Pricing To Habits and the Law of One Price: Appendix

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This appendix expands the model presented in Ravn, Schmitt-Grohé, and Uribe (2006b) to allow for government purchases. It also contains a detailed derivation of the equilibrium conditions, the deterministic steady state, the calibration, the functional forms, and the simulation results.

1 The Model

The model economy consists of two countries, the home country and the foreign country. Each country specializes in the production of a set of differentiated goods. We denote by a the class of goods produced by the home country and by b the class of goods produced by the foreign country.

1.1 Households

Consider an economy populated by a large number of households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^c, h_t). \tag{1}$$

The variable x_t^c is a composite defined as

$$x_t^c = \chi(x_{a,t}^c, x_{b,t}^c), \tag{2}$$

where the aggregator function χ is assumed to be increasing and homogenous of degree one in both arguments. The variable $x_{a,t}^c$ is a habit-adjusted composite consumption good of varieties of goods of type a. Following Ravn, Schmitt-Grohé, and Uribe (2006a), we introduce deep habits. Specifically, we assume that habits form at the level of each individual variety of goods instead of at the level of the aggregate consumption good. We assume that deep habits are external to the individual household (i.e., we model catching up with the Joneses good by good). Formally, $x_{a,t}^c$ is given by

$$x_{a,t}^c = \left[\int_0^1 (c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c)^{1-1/\eta} di\right]^{1/(1-1/\eta)}.$$

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Here $c_{i,a,t}$ denotes consumption of variety i of goods belonging to the set a in period t. The variable $s^c_{i,a,t}$ denotes the stock of external habit in consumption of variety i of good a. This habit stock is assumed to evolve according to the following law of motion:

$$s^c_{i,a,t} = \rho s^c_{i,a,t-1} + (1-\rho)\tilde{c}_{i,a,t},$$

where $\tilde{c}_{i,a,t}$ denotes the average per capita consumption of variety i of good a in the domestic country; that is, $\tilde{c}_{i,a,t}$ is the integral of $c_{i,a,t}$ over all domestic households. The parameter $\theta^c_a \in [0,1)$ measures the intensity of deep external habits for consumption goods of type a. When θ^c_a is equal to zero, preferences for goods of type a display no deep habit formation. The parameter $\eta > 0$ represents the intratemporal elasticity of substitution across varieties, and the parameter $\rho \in [0,1)$ denotes the rate at which the stock of external habits decays over time.

Similarly, $x^c_{b,t}$ is given by

$$x^c_{b,t} = \left[\int_0^1 (c_{i,b,t} - \theta^c_b s^c_{i,b,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)},$$

with

$$s^c_{i,b,t} = \rho s^c_{i,b,t-1} + (1-\rho)\tilde{c}_{i,b,t}.$$

For simplicity, we assume that the parameter θ^c is common across varieties the same type of good. We allow for the degree of habit formation to vary across types of goods a and b.

To characterize the household's demands for varieties of type-a and type-b goods, we consider a two-step problem. Suppose the household has determined its desired consumption of the aggregate goods a and b, that is, $x^c_{a,t}$ and $x^c_{b,t}$. Then it is optimal for the household to distribute its purchases of individual varieties so as to minimize costs, that is,

$$\min_{c_{i,a,t}} \int_0^1 P_{i,a,t} c_{i,a,t} di$$

subject to

$$x^c_{a,t} = \left[\int_0^1 (c_{i,a,t} - \theta^c_a s^c_{i,a,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)}.$$

The associated Lagrangian takes the form

$$\int_0^1 P_{i,a,t} c_{i,a,t} di + \lambda \left[x^c_{a,t} - \left[\int_0^1 (c_{i,a,t} - \theta^c_a s^c_{i,a,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)}\right].$$

The first-order conditions of this cost-minimization problem are

$$P_{i,a,t} = \lambda \left[\int_0^1 (c_{i,a,t} - \theta^c_a s^c_{i,a,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)-1} (c_{i,a,t} - \theta^c_a s^c_{i,a,t-1})^{-1/\eta}$$

and

$$P_{j,a,t} = \lambda \left[\int_0^1 (c_{i,a,t} - \theta^c_a s^c_{i,a,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)-1} (c_{j,a,t} - \theta^c_a s^c_{j,a,t-1})^{-1/\eta}.$$

Taking ratios yields

$$\frac{P_{j,a,t}}{P_{i,a,t}} = \frac{(c_{j,a,t} - \theta_a^c s_{j,a,t-1}^c)^{-1/\eta}}{(c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c)^{-1/\eta}}$$

Rearranging this expression we obtain

$$(c_{j,a,t} - \theta_a^c s_{j,a,t-1}^c)^{1-1/\eta} = \left(\frac{P_{j,a,t}}{P_{i,a,t}}\right)^{1-\eta} (c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c)^{1-1/\eta}.$$

Now integrate over j to obtain

$$\int_0^1 (c_{j,a,t} - \theta_a^c s_{j,a,t-1}^c)^{1-1/\eta} dj = \frac{(c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c)^{1-1/\eta}}{P_{i,a,t}^{1-\eta}} \int_0^1 (P_{j,a,t})^{1-\eta} dj.$$

Define a price index for goods of type a, denoted $P_{a,t}$, as

$$P_{a,t} = \left[\int_0^1 (P_{j,a,t})^{1-\eta} dj\right]^{1/(1-\eta)}.$$

Combining the last two expressions yields

$$\int_0^1 (c_{j,a,t} - \theta_a^c s_{j,a,t-1}^c)^{1-1/\eta} dj = (c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c)^{1-1/\eta} \left(\frac{P_{a,t}}{P_{i,a,t}}\right)^{1-\eta}.$$

Now elevate this expression to the power $1/(1-1/\eta)$ to get

$$\left[\int_0^1 (c_{j,a,t} - \theta_a^c s_{j,a,t-1}^c)^{1-1/\eta} dj\right]^{1/(1-1/\eta)} = (c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c) \left(\frac{P_{a,t}}{P_{i,a,t}}\right)^{\frac{1-\eta}{1-1/\eta}}.$$

Finally use the definition of $x_{a,t}^c$ to obtain

$$(c_{i,a,t} - \theta_a^c s_{i,a,t-1}^c) = \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x_{a,t}^c.$$

Rearranging terms we have

$$c_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x_{a,t}^c + \theta_a^c s_{i,a,t-1}^c.$$

Similarly, one can express the demand for variety i of good b as

$$c_{i,b,t} = \left(\frac{P_{i,b,t}}{P_{b,t}}\right)^{-\eta} x_{b,t}^c + \theta_b^c s_{i,b,t-1}^c,$$

where $P_{b,t}$ is a price index of goods of type b defined as

$$P_{b,t} = \left[\int_0^1 (P_{j,b,t})^{1-\eta} dj\right]^{1/(1-\eta)}.$$

Total expenditures on goods of type a in period t is given by

$$\int_0^1 P_{i,a,t} c_{i,a,t} di = P_{a,t} x^c_{a,t} + \theta^c_a \int_0^1 P_{i,a,t} s^c_{i,a,t-1} di.$$

Let $\omega_{a,t}$ and $\omega_{b,t}$ be defined, respectively, as

$$\omega_{a,t} \equiv \theta^c_a \int_0^1 P_{i,a,t} s^c_{i,a,t-1} di$$

and

$$\omega_{b,t} \equiv \theta^c_b \int_0^1 P_{i,b,t} s^c_{i,b,t-1} di.$$

Note that because habits are assumed to be external, the household takes both $\omega_{a,t}$ and $\omega_{b,t}$ as exogenously given. It follows that total expenditure on goods of type a and b, respectively, can be written as

$$\int_0^1 P_{i,a,t} c_{i,a,t} di = P_{a,t} x^c_{a,t} + \omega_{a,t}$$

and

$$\int_0^1 P_{i,b,t} c_{i,b,t} di = P_{b,t} x^c_{b,t} + \omega_{b,t}.$$

In each period $t \geq 0$, households have access to complete contingent claims markets. Let $r_{t,t+j}$ denote the stochastic discount factor such that $E_t r_{t,t+j} z_{t+j}$ is the period-t price of a random payment z_{t+j} of the (numeraire good) in period $t+j$. In addition, households are assumed to be entitled to the receipt of pure profits from the ownership of firms, Φ_t. Households pay lump-sum taxes in the amount T_t. Then, the representative household's period-by-period budget constraint can be written as

$$P_{a,t} x^c_{a,t} + \omega_{a,t} + P_{b,t} x^c_{b,t} + \omega_{b,t} + E_t r_{t,t+1} d_{t+1} + T_t = d_t + W_t h_t + \Phi_t. \tag{3}$$

The variable W_t denotes the wage rate. In addition, households are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games. The representative household's optimization problem consists in choosing processes $x^c_{a,t}$, $x^c_{b,t}$, h_t, and d_{t+1} so as to maximize the lifetime utility function (1) subject to (2), (3), and a no-Ponzi-game constraint, taking as given the processes for $\omega_{a,t}$, $\omega_{b,t}$, W_t, T_t, and Φ_t and initial asset holdings d_0.

