The Neo-Fisher Effect
Econometric Evidence from Empirical and Optimizing Models

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What is the effect of an interest-rate shock on inflation?

The answer depends on (a) whether the change in the interest rate is expected to be transitory or permanent; and (b) the time horizon.

### Effect of an Increase in the Nominal Interest Rate \((i)\) on Inflation \((\pi)\)

<table>
<thead>
<tr>
<th></th>
<th>Long Run Effect on (\pi)</th>
<th>Short Run Effect on (\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitory increase in (i)</td>
<td>0</td>
<td>↓</td>
</tr>
<tr>
<td>Permanent increase in (i)</td>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

Entry (2,1) is the Fisher effect.
Entry (2,2) is the Neo-Fisher effect.
This Paper presents an econometric investigation of the effects of permanent and temporary movements in the nominal interest rate on inflation, output, and the real interest rate.

- **Two Frameworks:**
  - An empirical model
  - A New-Keynesian model

- Both models estimated on (the same) postwar data.
Main Findings

• A permanent monetary shock causes inflation and the nominal interest rate to increase to their permanently higher levels in the short run (within a year) and entails no output loss.

• A temporary increase in the nominal interest rate causes a fall in inflation and output in the short run.
Preliminaries: Evidence on the Fisher Effect

• Let $i$, $r$, and $\pi$ denote average values of the nominal interest rate, the real interest rate, and the inflation rate. Then, assuming that on average expected inflation equals actual inflation, the Fisher equation says that

$$i = r + \pi.$$ 

• Further assuming that the average real interest rate is primarily determined by real factors (demographics, technology, etc.) and that these factors are more stable than monetary factors across time and space, the Fisher equation implies a positive relationship between the nominal interest rate and the rate of inflation.

• The following two figures provide cross-sectional and time series evidence consistent with the validity of the Fisher hypothesis in the long run.
Average Inflation and Nominal Interest Rates: Cross-Country Evidence

Notes. Each dot represents one country. The solid line is the 45-degree line. Average sample 1989 to 2012. Source: WDI.
Inflation and the Nominal Interest Rate in the United States

Notes. Quarterly frequency, annualized rates.
The Empirical Model

\[
\begin{bmatrix}
y_t \\
\pi_t \\
i_t \\
\end{bmatrix} = \begin{bmatrix}
\text{log of real output} \\
\text{inflation} \\
\text{policy rate} \\
\end{bmatrix}; \quad \begin{bmatrix}
\hat{y}_t \\
\hat{\pi}_t \\
\hat{i}_t \\
\end{bmatrix} = \begin{bmatrix}
y_t - X^m_t \\
\pi_t - X^m_t \\
i_t - X^m_t \\
\end{bmatrix}.
\]

\[
\begin{bmatrix}
\hat{y}_t \\
\hat{\pi}_t \\
\hat{i}_t \\
\end{bmatrix} = B(L) \begin{bmatrix}
\hat{y}_{t-1} \\
\hat{\pi}_{t-1} \\
\hat{i}_{t-1} \\
\end{bmatrix} + C \begin{bmatrix}
\Delta X^m_t \\
z^m_t \\
\Delta X^n_t \\
z^n_t \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta X^m_t \\
z^m_t \\
\Delta X^n_t \\
z^n_t \\
\end{bmatrix} = \rho \begin{bmatrix}
\Delta X^m_{t-1} \\
z^m_{t-1} \\
\Delta X^n_{t-1} \\
z^n_{t-1} \\
\end{bmatrix} + \psi \begin{bmatrix}
\epsilon^1_t \\
\epsilon^2_t \\
\epsilon^3_t \\
\epsilon^4_t \\
\end{bmatrix}
\]

where $X^m_t =$ permanent monetary shock; $X^n_t =$ permanent nonmonetary shock; $z^m_t =$ transitory monetary shock; and $z^n_t =$ transitory nonmonetary shock. Innovations $\epsilon^i_t \sim \text{iid} N(0,1)$, for $i = 1, 2, 3, 4$, and $\rho, \psi$ diagonal.
Observables and Observation Equations

- $\Delta y_t$, growth rate of real output per capita.

- $r_t \equiv i_t - \pi_t$, interest-rate-inflation differential.

- $\Delta i_t \equiv i_t - i_{t-1}$, time difference of the nominal interest rate.

