+r_t}{1+r} \right)$ , equally spaced
$n_d$	800	Grid points for $d_t$ , equally spaced
$[\ln y^T, \ln \bar{y}^T]$	$[-0.3706, 0.3706]$	Range for tradable output
$\left[ \ln \left( \frac{1+r}{1+r} \right), \ln \left( \frac{1+\bar{r}}{1+r} \right) \right]$	$[-0.2040, 0.2040]$	Range for interest rate
$[d, \bar{d}]$	$[-0.5, 1.5]$	Range for debt

Note. The time unit is one year.

## Interest-Rate Shocks and The Typical Boom-Bust Cycle



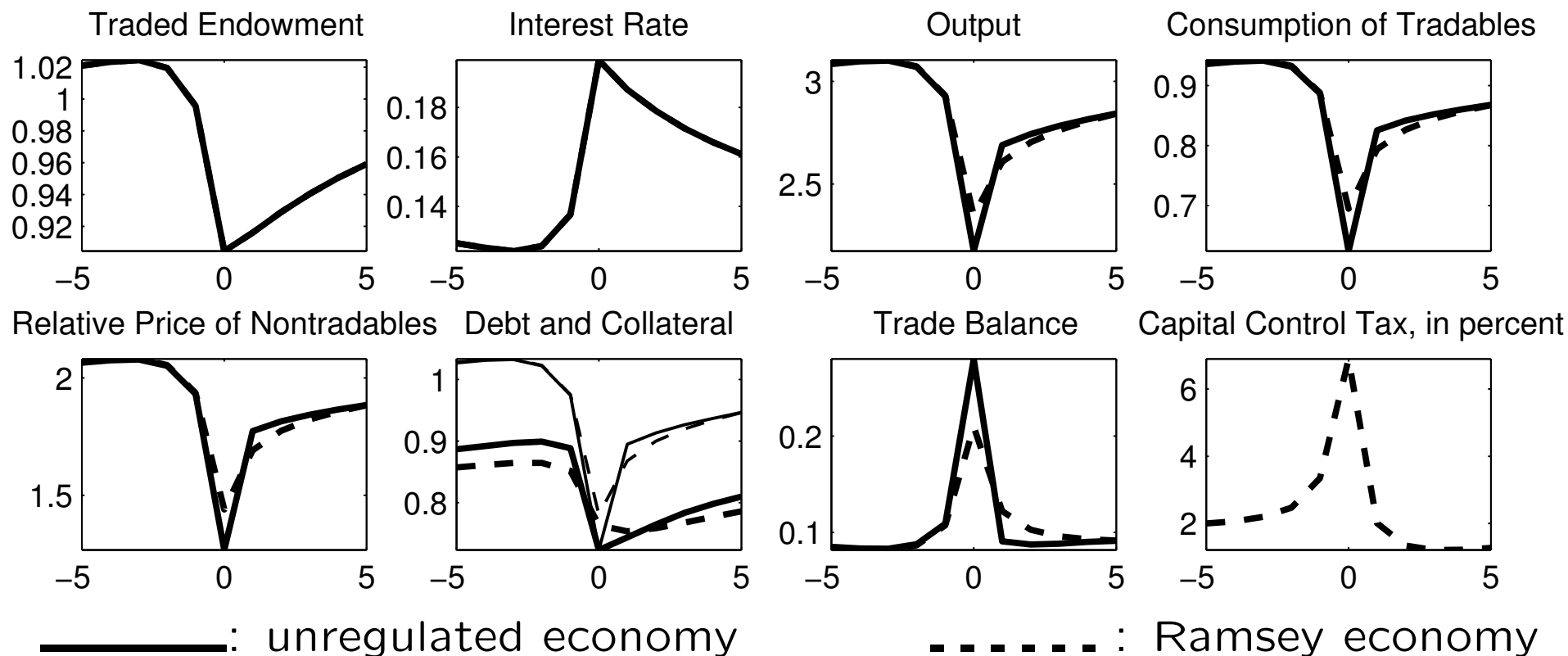
Note. Each line is the mean across all windows containing a boom-bust cycle in a time series of 1 million years. For the capital-control tax rate, the figure displays the median. Replication file `typical_boom_bust.m` in `sgu_rshocks.zip`, on the authors' websites.