Letting λ_t denote the Lagrange multiplier associated with constraint (3), the first-order conditions of the household's optimization problem are the constraints (2) and (3) and

$$U_x(x_t, h_t) \chi_a(x^c_{a,t}, x^c_{b,t}) = \lambda_t P_{a,t},$$

$$U_x(x_t, h_t) \chi_b(x^c_{a,t}, x^c_{b,t}) = \lambda_t P_{b,t},$$

$$-U_h(x_t, h_t) = \lambda_t W_t,$$

and

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1}.$$

Using the first of these optimality conditions to eliminate λ_t from the remaining conditions, we obtain

$$\frac{\chi_a(x^c_{a,t}, x^c_{b,t})}{\chi_b(x^c_{a,t}, x^c_{b,t})} = \frac{P_{a,t}}{P_{b,t}},$$

$$-\frac{U_h(x_t, h_t)}{U_x(x_t, h_t)\chi_a(x^c_{a,t}, x^c_{b,t})} = \frac{W_t}{P_{a,t}},$$

and

$$\frac{U_x(x_t, h_t)\chi_a(x^c_{a,t}, x^c_{b,t})}{P_{a,t}} r_{t,t+1} = \beta \frac{U_x(x_{t+1}, h_{t+1})\chi_a(x^c_{a,t+1}, x^c_{b,t+1})}{P_{a,t+1}}. \tag{4}$$

1.2 The Government

We assume that real government expenditures, denoted by g_t, are exogenous and stochastic and follow a univariate first-order autoregressive process of the form

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon^g_t, \tag{5}$$

where the innovation ϵ^g_t distributes i.i.d. with mean zero and standard deviation σ_{ϵ^g}. Public spending is assumed to be fully financed by lump-sum taxation, $P_t g_t = T_t$, where P_t is an index of consumer prices, which we will formally define shortly. Public consumption of individual varieties of goods of type a and b, which we denonte by $g_{i,a,t}$ and $g_{i,b,t}$, respectively, are constrained by the relationship

$$\int_0^1 (P_{i,a,t} g_{i,a,t} + P_{i,b,t} g_{i,b,t}) di \le P_t g_t. \tag{6}$$

Like the private consumer, the government aggtregates the invidual varieties of domestic and foreign goods to produce two intermediate composite goods denoted $x^g_{a,t}$ and $x^g_{b,t}$, respectively, according to the technologies

$$x^g_{a,t} = \left[\int_0^1 (g_{i,a,t} - \theta^g_a s^g_{i,a,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)} \tag{7}$$

and

$$x^g_{b,t} = \left[\int_0^1 (g_{i,b,t} - \theta^g_b s^g_{i,b,t-1})^{1-1/\eta} di\right]^{1/(1-1/\eta)}. \tag{8}$$

The parameters $\theta^g_a, \theta^g_b \in [0, 1)$ measure the degree of habit formation in government consumption of domestic and foreign goods, respectively. The variables $s^g_{i,a,t}$ and $s^g_{i,b,t}$ denote the stocks of habit in variety i of goods a and b, respectively, and are assumed to evolve over time as

$$s^g_{i,a,t} = \rho s^g_{i,a,t-1} + (1-\rho) g_{i,a,t}$$

and

$$s^g_{i,b,t} = \rho s^g_{i,b,t-1} + (1-\rho) g_{i,b,t},$$

where $\rho \in [0, 1)$ denotes the rate of depreciation of the stocks of habit. . We justify our specification of the aggregator function for government consumption by assuming that private households value government spending in goods in a way that is separable from private consumption and leisure and that households derive habits from consumption of government provided goods.

The govenment combines the intermediate goods $x^g_{a,t}$ and $x^g_{b,t}$ to produce a final, public good x^g_t according to the relationship

$$x^g_t = \chi(x^g_{a,t}, x^g_{b,t}). \tag{9}$$

Note that the aggregator function χ is the same as the one used by private consumers. The government's problem consists in choosing $g_{i,a,t}$ and $g_{i,b,t}$, $i \in [0,1]$, so as to maximize x^g_t subject to the budget constraint (6) and the aggregation restrictions (7), (8), and (9), taking as given the initial conditions $g_{i,a,-1} = g_{a,-1}$ and $g_{i,b,-1} = g_{b,-1}$ for all i. In solving this maximization problem, the government takes as given the effect of current public consumption on the level of next period's composite good—i.e., habits in government consumption are external. Conceivably, government habits could be treated as internal to the government even if they are external to their beneficiaries, namely, households. This alternative, however, is analytically less tractable and therefore not pursued here. The government's problem implies demand functions for individual varieties of goods a and b of the form

$$g_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x^g_{a,t} + \theta^g_a s^g_{i,a,t-1}$$

and

$$g_{i,b,t} = \left(\frac{P_{i,b,t}}{P_{b,t}}\right)^{-\eta} x^g_{b,t} + \theta^g_b s^g_{i,b,t-1}.$$

In converting nominal government expenditures into real values, we have used the consumer price deflator P_t. In the model economy under study, however, the presence of habit formation at a good-by-good level implies that there is no natural concept of either aggregate consumption or of an aggregate consumption price index. We therefore define the consumption price index as an expenditure weighted average of the price of final goods:

$$P_t = \gamma P_{a,t} + (1-\gamma) P_{b,t},$$

where γ is a fixed weight defined as

$$\gamma = \frac{P_a(c_a + g_a)}{P_a(c_a + g_a) + P_b(c_b + g_b)},$$

where variables without a time subscript represent the deterministic steady state value of their time-subscripted counterparts. We adopt a fixed-weight price index to mimick a common practice in developed countries, where consumer price indices take the Laspeyres form. We note that our definition of the consumer price index takes an arithmetic mean of prices in the broad categories a and b. Within each of these two categories, price indices are constructed as geometric means of individual prices. This convention is in line with the construction of the consumer price index in the United States where, since January 1999, a geometric mean formula has been used to average prices within item categories, while an arithmetic mean formula has been used to average prices across item categories.

The consumer price index in the foreign country is defined in a similar fashion.

$$P_t^* = \gamma^* P_{a,t}^* + (1-\gamma^*) P_{b,t}^*,$$

with

$$\gamma^* = \frac{P_a^*(c_a^* + g_a^*)}{P_a^*(c_a^* + g_a^*) + P_b^*(c_b^* + g_b^*)}.$$

1.3 Firms

Goods of type a are produced exclusively in the domestic country, and goods of type b are produced exclusively abroad. Each individual variety of good of type a or b is assumed to be produced by a monopolist. Each good $i \in [0,1]$ is manufactured using labor as the sole input with a linear production technology. Specifically domestic output of viariety i of type a, denoted $y_{i,a,t}$, is produced according to the relationship

$$y_{i,a,t} = z_t h_{i,a,t},$$

where $h_{i,a,t}$ denotes labor input in producing variety i of good a, and z_t denotes an aggregate technology shock in the domestic country.

The producer of variety i of good a faces demands from the private and public sectors in the domestic and foreign countries. The private and public domestic demand functions are given by

$$c_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x_{a,t}^c + \theta_a^c s_{i,a,t-1}^c,$$

and

$$g_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x_{a,t}^g + \theta_a^g s_{i,a,t-1}^g.$$

Letting an asterisk denote a foreign variable or parameter, the foreign private and public components of demand for variety i of type a are given by

$$c_{i,a,t}^* = \left(\frac{P_{i,a,t}^*}{P_{a,t}^*}\right)^{-\eta} x_{a,t}^{c*} + \theta_a^{c*} s_{i,a,t-1}^{c*}$$

and

$$g_{i,a,t}^* = \left(\frac{P_{i,a,t}^*}{P_{a,t}^*}\right)^{-\eta} x_{a,t}^{g*} + \theta_a^{g*} s_{i,a,t-1}^{g*}.$$

Implicit in the above demand functions is the assumption that firms do not price discriminate between the government and consumers residing in the same country.