We then have the following observation equations:

$$\begin{align*}
\Delta y_t &= \hat{y}_t - \hat{y}_{t-1} + \Delta X^n_t \\
 r_t &= \hat{i}_t - \hat{\pi}_t \\
\Delta i_t &= \hat{i}_t - \hat{i}_{t-1} + \Delta X^m_t
\end{align*}$$
Identification Assumptions

- Output \((y_t)\) is cointegrated with the permanent nonmonetary shock \((X_t^n)\).

- Inflation \((\pi_t)\) is cointegrated with the permanent monetary shock \((X_t^m)\).

- The nominal interest rate \((i_t)\) is cointegrated with the permanent monetary shock \((X_t^m)\).

- A transitory increase in the interest rate \((z_t^m \uparrow)\) has a nonpositive impact effect on inflation.

- A transitory increase in the interest rate \((z_t^m \uparrow)\) has a nonpositive impact effect on output.

◊ **Identifiability:** The model passes the Iskrev (2010) test.
The Neo-Fisher Effect in the Empirical Model

United States, 1954.Q4 to 2018.Q2
Impulse Responses to Interest-Rate Shocks: Empirical Model Estimated on U.S. Data
1954.Q4 to 2018.Q2

Permanent Interest–Rate Shock
Response of the Interest Rate and Inflation

Temporary Interest–Rate Shock
Response of the Interest Rate and Inflation

Permanent Interest–Rate Shock
Response of Output

Temporary Interest–Rate Shock
Response of Output
Response of the Real Interest Rate to Permanent and Transitory Interest-Rate Shock in the Empirical Model

Notes. Posterior mean estimates. The real interest rate is defined as $i_t - E_t \pi_{t+1}$.
U.S. Inflation and Its Permanent Component

$\pi_t$ and Inferred I Values of $X_t^m$

Note. Quarterly frequency. Smoothed using the Kalman filter. Initial value of $X_t^m$ normalized to match observed average inflation.
The Volcker Disinflation

The Neo-Fisher Effect: Econometric Evidence

Martín Uribe

The Volcker Disinflation

% per year


year

\( X_t^m \)

Federal Funds rate

inflation

1980:Q4
## Variance Decomposition: Empirical Model

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>$\Delta y_t$</th>
<th>$\Delta \pi_t$</th>
<th>$\Delta i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent Monetary Shock, $\Delta X^m_t$</td>
<td>9.1</td>
<td>44.6</td>
<td>21.9</td>
</tr>
<tr>
<td>Transitory Monetary Shock, $z^m_t$</td>
<td>2.1</td>
<td>6.2</td>
<td>10.9</td>
</tr>
<tr>
<td>Permanent Non-Monetary Shock, $\Delta X^n_t$</td>
<td>49.8</td>
<td>27.9</td>
<td>13.5</td>
</tr>
<tr>
<td>Transitory Non-Monetary Shock, $z^n_t$</td>
<td>39.1</td>
<td>21.4</td>
<td>53.7</td>
</tr>
</tbody>
</table>

Note. Posterior means. The variables $\Delta y_t$, $\Delta \pi_t$, and $\Delta i_t$ denote output growth, the change in inflation, and the change in the nominal interest rate, respectively.
Robustness Checks

(1) Truncating the sample at the beginning of the zero-lower-bound period.

(2) Estimating the empirical model on Japanese data.

(3) No cointegration of inflation with the nominal interest rate.

(4) Including the ten-year spread.

(4) Allowing for correlation between $X_t^m$ and $X_t^n$.

(5) CEE identification of transitory monetary shock: zero impact effect of $z_t^m$ on $\pi_t$ and $y_t$. 
Robustness Check 1

Truncating the Sample at the Beginning of the Zero-Lower-Bound Period
Impulse Responses to Interest-Rate Shocks: Empirical Model, Sample 1954.4 to 2008.4

Permanent Interest–Rate Shock Response of the Interest Rate and Inflation

Temporary Interest–Rate Shock Response of the Interest Rate and Inflation

Permanent Interest–Rate Shock Response of Output

Temporary Interest–Rate Shock Response of Output
Robustness Check 2

Estimating the Empirical Model on Japanese Data

Permanent Interest–Rate Shock Response of the Interest Rate and Inflation

Response of Output

Temporary Interest–Rate Shock Response of the Interest Rate and Inflation

Response of Output
Robustness Check 3

No Cointegration of Inflation with the Nominal Interest Rate
A simple regression of the nominal interest rate onto inflation yields:

\[ i_t = 1.42 + 1.053\pi_t + \epsilon_t \]

