On Quality Bias and Inflation Targets

Stephanie Schmitt-Grohé* Martín Uribe†

November 5, 2009

Abstract

This paper studies whether the central bank should adjust its inflation target to account for the systematic upward bias in measured inflation due to quality improvements in consumption goods. We show that the answer to this question depends on what prices are assumed to be sticky. If nonquality-adjusted prices are assumed to be sticky, then the inflation target should not be corrected. If, on the other hand, quality-adjusted (or hedonic) prices are assumed to be sticky, then the inflation target should be raised by the magnitude of the bias. *JEL Classification:* E52, E6.

Keywords: Quality Bias, Inflation Targets, Ramsey Policy.

*Columbia University, CEPR, and NBER. E-mail: ss3501@columbia.edu.

†Columbia University and NBER. E-mail: martin.uribe@columbia.edu.

1 Introduction

The existence of a positive quality bias in the consumer price index has led some to argue that an inflation target equal in size to the bias would be appropriate if the ultimate objective of the central bank is price stability. In this paper, we critically evaluate this argument. Specifically, we study whether a central bank implementing Ramsey-optimal policy should adjust its inflation target to account for the systematic upward bias in measured inflation due to quality improvements in consumption goods. We show that the answer to this question depends critically on what prices are assumed to be sticky. If nonquality-adjusted prices are assumed to be sticky, then the inflation target should not be corrected. If, on the other hand, quality-adjusted (or hedonic) prices are assumed to be sticky, then the inflation target should be raised by the magnitude of the bias.

In June 1995, the Senate Finance Committee appointed an advisory commission composed of five prominent economists (Michael Boskin, Ellen Dulberger, Robert Gordon, Zvi Griliches, and Dale Jorgenson) to study the magnitude of the measurement error in the consumer price index. The commission concluded that over the period 1995-1996, the U.S. CPI had an upward bias of 1.1 percent per year. Of the total bias, 0.6 percent was ascribed to unmeasured quality improvements (see Boskin et al., 1996). To illustrate the nature of the quality bias, consider the case of a personal computer. Suppose that between 1995 and 1996 the nominal price of a computer increased by 2 percent. Assume also that during this period the quality of personal computers, measured by attributes such as memory, processing speed, and video capabilities, increased significantly. If the statistical office in charge of constructing the consumer price index did not adjust the price index for quality improvement, then it would report two percent inflation in personal computers. However, because a personal computer in 1996 provides more services than does a personal computer in 1995, the quality-adjusted rate of inflation in personal computers should be recorded as lower than two percent. The difference between the reported rate of inflation and the quality-adjusted rate of inflation is called the quality bias in measured inflation.

We analyze the relationship between a quality bias in measured inflation and the Ramsey-optimal rate of inflation in the context of a neo-Keynesian model with Calvo (1983) and Yun (1996) price staggering. The key modification we introduce to that framework is that the quality of consumption goods is assumed to increase over time. This modification gives rise to an inflation bias if the statistical agency in charge of constructing the consumer price index fails to take quality improvements into account. The central question we entertain in the present study is whether under Ramsey optimal policy the inflation target should be adjusted by the presence of this bias.

The remainder of the paper is organized in four sections. Section 2 presents a simple neo-Keynesian model of quality bias in measured inflation. Section 3 characterizes Ramsey-optimal monetary policy when nonquality-adjusted prices are assumed to be sticky. Section 4 characterizes Ramsey-optimal monetary policy when quality-adjusted (or hedonic) prices are assumed to be sticky. Section 5 concludes.

2 A Simple Model of Quality Bias

In this section, we build a model of price staggering à la Calvo-Yun augmented to allow for quality improvement in consumption goods.

