On Quality Bias and Inflation Targets: Supplementary Material

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This document contains supplementary material to Schmitt-Grohé and Uribe (2011).

1 A Two Sector Model

We show that the presence of a quality bias in measured inflation does not justify raising the central bank's inflation target even if prices in sectors that experience quality improvements are fully flexible.

To this end, we modify the economy studied in Schmitt-Grohé and Uribe (2011) to allow for two sectors, a and b. In sector a prices are sticky and there is no quality improvement. In sector b, prices are fully flexible, and product quality improves over time.

1.1 Households

The economy is populated by a large number of households with preferences defined over consumption of good a, $c_{a,t}$, consumption of good b, $c_{b,t}$, and labor effort h_t, and described by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_{a,t} + \ln c_{b,t} + \theta \ln(1 - h_t)],$$

where the subjective discount factor β lies in the interval $(0, 1)$, and θ is a positive constant.

The good $c_{a,t}$ is a composite made of a continuum of intermediate goods indexed by $i \in [0, 1]$. Each unit of good i sells for $P_{i,a,t}$ dollars in period t. We denote the quantity of good of type a variety i purchased by the representative consumer in period t by $a_{i,t}$. The quality of good $a_{i,t}$ is denoted by $x_{i,a,t}$ and is assumed to evolve exogenously and to satisfy $x_{i,a,t} \geq x_{i,a,t-1}$. The good $c_{a,t}$ is obtained by the aggregation process

$$c_{a,t} = \left[\int_0^1 (x_{i,a,t} a_{i,t})^{1-1/\eta_a} di \right]^{1/(1-1/\eta_a)},$$

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where $\eta_a > 1$ denotes the elasticity of substitution across different good varieties. Given the amount of the composite good $c_{a,t}$ the household wishes to consume in period t, the demand for goods of variety $a_{i,t}$ is the solution to the following cost-minimization problem

$$\min_{\{a_{i,t}\}} \int_0^1 P_{i,a,t} a_{i,t} di$$

subject to

$$\left[\int_0^1 (x_{i,a,t} a_{i,t})^{1-1/\eta_a} di\right]^{1/(1-1/\eta_a)} \geq c_{a,t}.$$

The demand for good $a_{i,t}$ is then given by

$$a_{i,t} = \left(\frac{Q_{i,a,t}}{Q_{a,t}}\right)^{-\eta_a} \frac{c_{a,t}}{x_{i,a,t}},$$

where

$$Q_{i,a,t} \equiv \frac{P_{i,a,t}}{x_{i,a,t}}$$

denotes the quality-adjusted (or hedonic) price of good $a_{i,t}$, and $Q_{a,t}$, the quality-adjusted (or hedonic) price of good $c_{a,t}$, is given by

$$Q_{a,t} = \left[\int_0^1 Q_{i,a,t}^{1-\eta_a} di\right]^{1/(1-\eta_a)}.$$

The price index $Q_{a,t}$ has the property that the total cost of $c_{a,t}$ units of composite good is given by $Q_{a,t}c_{a,t}$, that is, $\int_0^1 P_{i,a,t}a_{i,t}di = Q_{a,t}c_{a,t}$.

Similarly, the good $c_{b,t}$ is a composite made of a continuum of intermediate goods $b_{i,t}$, which sell for $P_{i,b,t}$ dollars in period t. We denote the quantity of good of type b variety i purchased by the representative consumer in period t by $b_{i,t}$. The quality of good $b_{i,t}$ is denoted by $x_{i,b,t}$ and is assumed to evolve exogenously and to satisfy $x_{i,b,t} \geq x_{i,b,t-1}$. The good $c_{b,t}$ is obtained by the aggregation process

$$c_{b,t} = \left[\int_0^1 (x_{i,b,t} b_{i,t})^{1-1/\eta_b} di\right]^{1/(1-1/\eta_b)},$$

where $\eta_b > 1$ denotes the elasticity of substitution across different good varieties. The demand for goods of variety $b_{i,t}$ is given by

$$b_{i,t} = \left(\frac{Q_{i,b,t}}{Q_{b,t}}\right)^{-\eta_b} \frac{c_{b,t}}{x_{i,b,t}},$$

where

$$Q_{i,b,t} \equiv \frac{P_{i,b,t}}{x_{i,b,t}}$$

denotes the quality-adjusted (or hedonic) price of good $b_{i,t}$, and $Q_{b,t}$, the quality-adjusted (or hedonic) price of good $c_{b,t}$, is given by

$$Q_{b,t} = \left[\int_0^1 Q_{i,b,t}^{1-\eta_b} di \right]^{1/(1-\eta_b)}.$$

The price index $Q_{b,t}$ has the property that the total cost of $c_{b,t}$ units of composite good is given by $Q_{b,t}c_{b,t}$, that is, $\int_0^1 P_{i,b,t}b_{i,t}di = Q_{b,t}c_{b,t}$.