This result suggests some positive correlation between inflation and the real interest rate. To explore this issue more rigorously, consider modifying the empirical model by introducing the parameter \( \gamma \) such that

\[ i_t - \gamma X_t^m \quad \text{and} \quad \pi_t - X_t^m \]

are stationary. The baseline value assumes that \( \gamma = 1 \) (inflation cointegrated with the nominal interest rate).

Prior: Assume that \( (\gamma - 0.7)/0.6 \) has a beta distribution with mean 1/2 and standard deviation 1/4. Thus, \( \gamma \) has support \([0.7, 1.3]\), a mean of 1, and a standard deviation of 0.15.

Observables: We can no longer use \( r_t \equiv i_t - \pi_t \) as it is nonstationary when \( \gamma \neq 1 \). Instead, we use \( \Delta\pi_t \equiv \pi_t - \pi_{t-1} \).

Posterior: \( E(\gamma) = 1.088; \ \text{std}(\gamma) = 0.117, \ [5\%, 95\%] \) posterior interval \([0.876, 1.257]\).
Impulse Responses to Interest-Rate Shocks: Empirical Model

Noncointegrated $i_t$ and $\pi_t$
Robustness Check 4

Including the Ten-Year Spread
Expanding the Empirical Model

Stationary variables;
\[ y_t - X^n_t; \pi_t - \alpha \pi X^m_t, i_t - X^m_t, \text{ and } i_{10}^t - X^m_t, \] where \( i_{10}^t \) is the ten-year rate, and \( \alpha \pi \) is a parameter.

Observables:
\[ \Delta y_t, \Delta \pi_t, \Delta i_t, \text{ and } i_{10}^t - i_t. \]
Impulse Responses to Interest-Rate Shocks Including the Ten-Year Spread

Permanent Interest–Rate Shock
Response of the Interest Rate and Inflation

Temporary Interest–Rate Shock
Response of the Interest Rate and Inflation

Permanent Interest–Rate Shock
Response of Output

Temporary Interest–Rate Shock
Response of Output
Robustness Check 5

Christiano-Eichenboum-Evans Identification of Transitory Monetary Shock
Impulse Responses to Interest-Rate Shocks

CEE Identification

Permanent Interest–Rate Shock
Response of the Interest Rate and Inflation

Temporary Interest–Rate Shock
Response of the Interest Rate and Inflation

Permanent Interest–Rate Shock
Response of Output

Temporary Interest–Rate Shock
Response of Output
A Standard New-Keynesian Model
Households

$$\max E_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left\{ \frac{\left[ (C_t - \delta \tilde{C}_{t-1})(1 - e^{\theta_t h_t})^\chi \right]^{1-\sigma} - 1}{1-\sigma} \right\},$$

subject to

$$\int_0^1 P_{it} C_{it} di + \frac{B_{t+1}}{1+I_t} + T_t = B_t + W_t^n h_t + \Phi_t,$$

$$C_t = \left[ \int_0^1 C_{it}^{1-1/\eta} di \right]^{\frac{1}{1-1/\eta}},$$

where $C_{it} =$ consumption of variety $i$; $C_t =$ consumption of composite good; $\tilde{C}_t =$ cross-sectional average of $C_t$; $h_t =$ hours worked; $B_t =$ nominal bond; $I_t =$ nominal interest rate; $P_{it} =$ price of variety $i$; $W_t^n =$ nominal wage; $\Phi_t =$ nominal profit income; $T_t =$ nominal lump-sum taxes; $\xi_t =$ preference shock; $\theta_t =$ labor-supply shock.
Firms

$$\max \ E_0 \sum_{t=0}^{\infty} q_t \left[ \frac{P_{it}}{P_t} C_{it} - \frac{W^n_t}{P_t} h_{it} - \frac{\phi}{2} X^n_t \left( \frac{P_{it}}{\tilde{X}_t^m P_{it-1}} - 1 \right)^2 \right],$$

