Is the Monetarist Arithmetic Unpleasant?

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Starting Point

- In “Some Unpleasant Monetarist Arithmetic,” Sargent and Wallace (1981) warned that in fiscally dominant regimes ‘tighter money now can mean higher inflation eventually.’

- Why revisit this issue 35 years later?

(1) In spite of the qualifier ‘unpleasant,’ the monetarist arithmetic is purely positive in nature, and therefore provides no normative prediction as to under what circumstances it pays for governments in fiscally dominant regimes to delay inflation by financing deficits through debt creation.

(2) Fiscally dominant regimes represent a realistic description of the restrictions that central banks around the world have faced at different points in history. Current examples include Brazil, Argentina, and possibly the U.S. under Trump.
This Paper

• Addresses the question, When is it optimal in a welfare sense for a government in a fiscally dominant regime to delay inflation by financing part of the fiscal deficit through debt issuance.

• The analysis is conducted in the context of a model in which the monetarist arithmetic holds, in the sense that if the government finds it optimal to delay inflation, it does so knowing that it would result in higher inflation in the future.

• The central result of the paper is that delaying inflation is optimal when the fiscal deficit is expected to decline over time.
Intuition

Suppose that the real primary fiscal deficit is exogenous and follows a declining trajectory as shown in figure 1.

Figure 1: Exogenous Primary Fiscal Deficit
Intuition (continued)

Government solvency requires that the present discounted value of fiscal deficits must equal the present discounted value of seignorage revenue.

Thus, under a fiscally dominant regime, the central bank cannot control the overall magnitude of the inflation tax.

However, the central bank does have control over the timing of inflation. This is because there are an infinite number of inflation trajectories compatible with financing a given stream of fiscal deficits.

Consider, for example, the strategy of full monetization, that is, printing enough money to finance the deficit each period without issuing any interest-bearing debt.

If the fiscal deficit is expected to decline over time, full monetization requires relatively high money growth rates at the beginning and lower later, generating a declining path of inflation, as shown in figure 2 on the next slide.
Figure 2: The Inflation Rate Under Full Monetization
Intuition (continued)

An alternative possible strategy is partial monetization. Under this monetary policy, the central bank prints money to finance part of the deficit each period. The rest of the deficit is financed by issuing public debt.

Under partial monetization, initially the money growth rate is lower than under full monetization. As a result, the inflation rate is also initially lower.

Eventually the government must generate enough seignorage revenue to pay for the deficits plus the accumulated debt including interest. Thus, the long-run inflation rate is higher under partial monetization than under full monetization, as shown with a dashed line in figure 3 on the next slide. The figure illustrates the unpleasant monetarist arithmetic: ‘tighter money now means higher inflation eventually.’
Figure 3: Inflation Rate Under Partial and Full Monetization

![Diagram of inflation rate under partial and full monetization]

- **Full Monetization**
- **Partial Monetization**
Intuition (continued)

A benevolent government will pick the path of inflation that maximizes the welfare of households subject to the constraint that it be consistent with intertemporal government solvency.

What determines the best monetary policy?

In virtually all existing monetary models, inflation is a distortion.

A general principle of dynamic public finance is that it is optimal for the government to smooth distortions over time.

From figure 3, we see that partial monetization gives rise to a smoother path of inflation than full monetization.

Taken together, the above observations lead to the intuition that partial monetization can be preferred to full monetization.
Intuition (continued)
The optimal plan calls for delaying inflation even though the monetarist arithmetic is at work, that is, even though failing to fully monetize the deficit implies that in the future the rate of inflation will be higher than if the government had chosen to print money to finance the entire deficit period by period.

Under the optimal policy, the government issues debt to finance part of the fiscal deficits. As a result, public debt builds up, and converges to a higher long-run level, as shown in figure 4 in the next slide.
Figure 4: Public Debt Under the Optimal Monetary/Fiscal Policy
Formal Analysis
The Model

The main elements of the economic environment are:

- Infinite horizon

- Money in the utility function

- Flexible prices.

- Fiscal dominance, taking the form of an exogenous path for the primary fiscal deficit.