A number of important implications are evident from inspection of the above demand functions: First, in both the domestic and foreign markets, the price elasticity of demand is not constant, but rather an increasing function of aggregate demand (as measured by $x_{a,t}^c$, $x_{a,t}^g$, $x_{a,t}^{c*}$, and $x_{a,t}^{g*}$). This characteristic of the demand for individual varieties is the consequence of the presence of habit formation at the level of individual varieties. Second, the fact that the price elasticity is procyclical opens the possibility for markups to move countercyclically in equilibrium. Third, because the price elasticity of demand can in principle be different in the domestic and the foreign markets, it follows that firms have an incentive to charge different markups (via price discrimination) domestically and abroad. We refer to this incentive for price discrimination as 'pricing to habits' as it originates from the presence of a habitual demand for individual varieties of goods. Differences in markups across borders for the same type of goods give rise to cyclical deviations from the law of one price.

The firm's problem consists in choosing prices and quantities in the domestic and foreign markets to maximize

$$E_0 \sum_{t=0}^{\infty} r_{0,t} \left[P_{i,a,t}(c_{i,a,t} + g_{i,a,t}) + P^*_{i,a,t}(c^*_{i,a,t} + g^*_{i,a,t}) - W_t h_{i,a,t} \right]$$

subject to

$$c_{i,a,t} + g_{i,a,t} + c^*_{i,a,t} + g^*_{i,a,t} = z_t h_{i,a,t},$$

$$c_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x^c_{a,t} + \theta^c_a s^c_{i,a,t-1},$$

$$g_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}} \right)^{-\eta} x^g_{a,t} + \theta^g_a s^g_{i,a,t-1},$$

$$c^*_{i,a,t} = \left(\frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{-\eta} x^{c*}_{a,t} + \theta^{c*}_a s^{c*}_{i,a,t-1},$$

$$g^*_{i,a,t} = \left(\frac{P^*_{i,a,t}}{P^*_{a,t}} \right)^{-\eta} x^{g*}_{a,t} + \theta^{g*}_a s^{g*}_{i,a,t-1},$$

$$s^c_{i,a,t} = \rho s^c_{i,a,t-1} + (1-\rho) c_{i,a,t},$$

$$s^g_{i,a,t} = \rho s^g_{i,a,t-1} + (1-\rho) g_{i,a,t},$$

$$s^{c*}_{i,a,t} = \rho s^{c*}_{i,a,t-1} + (1-\rho) c^*_{i,a,t},$$

and

$$s^{g*}_{i,a,t} = \rho s^{g*}_{i,a,t-1} + (1-\rho) g^*_{i,a,t}.$$

The Lagrangian of the firm is:

$$\begin{aligned}
\mathcal{L} \;\; = \;\; & E_0 \sum_{t=0}^{\infty} r_{0,t} \Bigg\{ P_{i,a,t}(c_{i,a,t}+g_{i,a,t}) + P^*_{i,a,t}(c^*_{i,a,t}+g^*_{i,a,t}) - \frac{W_t}{z_t}(c_{i,a,t}+g_{i,a,t}+c^*_{i,a,t}+g^*_{i,a,t}) \\
& + P_{a,t}\nu^c_{i,a,t}\left[\left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x^c_{a,t} + \theta^c_a s^c_{i,a,t-1} - c_{i,a,t}\right] \\
& + P_{a,t}\nu^g_{i,a,t}\left[\left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x^g_{a,t} + \theta^g_a s^g_{i,a,t-1} - g_{i,a,t}\right] \\
& + P^*_{a,t}\nu^{c*}_{i,a,t}\left[\left(\frac{P^*_{i,a,t}}{P^*_{a,t}}\right)^{-\eta} x^{c*}_{a,t} + \theta^{c*}_a s^{c*}_{i,a,t-1} - c^*_{i,a,t}\right] \\
& + P^*_{a,t}\nu^{g*}_{i,a,t}\left[\left(\frac{P^*_{i,a,t}}{P^*_{a,t}}\right)^{-\eta} x^{g*}_{a,t} + \theta^{g*}_a s^{g*}_{i,a,t-1} - g^*_{i,a,t}\right] \\
& + P_{a,t}\lambda^c_{i,a,t}\left[s^c_{i,a,t} - \rho s^c_{i,a,t-1} - (1-\rho)c_{i,a,t}\right] \\
& + P_{a,t}\lambda^g_{i,a,t}\left[s^g_{i,a,t} - \rho s^g_{i,a,t-1} - (1-\rho)g_{i,a,t}\right] \\
& + P^*_{a,t}\lambda^{c*}_{i,a,t}\left[s^{c*}_{i,a,t} - \rho s^{c*}_{i,a,t-1} - (1-\rho)c^*_{i,a,t}\right] \\
& + P^*_{a,t}\lambda^{g*}_{i,a,t}\left[s^{g*}_{i,a,t} - \rho s^{g*}_{i,a,t-1} - (1-\rho)g^*_{i,a,t}\right]\Bigg\}
\end{aligned}$$

The optimality conditions associated with the firm's problem with respect to $P_{i,a,t}$, $P^*_{i,a,t}$, $c_{i,a,t}$, $g_{i,a,t}$, $c^*_{i,a,t}$, $g^*_{i,a,t}$, $s^c_{i,a,t}$, $s^g_{i,a,t}$, $s^{c*}_{i,a,t}$, $s^{g*}_{i,a,t}$, respectively, are

$$c_{i,a,t}+g_{i,a,t} = \eta\frac{P_{a,t}}{P_{i,a,t}}\left[\nu^c_{i,a,t}(c_{i,a,t}-\theta^c_a s^c_{i,a,t-1}) + \nu^g_{i,a,t}(g_{i,a,t}-\theta^g_a s^g_{i,a,t-1})\right]$$

$$c^*_{i,a,t}+g^*_{i,a,t} = \eta\frac{P^*_{a,t}}{P^*_{i,a,t}}\left[\nu^{c*}_{i,a,t}(c^*_{i,a,t}-\theta^{c*}_a s^{c*}_{i,a,t-1}) + \nu^{g*}_{i,a,t}(g^*_{i,a,t}-\theta^{g*}_a s^{g*}_{i,a,t-1})\right]$$

$$-P_{a,t}\nu^c_{i,a,t} + P_{i,a,t} - \frac{W_t}{z_t} - (1-\rho)P_{a,t}\lambda^c_{i,a,t} = 0$$

$$-P_{a,t}\nu^g_{i,a,t} + P_{i,a,t} - \frac{W_t}{z_t} - (1-\rho)P_{a,t}\lambda^g_{i,a,t} = 0$$

$$-P^*_{a,t}\nu^{c*}_{i,a,t} + P^*_{i,a,t} - \frac{W_t}{z_t} - (1-\rho)P^*_{a,t}\lambda^{c*}_{i,a,t} = 0$$

$$-P^*_{a,t}\nu^{g*}_{i,a,t} + P^*_{i,a,t} - \frac{W_t}{z_t} - (1-\rho)P^*_{a,t}\lambda^{g*}_{i,a,t} = 0$$

$$\theta^c_a E_t r_{t,t+1} P_{a,t+1}\nu^c_{i,a,t+1} + P_{a,t}\lambda^c_{i,a,t} - \rho E_t r_{t,t+1} P_{a,t+1}\lambda^c_{i,a,t+1} = 0$$

$$\theta^g_a E_t r_{t,t+1} P_{a,t+1}\nu^g_{i,a,t+1} + P_{a,t}\lambda^g_{i,a,t} - \rho E_t r_{t,t+1} P_{a,t+1}\lambda^g_{i,a,t+1} = 0$$

$$\theta^{c*}_a E_t r_{t,t+1} P^*_{a,t+1}\nu^{c*}_{i,a,t+1} + P^*_{a,t}\lambda^{c*}_{i,a,t} - \rho E_t r_{t,t+1} P^*_{a,t+1}\lambda^{c*}_{i,a,t+1} = 0$$

$$\theta^{g*}_a E_t r_{t,t+1} P^*_{a,t+1}\nu^{g*}_{i,a,t+1} + P^*_{a,t}\lambda^{g*}_{i,a,t} - \rho E_t r_{t,t+1} P^*_{a,t+1}\lambda^{g*}_{i,a,t+1} = 0$$

Similar optimality conditons can be derived for foreign firms producing varieties of good b.

Symmetric Equilibrium

We assume that given the type of good (a or b), the type of consumer (private or public), and the location of the consumer (domestic market or foreign market), initial habit stocks are identical across different varieties. In a symmetric equilibrium, all firms producing varieties of good a for the domestic market will charge the same price. That is, $P_{i,a,t} = P_{a,t}$ for all i. Similarly, all firms producing varieties of good a for the foreign market will charge the same price, or $P^*_{i,a,t} = P^*_{a,t}$ for all i. The same symmetry applies to the foreign produced goods (type b), that is, $P_{i,b,t} = P_{b,t}$ and $P^*_{i,b,t} = P^*_{b,t}$ for all i.

