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Financing COVID-19 Deficits in Fiscally Dominant Economies: Is The Monetarist Arithmetic Unpleasant?*

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The coronavirus pandemic of 2019-20 confronted fiscally dominant regimes around the world with the question of whether the large deficits caused by the health crisis should be monetized or financed by issuing debt. The unpleasant monetarist arithmetic of Sargent and Wallace (1981) states that in a fiscally dominant regime tighter money now can cause higher inflation in the future. In spite of the qualifier 'unpleasant,' this result is positive in nature, and, therefore, void of normative content. I analyze conditions under which it is optimal in a welfare sense for the central bank to delay inflation by issuing debt to finance part of the fiscal deficit. The analysis is conducted in the context of a model in which the aforementioned monetarist arithmetic holds, in the sense that if the government finds it optimal to delay inflation, it does so knowing that it would result in higher inflation in the future. The central result of the paper is that delaying inflation is optimal when the fiscal deficit is expected to decline over time.

Keywords: Optimal Monetary Policy, Inflation Tax, Fiscal Deficits, Public Debt

JEL Classification: E52, E61, E63

I. INTRODUCTION

In "Some Unpleasant Monetarist Arithmetic," Sargent and Wallace (1981) warned that 'tighter money now can mean higher inflation eventually.' They derived this conclusion in the context of a model with a policy regime characterized by fiscal dominance. Specifically, in their formulation the fiscal authority sets an exogenous

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path for real primary deficits, which must be passively finance by either printing money or issuing debt. In this environment, tightening current monetary conditions requires increasing the growth of interest-bearing debt. Because the government must pay its debts eventually, at some point it has to increase the money supply to pay not only for the primary deficits but also for the increase debt and accumulated interest, entailing higher inflation than if it had not tightened monetary conditions.

The fiscally dominant regime studied by Sargent and Wallace is a realistic description of the restrictions that central banks around the world have faced at different points in history. The most recent example is the burst of fiscal deficits caused by the covid-19 pandemic of 2020. Lock downs caused a collapse in tax revenue and a burst in government transfers and health-related government spending. A case in point from the pre-pandemic era is Argentina since the beginning of the Macri administration in late 2015. The government inherited a large fiscal deficit of about 5 percent of GDP and was quite limited in its ability to either cut spending or raise taxes. As a result, the fiscal authority adopted a gradual approach to reducing the deficit, quite independently of the monetary stance. The central bank chose not to fully monetize the fiscal deficit. This approach gave rise to a burst of central bank debt, known, in the local jargon, as quasi-fiscal deficits. The rise in central bank debt was the subject of much criticism by orthodox economist, who, on precisely the grounds laid out by Sargent and Wallace, warn about its consequences for future inflation.

I address the question of under what circumstances, if any, postponing inflation by failing to fully monetize the fiscal deficit can indeed be the optimal policy choice. This question is relevant not only because, as the above examples testifies, we do observe policymakers in fiscally dominant regimes resorting to debt issuance to finance fiscal deficits, but also because the term ‘unpleasant’ in the monetarist arithmetic Sargent and Wallace refer to ought not to be necessarily understood as meaning ‘welfare reducing.’ In fact, Sargent’s and Wallace’s analysis is purely positive and therefore void of explicit normative predictions. This paper extends their contribution by placing the choices of their passive monetary authority in a welfare framework. Specifically, I ask what is the welfare maximizing monetary policy in a fiscally dominant regime.

To ensure that the present analysis is conducted in a level playing field with that of Sargent and Wallace, I build a model in which the unpleasant monetarist arithmetic holds. In particular, in the model, the fiscal authority sets an exogenous path for the primary fiscal deficit, and the central bank is limited to choosing the mix of money creation and debt issuance. In this model, failing to monetize the fiscal deficit does result in higher inflation eventually, exactly as dictated by the unpleasant monetarist arithmetic. The key departure from the analysis of Sargent and Wallace is that the central bank chooses a monetary policy that maximizes the lifetime utility of the representative household.

The central result of this paper is that whether or not in a fiscally dominant regime it is optimal to delay inflation by issuing debt depends crucially on the expected path of fiscal deficits. If fiscal deficits are expected to follow a declining path, or, more generally, are temporarily high—as is likely to be the case for those taking place during the coronavirus pandemic—then it may be optimal for the central bank to fall short of full monetization of the fiscal deficit. In this case, public debt will initially rise and long-run inflation will be higher than if the central bank had refrained from initially restricting the pace of monetary expansion. If fiscal deficits are expected to grow over time, or, more generally, if fiscal deficits are temporarily low, it may indeed be optimal for the central bank to follow a monetary policy that is looser than the full monetization of the fiscal deficit would require. In this case, the long-run rate of inflation is lower than under the policy of monetizing the deficits period by period. Full monetization of the fiscal deficit emerges as the optimal policy outcome when the fiscal deficit is expected to be stable over time.

The intuition behind this result is as follows. In virtually all existing monetary models, inflation represents a distortion. Smoothing this distortion over time can be welfare increasing. In this case, the central bank will tend to set a smooth path of inflation subject to the restriction that the associated present discounted value of seignorage revenues be large enough to cover the lifetime liabilities of the government. Thus, the optimal inflation rate is dictated by the average fiscal deficit, rather than by the current one. As a result, if the current fiscal deficit is above its average value, seignorage will fall short of the fiscal deficit, and the government will need to issue debt to close the gap. This expansion in government liabilities implies higher future inflation than the alternative of printing money today to pay for the entire current fiscal deficit—the monetarist arithmetic—but is preferable because it renders a smoother path for the inflation tax.