2.1 Households

The economy is populated by a large number of households with preferences defined over a continuum of goods of measure one indexed by $i \in [0, 1]$. Each unit of good i sells for P_{it} dollars in period t. We denote the quantity of good i purchased by the representative consumer in period t by c_{it}. The quality of good i is denoted by x_{it} and is assumed to evolve exogenously and to satisfy $x_{it} > x_{it-1}$. The household cares about a composite good given by

$$\left[\int_0^1 (x_{it}c_{it})^{1-1/\eta} di\right]^{1/(1-1/\eta)},$$

where $\eta > 1$ denotes the elasticity of substitution across different good varieties. Note that the utility of the household increases with the quality content of each good. Let a_t denote the amount of the composite good the household wishes to consume in period t. Then, the demand for goods of variety i is the solution to the following cost-minimization problem

$$\min_{\{c_{it}\}} \int_0^1 P_{it}c_{it} di$$

subject to

$$\left[\int_0^1 (x_{it}c_{it})^{1-1/\eta} di\right]^{1/(1-1/\eta)} \geq a_t.$$

The demand for good i is then given by

$$c_{it} = \left(\frac{Q_{it}}{Q_t}\right)^{-\eta} \frac{a_t}{x_{it}},$$

where

$$Q_{it} \equiv P_{it}/x_{it}$$

denotes the quality-adjusted (or hedonic) price of good i, and Q_t is a quality-adjusted (or hedonic) price index given by

$$Q_t = \left[\int_0^1 Q_{it}^{1-\eta} di\right]^{1/(1-\eta)}.$$

The price index Q_t has the property that the total cost of a_t units of composite good is given by $Q_t a_t$, that is, $\int_0^1 P_{it}c_{it}di = Q_t a_t$. Because a_t is the object from which households derive utility, it follows from this property that Q_t, the unit price of a_t, represents the appropriate measure of the cost of living.

Households supply labor effort to the market for a nominal wage rate W_t and are assumed to have access to a complete set of financial assets. Their budget constraint is given by

$$Q_t a_t + E_t r_{t,t+1} D_{t+1} + T_t = D_t + W_t h_t + \Phi_t,$$

where $r_{t,t+j}$ is a discount factor defined so that the dollar price in period t of any random nominal payment D_{t+j} in period $t+j$ is given by $E_t r_{t,t+j} D_{t+j}$. The variable Φ_t denotes nominal profits received from the ownership of firms, and the variable T_t denotes lump-sum taxes.

The lifetime utility function of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(a_t, h_t),$$

where the period utility function U is assumed to be strictly increasing and strictly concave, and the subjective discount factor β lies in the interval $(0, 1)$. The household chooses processes $\{a_t, h_t, D_{t+1}\}$ to maximize this utility function subject to the sequential budget constraint and a no-Ponzi-game restriction of the form $\lim_{j\to\infty} E_t r_{t,t+j} D_{t+j} \geq 0$. The optimality conditions associated with the household's problem are the sequential budget constraint, the no-Ponzi-game restriction holding with equality, and

$$-\frac{U_2(a_t, h_t)}{U_1(a_t, h_t)} = \frac{W_t}{Q_t}$$

and

$$\frac{U_1(a_t, h_t)}{Q_t} r_{t,t+1} = \beta \frac{U_1(a_{t+1}, h_{t+1})}{Q_{t+1}}.$$

2.2 Firms

Each intermediate consumption good $i \in [0, 1]$ is produced by a monopolistically competitive firm via a linear production function $z_t h_{it}$, where h_{it} denotes labor input used in the production of good i, and z_t is an aggregate productivity shock. Profits of firm i in period t are given by

$$P_{it} c_{it} - W_t h_{it}(1 - \tau),$$

where τ denotes a subsidy per unit of labor received from the government. This subsidy is introduced so that under flexible prices the monopolistic firm would produce the competitive level of output. In this way, the only distortion remaining in the model is the one associated with sluggish price adjustment. While this assumption, which is customary in the neo-Keynesian literature, greatly facilitates the characterization of optimal monetary policy, it is not crucial in deriving the main results of this paper.