We define three aggregate variables that are important in descibing international business cycles: the terms of trade, the real exchange rate, and aggregate private consumption. The terms of trade, which we denote by τ_t, are defined as the relative price of foreign goods in terms of domestic goods, or

$$\tau_t = \frac{P_{b,t}}{P_{a,t}}.$$

We define two good-specific real exchange rates. One is the relative price of good a abroad in terms of units of good a at home, which we denote by $e_{a,t}$. The second is the relative price of good b abroad in terms of units of good b in the home market, denoted $e_{b,t}$. Formally, the real exchange rates for goods a and b, respectively, are given by

$$e_{a,t} = \frac{P^*_{a,t}}{P_{a,t}}$$

and

$$e_{b,t} = \frac{P^*_{b,t}}{P_{b,t}}.$$

Because firms can price discrimintate across domestic and foreing markets, good-specific real exchange rates need not be unity. When the real exchange rate for a particular good is different from one, we say that the law of one price for that good is violated.

At a more aggregate level, the real exchange rate, denoted e_t, is defined as the relative price of foreign consumption in terms of domestic consumption, or P^*_t/P_t. One can express the real exchange rate in terms of the good-specific real exchange rates and the terms of trade as:

$$e_t \equiv \frac{P^*_t}{P_t} = \frac{\gamma^* e_{a,t} + (1-\gamma^*) e_{b,t}\tau_t}{\gamma + (1-\gamma)\tau_t}.$$

Finally, we define aggregate domestic consumption as $c_t = (P_{a,t}c_{a,t} + P_{b,t}c_{b,t})/P_t$, or

$$c_t = \frac{c_{a,t} + \tau_t c_{b,t}}{\gamma + (1-\gamma)\tau_t}.$$

Similarly, we define foreign aggreate consumption as $c^*_t = (P^*_{a,t}c^*_{a,t} + P^*_{b,t}c^*_{b,t})/P^*_t$, or

$$c^*_t = \frac{e_{a,t}c^*_{a,t} + e_{b,t}\tau_t c^*_{b,t}}{\gamma^* e_{a,t} + (1-\gamma^*) e_{b,t}\tau_t}.$$

We assume that financial markets are complete and that financial capital can flow freely across countries. This means that domestic and foreign households face the same contingent-claim prices $r_{t,t+1}$. Combining the domestic Euler equation (4) with its foreign counterpart to eliminate $r_{t,t+1}$ yields

$$\frac{U_x(x^c_{t+1}, h_{t+1})\chi_a(x^c_{a,t+1}, x^c_{b,t+1})}{U_x(x^c_t, h_t)\chi_a(x^c_{a,t}, x^c_{b,t})}\frac{P_{a,t}}{P_{a,t+1}} = \frac{U_{x^*}(x^{c*}_{t+1}, h^*_{t+1})\chi^*_a(x^{c*}_{a,t+1}, x^{c*}_{b,t+1})}{U_{x^*}(x^{c*}_t, h^*_t)\chi^*_a(x^{c*}_{a,t}, x^{c*}_{b,t})}\frac{P^*_{a,t}}{P^*_{a,t+1}}.$$

Because this expression holds in every date and every state, it follows that $\frac{U_x(x^c_t,h_t)\chi_a(x^c_{a,t},x^c_{b,t})}{P_{a,t}}$ must be proportional to $\frac{U_{x^*}(x^{c*}_t,h^*_t)\chi^*_a(x^{c*}_{a,t},x^{c*}_{b,t})}{P^*_{a,t}}$. The factor of proportionality is determined by the relative wealth of the two countries. We consider a case in which both countries are equally wealthy so that the factor of proportionality is unity. It follows that

$$e_{a,t} = \frac{U_{x^*}(x^{c*}_t, h^*_t)\chi^*_a(x^{c*}_{a,t}, x^{c*}_{b,t})}{U_x(x^c_t, h_t)\chi_a(x^c_{a,t}, x^c_{b,t})}. \tag{10}$$

The complete set of equilibrium conditions consists of equation (10) and the following equaitons, which we group into a domestic block and a foreign block.

Domestic Block

$$\frac{\chi_b(x^c_{a,t}, x^c_{b,t})}{\chi_a(x^c_{a,t}, x^c_{b,t})} = \tau_t \tag{11}$$

$$x^c_t = \chi(x^c_{a,t}, x^c_{b,t}) \tag{12}$$

$$-\frac{U_h(x^c_t, h_t)}{U_x(x^c_t, h_t)\chi_a(x_{a,t}, x_{b,t})} = w_t \tag{13}$$

$$c_{a,t} + c^*_{a,t} + g_{a,t} + g^*_{a,t} = z_t h_t \tag{14}$$

$$x^g_{a,t} = g_{a,t} - \Theta^g_a s^g_{a,t-1} \tag{15}$$

$$x^c_{a,t} = c_{a,t} - \Theta^c_a s^c_{a,t-1} \tag{16}$$

$$x^g_{b,t} = g_{b,t} - \Theta^g_b s^g_{b,t-1} \tag{17}$$

$$x^c_{b,t} = c_{b,t} - \Theta^c_b s^c_{b,t-1} \tag{18}$$

$$1 - \frac{1}{\mu_{a,t}} = \nu^c_{a,t} + (1-\rho)\lambda^c_{a,t} \tag{19}$$

$$1 - \frac{1}{\mu_{a,t}} = \nu^g_{a,t} + (1-\rho)\lambda^g_{a,t} \tag{20}$$

$$1 - \frac{1}{\mu^*_{a,t}} = \nu^{c*}_{a,t} + (1-\rho)\lambda^{c*}_{a,t} \tag{21}$$

$$1 - \frac{1}{\mu^*_{a,t}} = \nu^{g*}_{a,t} + (1-\rho)\lambda^{g*}_{a,t} \tag{22}$$

$$c_{a,t} + g_{a,t} = \eta[\nu^c_{a,t}(c_{a,t} - \theta^c_a s^c_{a,t-1}) + \nu^g_{a,t}(g_{a,t} - \theta^g_a s^g_{a,t-1})] \tag{23}$$

$$c^*_{a,t} + g^*_{a,t} = \eta[\nu^{c*}_{a,t}(c^*_{a,t} - \theta^{c*}_a s^{c*}_{a,t-1}) + \nu^{g*}_{a,t}(g^*_{a,t} - \theta^{g*}_a s^{g*}_{a,t-1})] \tag{24}$$

$$\theta^c_a \beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\nu^c_{a,t+1} + \lambda^c_{a,t} = \rho\beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\lambda^c_{a,t+1} \tag{25}$$

$$\theta^g_a \beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\nu^g_{a,t+1} + \lambda^g_{a,t} = \rho\beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\lambda^g_{a,t+1} \tag{26}$$

$$\theta^{c*}_a \beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\nu^{c*}_{a,t+1} + \lambda^{c*}_{a,t} = \rho\beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\lambda^{c*}_{a,t+1} \tag{27}$$

$$\theta^{g*}_a \beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\nu^{g*}_{a,t+1} + \lambda^{g*}_{a,t} = \rho\beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\lambda^{g*}_{a,t+1} \tag{28}$$

$$s^c_{a,t} = \rho s^c_{a,t-1} + (1-\rho)c_{a,t} \tag{29}$$

$$s^g_{a,t} = \rho s^g_{a,t-1} + (1-\rho)g_{a,t} \tag{30}$$

$$s^{c*}_{a,t} = \rho s^{c*}_{a,t-1} + (1-\rho)c^*_{a,t} \tag{31}$$

$$s^{g*}_{a,t} = \rho s^{g*}_{a,t-1} + (1-\rho)g^*_{a,t} \tag{32}$$

$$\mu_{a,t} = \frac{z_t}{w_t} \tag{33}$$

$$\frac{\mu^*_{a,t}}{\mu_{a,t}} = e_{a,t} \tag{34}$$

$$[\gamma + (1-\gamma)\tau_t]g_t = g_{a,t} + \tau_t g_{b,t} \tag{35}$$

$$\frac{x^c_{a,t}}{x^c_{b,t}} = \frac{x^g_{a,t}}{x^g_{b,t}} \tag{36}$$

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \epsilon^g_t \tag{37}$$

where $w_t \equiv W_t/P_{a,t}$ denotes the real wage in terms of domestic goods. We denote by Θ^j_k denotes the superficial-habit parameter for agent $j = c, g$ and for good $k = a, b$. Lower case is used for the deep-habit parameter.

