Online Appendix to 'The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models,' by Martín Uribe

Section A contains a detailed exposition of the empirical model. Section B presents details of the optimizing model. And section C presents prior predictions of the empirical model.

A Detailed Exposition of the Empirical Model

Let Y_t be a vector collecting these three variables,

$$Y_t \equiv \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix},$$

where y_t denotes the logarithm of real output per capita, π_t denotes the inflation rate expressed in percent per year, and i_t denotes the nominal interest rate expressed in percent per year. Let $\tilde{Y}_t$

$$\tilde{Y}_t \equiv \begin{bmatrix} (y_t - X_t) \times 100 \\ \pi_t - X_t^m \\ i_t - X_t^m \end{bmatrix},$$

where X_t^m is a permanent monetary shock, z_t^m is a transitory monetary shock, X_t is a nonstationary nonmonetary shock, and z_t is a stationary nonmonetary shock. Let $\hat{Y}_t$ denote the deviation of $\tilde{Y}_t$ from its unconditional mean, that is,

$$\hat{Y}_t \equiv \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \end{bmatrix} \equiv \tilde{Y}_t - E\tilde{Y}_t,$$

where E denotes the unconditional expectations operator.

The law of motion of $\hat{Y}_t$ takes the autoregressive form

$$\hat{Y}_t = \sum_{i=1}^{L} B_i \hat{Y}_{t-i} + C u_t \tag{9}$$

where

$$u_t \equiv \begin{bmatrix} x_t^m \\ z_t^m \\ x_t \\ z_t \end{bmatrix},$$

$$x_t^m \equiv \Delta X_t^m - \Delta X^m$$

and

$$x_t \equiv (\Delta X_t - \Delta X) \times 100.$$

with Δ denoting the time-difference operator, $\Delta X^m \equiv E\Delta X_t^m$, and $\Delta X \equiv E\Delta X_t$. The variables x_t^m and x_t denote demeaned changes in the nonstationary shocks. The objects B_i, for $i = 1, \dots, L$, are 3-by-3 matrices of coefficients, C is a 3-by-4 matrix of coefficients, and L is a scalar denoting the lag length of the empirical model. The vector u_t is assumed to follow an AR(1) law of motion of the form

$$u_{t+1} = \rho u_t + \psi \epsilon_{t+1}, \tag{10}$$

where ρ and ψ are 4-by-4 diagonal matrices of coefficients, and ϵ_t is a 4-by-1 i.i.d. disturbance distributed $N(\emptyset, I)$.

The observable variables used in the estimation of the empirical model are output growth expressed in percent per quarter, the change in the nominal interest rate, and the interest-rate-inflation differential, defined as

$$r_t \equiv i_t - \pi_t.$$

The following equations link the observables to variables included in the unobservable system (9)-(10):

$$\begin{aligned} 100 \times \Delta y_t &= 100 \times \Delta X + \hat{y}_t - \hat{y}_{t-1} + x_t \\ r_t &= r + \hat{i}_t - \hat{\pi}_t \\ \Delta i_t &= \Delta X^m + \hat{i}_t - \hat{i}_{t-1} + x_t^m \end{aligned} \tag{11}$$

where $r \equiv Er_t$ represents the unconditional mean of the interest-rate-inflation differential. The variables Δy_t, r_t, and Δi_t are assumed to be observed with measurement error. Let o_t be the vector of variables observed in quarter t. Then

$$o_t = \begin{bmatrix} \Delta y_t \times 100 \\ r_t \\ \Delta i_t \end{bmatrix} + \mu_t \tag{12}$$

where μ_t is a 3-by-1 vector of measurement errors distributed i.i.d. $N(\emptyset, R)$, and R is a diagonal variance-covariance matrix.