Households supply labor effort to the market for a nominal wage rate W_t and are assumed to have access to a complete set of financial assets. Their budget constraint is given by

$$Q_{a,t}c_{a,t} + Q_{b,t}c_{b,t} + E_t r_{t,t+1}D_{t+1} + T_t = D_t + W_t h_t + \Phi_t,$$

where $r_{t,t+j}$ is a discount factor defined so that the dollar price in period t of any random nominal payment D_{t+j} in period $t+j$ is given by $E_t r_{t,t+j} D_{t+j}$. The variable Φ_t denotes nominal profits received from the ownership of firms, and the variable T_t denotes lump-sum taxes.

The household chooses processes $\{c_{a,t}, c_{b,t}, h_t, D_{t+1}\}$ to maximize its utility function subject to the sequential budget constraint and a no-Ponzi-game restriction of the form

$$\lim_{j \to \infty} E_t r_{t,t+j} D_{t+j} \geq 0.$$

The optimality conditions associated with the household's problem are the sequential budget constraint, the no-Ponzi-game restriction holding with equality, and

$$\frac{c_{b,t}}{c_{a,t}} = \frac{Q_{a,t}}{Q_{b,t}}$$

$$\frac{\theta c_{a,t}}{1-h_t} = \frac{W_t}{Q_{a,t}}$$

and

$$\frac{1}{Q_{a,t}c_{a,t}} r_{t,t+1} = \beta \frac{1}{Q_{a,t+1}c_{a,t+1}}.$$

1.2 Firms

Intermediate consumption goods $a_{i,t}$ and $b_{i,t}$ are produced by monopolistically competitive firms via linear production functions $z_t h_{i,a,t}$ and $z_t h_{i,b,t}$, where $h_{i,a,t}$ and $h_{i,b,t}$ denote labor input used in the production of goods $a_{i,t}$ and $b_{i,t}$, respectively, and z_t is an aggregate productivity shock. Profits are given by

$$P_{i,a,t}a_{i,t} - W_t h_{i,a,t}(1-\tau_a),$$

and

$$P_{i,b,t}b_{i,t} - W_t h_{i,b,t}(1-\tau_b),$$

where τ_a, τ_b denote subsidies per unit of labor received from the government in sectors a and b. These subsidies are introduced so that under flexible prices monopolistic firms produce the competitive level of output.

Firms must satisfy demand at posted prices. Formally, this requirement gives rise to the restrictions

$$z_t h_{i,a,t} \geq a_{i,t}$$

and

$$z_t h_{i,b,t} \geq b_{i,t}.$$

Let $MC_{i,a,t}$ and $MC_{i,b,t}$ denote the Lagrange multipliers on the above constraints. Then, the optimality condition of each firm's problem with respect to labor is given by

$$(1-\tau_a)W_t = MC_{i,a,t} z_t$$

and

$$(1-\tau_b)W_t = MC_{i,b,t} z_t.$$

It is clear from these first-order conditions that $MC_{i,a,t}$ and $MC_{i,b,t}$ must be identical across firms belonging to the same sector. We therefore drop the subscript i from these variables.

We assume that only prices in sector a are sticky. Prices in sector b are assumed to be fully flexible. In addition, we assume that there are no quality improvements in sector a. That is, we assume that

$$x_{i,a,t} = 1,$$

for all i, t.