- Benevolent central bank
**Households**

Households choose paths for consumption, $c_t$, money holdings, $M_t$, and bond holdings, $B_t$, to maximize 

$$
\int_0^\infty e^{-\rho t} [u(c_t) + v(M_t/P_t)] dt,
$$

subject to 

$$
c_t + \frac{\dot{M}_t + \dot{B}_t}{P_t} = y + \tau_t + i_t \frac{B_t}{P_t},
$$

$$
\lim_{t \to \infty} e^{-R_t} \frac{M_t + B_t l}{P_t} \geq 0.
$$

where $\rho > 0$ is the subjective discount factor; $u(\cdot)$ and $v(\cdot)$ are increasing and concave functions; $y$ is a constant endowment; $\tau_t$ is a real government transfer; $P_t$ is the price level; $i_t$ is the nominal interest rate; $R_t \equiv \int_0^t r_s ds$ is the market discount factor; $r_t \equiv i_t - \pi_t$ is the real interest rate; and $\pi_t \equiv \dot{P_t}/P_t$ is the inflation rate.
Optimality Conditions

\[
\frac{u''(c_t)}{u'(c_t)} \dot{c}_t = \rho - r_t
\]

This is an Euler equation, stating that consumption grows when the real interest rate exceeds the subjective discount factor.

\[
\frac{v'(m_t)}{u'(c_t)} = i_t,
\]

This expression gives rise to a money demand function of the type

\[
m_t = L(i_t, c_t),
\]

where \( m_t \equiv M_t/P_t \) denotes real money balances. I assume that \( i_t L(i_t, c_t) \) is increasing in \( i \). Finally, the following transversality condition must hold:

\[
\lim_{t \to \infty} e^{-R_t}(m_t + b_t) = 0,
\]

where \( b_t \equiv B_t/P_t \) denotes real bond holdings.
The Government
The government generates an exogenous flow of real primary fiscal deficits, $\tau_t$. It finances the (secondary) fiscal deficit, $\tau_t + i_t B_t/P_t$, by a combination of money creation, $\dot{M}_t$, and debt issuance, $\dot{B}_t$. Its flow budget constraint is

$$\frac{\dot{M}_t + \dot{B}_t}{P_t} = \tau_t + i_t \frac{B_t}{P_t}.$$  

Market Clearing
In equilibrium, the product market must clear, that is,

$$c_t = y.$$  

This expression and the consumer Euler equation imply that in equilibrium the real interest rate equals the subjective discount factor

$$r_t = \rho.$$
The Competitive Equilibrium

Combining the flow budget constraints of the household, the flow budget constraints of the government, the market clearing condition, and the transversality condition yields the following intertemporal restriction

\[ \frac{B_0 + M_0}{P_0} = \int_0^\infty e^{-\rho t} [i_t L(i_t, y) - \tau_t] dt, \tag{1} \]

It says that the present discounted value of seignorage revenues, \( i_t L(i_t, y) \), must be large enough to pay for the sum of the government’s initial liabilities, \((M_0 + B_0)/P_0\), and the present discounted value of primary deficits, \( \int_0^\infty e^{-\rho t} \tau_t dt \).

Definition 1 (Competitive Equilibrium) A competitive equilibrium is an initial price level \( P_0 \) and a time path of nominal interest rates \( \{i_t\} \) satisfying equation (1), given the initial level of nominal government liabilities \( B_0 + M_0 \) and the time path of real primary fiscal deficits \( \{\tau_t\} \).
The Ramsey Optimal Equilibrium

The Ramsey planner chooses a path for the nominal interest rate \( \{i_t\}_0^\infty \) to maximize the indirect utility function

\[
\int_0^\infty e^{-\rho t}[u(y) + v(L(i_t, y))]dt,
\]

subject to the restriction that \( \{i_t\}_0^\infty \) be consistent with a competitive equilibrium, that is, subject to

\[
\frac{B_0 + M_0}{P_0} = \int_0^\infty e^{-\rho t}[i_tL(i_t, y) - \tau_t]dt,
\]