subject to

$$Y_{it} \geq C_{it},$$

$$C_{it} = C_t \left( \frac{P_{it}}{P_t} \right)^{-\eta},$$

$$Y_{it} = e^{z_t} X_t^n h_{it}^\alpha,$$

where

$$\tilde{X}_t^m \equiv (X_t^m)^\gamma_m (\tilde{X}_{t-1}^m)^{1-\gamma_m} = \text{indexation factor}; \text{ and}$$

$$X_t^m = \text{permanent component of inflation, defined later}$$

and $P_t =$ price of composite consumption good; $h_{it} =$ hours employed by firm $i$; $q_t =$ discount factor; $Y_{it} =$ output of firm $i$; $X_t^n =$ permanent tech. shock; $z_t =$ transitory tech. shock.
Monetary Policy

\[
\frac{1 + I_t}{X_t^m} = \left( \frac{1 + \Pi_t}{X_t^m} \right)^{\alpha_{\pi}} \left( \frac{Y_t}{X_t^n} \right)^{\alpha_y} e^{z_t^m},
\]

where \( \Pi_t \equiv P_t/P_{t-1} - 1 \) = inflation rate; \( X_t^m \) = permanent monetary shock; \( z_t^m \) = transitory monetary shock.

Also allow for policy inertia by including \( I_{t-1} \) on the right-hand side.

**Fiscal Policy:** Passive (or Ricardian); no government consumption.
Estimation

- Same data and sample as in the estimation of the empirical model.
- Estimate a subset of the model’s parameters and calibrate the rest.
- Apply likelihood-based Bayesian techniques (same as in the estimation of the empirical model).
### Calibrated Parameters in the New Keynesian Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9982</td>
<td>subjective discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>inverse of intertemp. elast. subst.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>6</td>
<td>intratemporal elast. of subst.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>labor semielast. of output</td>
</tr>
<tr>
<td>$g$</td>
<td>0.004131</td>
<td>mean output growth rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.4055</td>
<td>preference parameter</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.625</td>
<td>preference parameter</td>
</tr>
</tbody>
</table>

Note. The time unit is one quarter.
## Prior and Posterior Parameter Distributions: New Keynesian Model

<table>
<thead>
<tr>
<th>Param.</th>
<th>Distrib.</th>
<th>Mean</th>
<th>Std</th>
<th>Mean</th>
<th>Std</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Gamma</td>
<td>50</td>
<td>20</td>
<td>159</td>
<td>31.3</td>
<td>111</td>
<td>214</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>Gamma</td>
<td>1.5</td>
<td>0.25</td>
<td>1.83</td>
<td>0.31</td>
<td>1.35</td>
<td>2.37</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>Gamma</td>
<td>0.125</td>
<td>0.1</td>
<td>0.687</td>
<td>0.2</td>
<td>0.386</td>
<td>1.03</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Uniform</td>
<td>0.5</td>
<td>0.289</td>
<td>0.464</td>
<td>0.195</td>
<td>0.201</td>
<td>0.851</td>
</tr>
<tr>
<td>$\gamma_I$</td>
<td>Uniform</td>
<td>0.5</td>
<td>0.289</td>
<td>0.579</td>
<td>0.108</td>
<td>0.366</td>
<td>0.722</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Uniform</td>
<td>0.5</td>
<td>0.289</td>
<td>0.294</td>
<td>0.0508</td>
<td>0.21</td>
<td>0.378</td>
</tr>
<tr>
<td>$\rho_\xi$</td>
<td>Beta</td>
<td>0.7</td>
<td>0.2</td>
<td>0.902</td>
<td>0.0259</td>
<td>0.856</td>
<td>0.941</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>Beta</td>
<td>0.7</td>
<td>0.2</td>
<td>0.673</td>
<td>0.201</td>
<td>0.305</td>
<td>0.954</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.7</td>
<td>0.2</td>
<td>0.667</td>
<td>0.206</td>
<td>0.289</td>
<td>0.954</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.2</td>
<td>0.403</td>
<td>0.0915</td>
<td>0.236</td>
<td>0.538</td>
</tr>
<tr>
<td>$\rho_{gm}$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.2</td>
<td>0.331</td>
<td>0.176</td>
<td>0.0553</td>
<td>0.625</td>
</tr>
<tr>
<td>$\rho_{zm}$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.2</td>
<td>0.195</td>
<td>0.126</td>
<td>0.0346</td>
<td>0.432</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>Gamma</td>
<td>0.01</td>
<td>0.01</td>
<td>0.0251</td>
<td>0.00393</td>
<td>0.0199</td>
<td>0.0325</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Gamma</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00164</td>
<td>0.0013</td>
<td>0.000119</td>
<td>0.00417</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Gamma</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00124</td>
<td>0.001</td>
<td>9.22e-05</td>
<td>0.00318</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>Gamma</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00626</td>
<td>0.000841</td>
<td>0.00492</td>
<td>0.00769</td>
</tr>
<tr>
<td>$\sigma_{gm}$</td>
<td>Gamma</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.00103</td>
<td>0.00032</td>
<td>0.000567</td>
<td>0.0016</td>
</tr>
<tr>
<td>$\sigma_{zm}$</td>
<td>Gamma</td>
<td>0.0025</td>
<td>0.0025</td>
<td>0.00155</td>
<td>0.000271</td>
<td>0.00107</td>
<td>0.00189</td>
</tr>
</tbody>
</table>
Observations on Estimation