This paper is related to a large literature aimed at understanding the role of monetary policy in fiscally dominant regimes. The primary motivation of the paper stems from the work of Sargent and Wallace (1981). The key result of the paper that if the expected path of primary deficts is decreasing the optimal monetary-fiscal regime calls for a flat rate of inflation and debt financing in the initial transitiion is reminicent of the tax smoothing result of Barro (1979) and Lucas and Stokey (1983). The paper is also related to papers in the tradition of the fiscal theory of the price level (Woodford, 1996; Sims, 1994; and Leeper, 1991). The key difference with this body of work is that in the present paper the government is assumed to be benevolent and to be able to commit not to allow the price level to jumpt. Thus, the present paper can be interpreted as providing a fiscal theory of the inflation rate, rather than of the price level. Manuelli and Vizcaino (2017) extend the analysis in the present paper by considering a government that lacks commitment.

The remainder of the paper is organized as follows. Section 2 presents an intertemporal model in which a demand for money is motivated by assuming that real balances produce utility. Section 3 characterizes the Ramsey equilibrium. Section 4 derives conditions under which it is optimal for the central bank to increase public debt instead of fully monetizing the fiscal deficit. Section 5 analyzes an economy with long-run growth. Section 6 provides a numerical example motivated by the Argentine 2016-2019 stabilization effort. Section 7 provides concluding remarks.

II. THE MODEL

The theoretical environment is an infinite-horizon, flexible-price, endowment economy with money in the utility function. The fiscal authority runs an exogenous stream of real primary fiscal deficits and finances them by a combination of debt issuance and money creation.

1. Households

Consider an economy populated by a large number of identical households with preferences for consumption and real money balances described by the following lifetime utility function

$$\int_0^\infty e^{-\rho t}[u(c_t) + v(m_t)]dt \tag{1}$$

where c_t denotes consumption of a perishable good, m_t denotes real money balances, and $\rho > 0$ is a parameter denoting the subjective rate of discount. The subutility functions $u(\cdot)$ and $v(\cdot)$ are assumed to be strictly increasing and strictly concave.[1]

Households are endowed with an exogenous and constant stream of goods denoted $y > 0$ and receive real lump-sum transfers from the government, denoted τ_t. In addition, households can hold two types of assets, money, denoted M_t, and interest-bearing nominal bonds, denoted B_t. Bonds pay the nominal interest rate i_t, and money bears no interest. The household's flow budget constraint is then given by

$$P_t c_t + \dot{M}_t + \dot{B}_t = P_t y + P_t \tau_t + i_t B_t,$$

where a dot over a variable denotes its time derivative. Dividing through by the price level, one can write the flow budget constraint as

$$c_t + \dot{m}_t + \pi_t m_t + \dot{b}_t + \pi_t b_t = y + \tau_t + i_t b_t,$$

where $m_t \equiv M_t/P_t$ denotes real money balances, $b_t \equiv B_t/P_t$ denotes real bond holdings, and $\pi_t \equiv \dot{P}_t/P_t$ denotes the rate of inflation. Now letting

$$w_t \equiv m_t + b_t$$

denote real financial wealth and

$$r_t \equiv i_t - \pi_t$$

denote the real interest rate, one can express the flow budget constraint as

$$c_t + \dot{w}_t = y + \tau_t + r_t w_t - i_t m_t. \tag{2}$$

The right-hand side of constraint (2) represents the sources of income, given by the sum of nonfinancial income, $y + \tau_t$, and financial income $r_t w_t - i_t m_t$. The left-hand side represents the uses of income, consumption, c_t, and savings, $\dot{w}_t$. Households

[1] The present study is not concerned with dynamics in which the economy falls into liquidity traps, so I need not impose weaker assumptions on $v(\cdot)$.

are also subject to the following terminal borrowing constraint that prevents them from engaging in Ponzi schemes

$$\lim_{t\to\infty} e^{-R_t} w_t \geq 0, \tag{3}$$

where $R_t \equiv \int_0^t r_s ds$ is the compounded interest rate from time 0 to time t.

The household chooses time paths $\{c_t, m_t, w_t\}$ to maximize the utility function (1), subject to the flow budget constraint (2) and the no-Ponzi-game constraint (3). Letting λ_t denote the multiplier associated with the flow budget constraint (2), the optimality conditions associated with this problem are

$$u'(c_t) = \lambda_t, \tag{4}$$

$$\frac{v'(m_t)}{u'(c_t)} = i_t, \tag{5}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_t, \tag{6}$$

$$c_t + \dot{w}_t = y + \tau_t + r_t w_t - i_t m_t, \tag{7}$$

and

$$\lim_{t\to\infty} e^{-R_t} w_t = 0. \tag{8}$$

The first condition says that in the optimal plan the marginal utility of consumption must equal the shadow value of wealth. The second condition is a demand for money. It says that the desired level of real money holdings is decreasing in the nominal interest rate and increasing in consumption. Solving that optimality condition for m_t one can write

$$m_t = L(i_t, c_t), \tag{9}$$

where $L(\cdot,\cdot)$is a liquidity preference function with partial derivatives $L_1 < 0$ and $L_2 > 0$. I assume that $iL(i,y)$ is increasing in i for the ranges of interest rates that are relevant in the present analysis. This assumption ensures that in equilibrium the government can increase seignorage revenue by raising the nominal interest rate.[2] The third optimality condition is the Euler equation associated with bond holdings. The fourth optimality condition is the flow budget constraint. And the fifth and last optimality condition is a transversality condition given by the no-Ponzi-game constraint holding with equality.

2. The Government

I assume that the primary fiscal deficit, τ_t, evolves exogenously over time. An environment of this type is said to display fiscal dominance. To finance the stream of fiscal deficits, the government can either print money, $\dot{M}_t > 0$, or issue interest-bearing bonds, $\dot{B}_t > 0$. The flow budget constraint of the government is therefore given by

$$\dot{M}_t + \dot{B}_t = P_t \tau_t + i_t B_t.$$

We can write this constraint in real terms as

$$\dot{w}_t = \tau_t + r_t w_t - i_t m_t. \tag{10}$$

This expression says that the government uses increases in its total liabilities, $\dot{w}_t$, and seignorage, $i_t m_t$, to pay for the primary deficit, τ_t, and to meet interest obligations on outstanding liabilities $r_t w_t$.