Consider the price setting problem of the monopolistically competitive firms in sector a. Suppose that with probability α firm $i \in [0,1]$ in sector a cannot reoptimize its price, $P_{i,a,t}$, in a given period. Consider the price-setting problem of a firm that has the chance to reoptimize its price in period t. Let $\tilde{P}_{i,a,t}$ be the price chosen by such firm. The portion of the Lagrangian associated with the firm's optimization problem that is relevant for the purpose of determining $\tilde{P}_{i,a,t}$ is given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} r_{t,t+j} \alpha^j \left[\tilde{P}_{i,a,t} - MC_{a,t+j}\right] \left(\frac{\tilde{P}_{i,a,t}}{P_{a,t+j}}\right)^{-\eta_a} c_{a,t+j},$$

where

$$P_{a,t} = \left[\int_0^1 P_{i,a,t}^{1-\eta_a} di\right]^{1/(1-\eta_a)}.$$

The first-order condition with respect to $\tilde{P}_{i,a,t}$ is given by

$$E_t \sum_{j=0}^{\infty} r_{t,t+j} \alpha^j \left[\left(\frac{\eta_a - 1}{\eta_a}\right) \tilde{P}_{i,a,t} - MC_{a,t+j}\right] \left(\frac{\tilde{P}_{i,a,t}}{P_{a,t+j}}\right)^{-\eta_a} c_{a,t+j} = 0.$$

It is clear from this expression that all firms in industry a that have the chance to reoptimize their price in a given period will choose the same price. We therefore drop the subscript i from the variable $\tilde{P}_{i,a,t}$.

The aggregate price level $P_{a,t}$ is related to the reoptimized price $\tilde{P}_{a,t}$ by the following familiar expression in the Calvo-Yun framework:

$$P_{a,t}^{1-\eta_a} = \alpha P_{a,t-1}^{1-\eta_a} + (1-\alpha)\tilde{P}_{a,t}^{1-\eta_a}.$$

Market clearing for good $a_{i,t}$ requires that

$$z_t h_{i,a,t} = \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta_a} c_{a,t}.$$

Integrating over $i \in [0,1]$ yields

$$z_t h_{a,t} = c_{a,t} \int_0^1 \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} di,$$

where $h_{a,t} \equiv \int_0^1 h_{i,a,t} di$. Letting $s_{a,t} \equiv \int_0^1 \left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta_a} di$, we can write the aggregate resource constraint in industry a as

$$z_t h_{a,t} = s_{a,t} c_{a,t},$$

where $s_{a,t} \geq 1$ measures the degree of price dispersion in industry a and can be shown to obey the law of motion

$$s_{a,t} = (1-\alpha)\tilde{p}_{a,t}^{-\eta_a} + \alpha \pi_{a,t}^{\eta_a} s_{a,t-1},$$

where $\tilde{p}_{a,t} \equiv \tilde{P}_{a,t}/P_{a,t}$ denotes the relative price of goods of type a whose price was reoptimized in period t, and $\pi_{a,t} \equiv P_{a,t}/P_{a,t-1}$ denotes the gross rate of inflation in sector a in period t.

The price-setting problem in firms belonging to industry b is simplified by the fact that all firms in sector b are assumed to have flexible prices. Specifically, firm i in sector b sets the price $P_{i,b,t}$ to maximize

$$(P_{i,b,t} - MC_{b,t}) \left(\frac{P_{i,b,t}}{x_{i,b,t} Q_{b,t}}\right)^{-\eta_b} \frac{c_{b,t}}{x_{i,b,t}}.$$

The optimality condition equalizes marginal cost to marginal revenue

$$\left(\frac{\eta_b - 1}{\eta_b}\right) P_{i,b,t} = MC_{b,t}.$$

It follows from this expression that every firm in sector b charges the same price. We therefore drop the index i from $P_{i,b,t}$.

To simplify aggregation, we assume that quality is homogeneous across firms in sector b. That is,

$$x_{i,b,t} = x_{b,t},$$

for all i. It follows that

$$Q_{i,b,t} = Q_{b,t} \equiv \frac{P_{b,t}}{x_{b,t}},$$

for all i. Let

$$\kappa_{b,t} \equiv \frac{x_{b,t}}{x_{b,t-1}}$$

denote the rate of quality improvement in sector b. We assume that $\kappa_{b,t}$ satisfies

$$\kappa_{b,t} > 1,$$

for all t.

1.3 Competitive Equilibrium

The resource constraint in industry i of sector b is

$$z_t h_{i,b,t} = \frac{c_{b,t}}{x_{b,t}},$$

which implies that all firms in sector b hire the same number of hours and thus we can drop the i subscript from $h_{i,b,t}$. Since all firms in sector b charge the same price and hire the same number of hours aggregate output in sector b is given by:

$$z_t h_{b,t} = \frac{c_{b,t}}{x_{b,t}},$$

where $h_{b,t} = \int_0^1 h_{i,b,t} di$.