- Parameters are estimated with significant uncertainty (common feature of estimated small optimizing macro models).

- Nonetheless, the estimation is successful along three dimensions:
  - The data speaks with a strong voice with respect to the degrees of price stickiness, $\phi$, and habit formation $\delta$, which define the propagation of nominal and real shocks.
  - The optimizing model recovers a permanent monetary shock, $X_t^m$, similar to the one inferred from the empirical model (see next slide).
  - The optimizing model predicts a contribution of permanent monetary shocks to inflation changes similar to that predicted by the empirical model (see the slide after the next).
Inflation and Its Permanent Component: New-Keynesian Model

![Graph showing inflation and its permanent component with New-Keynesian model and Empirical model.](image)
## Variance Decomposition: New Keynesian Model

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>$g_t^m$</th>
<th>$z_t^m$</th>
<th>$g_t$</th>
<th>$z_t$</th>
<th>$\xi_t$</th>
<th>$\theta_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent Monetary Shock, $g_t^m$</td>
<td>1.7</td>
<td>42.8</td>
<td>9.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transitory Monetary Shock, $z_t^m$</td>
<td>3.0</td>
<td>2.1</td>
<td>35.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permanent Productivity Shock, $g_t$</td>
<td>84.7</td>
<td>2.2</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transitory Productivity Shock, $z_t$</td>
<td>0.4</td>
<td>5.1</td>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preference Shock, $\xi_t$</td>
<td>9.7</td>
<td>42.8</td>
<td>46.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor-Supply Shock, $\theta_t$</td>
<td>0.4</td>
<td>5.1</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Permanent Interest–Rate Shock
Response of the Interest Rate and Inflation

Temporary Interest–Rate Shock
Response of the Interest Rate and Inflation

Permanent Interest–Rate Shock
Response of Output

Temporary Interest–Rate Shock
Response of Output
Response of the Real Interest Rate to Permanent and Transitory Interest-Rate Shock in the New-Keynesian Model

Notes. Posterior mean estimates. The real interest rate is defined as $i_t - E_t \pi_{t+1}$.
Observations on the Previous Three Figures

The main results from the empirical model carry over to the optimizing model:

- In response to a permanent increase in the interest rate, inflation converges to its higher long-run value in the short run.

- The adjustment takes place in the context of low real rates and does not cause output loss.

- A temporary increase in the nominal interest rate triggers a fall in inflation, an increase in real rates, and a contraction in real activity.
Robustness Checks

(1) Truncating the sample at the beginning of the zero-lower-bound period.

(2) Estimating the empirical model on Japanese data.

(3) Allowing for indexation to past inflation.

(4) Add a second transitory monetary shock with high persistence to compete for the data with the permanent monetary shock.