Note that: (a) The indirect utility function is decreasing in \( i_t \). (b) The initial price level, \( P_0 \), does not enter in the indirect utility function. (c) If \( M_0 + B_0 > 0 \) and \( \int_0^\infty e^{-\rho t}\tau_t dt > 0 \), then an increase in \( P_0 \) allows for a lower path for \( i_t \). This implies that the benevolent central bank has an incentive to engineer an initial hyperinflation, \( P_0 \to \infty \), to inflate away its initial real liabilities, \( (M_0 + B_0)/P_0 \). For this reason, following the related literature, I assume that the Ramsey planner takes the initial price level, \( P_0 \), as given.
The Ramsey Optimal Equilibrium (continued)

So the Ramsey problem is to choose a path for the nominal interest rate \(\{i_t\}_{0}^{\infty}\) to maximize

\[
\int_{0}^{\infty} e^{-\rho t} [u(y) + v(L(i_t, y))] dt,
\]

subject to

\[
\frac{B_0 + M_0}{P_0} = \int_{0}^{\infty} e^{-\rho t} [i_t L(i_t, y) - \tau_t] dt,
\]
given \(P_0, M_0 + B_0,\) and \(\int_{0}^{\infty} e^{-\rho t} \tau_t dt.\) Letting \(\eta\) be the Lagrange multiplier, the associated first-order condition is

\[
v'(L(i_t, y)) L_1(i_t, y) + \eta [L(i_t, y) + i_t L_1(i_t, y)] = 0, \tag{2}
\]

The optimal nominal interest rate is constant

\[i_t = i^*.\]

And so is the optimal rate of inflation

\[\pi_t = \pi^* \equiv i^* - \rho.\]
The Optimal Evolution of Public Debt

Taking into account that the optimal interest rate and the optimal inflation rate are constant, we can write the flow budget constraint of the government as

\[
\dot{b}_t = \rho b_t + \tau_t - \pi^* L(\pi^* + \rho, y)
\]

Integrating forward and using the transversality condition yields

\[
b_t = \frac{\pi^* L(\pi^* + \rho, y)}{\rho} - \int_0^\infty e^{-\rho s} \tau_{t+s} ds.
\]

The optimal path of debt depends on the expected trajectory of future primary fiscal deficits. Suppose that \(\tau_t\) follows the declining path \(\tau_t = \tau_0 e^{-\delta t}\), with \(\tau_0, \delta > 0\). Then, we have that

\[
b_t = \frac{\pi^* L(\pi^* + \rho, y)}{\rho} - \frac{\tau_0}{\rho + \delta} e^{-\rho t}.
\]

which says that if the primary fiscal deficit is expected to fall over time, then the central bank will not find it optimal to fully monetize the fiscal deficit. Instead, it will finance part of the deficit by issuing interest-bearing debt.
A Numerical Illustration: Fiscal Gradualism in Argentina Under Macri

- The Macri administration inherited a large fiscal deficit
- The government promises to reduce the deficit gradually.
- The central bank monetizes only a fraction of the fiscal deficit.
- The combination of large fiscal deficits and partial monetization has given rise to a significant increase of the public debt.
- This burst of public debt has been criticized on the grounds that it will eventually lead to more inflation than the alternative of full monetization.

Let’s introduce some numbers to characterize this environment more precisely and then ascertain what the present model has to say about the optimal path of debt.
### Calibration of the Argentine Monetary-Fiscal Regime

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_0$</td>
<td>0.05</td>
<td>Initial deficit-to-GDP ratio, $\tau_t = \tau_0 e^{-\delta t}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.3</td>
<td>Decay rate of fiscal deficit ($\frac{1}{2}$ life = 3 yrs)</td>
</tr>
<tr>
<td>$\frac{M_0 + B_0}{yP_0}$</td>
<td>0.389</td>
<td>Ratio of Initial gov’t liabilities to GDP</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0882</td>
<td>Money demand function $L(i, y) = Ay i^{-\alpha}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.13</td>
<td>Interest-rate elasticity of money demand</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.0392</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$g$</td>
<td>0.0198</td>
<td>Growth rate</td>
</tr>
</tbody>
</table>