Foreign Block

$$\frac{\chi_b^*(x_{a,t}^{c*}, x_{b,t}^{c*})}{\chi_a^*(x_{a,t}^{c*}, x_{b,t}^{c*})} = \tau_t \frac{e_{b,t}}{e_{a,t}} \tag{38}$$

$$x_t^{c*} = \chi(x_{a,t}^{c*}, x_{b,t}^{c*}) \tag{39}$$

$$-\frac{U_h(x_t^{c*}, h_t^*)}{U_x(x_t^{c*}, h_t^*)\chi_b^*(x_{a,t}^{c*}, x_{b,t}^{c*})} = w_t^* \tag{40}$$

$$c_{b,t} + g_{b,t} + c_{b,t}^* + g_{b,t}^* = z_t^* h_t^* \tag{41}$$

$$x_{a,t}^{c*} = c_{a,t}^* - \Theta_a^{c*} s_{a,t-1}^{c*} \tag{42}$$

$$x_{a,t}^{g*} = g_{a,t}^* - \Theta_a^{g*} s_{a,t-1}^{g*} \tag{43}$$

$$x_{b,t}^{c*} = c_{b,t}^* - \Theta_b^{c*} s_{b,t-1}^{c*} \tag{44}$$

$$x_{b,t}^{g*} = g_{b,t}^* - \Theta_b^{g*} s_{b,t-1}^{g*} \tag{45}$$

$$\frac{\mu_{b,t} - 1}{\mu_{b,t}} = \nu_{b,t}^c + (1-\rho)\lambda_{b,t}^c \tag{46}$$

$$\frac{\mu_{b,t} - 1}{\mu_{b,t}} = \nu_{b,t}^g + (1-\rho)\lambda_{b,t}^g \tag{47}$$

$$\frac{\mu_{b,t}^* - 1}{\mu_{b,t}^*} = \nu_{b,t}^{c*} + (1-\rho)\lambda_{b,t}^{c*} \tag{48}$$

$$\frac{\mu_{b,t}^* - 1}{\mu_{b,t}^*} = \nu_{b,t}^{g*} + (1-\rho)\lambda_{b,t}^{g*} \tag{49}$$

$$c_{b,t} + g_{b,t} = \eta[v_{b,t}^c(c_{b,t} - \theta_b^c s_{b,t-1}^c) + v_{b,t}^g(g_{b,t} - \theta_b^g s_{b,t-1}^g)] \tag{50}$$

$$c_{b,t}^* + g_{b,t}^* = \eta[v_{b,t}^{c*}(c_{b,t}^* - \theta_b^{c*} s_{b,t-1}^{c*}) + v_{b,t}^{g*}(g_{b,t}^* - \theta_b^{g*} s_{b,t-1}^{g*})] \tag{51}$$

$$\theta_b^c \beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)} \nu_{b,t+1}^c + \lambda_{b,t}^c = \rho\beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)} \lambda_{b,t+1}^c \tag{52}$$

$$\theta_b^g \beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)} \nu_{b,t+1}^g + \lambda_{b,t}^g = \rho\beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)} \lambda_{b,t+1}^g \tag{53}$$

$$\theta_b^{c*} \beta E_t \frac{U_x^*(t+1)\chi_b^*(t+1)}{U_x^*(t)\chi_b^*(t)} \nu_{b,t+1}^{c*} + \lambda_{b,t}^{c*} = \rho\beta E_t \frac{U_x^*(t+1)\chi_b^*(t+1)}{U_x^*(t)\chi_b^*(t)} \lambda_{b,t+1}^{c*} \tag{54}$$

$$\theta_b^{g*} \beta E_t \frac{U_x^*(t+1)\chi_b^*(t+1)}{U_x^*(t)\chi_b^*(t)} \nu_{b,t+1}^{g*} + \lambda_{b,t}^{g*} = \rho\beta E_t \frac{U_x^*(t+1)\chi_b^*(t+1)}{U_x^*(t)\chi_b^*(t)} \lambda_{b,t+1}^{g*} \tag{55}$$

$$s_{b,t}^c = \rho s_{b,t-1}^c + (1-\rho) c_{b,t} \tag{56}$$

$$s_{b,t}^g = \rho s_{b,t-1}^g + (1-\rho) g_{b,t} \tag{57}$$

$$s_{b,t}^{c*} = \rho s_{b,t-1}^{c*} + (1-\rho) c_{b,t}^* \tag{58}$$

$$s_{b,t}^{g*} = \rho s_{b,t-1}^{g*} + (1-\rho) g_{b,t}^* \tag{59}$$

$$\mu^*_{b,t} = \frac{z^*_t}{w^*_t} \tag{60}$$

$$\frac{\mu^*_{b,t}}{\mu_{b,t}} = e_{b,t} \tag{61}$$

$$g^*_{a,t} + \tau_t \frac{e_{b,t}}{e_{a,t}} g^*_{b,t} = \left[\gamma^* + (1-\gamma^*)\tau_t \frac{e_{b,t}}{e_{a,t}}\right] g^*_t \tag{62}$$

$$\frac{x^{c*}_{a,t}}{x^{c*}_{b,t}} = \frac{x^{g*}_{a,t}}{x^{g*}_{b,t}} \tag{63}$$

$$\log(g^*_t) = \rho_{g^*} \log(g^*_{t-1}) + \epsilon^{g^*}_t \tag{64}$$

where $w^*_t \equiv W^*_t / P^*_{b,t}$. A competitive equilibrium is a set of 55 stochastic difference equations in 55 unknowns. Specificlly, a competitive equilibrium is a set of processes $x^c_{a,t}$, $x^g_{a,t}$, $x^{c*}_{a,t}$, $x^{g*}_{a,t}$, $x^c_{b,t}$, $x^g_{b,t}$, $x^{c*}_{b,t}$, $x^{g*}_{b,t}$, $s^c_{a,t}$, $s^g_{a,t}$, $s^{c*}_{a,t}$, $s^{g*}_{a,t}$, $s^c_{b,t}$, $s^g_{b,t}$, $s^{c*}_{b,t}$, $s^{g*}_{b,t}$, $\lambda^c_{a,t}$, $\lambda^g_{a,t}$, $\lambda^{c*}_{a,t}$, $\lambda^{g*}_{a,t}$, $\lambda^c_{b,t}$, $\lambda^g_{b,t}$, $\lambda^{c*}_{b,t}$, $\lambda^{g*}_{b,t}$, $\nu^c_{a,t}$, $\nu^g_{a,t}$, $\nu^{c*}_{a,t}$, $\nu^{g*}_{a,t}$, $\nu^c_{b,t}$, $\nu^g_{b,t}$, $\nu^{c*}_{b,t}$, $\nu^{g*}_{b,t}$, $c_{a,t}$, $c_{b,t}$, $g_{a,t}$, $g_{b,t}$, $c^*_{a,t}$, $c^*_{b,t}$, $g^*_{a,t}$, $g^*_{b,t}$, w_t, w^*_t, h_t, h^*_t, x^c_t, x^{c*}_t, $\mu_{a,t}$, $\mu^*_{a,t}$, $\mu_{b,t}$, $\mu^*_{b,t}$, $e_{a,t}$, $e_{b,t}$, τ_t, g_t, and g^*_t, satisfying equations (10)-(64), given initial conditions $s^c_{a,-1}$, $s^g_{a,-1}$, $s^{c*}_{a,-1}$, $s^{g*}_{a,-1}$, $s^c_{b,-1}$, $s^g_{b,-1}$, $s^{c*}_{b,-1}$, $s^{g*}_{b,-1}$, g_{-1}, and g^*_{-1}, and the exogenous processes ϵ^g_t, $\epsilon^{g^*}_t$, z_t, and z^*_t.

It is clear from equation (34) that cyclical deviations from the law of one price in good a arise when domestic and foreign markups in this industry fail to move proportionally over the business cycle. A similar observation applies to the behavior of prices for good b. The presence of deep habits induces firms to price differently in the domestic and foreign markets whenever aggregate demand conditions differ across borders. This pricing behavior, to which we refer as pricing to habits, produces persistent and volatile cyclical deviations from the law of one price at a good-by-good level.

2 Functional Forms

$$U(x,h) = \frac{\left[x_t^{\phi}(1-h_t)^{1-\phi}\right]^{1-\sigma} - 1}{1-\sigma}$$

$$\chi(x_a, x_b) = \left[\omega x_a^{1-1/\xi} + (1-\omega) x_b^{1-1/\xi}\right]^{1/(1-1/\xi)}$$

$$\chi^*(x^*_a, x^*_b) = \left[(1-\omega) x_a^{*1-1/\xi} + \omega x_b^{*1-1/\xi}\right]^{1/(1-1/\xi)}$$

We note that the as the parameter σ approaches unity, the period utility funciton becomes log separable in consumption and leisure. The case of separable preferences in consumption and leisure is of particular interest because it highlights the fact that the pricing-to-habit mechanism does not depend on the assumption of nonseparabilites between leisure and consumption to deliver qualitatively realistic dynamics in response to aggregate demand shocks.