The state-space representation of the system composed of equations (9), (10), (11), and (12) can be written as follows:

$$\xi_{t+1} = F\xi_t + P\epsilon_{t+1}$$

$$o_t = A' + H'\xi_t + \mu_t,$$

where

$$\xi_t \equiv \begin{bmatrix} \hat{Y}_t \\ \hat{Y}_{t-1} \\ \vdots \\ \hat{Y}_{t-L+1} \\ u_t \end{bmatrix},$$

The matrices F, P, A, and H are known functions of B_i, $i = 1, \ldots L$, C, ρ, ψ, ΔX, ΔX^m, and r. Specifically, let

$$B \equiv [B_1 \cdots B_L],$$

and let I_j denote an identity matrix of order j, $\emptyset_j$ denote a square matrix of order j with all entries equal to zero, and $\emptyset_{i,j}$ denote a matrix of order i by j with all entries equal to zero. Also let L, S, and V denote, respectively, the number of lags, the number of shocks, and the number of endogenous variables included in the empirical model. Then, for $L \geq 2$ we have

$$F = \begin{bmatrix} B & C\rho \\ \begin{bmatrix} I_{V(L-1)} \, \emptyset_{V(L-1),V} \end{bmatrix} & \emptyset_{V(L-1),S} \\ \emptyset_{S,VL} & \rho \end{bmatrix}, P = \begin{bmatrix} C\psi \\ \emptyset_{V(L-1),S} \\ \psi \end{bmatrix};$$

$$A' = \begin{bmatrix} 100 \times \Delta X \\ r \\ \Delta X^m \end{bmatrix}, \text{ and } H' = \begin{bmatrix} M_\xi & \emptyset_{V,V(L-2)} & M_u \end{bmatrix},$$

where, in the specification considered in the body of the paper ($S = 4$, $V = 3$, and a particular ordering of the endogenous and exogenous variables in the vectors $\hat{Y}_t$ and u_t), the matrices M_ξ and M_u take the form

$$M_\xi = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \text{ and } M_u = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

The case $L = 1$ is a special case of $L = 2$ in which $B_2 = \emptyset_V$.

B Detailed Exposition of the Optimizing Model

B.1 Households

The economy is populated by households with preferences defined over streams of consumption and labor effort and exhibiting external habit formation. The household's lifetime utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left\{ \frac{\left[(C_t - \delta \widetilde{C}_{t-1})(1 - e^{\theta_t} h_t)^{\chi}\right]^{1-\sigma} - 1}{1-\sigma} \right\}, \tag{5}$$

where C_t denotes consumption, $\widetilde{C}_t$ denotes the cross sectional average of consumption, h_t denotes hours worked, ξ_t is a preference shock, θ_t is a labor-supply shock, and $\beta, \delta \in (0,1)$ and $\sigma, \chi > 0$ are parameters.

Households are subject to the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1+I_t} + T_t = B_t + W_t h_t + \Phi_t, \tag{6}$$

where P_t denotes the nominal price of consumption, B_{t+1} denotes a nominal bond purchased in t and paying the nominal interest rate I_t in $t+1$, T_t denotes nominal lump-sum taxes, W_t denotes the nominal wage rate, and Φ_t denotes nominal profits received from firms.

The consumption good C_t is assumed to be a composite of a continuum of varieties C_{it} indexed by $i \in [0,1]$ with aggregation technology

$$C_t = \left[\int_0^1 C_{it}^{1-1/\eta} di\right]^{\frac{1}{1-1/\eta}}, \tag{13}$$

where the parameter $\eta > 0$ denotes the elasticity of substitution across varieties.

Households choose processes $\{C_t, h_t, B_{t+1}\}_{t=0}^{\infty}$ to maximize the utility function (5) subject to the budget constraint (6) and to some borrowing limit that prevents them from engaging in Ponzi schemes. Letting $\beta^t \Lambda_t / P_t$ denote the Lagrange multiplier associated with the budget constraint, the first-order conditions of the household's optimization problem are

$$e^{\xi_t}(C_t - \delta\widetilde{C}_{t-1})^{-\sigma}(1 - e^{\theta_t}h_t)^{\chi(1-\sigma)} = \Lambda_t$$

$$\frac{\chi e^{\theta_t}(C_t - \delta\widetilde{C}_{t-1})}{1 - e^{\theta_t}h_t} = \frac{W_t}{P_t}$$

and

$$\Lambda_t = \beta(1 + I_t)E_t\left[\frac{\Lambda_{t+1}}{1 + \Pi_{t+1}}\right],$$

where $\Pi_t \equiv P_t/P_{t-1} - 1$ denotes the consumer-price inflation rate.

