On quality bias and inflation targets

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\textbf{A B S T R A C T}

Does Ramsey optimal policy call for adjusting the inflation target by the size of the quality bias in measured inflation? We find that if it is nonhedonic (or sticker) prices that are sticky, the conventional view, according to which it is optimal to adjust the inflation target upward by the size of the quality bias, is misguided. Furthermore, we establish that quality improvement is crucial for the determination of the optimal inflation target even in the absence of quality bias. In this case, if nonhedonic prices are sticky, sticker prices should fall at the rate of quality growth.

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1. Introduction

The existence of a positive quality bias in the consumer price index has led some to argue that an inflation target equal in size to the bias would be appropriate if the ultimate objective of the central bank is price stability. For early articulations of this view see Bernanke and Mishkin (1997) and Camba-Mendez (2003). Over the past ten years, the conventional wisdom in this regard has little changed. In a November 14, 2007 speech at the Cato Institute’s 25th Annual Monetary Conference, Ben Bernanke, Chairman of the Board of Governors of the Federal Reserve System, stated: “Were price stability the only objective mandated for the Federal Reserve, the FOMC presumably would strive to achieve zero inflation, properly measured— that is, the optimal measured inflation rate would deviate from zero on average only by the amount of the estimated measurement error in the preferred inflation index.”

This paper critically evaluates this argument. Specifically, it studies whether a central bank implementing Ramsey-optimal policy would adjust its inflation target to account for the systematic upward bias in measured inflation due to quality improvements in consumption goods. The paper shows that the answer to this question depends critically upon the nature of the assumed price stickiness. If nonquality-adjusted (or sticker) prices are sticky, then according to the Ramsey policy the inflation target should not be corrected; i.e., in this case the central bank should not target a quality-adjusted index such as a cost-of-living index (COLI). On the other hand, if quality-adjusted (or hedonic) prices are sticky, then the inflation target should be raised by the magnitude of the bias; i.e., in this case the central bank should target zero growth in a quality-adjusted index. The intuition is straightforward: optimal policy aims at keeping the assumed-sticky prices—whatever those prices are—constant over time, to avoid inefficient price dispersion.

The quality bias in the U.S. consumer price index has been studied and documented by both policymakers and academic researchers. In 1995, the Senate Finance Committee appointed an advisory commission composed of five prominent academic economists (Michael Boskin, Ellen Dulberger, Robert Gordon, Zvi Griliches, and Dale Jorgenson) to

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study the magnitude of the measurement error in the consumer price index. The commission concluded that over the period 1995–1996, the U.S. CPI had an upward bias of 1.1% per year. Of the total bias, 0.6% was ascribed to unmeasured quality improvements (see Boskin et al., 1996).

To illustrate the nature of the quality bias, consider the case of a personal computer. Suppose that between 1995 and 1996 the nominal price of a computer increased by 2%. Assume also that during this period the quality of personal computers, measured by attributes such as memory, processing speed, and video capabilities, increased significantly. If the statistical office in charge of constructing the consumer price index did not adjust the price index for quality improvement, then it would report 2% inflation in personal computers. However, because a personal computer in 1996 provides more services than does a personal computer in 1995, the quality-adjusted rate of inflation in personal computers should be recorded as lower than 2%. The difference between the reported rate of inflation and the quality-adjusted rate of inflation is called the quality bias in measured inflation.

The relationship between quality improvement and the Ramsey optimal rate of inflation is analyzed in the context of a simple model of quality improvement. The key modification introduced to that framework is that the quality of consumption goods is assumed to increase over time. This modification gives rise to an inflation bias if the statistical agency in charge of constructing the consumer price index fails to take quality improvements into account. The central question entertained in the present study is whether under Ramsey optimal policy the inflation target should be adjusted by the size of this bias.