Observation on the figure:

The optimal capital control tax is procyclical over the boom bust episode.



## The Typical Financial Crisis in the Interest-Rate Economy



Note. Each line is the mean across all 11-year windows containing a binding collateral constraint in the center in a one-million-year time series from the unregulated economy. For the capital-control tax rate, the figure displays the median instead of the mean across windows because this variable is highly skewed. Because the capital control tax rate is indeterminate when the collateral constraint binds under the Ramsey policy, in the figure this variable is given a number only if the collateral constraint is slack under the Ramsey policy. Replication file `typical_crisis.m` in `sgu_rshocks.zip`, on the authors' websites.

Observation on the figure:

The optimal capital control tax is procyclical in a financial crisis

## Conclusion

- Ramsey optimal capital control taxes make households fully internalize the collateral constraint induced pecuniary externality.
- Ramsey optimal capital control taxes are found to be procyclical, they are raised during recessions and lowered during booms. Therefore, the pecuniary externality does not support adoption of cyclical macroprudential policy.
- What drives the result?— Ramsey planner navigates a tradeoff between allowing agents to frontload consumption as much as possible and avoiding a binding collateral constraint.

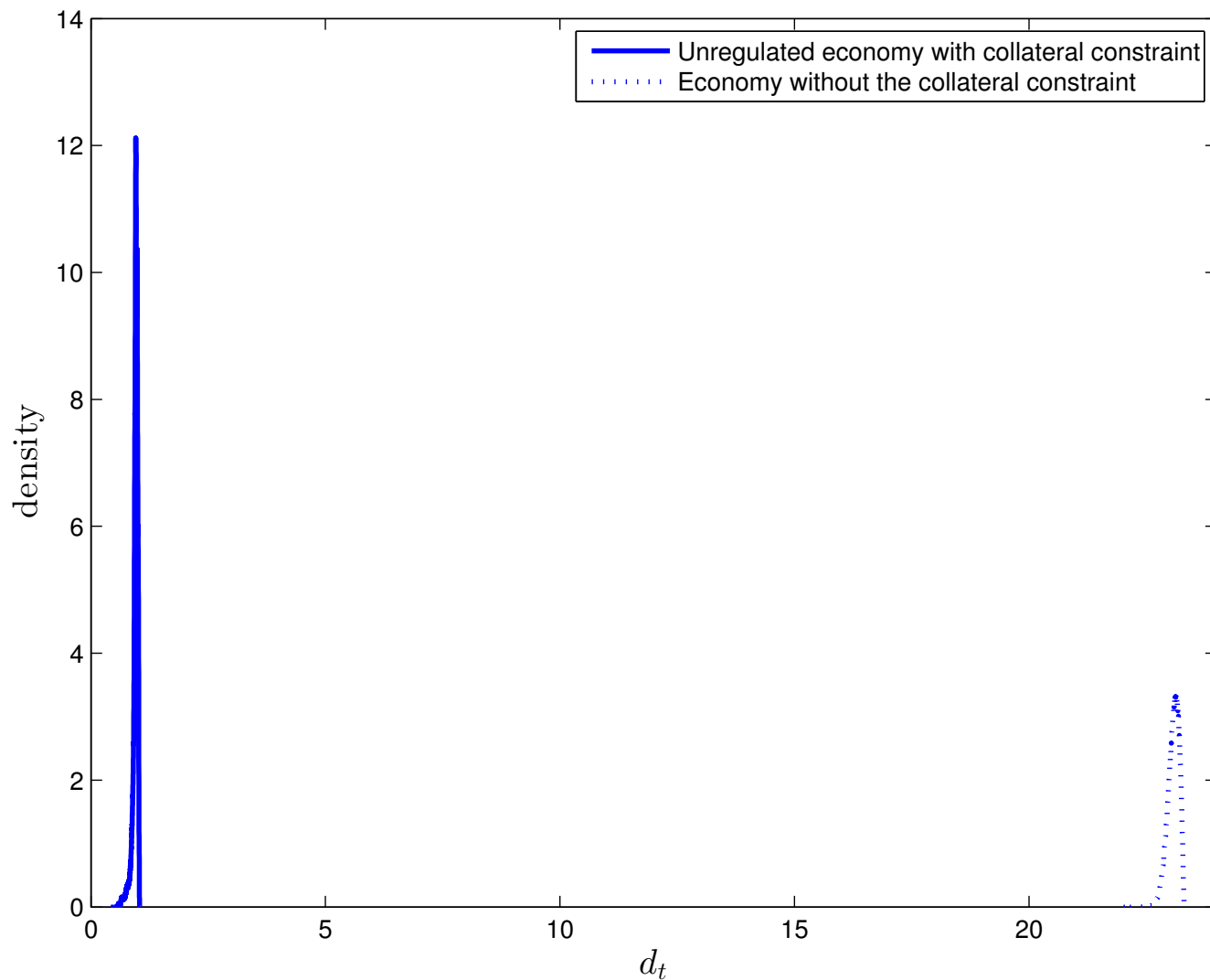
**EXTRAS**

## Debt, Frequency of Crises, and Optimal Capital Controls

Environment	Debt-to-Output Ratio		Frequency of Crises		Optimal Capital Controls	
	Unregulated	Ramsey	Unregulated	Ramsey	median ( $\tau_t$ )	corr( $\tau_t, y_t$ )
$y_t^T$ and $y_t^N$ shocks	29.2%	28.5%	12 years	26 years	2.5%	-0.8
$y_t^T$ and $r_t$ shocks	29.3%	28.3%	14 years	37 years	1.9%	-0.1

Note. The debt-to-output ratio is the unconditional mean of  $\frac{d_{t+1}/(1+r_t)}{y_t}$ . The variable  $y_t \equiv y_t^T + p_t y_t^N$  denotes output in terms of tradables. A crisis is defined as a period with a binding collateral constraint. Replication files: for line 1, table.m in sgu\_endowment\_shocks.zip, and for line 2, table.m in sgu\_rshocks.zip, both on the authors' websites.

## Debt Densities



Collateral constraint shifts mean of debt from 23.1 to 0.9.

The natural debt limit is 23.3.

With collateral constraint:

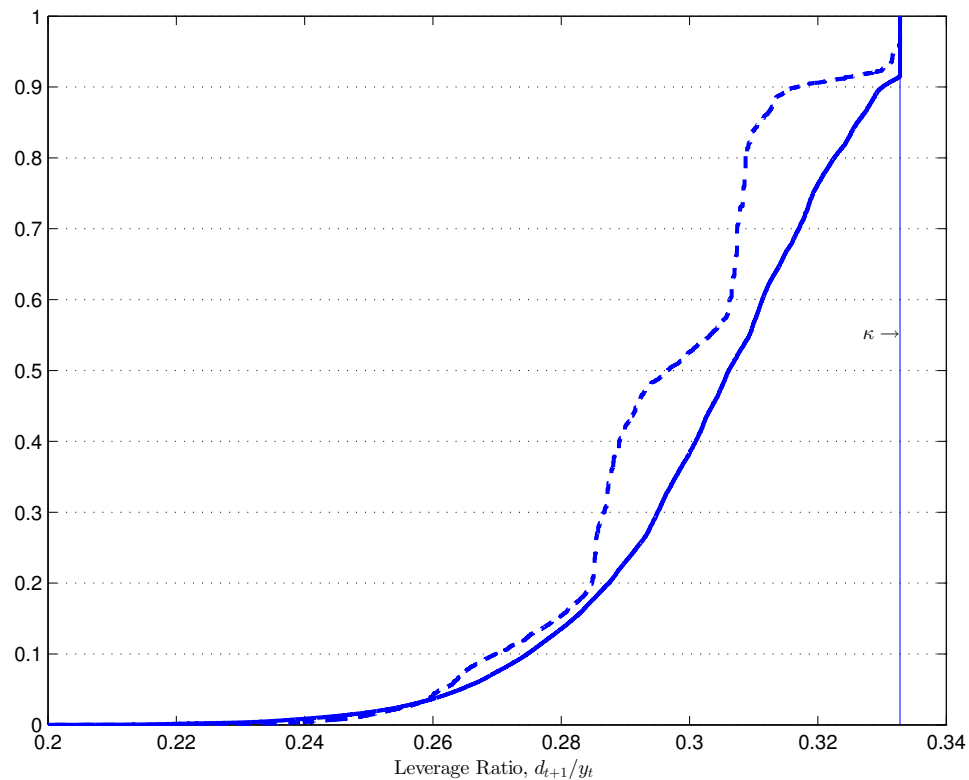
$$E\left(\frac{d_{t+1}}{(1+r)y_t}\right) = 29.2\%$$