Given C_t, the household chooses the consumption of varieties C_{it} to minimize total expenditure, $\int_0^1 P_{it}C_{it}di$, subject to the aggregation technology (13), where P_{it} denotes the nominal price of variety i. This problem delivers the following demand for individual varieties:

$$C_{it} = C_t\left(\frac{P_{it}}{P_t}\right)^{-\eta}, \tag{14}$$

where the price level P_t is given by

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta}di\right]^{\frac{1}{1-\eta}}, \tag{15}$$

and represents the minimum cost of one unit of the composite consumption good.

B.2 Firms

The firm producing variety i operates in a monopolistically competitive market and faces quadratic price adjustment costs à la Rotemberg (1982). The production technology uses labor and is buffeted by stationary and nonstationary productivity shocks. Specifically, output of variety i is given by

$$Y_{it} = e^{z_t}X_t h_{it}^{\alpha}, \tag{7}$$

where Y_{it} denotes output of variety i in period t, h_{it} denotes labor input used in the production of variety i, and z_t and X_t are stationary and nonstationary productivity shocks, respectively. The growth rate of the nonstationary productivity shock, $g_t \equiv \ln(X_t/X_{t-1})$, is assumed to be a stationary random variable. The expected present discounted value of real profits of the firm producing variety i expressed in units of consumption is given by

$$E_0 \sum_{t=0}^{\infty} q_t \left[\frac{P_{it}}{P_t} C_{it} - \frac{W_t}{P_t} h_{it} - \frac{\phi}{2} X_t \left(\frac{P_{it}/P_{it-1}}{1+\widetilde{\Pi}_t} - 1 \right)^2 \right], \tag{8}$$

where $1+\widetilde{\Pi}_t = (1+\widetilde{\Pi}_{t-1})^{\gamma_m}(1+\Pi_t)^{1-\gamma_m}$ denotes the average level of inflation around which price-adjustment costs are defined, and $\Pi_t \equiv P_t/P_{t-1} - 1$ denotes the inflation rate. The parameter $\phi > 0$ governs the degree of price stickiness, and the parameter $\gamma_m \in [0,1]$ the backward-looking component of the inflation measure at which price adjustment costs are centered. Both parameters are estimated. Allowing for a backward-looking component in firms' price-setting behavior is in order in the present context because, as pointed out by Garín, Lester, and Sims (2018), the larger is this component, the less likely it will be that stationary but persistent movements in the inflation target are implemented with rising interest rates and inflation in the short run. The variable $q_t \equiv \beta^t \frac{\Lambda_t}{\Lambda_0}$, denotes a pricing kernel reflecting the assumption that profits belong to households. The price adjustment cost in the profit equation (8) is scaled by the output trend X_t to keep nominal rigidity from vanishing along the balanced growth path.

The problem of the firm producing variety i is to choose processes $\{P_{it}, C_{it}, Y_{it}, h_{it}\}_{t=0}^{\infty}$ to maximize (8) subject to the demand equation (14), the production technology (7), and the requirement that demand be satisfied at the price set by the firm,[5]

$$Y_{it} \geq C_{it} \tag{16}$$

Letting $q_t P_{it}/(P_t \mu_t)$ be the Lagrange multiplier associated with the demand constraint (16), the first-order conditions associated with the firm's profit maximization problem are

$$\mu_t = \frac{P_{it}}{W_t/(\alpha e^{z_t} X_t h_{it}^{\alpha-1})}$$

$$\eta C_{it}\left(\frac{\eta-1}{\eta}-\frac{1}{\mu_t}\right) = -\phi X_t \frac{P_t/P_{it-1}}{1+\widetilde{\Pi}_t}\left(\frac{P_{it}/P_{it-1}}{1+\widetilde{\Pi}_t}-1\right) + \phi E_t \frac{q_{t+1}}{q_t} X_{t+1} \frac{P_{it+1}P_t/P_{it}^2}{1+\widetilde{\Pi}_{t+1}}\left(\frac{P_{it+1}/P_{it}}{1+\widetilde{\Pi}_{t+1}}-1\right)$$

The first optimality condition says that the multiplier μ_t represents the markup of prices over marginal cost. The second optimality condition says that, all other things equal, if the price markup is above its normal level, $\mu_t > \eta/(\eta-1)$, the firm will increase prices at a rate below normal, $P_{it}/P_{it-1} < 1+\widetilde{\Pi}_t$.