The paper finds that as long as nonhedonic (or sticker) prices are sticky, the conventional view, according to which it is optimal to adjust the inflation target upward by the size of the quality bias, is misguided. Furthermore, quality improvement is crucial for the determination of inflation targets even in the absence of quality bias in consumer prices. Specifically, in this case, if nonquality-adjusted (or sticker) prices are sticky, the optimal inflation target is indeed negative and equal in absolute value to the rate of quality growth.

This paper contributes to a body of work that is concerned with explaining the observed positive inflation targets of central banks. For a survey of this literature, see Schmitt-Grohé and Uribe (2011a). Among the reasons given in the existing literature for targeting positive inflation are (1) The zero bound on nominal interest rates, which creates the need for setting positive rates of inflation as a way to preserve the central bank’s ability to stimulate the economy (Summers, 1991; Reifschneider and Williams, 2000; Adam and Billi, 2006). (2) Downward price and wage rigidity. When nominal product or factor prices are downwardly rigid, any efficient change in relative prices calls for an increase in the price level (Olivera, 1964; Tobin, 1972). A related argument is presented in Wolman (2011). He shows that in a two-sector economy with different rates of long-run productivity growth, the optimal rate of inflation is positive if nominal prices are relatively more flexible in the sector with the lower rate of productivity growth. (3) Inflation as a levy on untaxed income. The sources of untaxed income may take different forms, including the underground economy (Nicolini, 1998), pure monopoly rents (Schmitt-Grohé and Uribe, 2004), and a foreign demand for money (Schmitt-Grohé and Uribe, forthcoming). These theories coexist with others that suggest the optimality of either zero inflation in the context of New-Keynesian frameworks (Goodfriend and King, 1997), or negative inflation, in the context of models with a demand for money (Friedman, 1969).

Our work suggests that quality change is a justification for a positive inflation target only if price stickiness affects mainly quality-adjusted prices and if the statistical agency fails to fully correct for the quality bias.

The remainder of the paper is organized in four sections. Section 2 presents a simple model of quality bias in measured inflation. Section 3 characterizes Ramsey-optimal monetary policy when nonquality-adjusted prices are assumed to be sticky. Section 4 characterizes Ramsey-optimal monetary policy when quality-adjusted (or hedonic) prices are assumed to be sticky. Section 5 concludes.

2. A simple model of quality bias

In this section, a model of price staggering à la Calvo–Yun augmented to allow for quality improvement in consumption goods is developed.

2.1. Households

The economy is populated by a large number of households with preferences defined over a continuum of goods of measure one indexed by $i \in [0, 1]$. Each unit of good $i$ sells for $P_i$ dollars in period $t$. Time is assumed to be discrete. The quantity of good $i$ purchased by the representative consumer in period $t$ is denoted by $C_i$. The quality of good $i$ is denoted by $x_i$ and is assumed to evolve exogenously and to satisfy

$$x_t > x_{t-1}.$$  \hfill (1)