The firm must satisfy demand at posted prices. Formally, this requirement gives rise to the restriction

$$z_t h_{it} \geq c_{it},$$

where, as derived earlier, c_{it} is given by $c_{it} = \left(\frac{Q_{it}}{Q_t}\right)^{-\eta} \frac{a_t}{x_{it}}$. Let MC_{it} denote the Lagrange multiplier on the above constraint. Then, the optimality condition of the firm's problem with respect to labor is given by

$$(1-\tau)W_t = MC_{it} z_t.$$

It is clear from this first-order condition that MC_{it} must be identical across firms. We therefore drop the subscript i from this variable.

Consider now the price setting problem of the monopolistically competitive firm. For the purpose of determining the optimal inflation target, it is crucial to be precise in regard to what prices are assumed to be costly to adjust. We distinguish two cases. In one case we assume that nonquality-adjusted prices, P_{it}, are sticky. In the second case, we assume that quality-adjusted (or hedonic) prices, Q_{it}, are sticky. Using again the example of the personal computer, the case of stickiness in nonquality-adjusted prices would correspond to a situation in which the price of the personal computer is costly to adjust. The case of stickiness in quality-adjusted prices results when the price of a computer per unit of quality is sticky, where in our example quality would be measured by an index capturing attributes such as memory, processing speed, video capabilities, etc.. We consider first the case in which stickiness occurs at the level of nonquality-adjusted prices.

3 Stickiness in Nonquality-Adjusted Prices

Suppose that with probability α firm $i \in [0,1]$ cannot reoptimize its price, P_{it}, in a given period. Consider the price-setting problem of a firm that has the chance to reoptimize its price in period t. Let $\tilde{P}_{it}$ be the price chosen by such firm. The portion of the Lagrangian associated with the firm's optimization problem that is relevant for the purpose of determining $\tilde{P}_{it}$ is given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} r_{t,t+j} \alpha^j \left[\tilde{P}_{it} - MC_{t+j}\right] \left(\frac{\tilde{P}_{it}}{x_{it+j} Q_{t+j}}\right)^{-\eta} \frac{a_{t+j}}{x_{it+j}}.$$

The first-order condition with respect to $\tilde{P}_{it}$ is given by

$$E_t \sum_{j=0}^{\infty} r_{t,t+j} \alpha^j \left[\left(\frac{\eta-1}{\eta}\right) \tilde{P}_{it} - MC_{t+j}\right] \left(\frac{\tilde{P}_{it}}{x_{it+j} Q_{t+j}}\right)^{-\eta} \frac{a_{t+j}}{x_{it+j}} = 0.$$

Although we believe that the case of greatest empirical interest is one in which quality varies across goods, maintaining such assumption complicates the aggregation of the model, as it adds another source of heterogeneity in addition to the familiar price dispersion stemming from Calvo-Yun staggering. Consequently, to facilitate aggregation, we assume that all goods are of the same quality, that is, we assume that $x_{it} = x_t$ for all i. We further simplify the exposition by assuming that x_t grows at the constant rate $\kappa > 0$, that is,

$$x_t = (1+\kappa) x_{t-1}.$$

In this case, the above first-order condition simplifies to

$$E_t \sum_{j=0}^{\infty} r_{t,t+j}\alpha^j \left[\left(\frac{\eta-1}{\eta} \right) \tilde{P}_{it} - MC_{t+j} \right] \left(\frac{\tilde{P}_{it}}{P_{t+j}} \right)^{-\eta} c_{t+j} = 0,$$

where

$$c_t \equiv \left[\int_0^1 c_{it}^{1-1/\eta} di \right]^{1/(1-1/\eta)}$$

and

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{1/(1-\eta)}.$$

It is clear from these expressions that all firms that have the chance to reoptimize their price in a given period will choose the same price. We therefore drop the subscript i from the variable $\tilde{P}_{it}$. We also note that the definitions of P_t and c_t imply that $P_t c_t = \int_0^1 P_{it} c_{it} di$. Thus P_t can be interpreted as the consumer price index unadjusted for quality improvements.