B.3 Monetary and Fiscal Policy

The monetary authority follows a Taylor-type interest-rate feedback rule with smoothing, as follows

$$\frac{1+I_t}{\Gamma_t} = \left[A\left(\frac{1+\Pi_t}{\Gamma_t}\right)^{\alpha_\pi}\left(\frac{Y_t}{X_t}\right)^{\alpha_y}\right]^{1-\gamma_I}\left(\frac{1+I_{t-1}}{\Gamma_{t-1}}\right)^{\gamma_I} e^{z_t^m}, \tag{17}$$

where Y_t denotes aggregate output, z_t^m denotes a stationary interest-rate shock, Γ_t is the inflation-target, and A, α_π, α_y and $\gamma_I \in [0,1)$ are parameters. The inflation target is assumed to have a permanent component denoted X_t^m and a transitory component denoted

[5] Strictly speaking, the right-hand side of this constraint must include the demand for goods of variety i by all firms for the purpose of generating the units of composite goods devoted to cover the price adjustment costs, which is given by $\frac{\phi}{2} X_t \left(\frac{P_{it}}{P_t}\right)^{-\eta} \int_0^1 \left(\frac{P_{jt}/P_{jt-1}}{1+\widetilde{\Pi}_t}-1\right)^2 dj$. However, because price adjustment costs are quadratic in $\frac{P_{jt}/P_{jt-1}}{1+\widetilde{\Pi}_t}-1$, which is zero along the deterministic balanced growth path, this source of demand for good i and all of its derivatives with respect to P_{it} are zero in equilibrium up to first order.

z_t^{m2}. Formally,

$$\Gamma_t = X_t^m e^{z_t^{m2}}.$$

The growth rate of the permanent component of the inflation target, $g_t^m \equiv \ln\left(\frac{X_t^m}{X_{t-1}^m}\right)$, is assumed to be stationary.

Government consumption is assumed to be nil at all times, and fiscal policy is assumed to be Ricardian.

The seven structural shocks driving the economy, ξ_t, θ_t, z_t, g_t, z_t^m, z_t^{m2}, and g_t^m are assumed to follow AR(1) processes of the form

$$x_t = \rho_x x_{t-1} + \epsilon_t^x,$$

for $x = \xi, \theta, z, g, z^m, z^{2m}, g^m$.

B.4 Market Clearing and Equilibrium

Clearing of the labor market requires that the demand for labor by firms equal the household's supply of labor, that is,

$$\int_0^1 h_{it} di = h_t. \tag{18}$$

Because all households are identical, so are individual and aggregate consumption per capita,

$$C_t = \widetilde{C}_t.$$

I focus attention on a symmetric equilibrium in which all firms charge the same nominal price and employ the same amount of labor, that is, an equilibrium in which h_{it} and P_{it} are the same for all $i \in [0, 1]$. We then have from equations (14), (15), (7), and (18) that $P_{it} = P_t$, $C_{it} = C_t$, $h_{it} = h_t$, and $Y_{it} = e^{z_t} X_t h_t^\alpha$, for all i. Output, measured in units of the final good is then given by $Y_t \equiv \left(\int_0^1 P_{it} Y_{it} di\right) / P_t = e^{z_t} X_t h_t^\alpha$. As long as the nominal wage is positive, the firm will choose to satisfy the demand constraint (16) with equality. By virtue of this condition, we have that in equilibrium

$$Y_t = C_t.$$

Finally, I express the model in terms of stationary variables by dividing all variables with stochastic trends by their respective permanent components. Thus, I create the variables $c_t \equiv C_t/X_t$, $y_t \equiv Y_t/X_t$, $w_t \equiv W_t/(P_t X_t)$, $\lambda_t \equiv \Lambda_t/X_t^{n-\sigma}$, $1+\pi_t \equiv (1+\Pi_t)/X_t^m$, $1+i_t \equiv (1+I_t)/X_t^m$, and $1+\widetilde{\pi}_t \equiv (1+\widetilde{\Pi}_t)/X_t^m$.