The assumption that quality evolves exogenously greatly facilitates the proceeding analysis but is clearly a simplification. In reality, quality change is likely to have a nontrivial endogenous component. For instance, in response to nominal rigidities, firms might find an incentive to change the quality of the product as a way to bring marginal costs closer to marginal revenue. Also, product changes could be used by firms as opportunities to alter nominal prices without alienating the customer base. A further assumption is that quality improvements are costless and that firms always adopt the highest available level of quality.
The household cares about a composite good, \( a_t \), given by
\[
a_t = \left[ \int_0^1 (x_i c_t)^{1-1/\eta} \, di \right]^{1/(1-1/\eta)},
\]
where \( \eta > 1 \) denotes the elasticity of substitution across different good varieties. Note that the utility of the household increases with the quality content of each good. Let \( a_t \) denote the amount of the composite good the household wishes to consume in period \( t \). Then, the demand for goods of variety \( i \) is the solution to the following cost-minimization problem:
\[
\min_{[0,1]} \int_0^1 P_{it} c_{it} \, di
\]
subject to
\[
\left[ \int_0^1 (x_i c_t)^{1-1/\eta} \, di \right]^{1/(1-1/\eta)} \geq a_t.
\]
The demand for good \( i \) is then given by
\[
c_{it} = \left( \frac{Q_{it}}{Q_t} \right)^{-\eta} \frac{a_t}{x_{it}},
\]
where
\[
Q_{it} = P_{it}/x_{it}
\]
denotes the quality-adjusted (or hedonic) price of good \( i \), and \( Q_t \) is a quality-adjusted (or hedonic) price index given by
\[
Q_t = \left[ \int_0^1 Q_t^{1-1/\eta} \, di \right]^{1/(1-1/\eta)}.
\]
The price index \( Q_t \) has the property that the total cost of \( a_t \) units of composite good is given by \( Q_t a_t \), that is, \( \int_0^1 P_{it} c_t di = Q_t a_t \). Because \( a_t \) is the object from which households derive utility, it follows from this property that \( Q_t \), the unit price of \( a_t \), represents the appropriate measure of the cost of living. In other words, \( Q_t \) is the Konus cost of living index.

Households supply labor effort to the market for a nominal wage rate \( W_t \) and are assumed to have access to a complete set of financial assets. Their budget constraint is given by
\[
Q_t a_t + E_t r_{t,t+1} D_{t+1} + T_t = D_t + W_t h_t + \Phi_t,
\]
where \( r_{t,t+j} \) is a discount factor defined so that the dollar price in period \( t \) of any random nominal payment \( D_{t+j} \) in period \( t+j \) is given by \( E_t r_{t,t+j} D_{t+j} \). The variable \( \Phi_t \) denotes nominal profits received from the ownership of firms, and the variable \( T_t \) denotes lump-sum taxes.

The lifetime utility function of the representative household is given by
\[
E_0 \sum_{t=0}^\infty \beta^t U(a_t, h_t),
\]
where the period utility function \( U \) is assumed to be strictly increasing and strictly concave, and the subjective discount factor \( \beta \) lies in the interval \((0,1)\). The household chooses processes \([a_t,h_t,D_{t+1}]\) to maximize this utility function subject to the sequential budget constraint and a no-Ponzi-game restriction of the form \( \lim_{t \to \infty} E_t r_{t,t+j} D_{t+j} \geq 0 \). The optimality conditions associated with the household’s problem are the sequential budget constraint, the no-Ponzi-game restriction holding with equality, and
\[
- \frac{U_2(a_t,h_t)}{U_1(a_t,h_t)} = \frac{W_t}{Q_t}
\]
and
\[
\frac{U_1(a_t,h_t)}{Q_t} r_{t,t+1} = \beta \frac{U_1(a_{t+1},h_{t+1})}{Q_{t+1}}.
\]

2.2. Firms

Each intermediate consumption good \( i \in [0,1] \) is produced by a monopolistically competitive firm via a linear production function \( z_t h_t \), where \( h_t \) denotes labor input used in the production of good \( i \), and \( z_t \) is an aggregate productivity shock. Profits of firm \( i \) in period \( t \) are given by
\[
P_{it} c_t - W_t h_t (1-\tau),
\]
where \( \tau \) denotes a subsidy per unit of labor received from the government. This subsidy is introduced so that under flexible prices the monopolistic firm would produce the competitive level of output. In this way, the only distortion remaining in the model is the one associated with sluggish price adjustment. This assumption, which is customary in the neo-Keynesian literature (see, for instance Woodford, 2003), greatly facilitates the characterization of optimal monetary policy, but is not crucial in deriving the main results of this paper.