(5) Allowing for long memory indexation, by drawing $\gamma_m$ from the lowest decile of its posterior distribution.
Robustness Check 1

Truncating the Sample at the Beginning of the Zero-Lower-Bound Period
Impulse Responses to Interest-Rate Shocks: New-Keynesian Model Estimated on U.S. Data 1954.4 to 2008.4

Permanent Interest−Rate Shock
Response of the Interest Rate and Inflation

Temporary Interest−Rate Shock
Response of the Interest Rate and Inflation

Permanent Interest−Rate Shock
Response of Output

Temporary Interest−Rate Shock
Response of Output
Robustness Check 2

Estimating the New-Keynesian Model on Japanese Date
Impulse Responses to Interest-Rate Shocks in the New Keynesian Model Estimated on Japanese Data 1955.Q3 to 2016.Q4

Permanent Interest–Rate Shock
Response of the Interest Rate and Inflation

- Interest Rate
- Inflation
- Inflation 95% band

Temporary Interest–Rate Shock
Response of the Interest Rate and Inflation

- Interest Rate
- Inflation
- Inflation 95% band

Permanent Interest–Rate Shock
Response of Output

- Output
- 95% band

Temporary Interest–Rate Shock
Response of Output

- Output
- 95% band
Robustness Check 3

Allowing for Indexation to Past Inflation
Allowing for Indexation to Past Inflation

Assume that the indexation factor $\tilde{X}_t^m$ now takes the form

$$\tilde{X}_t^m = \left[ X_t^m \gamma_{mm} (1 + \Pi_{t-1})^{1-\gamma_{mm}} \right]^{\gamma_m} (\tilde{X}_{t-1}^m)^{1-\gamma_m}$$

with the new parameter $\gamma_{mm} \in [0, 1]$.

Reestimating the model yields a mean posterior value of $\gamma_{mm}$ of 0.061 and a posterior standard deviation of 0.058, suggesting that indexation to past inflation is relevant. The next slide displays the implied impulse responses to permanent and temporary monetary shocks.
Impulse Responses to Interest-Rate Shocks: Allowing for Indexation to Past Inflation in the New Keynesian Model

Permanent Interest–Rate Shock
Response of the Interest Rate and Inflation

Temporary Interest–Rate Shock
Response of the Interest Rate and Inflation

Permanent Interest–Rate Shock
Response of Output

Temporary Interest–Rate Shock
Response of Output
Final Remarks
Discussions of how monetary policy can lift an economy out of chronic below-target inflation are almost always based on the logic of how transitory interest-rate shocks affect real and nominal variables.

Within this logic, a central bank trying to reflate a low-inflation economy will tend to set interest rates as low as possible.

Soon enough these economies find themselves with zero nominal rates and with the low-inflation problem not going away.

At some point, the Fisher effect kicks in, perpetuating the low-interest-rate low-inflation environment.

In this paper, I estimate an empirical model and an optimizing model with temporary and permanent monetary shocks using U.S. and Japanese data. The estimated models produce dynamics consistent with the neo-Fisherian prediction that a credible and gradual increase of nominal interest rates to normal levels can generate a quick reflation of the economy with low real interest rates and no output loss.
EXTRAS
Extras Empirical Model
Measurement Errors

I assume that $\Delta y_t$, $r_t$, and $\Delta i_t$ are observed with error. Letting $o_t$ be the vector of variables observed in quarter $t$, I assume that

$$
o_t = \begin{bmatrix} \Delta y_t \times 100 \\ r_t \\ \Delta i_t \end{bmatrix} + \mu_t \tag{2}
$$

where $\mu_t$ is a 3-by-1 vector of measurement errors distributed i.i.d. $N(\emptyset, R)$, with $R$ diagonal.
State-Space Form

Let

\[
\begin{align*}
\hat{Y}_t & \equiv \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \end{bmatrix}, \\
u_t & \equiv \begin{bmatrix} \Delta X^m_t \\ z^m_t \\ \Delta X^n_t \\ z^n_t \end{bmatrix}, \quad \text{and } \xi_t \equiv \begin{bmatrix} \hat{Y}_t \\ \hat{Y}_{t-1} \\ \vdots \\ \hat{Y}_{t-L+1} \\ u_t \end{bmatrix}.
\end{align*}
\]

Then the system can be written as follows:

\[
\begin{align*}
\xi_{t+1} &= F\xi_t + P\epsilon_{t+1} \\
o_t &= H'\xi_t + \mu_t,
\end{align*}
\]

where the matrices \(F, P,\) and \(H\) are known functions of \(B_i, i = 1, \ldots, L, C, \rho,\) and \(\psi.\)

This representation allows for the use of the Kalman filter to evaluate the likelihood function.
Data and Estimation Technique

- The data are quarterly observations of the U.S. growth rate of output per capita, the nominal-interest-rate-inflation differential, and the change in the nominal interest rate.