Without collateral constraint:

$$E\left(\frac{d_{t+1}}{(1+r)y_t}\right) = 1,993\%$$

## Leverage in the Endowment-Shock Economy

### Cumulative Probability Distribution of Leverage



$$\text{leverage ratio} = \frac{d_{t+1}}{y_t^T + p_t y_t^N}$$

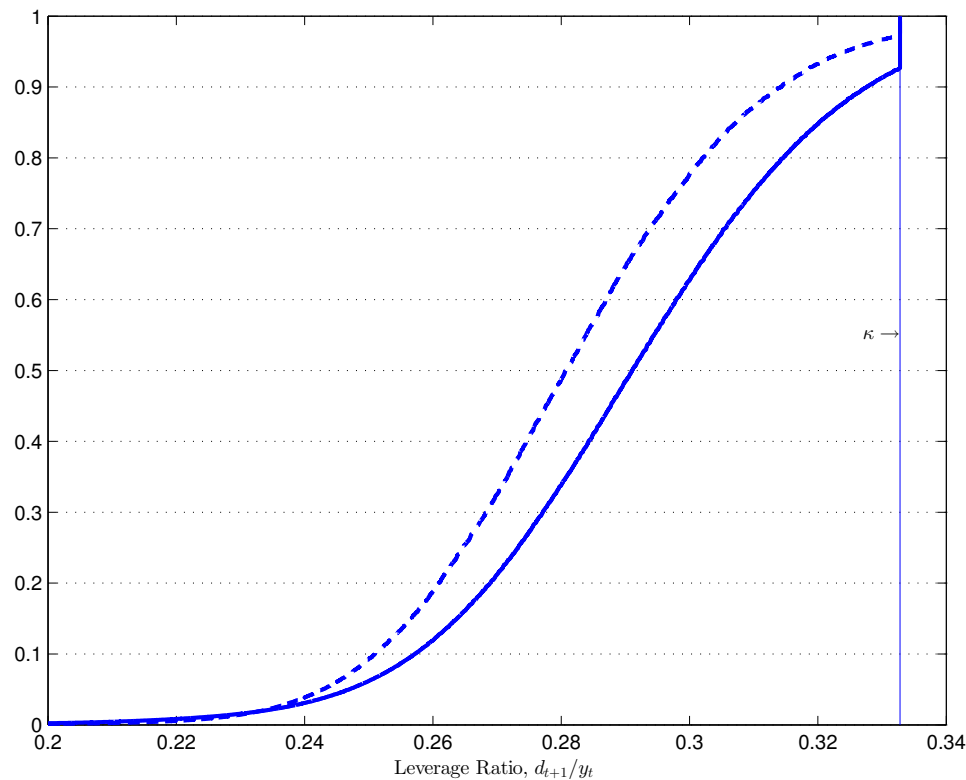
#### Unregulated Economy:

- More mass at  $\kappa$ .
- and always closer to the limit,  $\kappa$ .

—: unregulated economy    - - -: Ramsey economy

## Leverage in the Interest-Rate-Shock Economy

### Cumulative Probability Distribution of Leverage



$$\text{leverage ratio} = \frac{d_{t+1}}{y_t^T + p_t y_t^N}$$

#### Unregulated Economy:

- More mass at  $\kappa$ .
- and always closer to the limit,  $\kappa$ .