Note. The time unit is one year.
Calibration Details

- The time unit is a year.
- The Macri administration inherited a fiscal deficit of about 5% of GDP, so I set $\tau_0 = 0.05$.
- The government promises to reduce the deficit gradually to 1.5% of GDP in four years, which implies a half life of 2.3 years. The resulting law of motion of the primary deficit is
  \[ \tau_t = 0.05e^{-0.3t} \]
- I set the initial liabilities of the consolidated government to 38.9% of GDP, or $(M_0 + B_0)/P_0 = 0.389y$. (Monetary base 10%; central bank interest-bearing debt held by private agents, i.e., Lebacs, 6%; and treasury debt held by private agents 22.9%).
- I assume a demand for money of the form $L(i, c) = ca^{-\alpha}$, with $A = 0.089$ and $\alpha = 0.13$. (The interest-rate elasticity $\alpha$ is based on estimates by Kiguel and Neumeyer, JMCB, 1995; and the value of $A$ ensures a 10% monetary-base-to-GDP ratio when $i = 38\%$ as observed in early 2016.)
- I set the subjective discount factor to 4%, or $\rho = \ln(1.04)$, and the growth rate to 2%, or $g = \ln(1.02)$. 
Observation: The optimal transition to the steady state is characterized by significantly tight monetary conditions, as reflected by the fact that a deficit of 5% of GDP with a half life of 3 years is partly financed by an increase in public debt of more than 15 percentage points of GDP.
Full Monetization of Fiscal Deficits

In equilibrium, the evolution of total government liabilities, $w_t = \frac{M_t + B_t}{P_t}$, obeys the law of motion

$$\dot{w}_t = \rho w_t + \tau_t - i_t L(i_t, y).$$

Suppose that the central bank prints enough money to pay for the fiscal deficit and interest on the debt, so that government liabilities stay constant over time, $\dot{w}_t = 0$. I refer to this policy as full monetization of the fiscal deficit. What is the inflation rate associated with this policy? With $\dot{w}_t = 0$, the above expression implies that the inflation rate is given by

$$(\pi_t + \rho) L(\pi_t + \rho, y) = \rho w_0 + \tau_t$$

Under the assumed declining path for the fiscal deficit, $\tau_t = \tau_0 e^{-\rho t}$, inflation is high at the beginning and then falls over time. The figure on the next slide compares the equilibrium dynamics under full monetization with those associated with the Ramsey optimal policy.
Comparing Full Monetization with the Ramsey Optimal Policy

(a) Primary Fiscal Deficit, $\tau_t$

(b) Debt, $b_t$

(c) Inflation, $\pi_t$

(d) Monetary Base, $m_t$

Note. $\tau_t$, $b_t$, and $m_t$ are measured in percent of annual GDP, and $\pi_t$ is measured in percent per year.
Observations on the Figure

• The bottom left panel clearly illustrates that the monetarist arithmetic is at work, but is not unpleasant. Under the Ramsey policy inflation is lower at the beginning but higher in the long run compared to full monetization.

• The top right panel sheds light on the unpleasant monetarist arithmetic from a different perspective. Under full monetization, public debt is flat (indeed slightly decreasing) throughout the transition to the steady state. By contrast, under the optimal policy debt increases sharply at the beginning and continues to increase throughout the transition. A naive interpretation of the monetarist arithmetic would interpret the debt dynamics associated with full monetization as more healthy.
Concluding Remarks

- The unpleasant monetarist arithmetic states that under fiscal dominance tight money now implies higher inflation in the future.

- This paper does not quarrel with this dictum, but with the common interpretation—possibly suggested by the qualifier 'unpleasant'—that in a fiscally dominant regime tight monetary policy, understood as financing part of the fiscal deficit by issuing debt, is undesirable.

- In this paper I attempt to fill this gap, by characterizing the welfare maximizing path of public debt in a monetary economy characterized by fiscal dominance.

- The main result derived from this analysis is that in a fiscally dominant regime tight money may indeed be optimal.

- This result obtains when the fiscal deficit is expected to fall over time or is temporarily high.

- Extensions worth pursuing include introducing nominal rigidities, uncertainty about the duration and magnitude of the fiscal adjustment, and interest-rate smoothing.