3. Competitive Equilibrium

Combining the flow budget constraints of the household and the central bank (equations (2) and (10), respectively) yields the resource constraint

[2] Throughout this paper, I use the term seignorage revenue indistinctly to refer to $i_t m_t$ or to $\pi_t m_t$.

$$c_t = y, \tag{11}$$

which implies that consumption is constant over time. In turn, this result and equation (9) imply that in equilibrium the demand for money is given by

$$m_t = L(i_t, y).$$

Combining (4), (6), and (11) yields

$$r_t = \rho.$$

This expression says that in the present economy the equilibrium real interest rate is equal to the subjective discount factor.

Finally, the transversality condition (8) and the flow budget constraint of the government (10) are equivalent to the following intertemporal restriction

$$\frac{B_0 + M_0}{P_0} = \int_0^\infty e^{-\rho t}[i_t L(i_t, y) - \tau_t]dt, \tag{12}$$

where we are using the equilibrium conditions $c_t = y$ and $r_t = \rho$. Equation (12) says that the present discounted value of seignorage revenue must be equal to the sum of the present discounted value of primary deficits and the central bank's initial real liabilities. We are now ready to define the competitive equilibrium in this economy.

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is an initial price level P_0 and a time path of nominal interest rates $\{i_t\}$ satisfying equation (12), given the initial level of nominal government liabilities $B_0 + M_0$ and the time path of real primary fiscal deficits $\{\tau_t\}$.*

I am interested in economies in which the government is initially a net debtor and in which the fiscal authority runs a stream of primary deficits that is positive in present discounted value. Accordingly, we assume that

$$B_0 + M_0 > 0 \tag{13}$$

and

$$\int_0^{\infty} e^{-\rho t}\tau_t dt > 0. \tag{14}$$

These assumptions ensure that the central bank must generate a stream of seignorage income that is positive in present discounted value to meet its lifetime financial obligations. The question is what part of its obligations should it finance with seignorage and what part by issuing debt at any point in time.

III. RAMSEY OPTIMAL CENTRAL BANK POLICY

I assume that the central bank is benevolent and has the ability to commit to its promises. This means that among all the interest rate paths and initial price levels that are consistent with a competitive equilibrium, the monetary authority picks the one that maximizes the representative household's lifetime welfare. I refer to such equilibrium as the Ramsey optimal equilibrium. In equilibrium, welfare is given by the following indirect lifetime utility function:

$$\int_0^{\infty} e^{-\rho t}[u(y) + v(L(i_t, y))]dt. \tag{15}$$

Because $v(\cdot)$ is strictly increasing and $L(\cdot,\cdot)$ is decreasing in its first argument, lifetime utility is strictly decreasing in the nominal interest rate. It then follows from equations (12)-(15) that it is optimal for the central bank to implement a policy in which $P_0 \to \infty$, that is, it is optimal to cause a hyperinflation in period 0. By doing this, the central bank inflates away all of the government's initial real liabilities, $(B_0 + M_0)/P_0 \to 0$, reducing the need to generate seignorage revenue through the (distortionary) inflation tax. To avoid this unrealistic feature of optimal policy, it is typically assumed in the related literature (see, e.g., Schmitt-Grohé and Uribe, 2004, and the references therein) that the initial price level, P_0, is given. I follow this tradition. The Ramsey optimal equilibrium is then defined as follows:

Definition 2 (Ramsey Optimal Equilibrium) *A Ramsey optimal equilibrium is a path for the nominal interest rate* $\{i_t\}$ *that maximizes the indirect utility function (15)* *subject to the intertemporal constraint (12), given the initial level of real government liabilities* $(B_0 + M_0)/P_0$ *and the path of primary fiscal deficits* $\{\tau_t\}$.

The optimality conditions associated with the Ramsey problem are equation (12) and

$$v'(L(i_t, y))L_1(i_t, y) + \eta[L(i_t, y) + i_t L_1(i_t, y)] = 0, \quad (16)$$

where η denotes the Lagrange multiplier associated with the constraint (12). The Lagrange multiplier η is endogenously determined in period 0, but it is constant over time. This means that the Ramsey optimal nominal interest rate is also time invariant. Let i^* denote the Ramsey optimal nominal interest rate. Then we have that in the Ramsey equilibrium

$$i_t = i^*,$$

at all times $t \geq 0$. It follows immediately from the intertemporal constraint (12) and from the assumption that $i_t L(i_t, y)$ is increasing in i_t that i^* is increasing in both the initial level of real government liabilities, $(M_0 + B_0)/P_0$, and the present discounted value of fiscal deficits, $\int_0^\infty e^{-\rho t}\tau_t dt$. This implication is intuitive, the larger the present value of all government liabilities, the larger the amount of seignorage the central bank must generate to meet its obligations.

Because both the Ramsey optimal nominal interest rate and the equilibrium real interest rate are constant, we have that the Ramsey optimal inflation rate is also constant. Letting π^* be the optimal rate of inflation, we have that in the Ramsey equilibrium

$$\pi_t = \pi^* \equiv i^* - \rho,$$

for all $t \geq 0$. Similarly, in the Ramsey optimal equilibrium real money balances are constant and satisfy

$$m_t = m^* \equiv L(i^*, y),$$

for all $t \geq 0$.

IV. OPTIMAL PUBLIC DEBT DYNAMICS

We are now equipped with the necessary elements to characterize the optimal path of public debt, b_t. Recalling that $\dot{w}_t \equiv \dot{b}_t + \dot{m}_t$ and that m_t, i_t, and π_t are constant over time, we can write the government flow budget constraint given in (10) as

$$\dot{b}_t = \rho b_t + \tau_t - \pi^* m^*, \tag{17}$$

with the initial condition $b_0 = (B_0 + M_0)/P_0 - m^*$.[3] Intuitively, the Ramsey government uses a combination of debt creation, $\dot{b}_t$, and seignorage, $\pi^* m^*$, to pay the interest on the outstanding debt, ρb_t, and to finance the primary deficit, τ_t.