Steady State I

Complete-Market Condition

$$e_a = \frac{x^{c*\phi(1-\sigma)-1}(1-h^*)^{(1-\phi)(1-\sigma)}(1-\omega)(x^{c*}/x_a^*)^{1/\xi}}{x^{c\phi(1-\sigma)-1}(1-h)^{(1-\phi)(1-\sigma)}(\omega)(x^c/x_a)^{1/\xi}}$$

Domestic Block

$$\left(\frac{1-\omega}{\omega}\right)\left(\frac{x_b^c}{x_a^c}\right)^{-1/\xi} = \tau \tag{65}$$

$$x^c = \left[\omega x_a^{c\,1-1/\xi} + (1-\omega)x_b^{c\,1-1/\xi}\right]^{1/(1-1/\xi)} \tag{66}$$

$$\left(\frac{1-\phi}{\omega\phi}\right)\left(\frac{x^c}{1-h}\right)\left(\frac{x_a^c}{x^c}\right)^{1/\xi} = \frac{1}{\mu_a} \tag{67}$$

$$c_a + c_a^* + g_a + g_a^* = h \tag{68}$$

$$x_a^g = g_a(1-\Theta_a^g) \tag{69}$$

$$x_a^c = c_a(1-\Theta_a^c) \tag{70}$$

$$x_b^g = g_b(1-\Theta_b^g) \tag{71}$$

$$x_b^c = c_b(1-\Theta_b^c) \tag{72}$$

$$1 - \frac{1}{\mu_a} = \nu_a^c + (1-\rho)\lambda_a^c \tag{73}$$

$$1 - \frac{1}{\mu_a} = \nu_a^g + (1-\rho)\lambda_a^g \tag{74}$$

$$1 - \frac{1}{\mu_a^*} = \nu_a^{c*} + (1-\rho)\lambda_a^{c*} \tag{75}$$

$$1 - \frac{1}{\mu_a^*} = \nu_a^{g*} + (1-\rho)\lambda_a^{g*} \tag{76}$$

$$\frac{x_a^c}{1-\Theta_a^c} + \frac{x_a^g}{1-\Theta_a^g} = \eta\left[\nu_a^c x_a^c\left(\frac{1-\theta_a^c}{1-\Theta_a^c}\right) + \nu_a^g x_a^g\left(\frac{1-\theta_a^g}{1-\Theta_a^g}\right)\right] \tag{77}$$

$$\frac{x_a^{c*}}{1-\Theta_a^{c*}} + \frac{x_a^{g*}}{1-\Theta_a^{g*}} = \eta\left[\nu_a^{c*} x_a^{c*}\left(\frac{1-\theta_a^{c*}}{1-\Theta_a^{c*}}\right) + \nu_a^{g*} x_a^{g*}\left(\frac{1-\theta_a^{g*}}{1-\Theta_a^{g*}}\right)\right] \tag{78}$$

$$\theta_a^c\beta\nu_a^c = (\rho\beta-1)\lambda_a^c \tag{79}$$

$$\theta_a^g\beta\nu_a^g = (\rho\beta-1)\lambda_a^g \tag{80}$$

$$\theta_a^{c*}\beta\nu_a^{c*} = (\rho\beta-1)\lambda_a^{c*} \tag{81}$$

$$\theta_a^{g*}\beta\nu_a^{g*} = (\rho\beta-1)\lambda_a^{g*} \tag{82}$$

$$s_a^c = c_a \tag{83}$$

$$s_a^g = g_a \tag{84}$$

$$s_a^{c*} = c_a^* \tag{85}$$

$$s_a^{g*} = g_a^* \tag{86}$$

$$\mu_a = \frac{1}{w} \tag{87}$$

$$\frac{\mu_a^*}{\mu_a} = e_a \tag{88}$$

$$[\gamma + (1-\gamma)\tau]g = \frac{x_a^g}{1-\Theta_a^g} + \tau\frac{x_b^g}{1-\Theta_b^g} \tag{89}$$

$$\frac{x_a^c}{x_b^c} = \frac{x_a^g}{x_b^g} \tag{90}$$

$$g = \bar{g} \tag{91}$$

Foreign Block

$$\left(\frac{\omega}{1-\omega}\right)\left(\frac{x_b^{c*}}{x_a^{c*}}\right)^{-1/\xi} = \tau\frac{e_b}{e_a} \tag{92}$$

$$x^{c*} = \left[(1-\omega)x_a^{c*1-1/\xi} + \omega x_b^{c*1-1/\xi}\right]^{1/(1-1/\xi)} \tag{93}$$

$$\left(\frac{1-\phi}{\omega\phi}\right)\left(\frac{x^{c*}}{1-h^*}\right)\left(\frac{x_b^{c*}}{x^{c*}}\right)^{1/\xi} = w^* \tag{94}$$

$$c_b + g_b + c_b^* + g_b^* = h^* \tag{95}$$

$$x_a^{c*} = c_a^*(1-\Theta_a^{c*}) \tag{96}$$

$$x_a^{g*} = g_a^*(1-\Theta_a^{g*}) \tag{97}$$

$$x_b^{c*} = c_b^*(1-\Theta_b^{c*}) \tag{98}$$

$$x_b^{g*} = g_b^*(1-\Theta_b^{g*}) \tag{99}$$

$$\frac{\mu_b - 1}{\mu_b} = \nu_b^c + (1-\rho)\lambda_b^c \tag{100}$$

$$\frac{\mu_b - 1}{\mu_b} = \nu_b^g + (1-\rho)\lambda_b^g \tag{101}$$

$$\frac{\mu_b^* - 1}{\mu_b^*} = \nu_b^{c*} + (1-\rho)\lambda_b^{c*} \tag{102}$$

$$\frac{\mu_b^* - 1}{\mu_b^*} = \nu_b^{g*} + (1-\rho)\lambda_b^{g*} \tag{103}$$

$$\frac{x_b^c}{1-\Theta_b^c} + \frac{x_b^g}{1-\Theta_b^g} = \eta\left[\upsilon_b^c x_b^c\left(\frac{1-\theta_b^c}{1-\Theta_b^c}\right) + \upsilon_b^g x_b^g\left(\frac{1-\theta_b^g}{1-\Theta_b^g}\right)\right] \tag{104}$$

$$\frac{x_b^{c*}}{1-\Theta_b^{c*}} + \frac{x_b^{g*}}{1-\Theta_b^{g*}} = \eta\left[\nu_b^{c*} x_b^{c*}\left(\frac{1-\theta_b^{c*}}{1-\Theta_b^{c*}}\right) + \nu_b^{g*} x_b^{g*}\left(\frac{1-\theta_b^{g*}}{1-\Theta_b^{g*}}\right)\right] \tag{105}$$

$$\theta_b^c\beta\nu_b^c = (\rho\beta - 1)\lambda_b^c \tag{106}$$

$$\theta_b^g\beta\nu_b^g = (\rho\beta - 1)\lambda_b^g \tag{107}$$

$$\theta_b^{c*}\beta\nu_b^{c*} = (\rho\beta - 1)\lambda_b^{c*} \tag{108}$$

$$\theta_b^{g*}\beta\nu_b^{g*} = (\rho\beta - 1)\lambda_b^{g*} \tag{109}$$

$$s_b^c = c_b \tag{110}$$

$$s_b^g = g_b \tag{111}$$

$$s_b^{c*} = c_b^* \tag{112}$$

$$s_b^{g*} = g_b^* \tag{113}$$

$$\mu_b^* = \frac{1}{w^*} \tag{114}$$

$$\frac{\mu_b^*}{\mu_b} = e_b \tag{115}$$

$$g_a^* + \tau \frac{e_b}{e_a} g_b^* = \left[\gamma^* + (1 - \gamma^*) \tau \frac{e_b}{e_a} \right] g^* \tag{116}$$

$$\frac{x_a^{c*}}{x_b^{c*}} = \frac{x_a^{g*}}{x_b^{g*}} \tag{117}$$

$$g^* = \bar{g}^* \tag{118}$$

Steady State II

Eliminate ν, λ, c, and g in both countries.