A competitive equilibrium is then a set of process $\{y_t, h_t, \lambda_t, \pi_t, i_t, w_t, \mathrm{mc}_t, \widetilde{\pi}_t\}$ satisfying

$$e^{\xi_t}\left(y_t - \delta\frac{y_{t-1}}{e^{g_t}}\right)^{-\sigma}\left(1-e^{\theta_t}h_t\right)^{\chi(1-\sigma)} = \lambda_t$$

$$\frac{\chi e^{\theta_t}\left(y_t - \delta\frac{y_{t-1}}{e^{g_t}}\right)}{1-e^{\theta_t}h_t} = w_t$$

$$\lambda_t = \beta(1+i_t)E_t\left[\frac{\lambda_{t+1}}{1+\pi_{t+1}}e^{-g^m_{t+1}-\sigma g_{t+1}}\right],$$

$$y_t = e^{z_t}h_t^{\alpha}$$

$$\mathrm{mc}_t = \frac{w_t}{\alpha e^{z_t}h_t^{\alpha-1}}$$

$$\frac{1+\pi_t}{1+\widetilde{\pi}_t}\left(\frac{1+\pi_t}{1+\widetilde{\pi}_t}-1\right) = \beta E_t e^{(1-\sigma)g_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}\frac{1+\pi_{t+1}}{1+\widetilde{\pi}_{t+1}}\left(\frac{1+\pi_{t+1}}{1+\widetilde{\pi}_{t+1}}-1\right) + \frac{1}{\phi(\mu-1)}\left(\mu\mathrm{mc}_t - 1\right)y_t \tag{19}$$

$$1+i_t = \left[A\left(1+\pi_t\right)^{\alpha_\pi} y_t^{\alpha_y}\right]^{1-\gamma_I}\left(1+i_{t-1}\right)^{\gamma_I} e^{z_t^m + (1-\alpha_\pi)z_t^{m2} - \gamma_I z_{t-1}^{m2}},$$

$$1+\widetilde{\pi}_t = e^{-\gamma_m g_t^m}(1+\widetilde{\pi}_{t-1})^{\gamma_m}(1+\pi_t)^{1-\gamma_m}$$

where $\mathrm{mc}_t \equiv 1/\mu_t$ and $\mu \equiv \eta/(\eta-1)$ denote, respectively, the equilibrium real marginal cost and the steady-state product markup. Equation (19) is a Phillips curve and says that all other things equal, current inflation is increasing in the marginal cost.