The optimal dynamics of public debt depend crucially on the expected future path of fiscal deficits. To see this, consider first a situation in which the primary fiscal deficit is expected to fall over time. To fix ideas, assume that the fiscal deficit evolves according to a first-order autoregressive process of the type

$$\dot{\tau}_t = -\delta \tau_t, \tag{18}$$

with $\tau_0 > 0$ and $\delta > 0$. Then, we can write the equilibrium law of motion of public debt given in equation (17) as

$$\dot{b}_t = \rho b_t + \tau_0 e^{-\delta t} - \pi^* m^*, \tag{19}$$

Because $\rho > 0$, for arbitrary values of $\pi^* m^*$ the differential equation (19) is mathematically unstable. However, it is economically stable, because the central bank chooses the level of seignorage $\pi^* m^*$ to guarantee the satisfaction of the transversality condition (8), which implies that w_t, and therefore also b_t since m_t is constant, grows at a rate less than ρ. To see this, solve the difference equation (19) to obtain

[3] The implementation of the Ramsey optimal plan, if unanticipated, in general gives rise to a portfolio recomposition at time 0, because households may change their desired money holdings (and therefore decrease their desired bond holdings) in a discrete fashion.

$$b_t = \left[b_0 + \frac{\tau_0}{\rho+\delta} - \frac{\pi^* m^*}{\rho}\right] e^{\rho t} - \frac{\tau_0}{\rho+\delta} e^{-\delta t} + \frac{\pi^* m^*}{\rho}$$

Using equation (12) one can show that the expression within square brackets is zero,

$$b_0 + \frac{\tau_0}{\rho+\delta} - \frac{\pi^* m^*}{\rho} = 0,$$

which eliminates the unstable branch of the solution. It follows that the optimal equilibrium dynamics of public debt is given by

$$b_t = \frac{\pi^* m^*}{\rho} - \frac{\tau_0}{\rho+\delta} e^{-\delta t}. \tag{20}$$

Equation (20) delivers the main result of this paper, namely, that if the fiscal deficit is expected to fall over time, it is optimal for the government to finance it partly by issuing debt, instead of by money creation alone. This result is quite intuitive. The central bank finds it optimal to smooth seignorage revenue over time. As a result, if initially the fiscal deficit exceeds seignorage revenue, the government finances the difference by issuing new debt. Over time, the primary deficits fall, but interest obligations increase. On net, however, the sum of these two sources of outlays fall, converging to zero asymptotically. In the limit, the primary fiscal deficit is nil $\tau_t \to 0$, and interest obligations are exactly equal to seignorage revenue $\rho b_t \to \pi^* m^*$. This means that asymptotically, public debt converges to a constant, given by the present discounted value of seignorage revenue.

It is straightforward to show that if the primary fiscal deficit is temporarily low or follows an increasing path over time, as in the autoregressive form

$$\tau_t = \bar{\tau} - (\bar{\tau} - \tau_0) e^{-\delta t},$$

with $\bar{\tau} > \tau_0 > 0$ and $\delta > 0$, then the optimal path of debt is decreasing. In this case, the central bank finds it optimal to create more money than is necessary to cover the fiscal deficit, and it uses the excess seignorage to retire some debt. As time goes by, the primary deficit increases, and a larger fraction of the constant seignorage revenue is devoted to paying for it. Finally, in the intermediate case in which the primary deficit is expected to be constant over time, the government does not resort to debt issuance to finance the primary deficit.

For more general laws of motion of the primary fiscal deficit, one can establish that the optimal path of public debt depends on the expected trajectory of the present discounted value of future primary fiscal deficits. To see this, multiply the expression $\dot{b}_s = \rho b_s + \tau_s - \pi^* m^*$ (which is equation (17) evaluated at time s) by $e^{-\rho s}$, integrate over the interval (t, ∞), and apply the transversality condition to obtain

$$b_t = \frac{\pi^* L(\pi^* + \rho, y)}{\rho} - \int_0^\infty e^{-\rho s} \tau_{t+s} ds.$$

This expression says that debt will increase, decrease, or stay constant over time depending on whether the present discounted value of future primary fiscal deficits is expected to fall, increase, or stay constant over time, respectively.

V. A GROWING ECONOMY

Thus far, I have limited the analysis to a stationary economy. It is of interest to ascertain how the conditions under which it is optimal to delay inflation change in an environment with long-run growth. To this end, here I generalize the law of motion of the endowment to allow for secular growth as follows,

$$y_t = y\, e^{gt},$$

where $g > 0$ is a parameter defining the growth rate of output, and $y > 0$ is a parameter defining the detrended level of output. In a balanced growth path, consumption and real money holdings grow at the same rate as output in the long run. To make this possible, I assume that the subutility functions $u(\cdot)$ and $v(\cdot)$ are both homogeneous of the same degree, as in the utility function

$$u(c) + v(m) = \frac{c^{1-1/\alpha} + A^{1/\alpha} m^{1-1/\alpha}}{1 - 1/\alpha}, \tag{21}$$

where $A, \alpha > 0$ are parameters.