Market clearing in the labor market requires that

$$h_t = h_{a,t} + h_{b,t}$$

A competitive equilibrium is a set of processes $\{h_t,\ h_{a,t},\ h_{b,t},\ c_{a,t},\ c_{b,t},\ P_{a,t},\ P_{b,t},\ Q_{a,t},\ Q_{b,t},\ MC_{a,t},\ MC_{b,t},\ W_t,\ \tilde{p}_{a,t}$, and $s_{a,t}\}_{t=0}^{\infty}$ satisfying

$$\frac{c_{b,t}}{c_{a,t}} = \frac{Q_{a,t}}{Q_{b,t}} \tag{1}$$

$$\frac{\theta c_{a,t}}{1-h_t} = \frac{W_t}{Q_{a,t}} \tag{2}$$

$$z_t h_{a,t} = s_{a,t} c_{a,t} \tag{3}$$

$$z_t h_{b,t} = c_{b,t}/x_{b,t} \tag{4}$$

$$s_{a,t} = (1-\alpha)\tilde{p}_{a,t}^{-\eta_a} + \alpha(P_{a,t}/P_{a,t-1})^{\eta_a} s_{a,t-1}, \tag{5}$$

$$1 = \alpha(P_{a,t}/P_{a,t-1})^{\eta_a - 1} + (1-\alpha)\tilde{p}_{a,t}^{1-\eta_a} \tag{6}$$

$$E_t \sum_{j=0}^{\infty} \frac{P_{a,t} c_{a,t}}{P_{a,t+j} c_{a,t+j}} (\alpha\beta)^j \left[\left(\frac{\eta_a - 1}{\eta_a} \right) \tilde{p}_{a,t} - \frac{MC_{a,t+j}}{P_{a,t}} \right] \left(\tilde{p}_{a,t} \frac{P_{a,t}}{P_{a,t+j}} \right)^{-\eta_a} c_{a,t+j} = 0. \tag{7}$$

$$(1-\tau_a) W_t = MC_{a,t} z_t \tag{8}$$

$$\frac{1-\tau_a}{1-\tau_b} = \frac{MC_{a,t}}{MC_{b,t}} \tag{9}$$

$$h_t = h_{a,t} + h_{b,t} \tag{10}$$

$$\frac{\eta_b - 1}{\eta_b} P_{b,t} = MC_{b,t} \tag{11}$$

$$Q_{b,t} = P_{b,t}/x_{b,t} \tag{12}$$

$$Q_{a,t} = P_{a,t}, \tag{13}$$

given $P_{a,-1}$, $s_{a,-1}$, and a monetary/fiscal regime.

1.4 Optimal Policy

We assume that the economy starts with no price dispersion in sector a, that is, we assume that $s_{a,-1} = 1$. Consider the policy $P_{a,t}/P_{a,t-1} = 1$, $1 - \tau_a = (\eta_a - 1)/\eta_a$, and $1 - \tau_b = (\eta_b - 1)/\eta_b$. Now conjecture that a competitive equilibrium under this policy features the relationship

$$\frac{MC_{a,t}}{P_{a,t}} = \frac{\eta_a - 1}{\eta_a}.$$

Then, the system (1)-(13) reduces to

$$\frac{c_{a,t}}{c_{b,t}} = x_{b,t}$$

$$\frac{\theta c_{a,t}}{1 - h_t} = z_t$$

$$z_t h_{a,t} = c_{a,t}$$

$$z_t h_{b,t} = \frac{c_{b,t}}{x_{b,t}}$$

$$h_{a,t} + h_{b,t} = h_t.$$

These equations coincide with the necessary and sufficient conditions of the following Pareto optimality problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_{a,t} + \ln c_{b,t} + \theta \ln(1 - h_t)],$$

subject to

$$c_{a,t} = \left[\int_0^1 (a_{i,t})^{1-1/\eta_a} di \right]^{1/(1-1/\eta_a)}$$

$$c_{b,t} = \left[\int_0^1 (x_{i,b,t} b_{i,t})^{1-1/\eta_b} di \right]^{1/(1-1/\eta_b)},$$

$$a_{i,t} = z_t h_{i,a,t}$$

$$b_{i,t} = z_t h_{i,b,t}$$

$$\int_0^1 [h_{i,a,t} + h_{i,b,t}] di = h_t.$$

This shows that setting the inflation rate equal to zero in the sector with price stickiness and no quality improvement, the a sector, results in a competitive equilibrium that is Pareto optimal.