—: unregulated economy    - - -: Ramsey economy

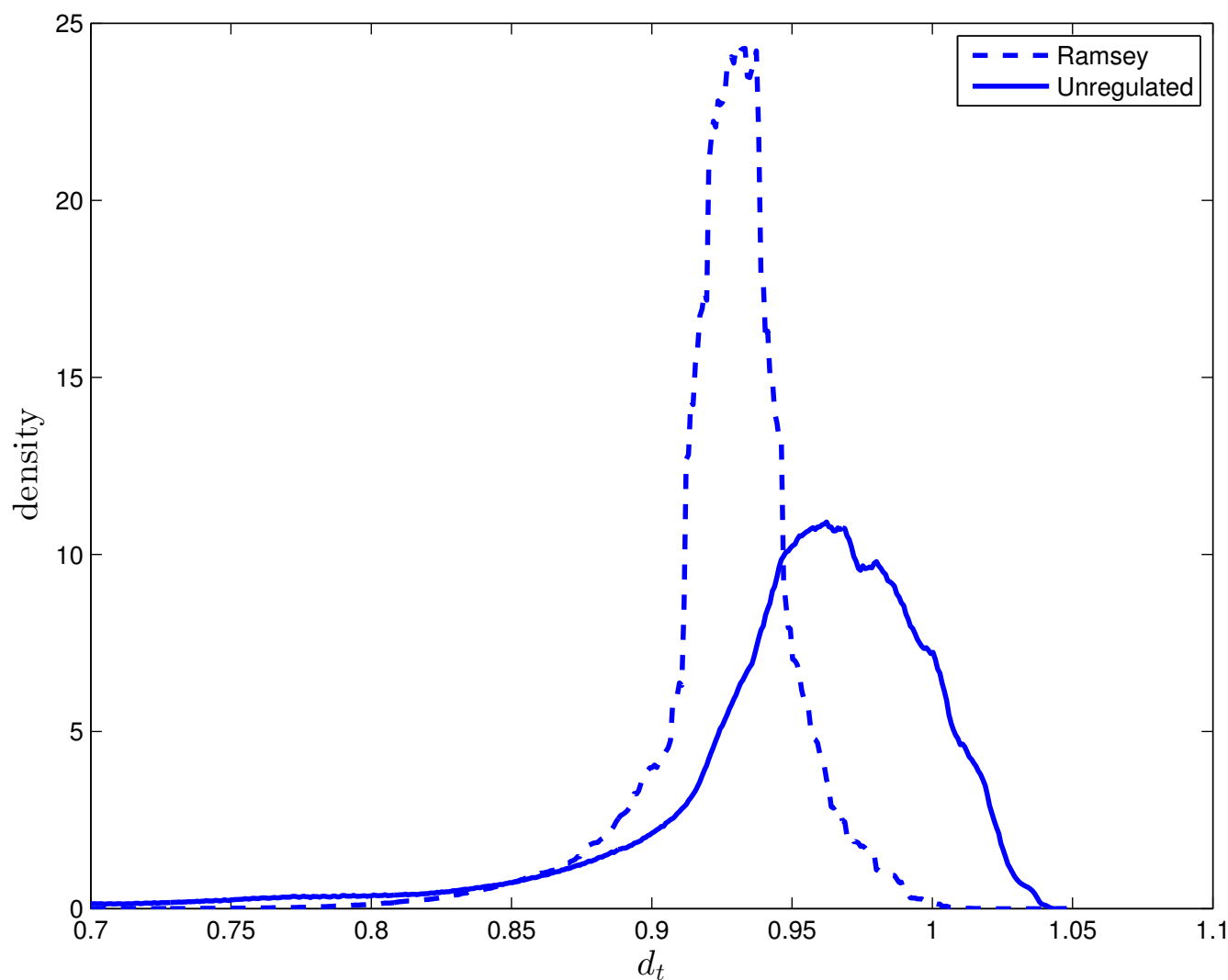


## Overborrowing

Definition: the unregulated economy is said to **overborrow** if its average level of external debt is **higher** than that of the Ramsey economy.

Comment: To our knowledge there does not exist an analytical proof that economies with a pecuniary externality due to a flow collateral constraint of the type analyzed here **must** display overborrowing. Overborrowing seems to be calibration dependent. [We have shown elsewhere (Schmitt-Grohé and Uribe, 2016) that economies of the type studied here may display underborrowing. Our analytical proof was for an economy without uncertainty and with  $\beta(1 + r) = 1$ . There we also show underborrowing in a calibrated stochastic economy with  $\beta(1 + r) < 1$ .]

## Modest Amount of Overborrowing under the Bianchi Calibration



Under Ramsey Optimal Capital Controls:

$$E\left(\frac{d_{t+1}}{(1+r)y_t}\right) = 28.5\%$$

With collateral constraint:

$$E\left(\frac{d_{t+1}}{(1+r)y_t}\right) = 29.2\%$$

⇒ Pecuniary externality leads to overborrowing of 0.7 percentage points of output

(These results are our replication of those reported in Bianchi. He reports, 28.6 and 29.2)