The firm must satisfy demand at posted prices. Formally, this requirement gives rise to the restriction

\[
z_t h_t \geq c_t.
\]

where, as derived earlier, \( c_t = (Q_t / Q_t^*) - \eta (a_t / x_t) \). Let \( MC \) denote the Lagrange multiplier on the above constraint. Then, the optimality condition of the firm’s problem with respect to labor is given by

\[
(1-\tau) W_t = MC_t z_t.
\]

It is clear from these expressions that all firms that have the chance to reoptimize their price in a given period will choose \( P_{it} \). Consider now the price setting problem of the monopolistically competitive firm. For the purpose of determining the optimal inflation target, it is crucial to be precise about the nature of the price stickiness. Under quality change, price stickiness takes a different character: if nonquality adjusted (or sticker) prices are sticky, then quality-adjusted (or hedonic) prices are not, and vice versa. Two cases are distinguished. In one case, it is assumed that nonquality-adjusted (or sticker) prices are sticky. (The latest-model PC always costs $2000.) In the other case, it is assumed that quality-adjusted (or hedonic) prices, \( Q_{it} \), are sticky. (Cars cost more this year because they have airbags and anti-lock brakes.) The case of sticky sticker prices is considered first.

### 3. Stickiness in nonquality-adjusted prices

Suppose that with probability \( x \) firm \( i \in [0,1] \) cannot reoptimize its price, \( P_{it} \), in a given period. Consider the price-setting problem of a firm that has the chance to reoptimize its price in period \( t \). Let \( \tilde{P}_{it} \) be the price chosen by such firm. The portion of the Lagrangian associated with the firm’s optimization problem that is relevant for the purpose of determining \( \tilde{P}_{it} \) is given by

\[
\mathcal{L} = E_t \sum_{j=0}^\infty r_{t,j} x^{j} \left[ \tilde{P}_{it} - MC_{it+j} \right] \left( \frac{-\eta}{\eta} \tilde{P}_{it} \right) \left( \frac{-\eta}{\eta} \tilde{P}_{it} \right) \frac{a_{t+j}}{x_{it+j}}.
\]

The first-order condition with respect to \( \tilde{P}_{it} \) is given by

\[
E_t \sum_{j=0}^\infty r_{t,j} x^{j} \left[ \left( \frac{\eta-1}{\eta} \right) \tilde{P}_{it} - MC_{it+j} \right] \left( \frac{\tilde{P}_{it}}{x_{it+j} Q_{it+j}} \right) \frac{-\eta}{\eta} \frac{a_{t+j}}{x_{it+j}} = 0.
\]

Although we believe that the case of greatest empirical interest is one in which quality varies across goods, maintaining such assumption complicates the aggregation of the model, as it adds another source of heterogeneity in addition to the familiar price dispersion stemming from Calvo–Yun staggering. Consequently, to facilitate aggregation, it is assumed that all goods are of the same quality, that is, it is assumed that \( x_{it} = x_t \) for all \( i \). The exposition is further simplified by assuming that \( x_t \) grows at the constant rate \( \kappa > 0 \), that is,

\[
x_t = (1+\kappa)x_{t-1}.
\]

In this case, the above first-order condition simplifies to

\[
E_t \sum_{j=0}^\infty r_{t,j} x^{j} \left[ \left( \frac{\eta-1}{\eta} \right) \tilde{P}_{it} - MC_{it+j} \right] \left( \frac{\tilde{P}_{it}}{P_{t+j}} \right) \frac{-\eta}{\eta} c_{t+j} = 0,
\]

where

\[
c_t = \left[ \int_0^1 c_{it}^{1-\eta} dt \right]^{1/(1-\eta)}
\]

and

\[
P_t = \left[ \int_0^1 P_{it}^{1-\eta} dt \right]^{1/(1-\eta)}.
\]