Let $\tilde{x}_t \equiv x_t e^{-gt}$ be the detrended version of x_t, for $x_t = c_t, m_t, \tau_t, w_t, b_t$ and let $\tilde{\lambda}_t \equiv \lambda_t e^{g\alpha t}$ be the detrended version of λ_t. We can then write the first-order conditions associated with the household's utility maximization problem, given in equations (4)-(8), in terms of detrended variables as

$$u'(\tilde{c}_t) = \tilde{\lambda}_t, \tag{22}$$

$$\frac{v'(\widetilde{m}_t)}{u'(\tilde{c}_t)} = i_t,$$

$$\frac{\dot{\tilde{\lambda}}_t}{\tilde{\lambda}_t} = \rho + \frac{g}{\alpha} - r_t, \tag{23}$$

$$\tilde{c}_t + \dot{\tilde{w}}_t = y + \tilde{\tau}_t + (r_t - g)\widetilde{w}_t - i_t\widetilde{m}_t,$$

and

$$\lim_{t\to\infty} e^{-(R_t - gt)}\widetilde{w}_t = 0.$$

Similarly, after expressing variables in detrended form, the government flow budget constraint (10) becomes

$$\dot{\tilde{w}}_t = \tau_t + (r_t - g)\widetilde{w}_t - i_t\widetilde{m}_t. \tag{24}$$

In equilibrium, detrended consumption must equal detrended output

$$\tilde{c}_t = y.$$

This result together with optimality conditions (22) and (23) implies that the real interest rate is constant and given by

$$r_t = \tilde{\rho} \equiv \rho + \frac{g}{\alpha}.$$

According to this expression, the real interest rate is higher in the growing economy. This is intuitive, because growth makes the marginal utility of consumption fall faster over time, causing agents to demand higher compensation for sacrificing current consumption in exchange for future consumption.

By an analysis similar to that applied in the economy without growth, we can deduce that a competitive equilibrium in the economy with long-run growth is an initial price level P_0 and a time path of nominal interest rates $\{i_t\}$ satisfying the intertemporal constraint

$$\frac{B_0 + M_0}{P_0} = \int_0^\infty e^{-(\tilde{\rho}-g)t}[i_t L(i_t, y) - \tilde{\tau}_t]dt, \tag{25}$$

given the initial level of nominal government liabilities $B_0 + M_0$ and the time path of real detrended primary fiscal deficits $\{\tilde{\tau}_t\}$.

With long-run growth, the indirect utility function (15) takes the form

$$\int_0^\infty e^{-(\tilde{\rho}-g)t}[u(y) + v(L(i_t, y))]dt. \tag{26}$$

A Ramsey optimal equilibrium in the growing economy is then a path for the nominal interest rate $\{i_t\}$ that maximizes the indirect utility function (26) subject to the intertemporal constraint (25), given the initial level of real government liabilities $(B_0 + M_0)/P_0$ and the path of real detrended primary fiscal deficits $\{\tilde{\tau}_t\}$. The first-order condition with respect to i_t associated with this optimization problem is identical to its counterpart in the stationary economy, namely, equation (16). This implies, in particular, that the Ramsey optimal nominal interest rate is constant over time in the growing economy.

Finally, assume, as we did in the stationary economy, that the primary fiscal deficit obeys law of motion

$$\dot{\tilde{\tau}}_t = -\delta \tilde{\tau}_t,$$

with $\tilde{\tau}_0 = \tau_0 > 0$ and $\delta > 0$. That is, the primary fiscal deficit as a fraction of output falls gradually over time. Following the same steps as in the economy without growth, we can deduce that the Ramsey optimal path of public debt is given by

$$\tilde{b}_t = \frac{(\pi^* + g)m^*}{\tilde{\rho} - g} - \frac{\tau_0}{\tilde{\rho} - g + \delta} e^{-\delta t},$$

which says that if the detrended primary fiscal deficit is expected to fall over time, then it is optimal for the government not to fully monetize the fiscal deficit and instead allow the public debt to grow over time as a fraction of output. We therefore conclude that the central result of this paper is robust to allowing for secular growth.

VI. AN APPLICATION: FISCAL GRADUALISM IN ARGENTINA 2016-2019

To illustrate the implications of optimal policy for the dynamics of public debt, consider the following numerical example, motivated by developments in Argentina over the period 2016-2019. The time unit is a year. The Argentine fiscal authority inherited a large fiscal deficit of about 5 percentage points of GDP at the end of 2015. Thus, I set the initial condition $\tau_0 = 0.05\, y_0$. The government was committed to reducing the fiscal deficit at a gradual pace. Specifically, it targeted a reduction of the primary deficit to 1.5 percent of GDP in four years. Assuming that the deficit declines at a constant rate, this target implies that the parameter δ in the law of motion (18) takes the value $0.3 \approx -\ln(0.015/0.05)/4$. The resulting path of primary fiscal deficits has a half life of 2.3 years. With these parameter values, the primary fiscal deficit as a fraction of output evolves according to the expression

$$\tau_t = 0.05\, e^{-0.3t}.$$

I set the initial total government liabilities equal to 38.9 percent of GDP, or[4]

[4] This figure is composed of treasury liabilities with the private sector of 22.9 percent of GDP and central bank liabilities with the private sector of 16 percent of GDP at the beginning of 2016. I calculate the liabilities of the treasury with the private sector as the difference between the total debt of the treasury of 53.6 percent of GDP and the debt of the treasury with other government agencies of 30.7 percent of GDP (Informe Ministerio de Hacienda y Finanzas, first quarter 2016). I measure the liabilities of the central bank with the private sector by the sum of the monetary base and the stock of Lebac bonds outstanding. These two aggregates stood at 600 billion pesos and 360 billion pesos, respectively, at the beginning of 2016 (Informe Diario del BCRA, 2016). At the beginning of 2016, Annual GDP in Argentina was estimated to be about 400 billion dollars, and

$$w_0 \equiv \frac{M_0 + B_0}{P_0} = 0.389\, y_0.$$

I assume a money demand function of the form

$$L(i,c) = A\, c\, i^{-\alpha},$$

where $A, \alpha > 0$ are parameters. This liquidity preference specification is implied by the period utility function given in equation (21). I set $\alpha = 0.13$ using the estimate of the interest-rate semielasticity of the demand for money in Argentina produced by Kiguel and Neumeyer (1995).[5] This value is in line with estimates for other countries including low-inflation ones (see, for example, Inagaki, 2009, for the United States and Japan). I calibrate the scale parameter A to match a monetary-base-to-GDP ratio of about 10 percent of GDP and an interest rate of 38 percent as observed in Argentina at the beginning of 2016 (see footnotes 6 and 6). Thus, I set A to satisfy $0.1 = A \cdot 0.38^{-0.13}$, or $A = 0.0882$.

