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Evaluating the sample likelihood of linearized DSGE models without the use of the Kalman filter

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ABSTRACT

This paper derives a method for constructing the likelihood function of a general class of linearized dynamic general equilibrium models that does not require the application of the Kalman filter. The method easily handles models in which variables are observed with error.

We derive a method for constructing the likelihood function of a general class of linearized dynamic general equilibrium models that does not require the application of the Kalman filter. The standard approach is based on a prediction-error decomposition, which expresses the likelihood as a function of unobservable states. By contrast, we view the observed sample as a single draw from a multivariate density, which allows for a representation of the likelihood in terms of observables alone.

Our proposed approach for evaluating the likelihood function of DSGE models is of use in instances in which the data is filtered using a two-sided filter (such as the HP filter or a BP filter) before estimation. In this case, consistency between data and model predictions requires applying the same filter to the model predictions, which makes it impossible to apply a recursive approach (such as the Kalman filter) to evaluate the likelihood function.

Following Schmitt-Grohé and Uribe (2004), we consider a general class of linearized DSGE models where an $n_x \times 1$ state vector x_t and an $n_y \times 1$ control vector y_t evolve according to the law of motion

$$x_{t+1} = h(\theta)x_t + \eta(\theta)\epsilon_{t+1} \tag{1}$$

$$y_t = g(\theta)x_t, \tag{2}$$

where θ is an $n_\theta \times 1$ vector of deep structural parameters, which the econometrician wishes to estimate, $h(\theta)$ is an $n_x \times n_x$ transition matrix with roots inside the unit circle, $\eta(\theta)$ is an $n_x \times n_\epsilon$ matrix, and ϵ_t is an $n_\epsilon \times 1$ Gaussian vector with mean zero and variance–covariance matrix equal to an identity matrix of size $n_\epsilon \times n_\epsilon$. Assume that $n_y \leq n_\epsilon$. The vector x_t may contain observable and unobservable endogenous and exogenous state variables. The vector y_t is assumed to be observable.

Suppose that the sample consists of T observations of the vector y_t. Let Y denote the $n_y T \times 1$ vector

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}.$$

We can interpret Y as a single draw from a $N(\mu, \Omega)$ distribution, where μ is a vector of order $n_y T \times 1$ and Ω is a matrix of order $n_y T \times n_y T$. Clearly,

$$\mu = \emptyset.$$

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In turn,

$$\Omega = E(YY') = E\begin{bmatrix} y_1y_1' & y_1y_2' & \cdots & y_1y_T' \\ y_2y_1' & y_2y_2' & \cdots & y_2y_T' \\ \vdots & \vdots & \vdots & \vdots \\ y_Ty_1' & y_Ty_2' & \cdots & y_Ty_T' \end{bmatrix}.$$

We next show how to compute Ω for a given value of θ. Start with

$$\begin{aligned} Ey_1y_1' &= Eg(\theta)x_1x_1'g(\theta)' \\ &= g(\theta)Ex_1x_1'g(\theta)' \\ &= g(\theta)\Sigma_x g(\theta)', \end{aligned}$$

where Σ_x is the covariance matrix of x_t, which, from the law of motion of x_t, must satisfy

$$\Sigma_x = h(\theta)\Sigma_x h(\theta)' + \eta(\theta)\eta(\theta)'.$$

Given θ, Σ_x can be readily computed.[1] In general,

$$Ey_iy_j' = \begin{cases} g(\theta)\Sigma_x[h(\theta)']^{j-i}g(\theta)' & \text{if } i \le j \\ g(\theta)[h(\theta)]^{i-j}\Sigma_x g(\theta)' & \text{if } i > j \end{cases},$$

for $i,j = 1,\ldots,T$. It follows that, given θ, the covariance matrix Ω can be readily computed. The sample log-likelihood can then be written immediately as

$$\mathcal{L}(\theta|Y) = (-Tn_y/2)ln(2\pi) + \frac{1}{2}ln|\Omega^{-1}| - \frac{1}{2}(Y-\mu)'\Omega^{-1}(Y-\mu).$$

This completes a procedure for evaluating the sample log-likelihood for a linearized DSGE model with unobservable states without use of the Kalman filter.

1. Handling measurement error

Suppose y_t is observed with measurement error. Specifically, suppose that the econometrician observes a vector y_t^o given by

$$y_t^o = y_t + mw_t,$$

where the measurement error vector w_t is an autoregressive process of the form

$$w_t = nw_{t-1} + \nu\mu_t,$$

where μ_t is a Gaussian random vector with mean zero and identity variance–covariance matrix. Note that variables in y_t that are observed without error give rise to rows of m made up of zeros. Let $\tilde\theta$ be a new vector of parameters to be estimated which includes all of the elements of θ plus some elements of m, n, and ν that the econometrician wishes to estimate. Define

$$\tilde{x}_t = \begin{bmatrix} x_t \\ w_t, \end{bmatrix}; \quad \tilde{h}(\tilde\theta) = \begin{bmatrix} h(\theta) & \emptyset \\ \emptyset & m \end{bmatrix}; \quad \tilde\eta(\tilde\theta) = \begin{bmatrix} \eta & \emptyset \\ \emptyset & \nu \end{bmatrix};$$

$$\tilde{\epsilon}_t = \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}; \quad \tilde{g}(\tilde\theta) = [g(\theta) \quad m].$$

Then one can write

$$\tilde{x}_{t+1} = \tilde{h}(\tilde\theta)\tilde{x}_t + \tilde\eta(\tilde\theta)\tilde\epsilon_{t+1}$$

$$y_t^o = \tilde{g}(\tilde\theta)\tilde{x}_t.$$

This system has the same structure as Eqs. (1) and (2), so its associated sample likelihood can be constructed applying the procedure described in the previous section.

Reference

Schmitt-Grohé, Stephanie, Uribe, Martín, 2004. Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics and Control 28, 755–775 January.

[1] For example, by $\text{vec}(\Sigma_x) = (I - h(\theta) \otimes h(\theta))^{-1}\,\text{vec}(\eta(\theta)\eta(\theta)')$. For more efficient algorithms for computing Σ_x, see the program mom.m on our web sites.