- Sample 1954.4 to 2018.2. Output is proxied by real GDP per capita. Inflation is measured by the growth rate of the Implicit GDP Deflator. The nominal interest rate is the Effective Federal Funds Rate.


- The model is estimated with 4 lags using Bayesian techniques.
Priors

• In the spirit of the Minnesota Prior (MP), I assume that at the prior mean the elements of \( \hat{Y}_t \) follow univariate AR(1) processes \((B_1(j, k) = 0 \ \forall j \neq k, \ B_i = 0 \ \forall i > 1)\).

• Also as in the MP, I impose higher prior standard deviations on the diagonal elements of \( B_1 \) than on the remaining elements of \( B_i \) for \( i = 1, \ldots, L \).

• I assume that the prior distribution of \( C_{21} \), governing the impact effect of a permanent interest-rate shock on inflation, is \( N(-1, 1) \). The mean of -1 implies a prior belief that the impact effect of a permanent interest rate shock on inflation, given by \( 1 + C_{21} \), can be positive or negative with equal probability.

• I impose nonnegative serial correlations on exogenous shocks \( \rho_{ii} \geq 0 \), with beta distributions.

• The table on the next slide provides a full description of the assumed prior distributions.
## Prior Distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean.</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main diagonal elements of $B_1$</td>
<td>Normal</td>
<td>0.95</td>
<td>0.5</td>
</tr>
<tr>
<td>Other elements of $B$</td>
<td>Normal</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>$C_{21}, C_{31}$</td>
<td>Normal</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$-C_{12}, -C_{22}$</td>
<td>Gamma</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Other elements of $C$</td>
<td>Normal</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_{ii}, i = 1, 2, 3, 4$</td>
<td>Gamma</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\rho_{ii}, i = 1, 2, 3$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_{44}$</td>
<td>Beta</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>$R_{ii}$</td>
<td>Uniform</td>
<td>$\frac{\text{var}(o_t)}{10 \times 2}$</td>
<td>$\frac{\text{var}(o_t)}{10 \times \sqrt{12}}$</td>
</tr>
</tbody>
</table>
Unit Root Tests

• The Augmented Dickey-Fuller (ADF) test fails to reject the null hypothesis that $y_t$, $i_t$, and $\pi_t$ have a unit root at standard confidence levels ($p$ values of 0.60, 0.13, and 0.14, respectively).

• It rejects the hypothesis that $i_t - \pi_t$ has a unit root at standard confidence levels ($p$ value of 0.04.).
Impulse Responses

- Point estimates are means of a random sample of size 100 thousand with replacement from an MCMC chain of length 1 million of draws from the posterior distribution of impulse responses.

- 95-percent asymmetric error bands are computed using the Sims-Zha method.

- **Transitory Interest-Rate Shock:** Initial shock is set so that the impact effect on the nominal interest rate is 1 annual percentage point.

- **Permanent Interest-Rate Shock:** Initial shock is set so that the posterior-mean long-run increase in the nominal interest rate is 1 percent.
Robustness Check in the Empirical Model

Correlated Monetary and Real Permanent Components
Evolution of the Driving Forces

\[
\begin{bmatrix}
\Delta X^m_t \\
\Delta X^n_t \\
z^m_t \\
z^n_t
\end{bmatrix}
= \rho
\begin{bmatrix}
\Delta X^m_{t-1} \\
\Delta X^n_{t-1} \\
z^m_{t-1} \\
z^n_{t-1}
\end{bmatrix}
+ \Gamma
\begin{bmatrix}
\epsilon^1_t \\
\epsilon^2_t \\
\epsilon^3_t \\
\epsilon^4_t
\end{bmatrix}
\]

In the baseline specification, \( \rho \) is restricted to be diagonal. Assume now that \( \rho_{1,3} \rho_{3,1} \) may be nonzero.

Assume Beta prior distributions for \( 0.5 + \rho_{1,3} \) and \( 0.5 + \rho_{3,1} \) with mean 0.5 and standard deviation 0.2.