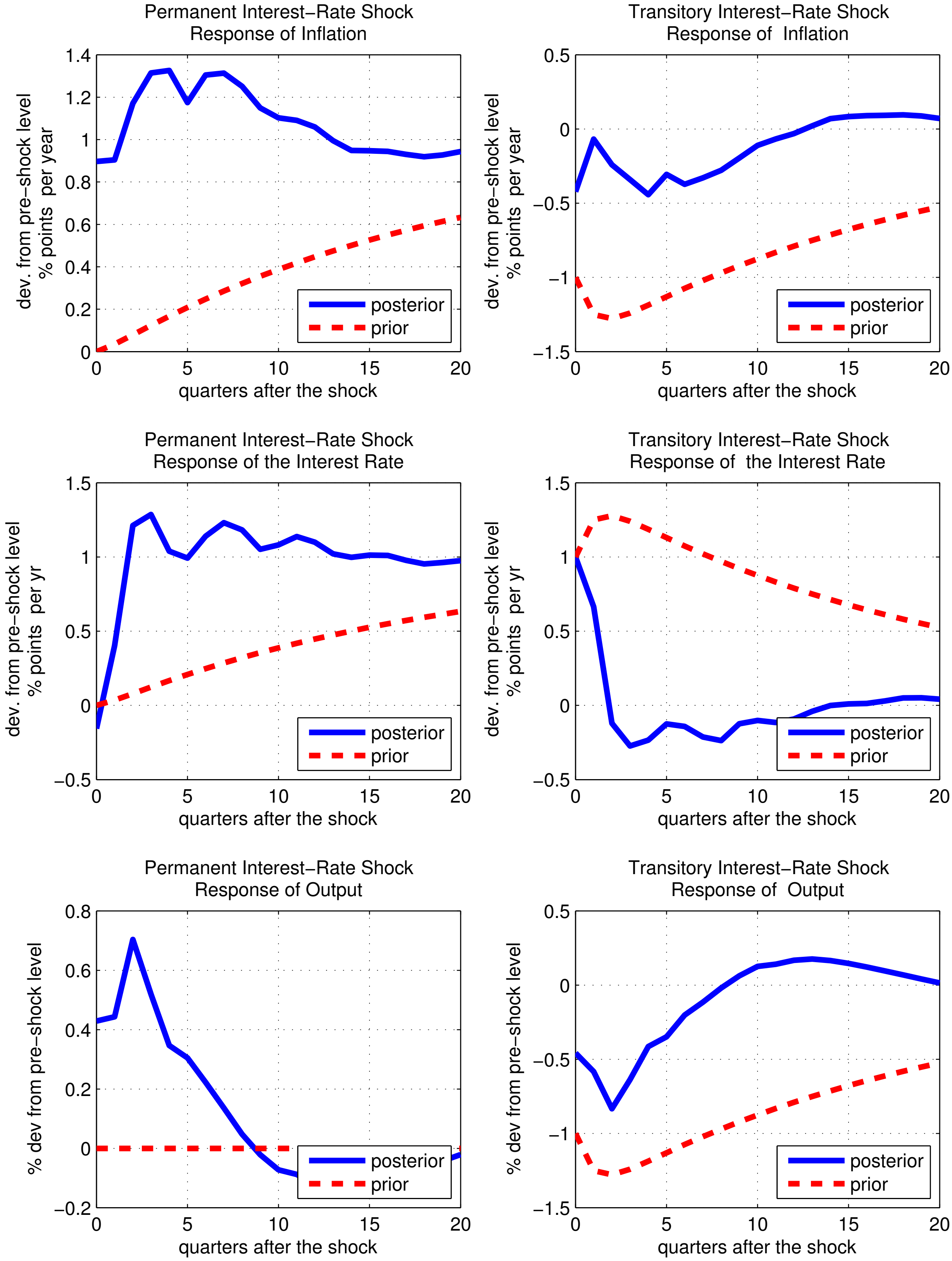
A first-order approximation of the Phillips curve around $\pi_t = \pi = 0$ yields

$$\widehat{\pi}_t - \widehat{\widetilde{\pi}}_t = \widetilde{\beta} E_t(\widehat{\pi}_{t+1} - \widehat{\widetilde{\pi}}_{t+1}) + \kappa \widehat{\mathrm{mc}}_t, \tag{20}$$

where $\widetilde{\beta} \equiv \beta e^{(1-\sigma)g}$, $\kappa \equiv \frac{(\eta-1)y}{\phi}$, $\widehat{\pi}_t \approx \pi_t - \pi$, $\widehat{\widetilde{\pi}}_t \approx \widetilde{\pi}_t - \pi$, $\widehat{\mathrm{mc}}_t \approx \ln(\mathrm{mc}_t/\mathrm{mc})$, and $\mathrm{mc} = 1/\mu$. This is a familiar expression of a linear Phillips curve, except that it is cast in terms of deviations of the cyclical component of inflation, $\widehat{\pi}_t$ from the cyclical component of a weighted average of past inflations, $\widehat{\widetilde{\pi}}_t$.