The aggregate price level P_t is related to the reoptimized price $\tilde{P}_t$ by the following familiar expression in the Calvo-Yun framework:

$$P_t^{1-\eta} = \alpha P_{t-1}^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}.$$

Market clearing for good i requires that

$$z_t h_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t.$$

Integrating over $i \in [0,1]$ yields

$$z_t h_t = c_t \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\eta} di,$$

where $h_t \equiv \int_0^1 h_{it} di$. Letting $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\eta} di$, we can write the aggregate resource constraint as

$$z_t h_t = s_t c_t,$$

where $s_t \geq 1$ measures the degree of price dispersion in the economy and can be shown to obey the law of motion

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \pi_t^{\eta} s_{t-1},$$

where $\tilde{p}_t \equiv \tilde{P}_t / P_t$ denotes the relative price of goods whose price was reoptimized in period t, and $\pi_t \equiv P_t / P_{t-1}$ denotes the gross rate of inflation in period t not adjusted for quality improvements.

A competitive equilibrium is a set of processes c_t, h_t, mc_t, s_t, and $\tilde{p}_t$ satisfying

$$-\frac{U_2(x_t c_t, h_t)}{U_1(x_t c_t, h_t)} = \frac{\mathrm{mc}_t z_t x_t}{1-\tau}, \tag{1}$$

$$z_t h_t = s_t c_t, \tag{2}$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \pi_t^{\eta} s_{t-1}, \tag{3}$$

$$1 = \alpha \pi_t^{\eta-1} + (1-\alpha)\tilde{p}_t^{1-\eta}, \tag{4}$$

and

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_1(x_s c_s, h_s)}{U_1(x_t c_t, h_t)} \left(\prod_{k=t+1}^{s} \pi_k^{-1} \right)^{-\eta} x_s c_s \left[\mathrm{mc}_s - \left(\frac{\eta-1}{\eta} \right) \tilde{p}_t \left(\prod_{k=t+1}^{s} \pi_k^{-1} \right) \right] = 0, \tag{5}$$

given exogenous processes z_t and x_t, a policy regime π_t, and the initial condition s_{-1}. We assume that $s_{-1} = 1$, so that there is no inherited price dispersion in period 0. The variable $mc_t \equiv MC_t/P_t$ denotes the marginal cost of production in terms of the composite good c_t.

We next establish that the optimal monetary policy consists in constant nonquality-adjusted prices. This policy coincides with the one that would be Ramsey optimal in the absence of quality improvements. That is, we show that when nonquality-adjusted prices are sticky, the Ramsey optimal monetary policy calls for not incorporating the quality bias into the inflation target. To this end, set $\pi_t = 1$ for all t and $1 - \tau = (\eta - 1)/\eta$. Then, by equation (4) $\tilde{p}_t = 1$ for all t and by (3) $s_t = 1$ for all t. Letting $mc_t = (\eta - 1)/\eta$, it is easy to very that equation (4) is satisfied. Equilibrium conditions (1) and (2) then become identical to those associated with the problem of maximizing $E_0 \sum_{t=0}^{\infty} \beta^t U(x_t c_t, h_t)$, subject to $z_t h_t = c_t$. We have therefore demonstrated that setting π_t equal to unity is not only Ramsey optimal but also Pareto efficient.

Importantly, π_t is the rate of inflation that results from measuring prices without adjusting for quality improvement. The inflation rate that takes into account improvements in the quality of goods is given by Q_t/Q_{t-1}, which equals $\pi_t/(1+\kappa)$ and is less than π_t by our maintained assumption that quality improves over time at the rate $\kappa > 0$. Therefore, although there is a quality bias in the measurement of inflation, given by the rate of quality improvement κ, the central bank should not target a positive rate of inflation.