Complete-Market Condition

$$e_a = \frac{x^{c*\phi(1-\sigma)-1}(1-h^*)^{(1-\phi)(1-\sigma)}(1-\omega)(x^{c*}/x_a^*)^{1/\xi}}{x^{c\phi(1-\sigma)-1}(1-h)^{(1-\phi)(1-\sigma)}(\omega)(x^c/x_a)^{1/\xi}}$$

Domestic Block

$$\left(\frac{1-\omega}{\omega}\right)\left(\frac{x_b^c}{x_a^c}\right)^{-1/\xi} = \tau \tag{119}$$

$$x^c = \left[\omega x_a^{c\,1-1/\xi} + (1-\omega)x_b^{c\,1-1/\xi}\right]^{1/(1-1/\xi)} \tag{120}$$

$$\left(\frac{1-\phi}{\omega\phi}\right)\left(\frac{x^c}{1-h}\right)\left(\frac{x_a^c}{x^c}\right)^{1/\xi} = \frac{1}{\mu_a} \tag{121}$$

$$\frac{x_a^c}{1-\Theta_a^c} + \frac{x_a^{c*}}{1-\Theta_a^{c*}} + \frac{x_a^g}{1-\Theta_a^g} + \frac{x_a^{g*}}{1-\Theta_a^{g*}} = h \tag{122}$$

$$1 - \frac{1}{\mu_a} = \frac{\frac{x_a^c}{1-\Theta_a^c} + \frac{x_a^g}{1-\Theta_a^g}}{\eta\left[\frac{x_a^c\left(\frac{1-\theta_a^c}{1-\Theta_a^c}\right)}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{x_a^g\left(\frac{1-\theta_a^g}{1-\Theta_a^g}\right)}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right]} \tag{123}$$

$$1 - \frac{1}{\mu_a^*} = \frac{\frac{x_a^{c*}}{1-\Theta_a^{c*}} + \frac{x_a^{g*}}{1-\Theta_a^{g*}}}{\eta\left[\frac{x_a^{c*}\left(\frac{1-\theta_a^{c*}}{1-\Theta_a^{c*}}\right)}{1-\beta\theta_a^{c*}(1-\rho)/(1-\beta\rho)} + \frac{x_a^{g*}\left(\frac{1-\theta_a^{g*}}{1-\Theta_a^{g*}}\right)}{1-\beta\theta_a^{g*}(1-\rho)/(1-\beta\rho)}\right]} \tag{124}$$

$$\frac{\mu_a^*}{\mu_a} = e_a \tag{125}$$

$$[\gamma + (1-\gamma)\tau]g = \frac{x_a^g}{1-\Theta_a^g} + \tau\frac{x_b^g}{1-\Theta_b^g} \tag{126}$$

$$\frac{x_a^c}{x_b^c} = \frac{x_a^g}{x_b^g} \tag{127}$$

Foreign Block

$$\left(\frac{\omega}{1-\omega}\right)\left(\frac{x_b^{c*}}{x_a^{c*}}\right)^{-1/\xi} = \tau\frac{e_b}{e_a} \tag{128}$$

$$x^{c*} = \left[(1-\omega)x_a^{c*1-1/\xi} + \omega x_b^{c*1-1/\xi}\right]^{1/(1-1/\xi)} \tag{129}$$

$$\left(\frac{1-\phi}{\omega\phi}\right)\left(\frac{x^{c*}}{1-h^*}\right)\left(\frac{x_b^{c*}}{x^{c*}}\right)^{1/\xi} = \frac{1}{\mu_b^*} \tag{130}$$

$$\frac{x_b^c}{1-\Theta_b^c} + \frac{x_b^g}{1-\Theta_b^g} + \frac{x_b^{c*}}{1-\Theta_b^{c*}} + \frac{x_b^{g*}}{1-\Theta_b^{g*}} = h^* \tag{131}$$

$$\frac{\mu_b - 1}{\mu_b} = \frac{\frac{x_b^c}{1-\Theta_b^c} + \frac{x_b^g}{1-\Theta_b^g}}{\eta\left[\frac{x_b^c\left(\frac{1-\theta_b^c}{1-\Theta_b^c}\right)}{1-\theta_b^c(1-\rho)/(1-\beta\rho)} + \frac{x_b^g\left(\frac{1-\theta_b^g}{1-\Theta_b^g}\right)}{1-\theta_b^g(1-\rho)/(1-\beta\rho)}\right]} \tag{132}$$

$$\frac{\mu_b^* - 1}{\mu_b^*} = \frac{\frac{x_b^{c*}}{1-\Theta_b^{c*}} + \frac{x_b^{g*}}{1-\Theta_b^{g*}}}{\eta\left[\frac{x_b^{c*}\left(\frac{1-\theta c*_b}{1-\Theta c*_b}\right)}{1-\theta_b^{c*}(1-\rho)/(1-\beta\rho)} + \frac{x_b^{g*}\left(\frac{1-\theta g*_b}{1-\Theta g*_b}\right)}{1-\theta_b^{g*}(1-\rho)/(1-\beta\rho)}\right]} \tag{133}$$

$$\frac{\mu_b^*}{\mu_b} = e_b \tag{134}$$

$$g_a^* + \tau\frac{e_b}{e_a}g_b^* = \left[\gamma^* + (1-\gamma^*)\tau\frac{e_b}{e_a}\right]g^* \tag{135}$$

$$\frac{x_a^{c*}}{x_b^{c*}} = \frac{x_a^{g*}}{x_b^{g*}} \tag{136}$$

pick up from here

Steady State III

We impose the following symmetry assumptions: $\theta_a^c = \theta_b^{c*}$, $\theta_b^c = \theta_a^{c*}$, $\theta_a^g = \theta_b^{g*}$, and $\theta_b^g = \theta_a^{g*}$. These assumptions imply that $x_a^c = x_b^{c*}$, $x_b^c = x_a^{c*}$, $x_a^g = x_b^{g*}$, $x_b^g = x_a^{g*}$, $x^c = x^{c*}$, $h = h^*$, $\mu_b = \mu_a^*$, $\mu_b^* = \mu_a$, and $e_a = 1/e_b$. We can then drop the foreign block and work only with the domestic portion of the model together with the complete-asset-market condition.

Complete-Market Condition

$$e_a = \frac{1-\omega}{\omega}\left(\frac{x_a^c}{x_b^c}\right)^{1/\xi} \tag{137}$$

Domestic Block

$$e_a = \tau \tag{138}$$

$$x^c = \left[\omega x_a^{c\,1-1/\xi} + (1-\omega) x_b^{c\,1-1/\xi}\right]^{1/(1-1/\xi)} \tag{139}$$

$$\left(\frac{1-\phi}{\omega\phi}\right)\left(\frac{x^c}{1-h}\right)\left(\frac{x_a^c}{x^c}\right)^{1/\xi} = \frac{1}{\mu_a} \tag{140}$$

$$\frac{x_a^c}{1-\Theta_a^c} + \frac{x_b^c}{1-\Theta_b^c} + \frac{x_a^g}{1-\Theta_a^g} + \frac{x_b^g}{1-\Theta_b^g} = h \tag{141}$$

$$1 - \frac{1}{\mu_a} = \frac{\frac{x_a^c}{1-\Theta_a^c} + \frac{x_a^g}{1-\Theta_a^g}}{\eta\left[\frac{x_a^c\left(\frac{1-\theta_a^c}{1-\Theta_a^c}\right)}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{x_a^g\left(\frac{1-\theta_a^g}{1-\Theta_a^g}\right)}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right]} \tag{142}$$

$$1 - \frac{1}{\mu_a^*} = \frac{\frac{x_b^c}{1-\Theta_b^c} + \frac{x_b^g}{1-\Theta_b^g}}{\eta\left[\frac{x_b^c\left(\frac{1-\theta_b^c}{1-\Theta_b^c}\right)}{1-\beta\theta_b^c(1-\rho)/(1-\beta\rho)} + \frac{x_b^g\left(\frac{1-\theta_b^g}{1-\Theta_b^g}\right)}{1-\beta\theta_b^g(1-\rho)/(1-\beta\rho)}\right]} \tag{143}$$

$$\frac{\mu_a^*}{\mu_a} = e_a \tag{144}$$

$$[\gamma + (1-\gamma)\tau]g = \frac{x_a^g}{1-\Theta_a^g} + \tau\frac{x_b^g}{1-\Theta_b^g} \tag{145}$$

$$\frac{x_a^c}{x_b^c} = \frac{x_a^g}{x_b^g} \tag{146}$$

Steady State IV

We proceed to make further symmetry assumptions to make the steady-state system more tractable. Specifically, we assume that $\theta_a^c = \theta_b^c$ and that $\theta_a^g = \theta_b^g$. It can then be readily seen that $\tau = e_a = 1$.

$$x_b^c = \left(\frac{1-\omega}{\omega}\right)^{\xi} x_a^c \tag{147}$$

Combine this expression and (139)

$$x^c = x_a^c \omega^{-\xi} \left[\omega^{\xi} + (1-\omega)^{\xi}\right]^{1/(1-1/\xi)} \tag{148}$$

$$\left(\frac{1-\phi}{\phi}\right)\left(\frac{x_a^c}{1-h}\right)\frac{\left[\omega^{\xi} + (1-\omega)^{\xi}\right]}{\omega^{\xi}} = \frac{1}{\mu_a} \tag{149}$$

$$\frac{x_a^c}{1-\Theta_a^c} + \frac{x_b^c}{1-\Theta_b^c} + \frac{x_a^g}{1-\Theta_a^g} + \frac{x_b^g}{1-\Theta_b^g} = h \tag{150}$$

$$1 - \frac{1}{\mu_a} = \frac{\frac{x_a^c}{1-\Theta_a^c} + \frac{x_a^g}{1-\Theta_a^g}}{\eta\left[\frac{x_a^c\left(\frac{1-\theta_a^c}{1-\Theta_a^c}\right)}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{x_a^g\left(\frac{1-\theta_a^g}{1-\Theta_a^g}\right)}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right]} \tag{151}$$

Combine (147), (145), and (146), and recall that $\tau = 1$ to get

$$x_a^g = \frac{\omega^{\xi}}{\omega^{\xi} + (1-\omega)^{\xi}}(1-\Theta_a^g)g \tag{152}$$

and

$$x_b^g = \frac{(1-\omega)^{\xi}}{(1-\omega)^{\xi} + \omega^{\xi}}(1-\Theta_a^g)g$$

The sum of these two expressions equals $(1-\Theta_a^g)g$.