It is clear from these expressions that all firms that have the chance to reoptimize their price in a given period will choose the same price. The subscript \( i \) is therefore dropped from the variable \( P_{it} \). Note that the definitions of \( P_t \) and \( c_t \) imply that \( P_t c_t = \int_0^1 P_{it} c_{it} dt \). Thus \( P_t \) can be interpreted as the consumer price index (or a cost-of-goods index) unadjusted for quality improvements.
The aggregate price level \( P_t \) is related to the reoptimized price \( \tilde{P}_t \) by the following familiar expression in the Calvo–Yun framework:

\[
P_t^{1-\eta} = \alpha P_{t-1}^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}.
\]  

(21)

Market clearing for good \( i \) requires that

\[
z_i h_t = \left( \frac{P^*_i}{\tilde{P}_t} \right)^{-\eta} c_t.
\]  

(22)

Integrating over \( i \) in \([0, 1]\) yields

\[
z_i h_t = c_t \int_0^1 \left( \frac{P^*_i}{\tilde{P}_t} \right)^{-\eta} \, di,
\]  

(23)

where \( h_t \equiv \int_0^1 h_{it} \, di \). Letting \( s_t \equiv \int_0^1 (P^*_i/P_t)^{-\eta} \, di \), one can write the aggregate resource constraint as

\[
z_i h_t = s_t c_t,
\]  

(24)

where \( s_t \geq 1 \) measures the degree of price dispersion in the economy and can be shown to obey the law of motion:

\[
s_t = (1-\alpha)\tilde{P}_t^{-\eta} + \alpha (\pi_t/\tau) s_{t-1},
\]  

(25)

where \( \tilde{P}_t \equiv \tilde{P}_t/P_t \) denotes the relative price of goods whose price was reoptimized in period \( t \), and \( \pi_t \equiv P_t/P_{t-1} \) denotes the gross rate of inflation in period \( t \) not adjusted for quality improvements.

Let

\[
m_i c_t = \frac{MC_i}{\tilde{P}_t}
\]  

(26)

denote the marginal cost of production in terms of the composite good \( c_t \). Then, a competitive equilibrium is a set of processes \( c_t, h_t, m_t, s_t \), and \( \tilde{P}_t \) satisfying

\[
\frac{U_j(x_t c_t, h_t)}{U_i(x_t c_t, h_t)} = \frac{mc_t z_t x_t}{1-\tau},
\]  

(27)

\[z_i h_t = s_t c_t,
\]  

(28)

\[s_t = (1-\alpha)\tilde{P}_t^{-\eta} + \alpha (\pi_t/\tau) s_{t-1},
\]  

(29)

\[1 = \alpha (\pi_t^{1-\eta}) + (1-\alpha)\tilde{P}_t^{1-\eta},
\]  

(30)

and

\[
E_t \sum_{s=t}^{\infty} (x^\beta) \frac{U_j(x_t c_t, h_t)}{U_i(x_t c_t, h_t)} \left( \prod_{k=t+1} \pi_k^{-1} \right)^{-\eta} x_t c_t \left[ mc_t - (\frac{\eta-1}{\eta}) \tilde{P}_t \left( \prod_{k=t+1} \pi_k^{-1} \right) \right] = 0
\]  

(31)

given exogenous processes \( z_t \) and \( x_t \), a policy regime \( \pi_t \), and the initial condition \( s_{-1} \). Assume that \( s_{-1} = 1 \), so that there is no inherited price dispersion in period 0.