Finally, I set the growth-adjusted subjective discount factor $\tilde{\rho}$ to 4 percent, or $\tilde{\rho} = \ln(1.04)$, and the long-run growth rate of output per capita at 2 percent, or $g = \ln(1.02)$. Table 1 summarizes the calibration of the model.
Under this calibration, the model predicts an optimal inflation rate of 4.8 percent per year $(\pi^* = 0.048)$. The associated optimal nominal interest rate is 8.8 percent $(i^* = 0.088)$. The optimal rate of inflation generates seignorage revenue equal to 0.6 percent of GDP $(\pi^* m^* = 0.006)$.

the nominal exchange fluctuated around 15 pesos per dollar. Taken together, these figures imply liabilities of the central bank of 16 percent of GDP.

5 The interest-rate semielasticity of the demand for money is defined as $\partial \ln L(i,y)/\partial i$. Kiguel and Neumeyer denote this object by a_1. They report an average estimate of a_1 of -0.041 (arithmetic mean of all the estimates reported in their table 2). To derive the value of α implied by this estimate of a_1, one must apply two transformations. First, in their specification, the opportunity cost of money is measured in percent per month, so one must rescale a_1 by the factor 100/12. Second, to convert the semielasticity a_1 into an elasticity, one must multiply by the opportunity cost of money, i. For this purpose, I apply the interest rate on Lebacs prevailing in Argentina at the beginning of 2016 of 38 percent per year (Informe Diario del BCRA, 2016). This yields $\alpha = 0.1298 = 0.041 \times (100/12) \times 0.38$.

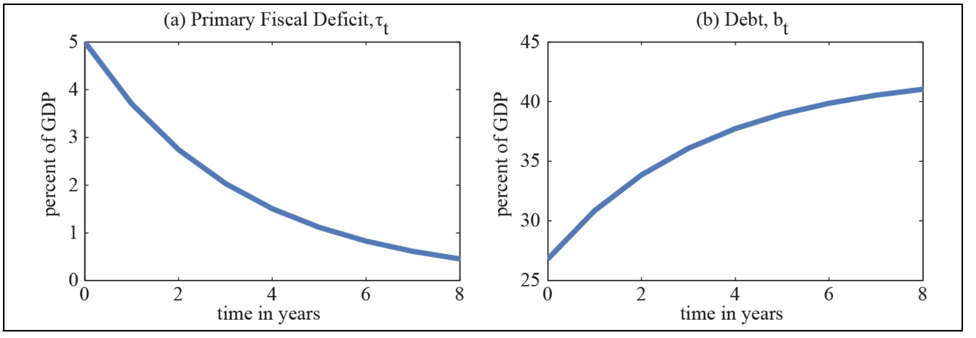
Table 1. Calibration of the Argentine Monetary-Fiscal Regime

Parameter	Value	Description
τ_0	0.05	Initial deficit-to-GDP ratio, $\tau_t = \tau_0 e^{-\delta t}$
δ	0.3	Decay rate of fiscal deficit ($\frac{1}{2}$ life $= 2.3$ yrs)
$\frac{M_0 + B_0}{yP_0}$	0.389	Ratio of initial gov't liabilities to GDP
A	0.0882	Money demand function $L(i, y) = A\, y\, i^{-\alpha}$
α	0.13	Interest-rate elasticity of money demand
$\tilde{\rho}$	0.0392	Growth-adjusted subjective discount factor
g	0.0198	Growth rate

Note. The time unit is one year.

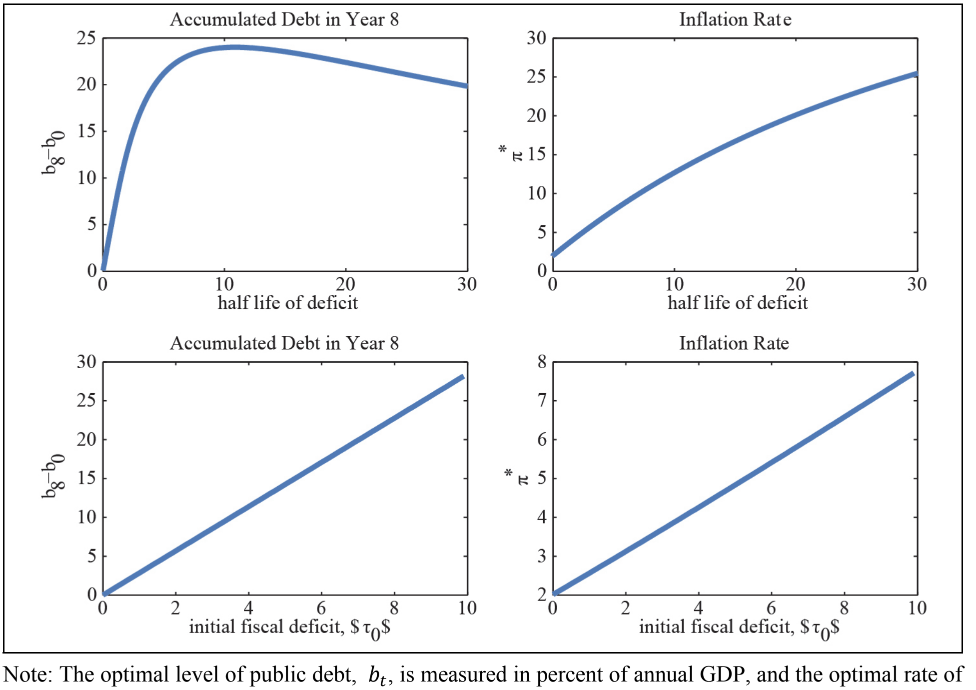
Evaluating equation (20) at the parameter values given in table 0, one can trace out the path of public debt under the Ramsey optimal policy. Panel (b) of figure 0 displays the equilibrium dynamics of b_t, as a fraction of output. Under the optimal monetary policy, public debt increases significantly along the initial transition. It starts at 27 percent of GDP at time 0 and reaches 41 percent of GDP by the end of year 8. This result shows that, seen through the lens of the present model, the optimality of delaying inflation by financing part of the fiscal deficit with public debt may not be a mere theoretical curiosity. In the present example, the government increases public debt by 15 percentage points of GDP in a relatively short period of time, as a way to smooth the distortions created by inflation. Under the optimal policy, the initial financing needs of the government, $\tilde{\rho} w_0 + \tau_0$, is 6.5 percent of GDP, whereas seignorage revenue, $\pi^* m^*$, is only 0.6 percent of GDP. The government covers this gap, which falls only gradually over time, by issuing public debt. In the long run, the debt stabilizes at 42.5 percent of output. Comparing this figure with the level of debt in year 8, it follows that almost all of the increase in debt occurs in the first 8 years.