This result runs contrary to the usual argument that in the presence of a quality bias in the aggregate price level, the central bank should adjust its inflation target upwards by the magnitude of the quality bias. For instance, suppose that, in line with the findings of the Boskin Commission, the quality bias in the rate of inflation was 0.6 percent (or $\kappa = 0.006$). Then, the conventional wisdom would suggest that the central bank target a rate of inflation of about 0.6 percent. We have shown, however, that such policy would be suboptimal. Rather, optimal policy calls for a zero inflation target. The key to understanding this result is to identify exactly which prices are sticky. For optimal policy aims at keeping the price of goods that are sticky constant over time to avoid inefficient price dispersion. Here we are assuming that stickiness originates in non-quality adjusted prices. Therefore, optimal policy consists in keeping these prices constant over time. At the same time, because quality-adjusted (or hedonic) prices are flexible, the monetary authority lets them decline at the rate κ without creating distortions.

Suppose now that the statistical agency responsible for constructing the consumer price index decided to correct the index to reflect quality improvements. For example, in response to the publication of the Boskin Commission report, the U.S. Bureau of Labor Statistics reinforced its use of hedonic prices in the construction of the CPI (Gordon, 2006). In the ideal case in which all of the quality bias is eliminated from the CPI, the statistical

Table 1: The Optimal Rate of Inflation under Quality Bias

	Statistical Agency Corrects Quality Bias	
Stickiness in	No	Yes
Nonquality-Adjusted Prices	0	$-\kappa$
Quality-Adjusted (or Hedonic) Prices	κ	0

Note. The parameter $\kappa > 0$ denotes the rate of quality improvement.

agency would publish data on Q_t rather than on P_t. How should the central bank adjust its inflation target in response to this methodological advancement? The goal of the central bank continues to be the complete stabilization of the nonquality-adjusted price, P_t, for this is the price that suffers from stickiness. To achieve this goal, the published price index, Q_t, would have to be falling at the rate of quality improvement, κ. This means that the central bank would have to target deflation at the rate κ.

To summarize, when nonquality-adjusted prices are sticky, the optimal inflation target of the central bank is either zero (when the statistical agency does not correct the price index for quality improvements) or negative at the rate of quality improvement (when the statistical agency does correct the price index for quality improvements). See table 1.

4 Stickiness in Quality-Adjusted Prices

Assume now that quality-adjusted (or hedonic) prices, Q_{it}, are costly to adjust. Consider the price-setting problem of a firm, i say, that has the chance to reoptimize Q_{it} in period t. Let $\tilde{Q}_{it}$ be the quality-adjusted price chosen by such firm. The portion of the Lagrangian associated with the firm's profit maximization problem that is relevant for the purpose of determining the optimal level of $\tilde{Q}_{it}$ is given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} r_{t,t+j}\alpha^j \left[\tilde{Q}_{it}x_{t+j} - MC_{t+j}\right] \left(\frac{\tilde{Q}_{it}}{Q_{t+j}}\right)^{-\eta} c_{t+j}.$$

The first order condition with respect to $\tilde{Q}_{it}$ is given by

$$E_t \sum_{j=0}^{\infty} r_{t,t+j}\alpha^j \left[\left(\frac{\eta-1}{\eta}\right)\tilde{Q}_{it}x_{t+j} - MC_{t+j}\right] \left(\frac{\tilde{Q}_{it}}{Q_{t+j}}\right)^{-\eta} c_{t+j} = 0.$$

A competitive equilibrium in the economy with stickiness in quality-adjusted prices is a set of processes c_t, h_t, mc_t, s_t, and $\tilde{p}_t$ that satisfy

$$-\frac{U_2(x_t c_t, h_t)}{U_1(x_t c_t, h_t)} = \frac{\mathrm{mc}_t z_t x_t}{1-\tau}$$

$$z_t h_t = s_t c_t$$

$$s_t = (1-\alpha)(\tilde{p}_t)^{-\eta} + \alpha \left(\pi_t \frac{x_{t-1}}{x_t}\right)^{\eta} s_{t-1},$$

$$1 = \alpha \pi_t^{\eta-1} \left(\frac{x_t}{x_{t-1}}\right)^{1-\eta} + (1-\alpha)(\tilde{p}_t)^{1-\eta},$$