The fact that in the competitive equilibrium $P_{b,t} = P_{a,t}$ implies a zero inflation in non-quality-adjusted prices in the b sector. Since the quality of goods in sector b grows at the positive rate $\kappa_{b,t}$, we have that the hedonic price of type-b goods falls over time at the rate $\kappa_{b,t}$. That is, the measured inflation rate in sector b has a positive bias equal to $\kappa_{b,t}$. Furthermore, the overall rate of inflation also has a positive quality bias equal to $\kappa_{b,t}$ times the share of expenditure in goods of type b.

The important result, for the purpose of this paper, is that it is optimal for the central bank not to incorporate the quality bias in its inflation target. The optimal rate of inflation is zero even though the consumer price index contains a positive quality bias.

2 The Welfare Effects of Adjusting Inflation Targets to Quality Bias

In this section, we explore the welfare effects of a policy of adjusting the inflation target upwards by an amount equal to the quality bias in measured inflation, κ. We do this for the economy in which sticker prices, that is, non-hedonic prices are sticky, which is the economy analyzed in section 3 of Schmitt-Grohé and Uribe (2011). As shown there, the optimal policy would be not to adjust the inflation target by the quality bias. We consider values of the quality bias between 0.2 percent and 2 percent per year and assume that the central bank sets the inflation target equal to the quality bias, that is, it sets $\pi = 1 + \kappa$. We then compute the level of welfare in the non-stochastic steady state for a particular calibration of the remaining structural parameters of the model. Specifically, we calibrate $\alpha = 0.8$, which corresponds to the case that firms adjust prices on average every five quarters, θ so that under the optimal policy households spend 20 percent of their time working, η so that the (pre-tax) markup is 20 percent, β so that the quarterly discount factor is 1 percent, σ so that the intertemporal elasticity of substitution is 0.5, and we assume quality bias of 1 percent per year, $\kappa = 1.01^{1/4} - 1$. We then compute the welfare cost as the fraction of steady state consumption that households living in the economy in which the inflation target is adjusted by quality bias would demand to be as well off as households living in an economy with the optimal inflation target, that is, with ($\pi = 1$). This welfare cost is about four one hundredth of one percent of consumption. It is a small number, but similar in magnitude to those found in other studies on the welfare costs of suboptimal monetary policy in the Calvo-Yun model.

Figure 1 shows pairs (α, κ) for which the welfare costs of adjusting the inflation target for the quality bias is constant and equal to the one associated with the baseline calibration, i.e., $\alpha = 0.8$ and κ equal to one percent per year. As the quality bias increases, the monetary authority mistakenly incorporates this bias into its inflation target, deviating further from the optimal rate of inflation, which in this economy is equal to zero regardless of the rate at which product quality improves over time. As a consequence of this misguided choice of inflation target, the level of welfare of the representative household falls with κ. To offset these welfare losses prices must become more flexible, that is, α must decline. The figure shows that this tradeoff is quite pronounced. An increase in κ from one to two percent per year requires an increase in the frequency of price reoptimizations from 5 quarters to less than 3 quarters.

Figure 2 presents the combinations of values of η and κ that produce the same welfare cost of inflation as does the baseline calibration. As the inflation target rises with κ, the welfare level of the representative household falls. This is so primarily because of an elevated level of price dispersion across varieties of goods which causes dispersion in production and consumption. This dispersion is suboptimal because it takes place in spite of the fact that all firms face identical marginal cost functions. To offset the loss of welfare caused by