Steady State V

$$\left(\frac{1-\phi}{\phi}\right)\left(\frac{x_a^c}{1-h}\right)\frac{\left[\omega^{\xi} + (1-\omega)^{\xi}\right]}{\omega^{\xi}} = \frac{1}{\mu_a} \tag{153}$$

$$x_a^c = \frac{\omega^{\xi}}{\omega^{\xi} + (1-\omega)^{\xi}}(h-\bar{g})(1-\Theta_a^c) \tag{154}$$

$$1 - \frac{1}{\mu_a} = \frac{h}{\eta\left(\frac{(1-\theta_a^c)(h-\bar{g})}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{(1-\theta_a^g)\bar{g}}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right)} \tag{155}$$

Steady State VI

$$\left(\frac{h-\bar{g}}{1-h}\right)\left(\frac{1-\phi}{\phi}\right)(1-\Theta_a^c) = 1 - \frac{h}{\eta\left(\frac{(1-\theta_a^c)(h-\bar{g})}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{(1-\theta_a^g)\bar{g}}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right)} \tag{156}$$

Steady State VII

If one assumes that $\bar{g}$ is given, then the steady-state value of hours is computed as follows: Let

$$b_1 = \left(\frac{1-\phi}{\phi}\right)(1-\Theta_a^c)$$

$$a_1 = \eta \frac{(1-\theta_a^c)}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)}$$

$$a_2 = \eta\bar{g}\left(\frac{-(1-\theta_a^c)}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{(1-\theta_a^g)}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right)$$

Then equation (156) can be written as

$$\left(\frac{h-\bar{g}}{1-h}\right)b_1 = 1 - \frac{h}{a_1 h + a_2} \tag{157}$$

Now let

$$c_1 = b_1 a_1 + a_1 - 1$$

$$c_2 = b_1 a_2 - b_1 a_1 \bar{g} - a_1 + a_2 + 1$$

$$c_3 = -\bar{g} b_1 a_2 - a_2$$

Then

$$c_1 h^2 + c_2 h + c_3 = 0$$

Therefore,

$$h = \frac{-c_2 \pm \sqrt{c_2^2 - 4c_1c_3}}{2c_1}$$

If, insteady, one assumes that the steady-state share of government spending in output, which we denote by s_g, is constant, then the steady-state value of hours is computed as follows:

Let

$$b_1 = \left(\frac{1-\phi}{\phi}\right)(1-\Theta_a^c)(1-s_g)$$

$$a_1 = 1 - \frac{1}{\eta\left[\frac{(1-\theta_a^c)(1-s_g)}{1-\beta\theta_a^c(1-\rho)/(1-\beta\rho)} + \frac{(1-\theta_a^g)s_g}{1-\beta\theta_a^g(1-\rho)/(1-\beta\rho)}\right]}$$

Then equation (156) can be written as

$$\left(\frac{h}{1-h}\right)b_1 = a_1 \tag{158}$$

Solving for h, we obtain

$$h = \frac{a_1}{a_1 + b_1}$$

Table 1: Structural Parameters

Parameter	Value	Description
β	0.99	Subjective discount factor (quarterly)
σ	1	Intertemporal elasticity of subsitution
ϕ	0.15	Preference parameter
ω	0.75	Preference parameter
ξ	1.5	Elasticity of substitution between home and foreign goods
η	6	Price-elasticity of demand for a specific good variety
θ_k^j	0.6	Degree of habit persistence ($j = c, g, c^*, g^*,\ k = a, b$)
ρ	0.85	Persistence of habit stock
$\bar{g}, \bar{g}^*$	0.0622	Steady-state level of government consumption
ρ_g, ρ_{g^*}	0.87	Serial correlation of the log of government spending
$\sigma_{\epsilon^g}, \sigma_{\epsilon^{g^*}}$	0.016	Std. dev. of the innovation to log of gov. consumption
$\rho_{11}^Z = \rho_{22}^Z$	0.906	Parameters of technology shock process
$\rho_{12}^Z = \rho_{21}^Z$	0.088	Parameters of technology shock process
$\sigma_{\epsilon^z}, \sigma_{\epsilon^{z^*}}$	0.00852	Std. dev. of innovation to technology shock
$\mathrm{Corr}(\epsilon_t^z, \epsilon_t^{z^*})$	0.258	correlation of innovations to technology shock

3 Calibration

Table 1 displays the values we assign to the structural parameters in in the baseline calibration of the model. The time unit is meant to be one quarter. The discount factor β is set at a value consistent with an interest rate of 4 percent per year. The curvature of the period utility function, σ, is set at 1, which implies that preferences are separame in leisure and consumption, a value commonly used in business-cycle studies. We pick the parameter ϕ of the utility funciton so that households devote about one third of their time to paid work in the deterministic steady state. The parameter ω of the aggregator function of domestic and foreign goods is set to 0.75. This value implies a steady-state share of imports to GDP of about 15 percent as in Backus et al. (1995). We also follow Backus et al. in setting the elasticity of substitution between home and foreign goods, ξ, equal to 1.5. We set the parameter η, defining the eslaticity of substitution across habit-adjusted consumption of individual varieties of goods of the same type (a or b), at 6. This value is consistent with a steady-state markup of 20 percent in the absence of deep habits an of 22 percent under our calibration of the deep habit mechanism. We assume that the habit parameter is common across types of goods, countries, and final consumers and equal to 0.6. That is, we impose $\theta_a^c = \theta_b^c = \theta_a^g = \theta_b^g = \theta_a^{c*} = \theta_b^{c*} = \theta_a^{g*} = \theta_b^{g*} = 0.6$. This value falls in the lower range of available empirical estimates. In earlier work (Ravn, Schmitt-Grohé, and Uribe, 2006), we estimate the deep habit model on U.S. data and find a point estimate of the degree of habit formation of 0.86. High degrees of habit formation have also been estimated in the context of superficial-habit models. Of these empirical estimates, those based on Euler equaitons are directly applicable to the calibration of the deep habit model. (For further discussion of this issue, see Ravn, Schmitt-Grohé, and Uribe, 2004.) We draw from our own estimates (Ravn, Schmitt-Grohé, and Uribe, 2006) to set the rate of decay of the stock of habit, $1-\rho$, equal to 0.15.

We assume that in the nonstochastic steady state government consumption represents 20 percent of value added. This figure is in line with empricial evidence form OECD countires. We estimate the stochastic process for government purchases using HP-filtered U.S. quarterly data from 1947-Q1 to 2004-Q3. We estimate an AR(1) with a serial correlation parameter (ρ_g) of 0.87 and an innovation standard deviation of 0.016 (i.e., $\sigma_{\epsilon^g} = \sigma_{\epsilon^{g*}} = 0.016$).

Finally, we follow Backus et al. (1995) in calibration the joint stochastic process for the domestic and foreign productivity shocks of the form

$$\begin{bmatrix} \log z_{t+1} \\ \log z^*_{t+1} \end{bmatrix} = \begin{bmatrix} \rho^Z_{11} & \rho^Z_{12} \\ \rho^Z_{21} & \rho^Z_{22} \end{bmatrix} \begin{bmatrix} \log z_t \\ \log z^*_t \end{bmatrix} + \begin{bmatrix} \epsilon^z_{t+1} \\ \epsilon^{z^*}_{t+1} \end{bmatrix}.$$

The variance/covariance matrix of the innovation vector is given in table 1. Notice that the innovations to technology are correlated contemporaneously across countries.

References

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