and

$$E_t \sum_{s=t}^{\infty} (\alpha\beta)^s \frac{U_1(x_s c_s, h_s)}{U_1(x_t c_t, h_t)} \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right)^{-\eta} x_s c_s \left[\mathrm{mc}_s - \left(\frac{\eta-1}{\eta}\right) \tilde{p}_t \left(\prod_{k=t+1}^{s} \pi_k^{-1}\right) \frac{x_s}{x_t}\right] = 0,$$

given exogenous processes z_t and x_t, a policy regime π_t, and the initial condition s_{-1}. As before, we assume no initial dispersion of relative prices by setting $s_{-1} = 1$.

We wish to demonstrate that when quality-adjusted prices are sticky, the optimal rate of inflation is positive and equal to the rate of quality improvement, κ. Setting $\pi_t = x_t/x_{t-1}$, we have that in the competitive equilibrium $\tilde{p}_t = 1$ and $s_t = 1$ for all t. Assuming further that the fiscal authority sets $1 - \tau = (\eta - 1)/\eta$, we have that the set of competitive equilibrium conditions becomes identical to the set of optimality conditions associated with the social planner's problem of maximizing $E_0 \sum_{t=0}^{\infty} \beta^t U(x_t c_t, h_t)$, subject to $z_t h_t = c_t$.

We have therefore proven that when quality-adjusted prices are sticky, a positive inflation target equal to the rate of quality improvement ($\pi_t = 1 + \kappa$) is Ramsey optimal and Pareto efficient. In this case, the optimal adjustment in the inflation target conforms to the conventional wisdom, according to which the quality bias in inflation measurement justifies an upward correction of the inflation target equal in size to the bias itself. The intuition behind this result is that in order to avoid relative price dispersion, the monetary authority must engineer a policy whereby firms have no incentives to change prices that are sticky. In the case considered here the prices that are sticky happen to be the quality-adjusted prices. At the same time, non-quality adjusted prices are fully flexible and therefore under the optimal policy they are allowed to grow at the rate κ without creating inefficiencies.

Finally, suppose that the statistical agency in charge of preparing the consumer price index decided to correct the quality bias built into the price index. In this case, the central bank should revise its inflation target downward to zero in order to accomplish its goal of price stability in (sticky) quality-adjusted prices. The second row of table 1 summarizes the results of this section.

5 Conclusion

We interpret the results derived in this paper as suggesting that if the case of greatest empirical relevance is one in which nonquality-adjusted prices (the price of the personal computer in the example we have been using throughout) is sticky, then the conventional wisdom that quality bias justifies an upward adjustment in the inflation target is misplaced. Applying this conclusion to the case of the United States, it would imply that no fraction of the 2 percent inflation target implicit in Fed policy is justifiable on the basis of the quality bias in the U.S. consumer price index. Moreover, the corrective actions taken by the Bureau of Labor Statistics in response to the findings of the Boskin commission, including new hedonic indexes for television sets and personal computers as well as an improved treatment-based methodology for measuring medical care prices, would actually justify setting negative inflation targets. If, on the other hand, the more empirically relevant case is the one in which hedonic prices are sticky, then the conventional view that the optimal inflation target should be adjusted upward by the size of the quality bias is indeed consistent with the predictions of our model. The central empirical question raised by the theoretical analysis presented in this paper is therefore whether regular or hedonic prices are more sticky. The existing empirical literature on nominal price rigidities has yet to address this matter.

References

Boskin, Michael J., E. Dulberger, R. Gordon, Z. Griliches, and D. Jorgenson, "Toward a More Accurate Measure of the Cost of Living," *Final Report to the Senate Finance Committee,* December 4, 1996.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics 12,* September 1983, 383-98.

Gordon, Robert J., "The Boskin Commission Report: A Retrospect One Decade Later," *International Productivity Monitor 12,* Spring 2006, 7-22.

Yun, Tack, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics 37,* July 1996, 345-70.