3.1. The optimal rate of inflation

Next it is established that the optimal monetary policy consists in constant nonquality-adjusted prices. This policy coincides with the one that would be Ramsey optimal in the absence of quality improvements. That is, it is shown that when nonquality-adjusted prices are sticky, the Ramsey optimal monetary policy calls for not incorporating the quality bias into the inflation target. To this end, set \( \pi_t = 1 \) for all \( t \) and \( 1-\tau = (\eta-1)/\eta \). Then, by Eq. (30) \( \tilde{P}_t = 1 \) for all \( t \) and by (29) \( s_t = 1 \) for all \( t \). Letting \( m_t = (\eta-1)/\eta \), it is easy to verify that Eq. (31) is satisfied. Equilibrium conditions (27) and (28) then become identical to those associated with the problem of maximizing \( E_t \sum_{s=t}^{\infty} (x^\beta) \frac{U_j(x_t c_t, h_t)}{U_i(x_t c_t, h_t)} \left( \prod_{k=t+1} \pi_k^{-1} \right)^{-\eta} x_t c_t \left[ mc_t - (\frac{\eta-1}{\eta}) \tilde{P}_t \left( \prod_{k=t+1} \pi_k^{-1} \right) \right] = 0 \) given exogenous processes \( z_t \) and \( x_t \), a policy regime \( \pi_t \), and the initial condition \( s_{-1} \). Assume that \( s_{-1} = 1 \), so that there is no inherited price dispersion in period 0.

Importantly, \( \pi_t \) is the rate of inflation that results from measuring prices without adjusting for quality improvement. The inflation rate that takes into account improvements in the quality of goods is given by the rate of quality improvement \( \kappa > 0 \). Therefore, although there is a quality bias in the measurement of inflation, given by the rate of quality improvement \( \kappa \), the central bank should not target a positive rate of inflation.

This result runs contrary to the usual argument that in the presence of a quality bias in the aggregate price level, the central bank should adjust its inflation target upwards by the magnitude of the quality bias. For instance, suppose that, in line with the findings of the Boskin Commission, the quality bias in the rate of inflation was 0.6% (or \( \kappa = 0.006 \)). Then, the conventional wisdom would suggest that the central bank target a rate of inflation of about 0.6%. It has been shown, however, that such policy would be suboptimal. Rather, optimal policy calls for a zero inflation target. The key to understanding this result is to identify exactly which prices are sticky. For optimal policy aims at keeping the price of
goods that are sticky constant over time to avoid inefficient price dispersion. Here it is assumed that stickiness originates in non-quality adjusted prices. Therefore, optimal policy consists in keeping these prices constant over time. At the same time, because quality-adjusted (or hedonic) prices are flexible, the monetary authority lets them decline at the rate \( \kappa \) without creating distortions.

Suppose now that the statistical agency responsible for constructing the consumer price index decided to correct the index to reflect quality improvements. For example, in response to the publication of the Boskin Commission report, the U.S. Bureau of Labor Statistics reinforced its use of hedonic prices in the construction of the CPI (Gordon, 2006). As a result, estimates of the quality bias built into the U.S. consumer price index have been falling over time. For example Lebow and Rudd (2003), using data for 2001, estimate a quality bias of 37 basis points per year, which is slightly above half of the 60 basis point estimate reported by the Boskin Commission, which used data from 1995 to 1996. In the ideal case in which all of the quality bias is eliminated from the CPI, the statistical agency would publish data on \( Q_t \) rather than on \( P_t \). How should the central bank adjust its inflation target in response to this methodological advancement? The goal of the central bank continues to be the complete stabilization of the nonquality-adjusted price, \( Qt \). How should the central bank adjust its inflation target in response to this methodological advancement? The goal of the central bank continues to be the complete stabilization of the nonquality-adjusted price, \( Qt \). To achieve this goal, the published price index, \( Q_t \), would have to be falling at the rate of quality improvement, \( \kappa \). This means that the central bank would have to target deflation at the rate \( \kappa \).