Posterior means: \( \rho_{13} = -0.057 \) and \( \rho_{31} = 0.12 \).
Impulse Responses to Interest-Rate Shocks: Empirical Model, Correlated $X_t^m$ and $X_t^n$
Robustness Check in the New-Keynesian Model

Two Transitory Shocks
Monetary Policy

\[ 1 + I_t = A \left( \frac{1 + \pi_t}{X_t^m} \right)^{\alpha_{\pi}} \left( \frac{Y_t}{X_t^n} \right)^{\alpha_y} X_t^m \right]^{1-\gamma I} (1 + I_{t-1})^{\gamma I} e^{z_{1t}^m + z_{2t}^m} \]

Identification by Restrictions on Prior Distributions: Assume that \(0.5 z_{m1}\) and \(0.5 + 0.5 z_{m2}\) distribute Beta with mean 0.3 and standard deviation 0.2.
Impulse Responses to Interest-Rate Shocks: NK Model, Two Transitory Monetary Monetary Shocks

Perm. Interest–Rate Shock

Response of the Interest Rate and Inflation

Low–ρ Temp. Interest–Rate Shock

Response of the Interest Rate and Inflation

High–ρ Temp. Interest–Rate Shock

Response of the Interest Rate and Inflation

Perm. Interest–Rate Shock

Response of Output

Low–ρ Temp. Interest–Rate Shock

Response of Output

High–ρ Temp. Interest–Rate Shock

Response of Output
**Variance Decomposition: New Keynesian Model with Two Transitory Monetary Shocks**

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>$\Delta y_t$</th>
<th>$\Delta \pi_t$</th>
<th>$\Delta i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent Monetary Shock, $g^m_t$</td>
<td>1.5</td>
<td>39.3</td>
<td>9.0</td>
</tr>
<tr>
<td>Low-$\rho$ Transitory Monetary Shock, $z^m_1t$</td>
<td>2.4</td>
<td>1.6</td>
<td>30.1</td>
</tr>
<tr>
<td>High-$\rho$ Transitory Monetary Shock, $z^m_2t$</td>
<td>1.5</td>
<td>6.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Permanent Productivity Shock, $g_t$</td>
<td>85.1</td>
<td>2.0</td>
<td>4.5</td>
</tr>
<tr>
<td>Transitory Productivity Shock, $z_t$</td>
<td>0.4</td>
<td>5.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Preference Shock, $\xi_t$</td>
<td>8.7</td>
<td>41.3</td>
<td>48.9</td>
</tr>
<tr>
<td>Labor-Supply Shock, $\theta_t$</td>
<td>0.4</td>
<td>4.9</td>
<td>2.2</td>
</tr>
</tbody>
</table>
Robustness Check in the New-Keynesian Model

Long-Memory Indexation

\( \gamma_m \in [0.00014, 0.241] \)
The price adjustment cost is

\[
\left( \frac{P_{it}}{\tilde{X}_t^m P_{it-1}} - 1 \right)^2
\]

and \( \tilde{X}_t^m \) obeys the law of motion

\[
\tilde{X}_t^m \equiv (X_t^m)^{\gamma_m} (\tilde{X}_{t-1}^m)^{1-\gamma_m}
\]

Posterior mean of \( \gamma^m = 0.46 \)

Lowest Decile: \( \gamma_m \in [0.00014, 0.241] \)
Impulse Responses to Interest-Rate Shocks Conditional on $\gamma_m$ in Lowest Decile: NK Model,

**Permanent Interest-Rate Shock**
Response of the Interest Rate and Inflation

<table>
<thead>
<tr>
<th>quarters after the shock</th>
<th>deviation from pre-shock level percentage points per year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
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<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**Temporary Interest-Rate Shock**
Response of the Interest Rate and Inflation

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<thead>
<tr>
<th>quarters after the shock</th>
<th>deviation from pre-shock level percentage points per year</th>
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<tr>
<td></td>
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<td>2</td>
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</table>

**Permanent Interest-Rate Shock**
Response of Output

<table>
<thead>
<tr>
<th>quarters after the shock</th>
<th>percent deviation from pre-shock level</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
</tr>
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<td></td>
<td>0.4</td>
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**Temporary Interest-Rate Shock**
Response of Output

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<th>quarters after the shock</th>
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<td>0.4</td>
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