Figure 1. Ramsey Optimal Debt Dynamics



Note: τ_t and b_t are measured in percent of annual GDP.

Figure 2. Sensitivity to Changes in the Half Life and Initial Level of the Primary Fiscal Deficit



Note: The optimal level of public debt, b_t, is measured in percent of annual GDP, and the optimal rate of inflation, π^*, is measured in percent per year. The half life of the primary deficit is measured in years, and the initial primary fiscal deficit, τ_0, is measured in percent of GDP. For each figure, all parameter values, except the one displayed on the x axis are set at the values displayed in table 1.

Figure 2 displays the accumulated public debt up to year 8 and the rate of inflation under the optimal monetary policy as functions of the half life of the primary fiscal deficit (top panel) and of the initial primary fiscal deficit (bottom panel). The optimal accumulation of debt is a nonmonotonic function of the half life of the fiscal deficit taking the value 0 when the half life is 0 or infinity. This is intuitive. If the fiscal deficit lasts for an infinitesimal period of time, it has no budgetary consequences, and therefore requires no financing. At the other extreme, if the deficit is infinitely lived, then the optimal policy is to generate a constant path of inflation that generates enough seignorage to finance the entire fiscal deficit. Since in this case the resulting optimal inflation rate is constant, it requires no smoothing through debt creation. In between these two extremes, positive debt accumulation is optimal. The maximum debt accumulation occurs at a half life of the deficit of 11 years, and the associated debt accumulation is 24 percent of output. The optimal inflation rate is highly sensitive to changes in the half life of the deficit, increasing from 2 percent to 25 percent as the half life increases from 0 to 30. Naturally, as shown by the bottom panels of figure 1, both optimal debt accumulation and the optimal rate of inflation are strictly increasing in the initial magnitude of the fiscal deficit, τ_0. As the initial primary fiscal deficit increases from 0 to 10 percent of output, debt accumulation increases from 0 to 28 percent of output, and inflation increases from 2 to 7.7 percent.

1. Optimal Policy Versus Full Monetization

It is of interest to compare the Ramsey optimal dynamics with those implied by a monetary policy that, through money creation, keeps total government liabilities from growing over time, $\dot{w}_t = 0$. I refer to this policy as full monetization of the fiscal deficit. This is the policy that results from interpreting the adjective ‘unpleasant’ attached to the monetarist arithmetic as meaning that in a fiscally dominant regime it is counterproductive to delay inflation by financing the fiscal deficit with debt. Formally, setting $\dot{w}_t = 0$, we can rewrite the central bank’s flow budget constraint (24) as

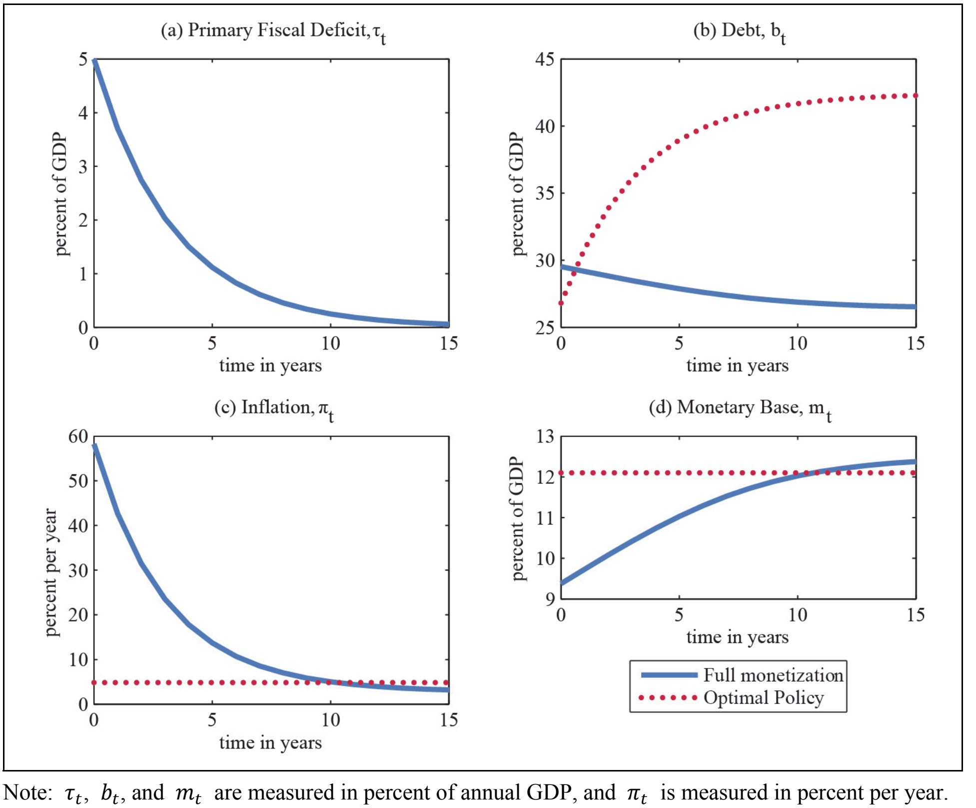
$$i_t L(i_t, y) = \tilde{\tau}_0 e^{-\delta t} + (\tilde{\rho} - g)\widetilde{w}_0. \tag{27}$$