More generally, the important result is that the issue of a misguided inflation target might remain even if the statistical agency partially or fully corrected the price index for quality improvement. To see this, suppose that the statistical agency captures a rate \( 0 < \omega < \kappa \) of quality improvement. Then, the quality bias in the CPI would be \( \kappa - \omega \). As \( \omega \) approaches \( \kappa \), the quality bias disappears. Accordingly, the conventional wisdom, which advocates an inflation target equal to the quality bias, would call for an inflation target equal to \( \kappa - \omega \). However, the optimal CPI inflation target would be \( -\omega \), which would be consistent with zero inflation in nonquality-adjusted prices. It follows that the information that is most valuable for the central bank in designing an inflation target is not the quality bias \( \kappa - \omega \), which is the overwhelming focus of the existing empirical literature, but the amount of quality-bias correction \( \omega \). The curious result is that regardless of the degree of bias correction in the CPI, the difference between the optimal CPI inflation target (i.e., zero inflation in nonquality-adjusted prices) and that recommended by conventional wisdom (i.e., inflation equal to the quality bias) is constant and equal to \( \kappa \). That is, the monetary-policy error implicit in the conventional view does not disappear as the quality bias disappears (i.e., as \( \omega \) approaches \( \kappa \)). The top row of Table 1 summarizes the main results of this section.

### 4. Stickiness in non-quality-adjusted prices

Assume now that quality-adjusted (or hedonic) prices, \( Q_t \), are sticky. Consider the price-setting problem of a firm \( i \) that has the chance to reoptimize \( Q_t \) in period \( t \). Let \( Q_{it} \) be the quality-adjusted price chosen by such firm. The portion of the Lagrangian associated with the firm’s profit maximization problem that is relevant for the purpose of determining the optimal level of \( Q_{it} \) is given by

\[
\mathcal{L} = E_t \sum_{j=0}^{\infty} r_{t,t+j} z^j [Q_{it} x_{t+j} - MC_{t+j}] \left( \frac{Q_{it}}{Q_{t+j}} \right)^{-\eta} c_{t+j}.
\]  

(32)

The first-order condition with respect to \( Q_{it} \) is given by

\[
E_t \sum_{j=0}^{\infty} r_{t,t+j} z^j \left[ \left( \frac{\eta - 1}{\eta} \right) Q_{it} x_{t+j} - MC_{t+j} \right] \left( \frac{Q_{it}}{Q_{t+j}} \right)^{-\eta} c_{t+j} = 0.
\]

(33)

A competitive equilibrium in the economy with stickiness in quality-adjusted prices is a set of processes \( c_t, h_t, mc_t, s_t \), and \( p_t \) that satisfy

\[
\frac{U_2(x_t c_t, h_t)}{U_1(x_t c_t, h_t)} = \frac{mc_t z_t x_t}{1 - \tau},
\]

(34)

\[
z_t h_t = s_t c_t.
\]

(35)
\[ S_t = (1-\alpha)(\hat{P}_t)^{-\eta} + \alpha \left( \frac{X_{t+1}}{X_t} \right)^{\eta} S_{t-1}, \]  
\[ 1 = \alpha \pi_t^{\eta-1} \left( \frac{X_t}{X_{t-1}} \right)^{1-\eta} + (1-\alpha)(\hat{P}_t)^{1-\eta}, \]  
and

\[ E_t \sum_{j=t}^{\infty} (\beta j) U_t(x_t c_t) \left( \prod_{k=1+1}^{\infty} \pi_k^{-1} \right)^{-\eta} x_t c_t \left[ m_c - (\eta-1) \hat{P}_t \left( \prod_{k=1+1}^{\infty} \pi_k^{-1} \right) \frac{X_t}{X_0} \right] = 0, \]

given exogenous processes \( x_t \) and \( x_0 \), a policy regime \( \pi_t \), and the initial condition \( s_{-1} = 1 \).

### 4.1. The optimal rate of inflation

This subsection demonstrates that when quality-adjusted prices are sticky, the optimal rate of inflation is positive and equal to the rate of quality improvement, \( \kappa \). Setting \( \pi_t = x_t / x_{t-1}, \) it follows that in the competitive equilibrium \( \hat{P}_t = 1 \) and \( s_t = 1 \) for all \( t \). Assuming further that the fiscal authority sets \( 1 - \tau = (\eta-1)/\eta \), one has that the set of competitive equilibrium conditions becomes identical to the set of optimality conditions associated with the social planner’s problem of maximizing

\[ \sum_{j=0}^{\infty} \beta^j U(x_t c_t h_t) \]

subject to \( z_t h_t = c_t \).