Figure 1: The α-κ Tradeoff

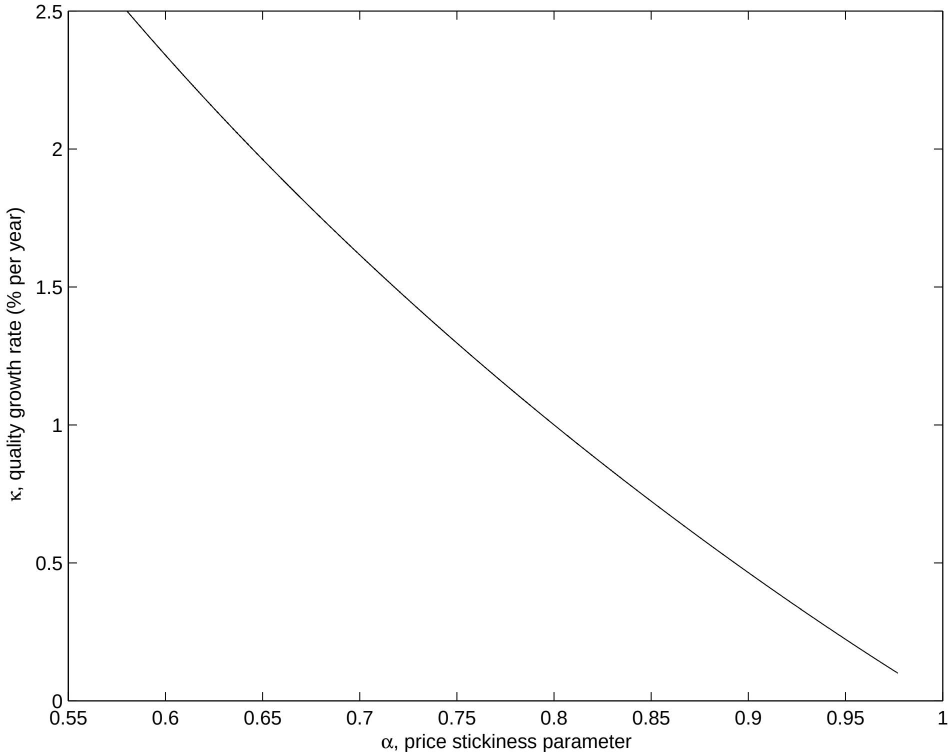
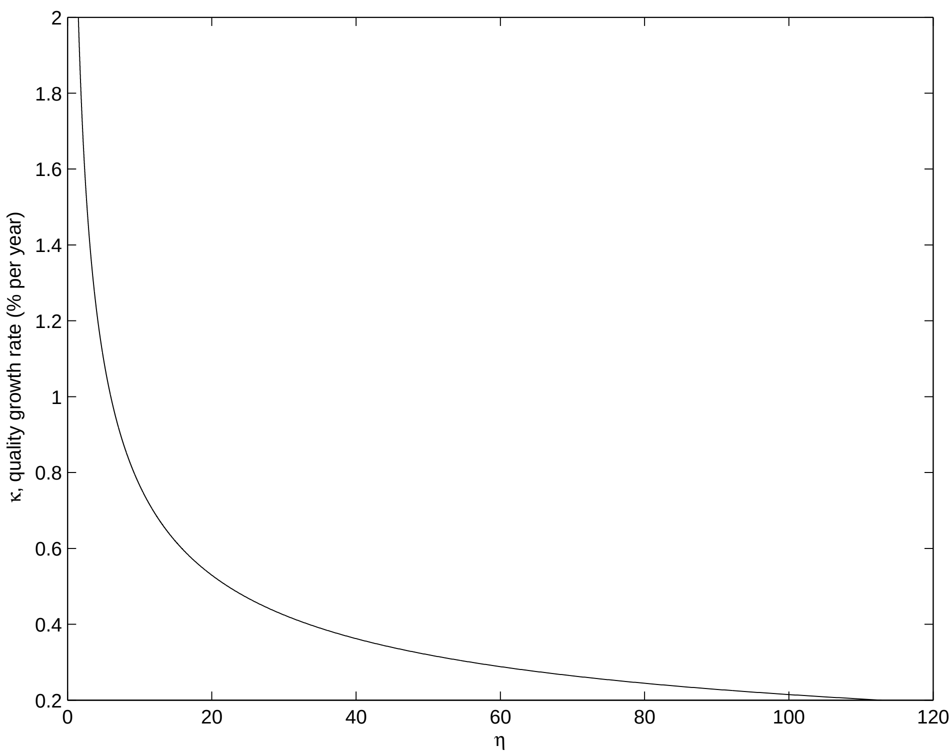


Figure 2: The η-κ Tradeoff



the quality-bias induced increase in the inflation target, the value of η must fall, that is, the elasticity of substitution across varieties must decline. The intuition for this tradeoff between κ and η is that as good varieties become less substitutable, the same amount of price dispersion produces a smaller amount of quantity-dispersion. Of course, smaller values of η also imply an elevation in the average degree of market power. However, this distortion is muted by our maintained assumption of a fiscal instrument that subsidizes production to the level that would be optimal under zero inflation. In the case considered in the figure the subsidy is assumed to be appropriately adjusted as η changes.

References

Schmitt-Grohé and Martín Uribe, "On Quality Bias and Inflation Targets," Columbia University, July 2011.