According to equation (27), under full monetization the government pays for the primary fiscal deficit and interest on its outstanding liabilities with seignorage revenue as they accrue. Using the assumed functional form for $L(\cdot,\cdot)$, equation (27) can be solved for the nominal interest rate to get

$$i_t = \left(\frac{\tau_0 e^{-\delta t} + (\tilde{\rho} - g)\tilde{w}_0}{A}\right)^{\frac{1}{1-\alpha}}.$$

Since $\alpha < 1$, we have that under full monetization the nominal interest rate is increasing in the current level of the primary deficit. With the path of i_t at hand, one can readily derive the equilibrium paths of all other endogenous variables of the model.

Figure 3. Equilibrium Dynamics Under Full Monetization



Note: τ_t, b_t, and m_t are measured in percent of annual GDP, and π_t is measured in percent per year.

Figure 3 displays with solid lines the equilibrium dynamics of debt, inflation, and money holdings under full monetization. For comparison, the figure also displays with dashed lines the Ramsey optimal dynamics. Under full monetization, initially the central bank must generate large seignorage revenues to finance the elevated primary deficit. This results in a high initial inflation rate of 58 percent per year, more than ten times as high as the Ramsey optimal rate of inflation of 4.8 percent (panel (c) of figure 3). Because under full monetization the central bank prints money instead of issuing debt to pay for both the primary deficit and the interest on the government's outstanding liabilities, the path of interest-bearing debt stays relatively flat at about 28 percent of GDP (panel (b) of figure 3).[6] By contrast, under the Ramsey policy debt increases continuously, reaching 42 percent of output in the long run.[7] As a result, in the long run, the inflation tax necessary to pay the interest on the debt is lower under full monetization than under the optimal policy. The long-run inflation rate is 2.6 percent under full monetization versus 4.8 percent under the optimal policy. This shows that the monetarist arithmetic is at work: the Ramsey policy delivers a lower rate of inflation than full monetization in the short run, but a higher one in the long run. However, in this case the monetarist arithmetic is not unpleasant. Panel (d) of figure 3 shows why. Under the Ramsey policy real money balances display a much smoother pattern than under full monetization. Because the period utility function is concave in real money balances, households prefer the smoother path of money.

VII. CONCLUDING REMARKS

The coronavirus pandemic of 2019-2020 has led to an unprecedented burst of fiscal deficits around the world, as tax revenue collapsed and transfers and health-related expenditures soared. In countries with monetary dominance, this situation causes the

[6] Indeed b_t displays a slightly downward sloping path under full monetization. The reason is that the sum of debt and money, $b_t + m_t$, is constant for all t and equal to w_0, while m_t increases steadily over time, as inflation falls. Thus, along the transition to the steady state, households continuously substitute money for bonds in their portfolios.

[7] At time 0, debt is higher under full monetization than under the Ramsey policy (panel (b) of figure 3). This is because at time 0 total government liabilities of the government, w_0, are fixed and equal in both regimes. But their compositions (money and bonds) are endogenously determined. Because at time 0 the nominal interest rate is higher under full monetization, in this regime households hold less money and more bonds than under the Ramsey optimal policy.

expectation of future increases in taxes and cuts in government spending, with monetary policy going about its business of maintaining price stability and full employment. By contrast, in economies in which the policy regime is fiscally dominant, policymakers are confronted with the question of how much of the fiscal imbalances they should monetize and how much they should finance with public debt. In other words, these governments must negotiate a tradeoff between current and future inflation. The resolution of this tradeoff is the central focus of the present investigation.

In a fiscally dominant economy, a central bank that commits not to let the price level jump does not control the average level of inflation. This is because under this policy regime money creation must passively finance the present discounted value of fiscal deficits plus the government's initial liabilities. By controlling when to print money, the central bank can, however, choose the time distribution of inflation. The unpleasant monetarist arithmetic states that tight current monetary conditions imply higher inflation in the future. This paper does not quarrel with this dictum, but with the conclusion—implicit in the qualifier 'unpleasant'—that in a fiscally dominant regime tight monetary policy, understood as financing part of the fiscal deficit by issuing debt, is undesirable. Arriving at such a conclusion requires a normative analysis. In this paper I attempt to fill this gap, by characterizing the welfare maximizing path of public debt in a monetary economy characterized by fiscal dominance.

The main result derived from this analysis is that in a fiscally dominant regime tight money may not have unpleasant consequences, but, on the contrary, be optimal. This result obtains when the fiscal deficit is expected to fall over time or is temporarily high. The fiscal imbalances observed during the covid-19 pandemic clearly fit this description. The intuition is simple. Among all the inflation paths that generate enough seignorage revenue to pay for the present discounted value of the government's liabilities, the monetary authority prefers a flat one to smooth out the distortions created by the inflation tax. In turn, a flat path of inflation induces a flat path of seignorage revenue. This means that in periods in which fiscal deficits are above average, the central bank will find it optimal to issue debt to finance part of the fiscal deficit. The central bank prefers to issue debt even though it knows that it will cause higher inflation in the future than the alternative of financing the entire current deficit by printing money. In this case, the monetarist arithmetic obtains, but is not unpleasant.

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