It has therefore been established that when quality-adjusted prices are sticky, a positive inflation target equal to the rate of quality improvement (\( \pi_t = 1 + \kappa \)) is Ramsey optimal and Pareto efficient. In this case, the optimal adjustment in the inflation target conforms to the conventional wisdom, according to which the quality bias in inflation measurement justifies an upward correction of the inflation target equal in size to the bias itself. The intuition behind this result is that in order to avoid relative price dispersion, the monetary authority must engineer a policy whereby firms have no incentives to change prices that are sticky. In the case considered here the prices that are sticky happen to be the quality-adjusted prices. At the same time, non-quality adjusted prices are fully flexible and therefore under the optimal policy they are allowed to grow at the rate \( \kappa \) without creating inefficiencies.

Finally, suppose that the statistical agency in charge of preparing the consumer price index corrects the quality bias built into the price index. In this case, the central bank should revise its inflation target downward to zero in order to accomplish its goal of price stability in (sticky) quality-adjusted prices. The second row of Table 1 summarizes the results of this section.

### 5. Conclusion

The results derived in this paper suggest that if the case of greatest empirical relevance is one in which non-quality-adjusted prices (sticker price, i.e., the price of the personal computer in the example appealed to throughout) are sticky, then the conventional wisdom that quality bias justifies an upward adjustment in the inflation target is misplaced. Applying this conclusion to the case of the United States, it would imply that no fraction of the 2% inflation target implicit in Fed policy is justifiable on the basis of the quality bias in the U.S. consumer price index. Moreover, the corrective actions taken by the Bureau of Labor Statistics in response to the findings of the Boskin commission, including new hedonic indexes for television sets and personal computers as well as an improved treatment-based methodology for measuring medical care prices, would actually justify setting negative inflation targets.

If, on the other hand, the more empirically relevant case is the one in which hedonic prices are sticky, then the conventional view that the optimal inflation target should be adjusted upward by the size of the quality bias is indeed consistent with the predictions of our model. But if the statistical agency in charge of producing the price index does correct prices for quality improvements, then the central bank should target stability in the published measure of the consumer price index. In this case, there is no quality bias to begin with, and thus no justification for a positive inflation target on the basis of unmeasured quality changes.

Throughout the paper, it has been assumed that all goods in the economy experience quality improvements over time and that all prices are sticky. It is also interesting to consider the case in which prices are sticky for goods whose quality does not improve and are flexible for goods whose quality does improve. In a separate appendix (Schmitt-Grohé and Uribe, 2011b), we show that in this case quality bias in measured inflation does not justify a positive inflation target either. Specifically, the economy studied here is modified to allow for two sectors, \( a \) and \( b \). In sector \( a \) prices are sticky and there is no quality improvement. In sector \( b \), prices are fully flexible, and product quality improves over time. The Ramsey optimal monetary policy is shown to consist in targeting zero inflation in sector \( a \), where prices are sticky. Under the optimal policy regime equilibrium prices in sector \( b \) are also constant over time. Since the quality of goods in sector \( b \) grows at a positive rate, it follows that the hedonic prices of type-\( b \) goods fall over time. This implies that the measured inflation rate in sector \( b \) has a positive quality bias. Furthermore, the overall rate of inflation also has a positive quality bias equal to the quality bias in sector \( b \) times the share of expenditure in goods of type \( b \). The important result, for the purpose of this paper, is that it is optimal for the central bank not to incorporate the quality bias in its inflation target. The optimal rate of
inflation is zero even though the consumer price index contains a positive quality bias. Nakamura and Steinsson (2007) arrive at a similar conclusion using an example in which improvements in quality give firms the opportunity to change prices.

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Appendix A. Supplementary data

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References