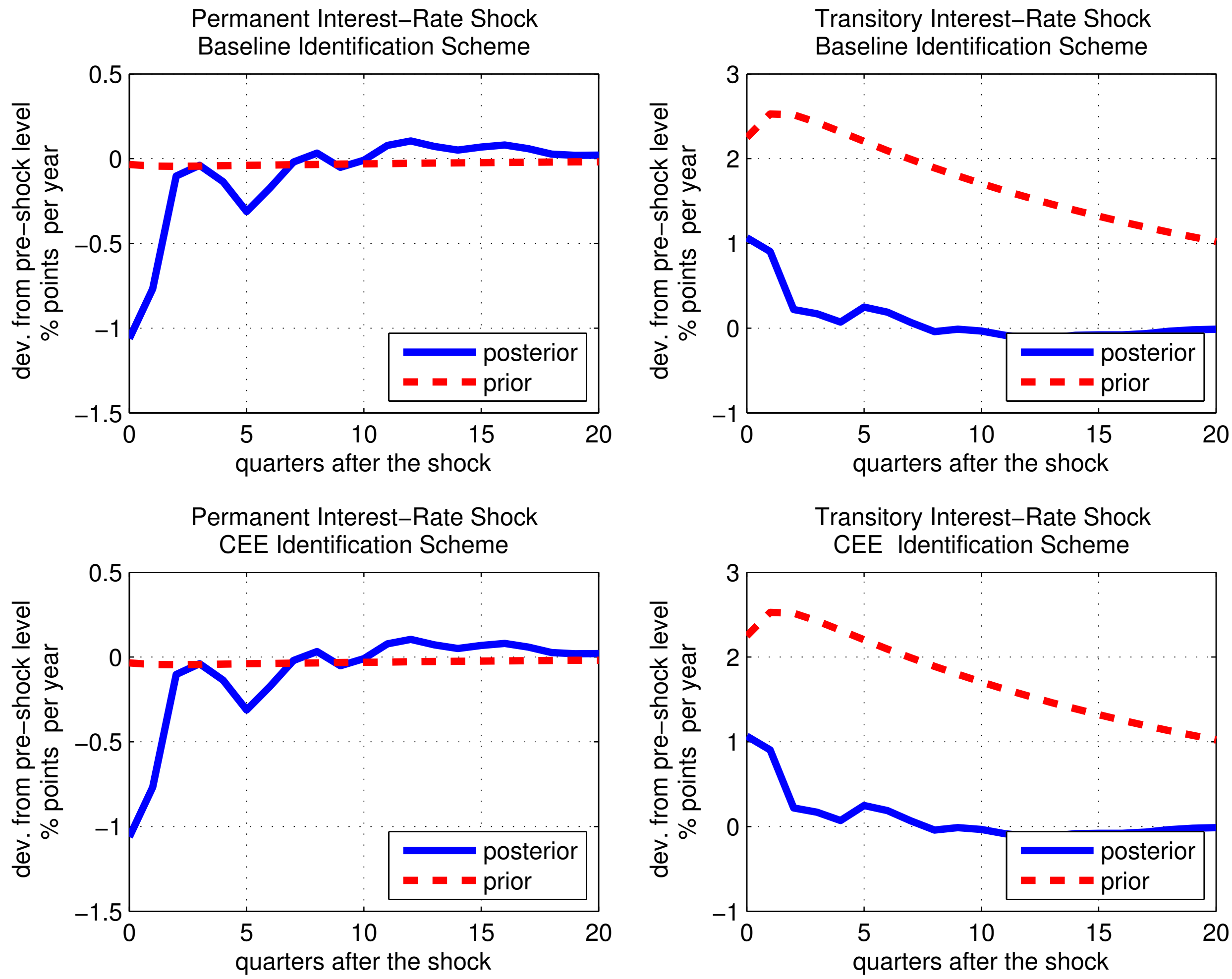
C Prior Predictions

Figure 13: Prior and Posterior Impulse Responses: Empirical Model



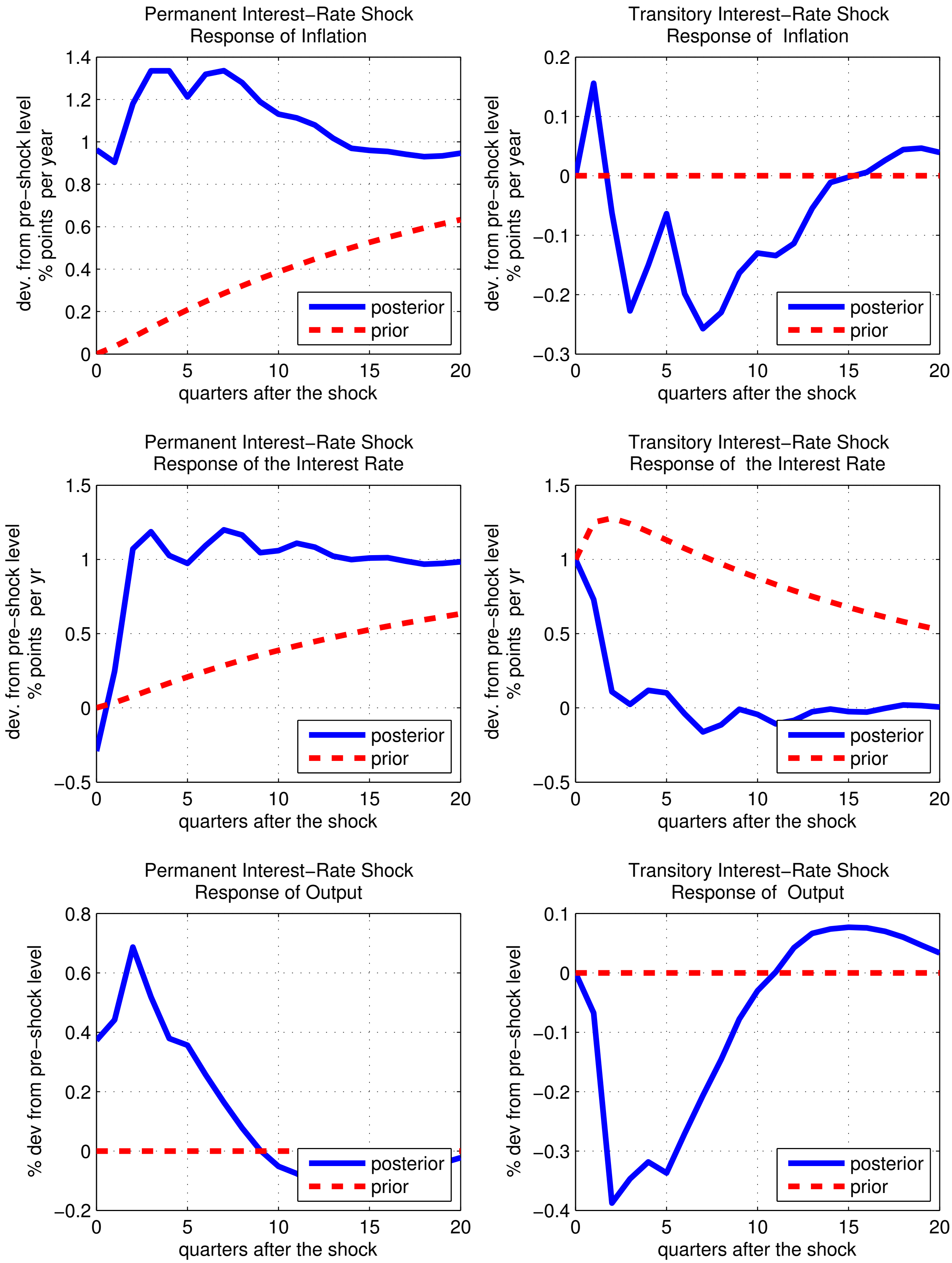
Notes. Impulse responses are computed using the posterior mean (solid lines) and prior mean of the vector of estimated parameters. Replication code: `plot_prior_predictions.m` in replication folder `empirical_model`.

Figure 14: Prior and Posterior Responses of the Real Rate: Empirical Model



Notes. Impulse responses are computed using the posterior mean (solid lines) and prior mean of the vector of estimated parameters.

Figure 15: Prior and Posterior Impulse Responses Under CEE Identification Restrictions: Empirical Model



Notes. Impulse responses are computed using the posterior mean (solid lines) and prior mean of the vector of estimated parameters. The CEE identification restrictions are $C_{12} = C_{22} = 0$.