Comparing Two Variants of Calvo-Type Wage Stickiness*

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Abstract

We compare two ways of modeling Calvo-type wage stickiness. One in which each household is the monopolistic supplier of a differentiated type of labor input (as in Erceg, et al., 2000) and one in which households supply a homogenous labor input that is transformed by monopolistically competitive labor unions into a differentiated labor input (as in Schmitt-Grohé and Uribe, 2006a,b). We show that up to a log-linear approximation the two variants yield identical equilibrium dynamics, provided the wage stickiness parameter is in each case calibrated to be consistent with empirical estimates of the wage Phillips curve. It follows that econometric estimates of New Keynesian models that rely on log-linearizations of the equilibrium dynamics are mute about which type of wage stickiness fits the data better. In the context of a medium-scale macroeconomic model, we show that the two variants of the sticky-wage formulation give rise to the same Ramsey-optimal dynamics, which call for low volatility of price inflation.

Keywords: Nominal Wage Rigidity, Wage Phillips Curve, Optimal Monetary Policy.

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1 Introduction

Nominal wage stickiness is a central characteristic of postwar U.S. aggregate data. Existing estimates of the wage Phillips curve document a low sensitivity of changes in wage inflation with respect to deviations of real wages from the marginal disutility of labor. A number of recent studies model wage stickiness as arising from Calvo-type staggering. The purpose of this paper is to compare two variants of the Calvo-type wage-stickiness model.

The first variant we consider is the one due to Erceg, Henderson, and Levin (2000), hereafter EHL. In this model, each household is the monopolistic supplier of a differentiated type of labor input and equilibrium effort intensity varies across households. The other version of wage stickiness we consider is the one developed in Schmitt-Grohé and Uribe (2006a,b), hereafter SGU. In this variant, households supply a homogeneous labor input that is transformed by monopolistically competitive labor unions into a differentiated labor input and every household works equal hours in equilibrium.

We embed both versions of wage stickiness into the medium-scale macroeconomic model of Altig et al. (2005). This model features a large number of nominal and real rigidities, including price stickiness, money demand by households and firms, habit formation, variable capacity utilization, investment adjustment costs, and imperfect competition in product and labor markets. The Altig et al. model is of particular empirical interest because it has been shown to account well for the observed effects of monetary and supply-side shocks.

We first establish analytically that up to a log-linear approximation the SGU and EHL variants of wage stickiness yield identical expectations-augmented wage Phillips curves. It follows that econometric estimates of this relationship are necessarily mute about which type of wage stickiness fits the data better. We derive the precise mapping from the wage Phillips curve coefficient to the deep structural parameter governing the degree of wage stickiness in the SGU and EHL models. We find that in the context of the EHL framework, the available empirical estimates of linear wage Phillips curves imply that nominal wages are reoptimized every 3 to 4 quarters. At the same time, we find that according to the SGU model, available estimates of the relationship between wage inflation and wage markups imply that nominal wages are reoptimized much less frequently, only about every 10 to 12 quarters.

We demonstrate that up to a log-linear approximation the SGU and EHL variants of wage stickiness yield identical equilibrium conditions, provided the wage stickiness parameter is in each case calibrated to be consistent with empirical estimates of the wage Phillips curve. This result implies that, for a given policy regime, both variants of wage stickiness give rise to identical equilibrium dynamics up to first order. A further consequence of this equivalence result is that the impossibility of identifying whether wage stickiness stems from the SGU or the EHL mechanism is not limited to econometric studies estimating the wage Phillips curve in isolation, but extends to studies estimating the deep structural parameters of the model from the complete set of log-linearized equilibrium conditions (such as Altig et al., 2005, Levin et al., 2006, and Smets and Wouters, 2004).

We characterize Ramsey optimal policy under SGU and EHL wage stickiness. The fact that the SGU and the EHL models induce the same equilibrium dynamics up to first order does not imply that Ramsey optimal policy under both types of wage stickiness is also identical. This is because the first-order conditions of the Ramsey problem include not only the complete set of equilibrium conditions, but also additional constraints involving the derivatives of the equilibrium conditions with respect to all endogenous variables. We find, however, that Ramsey dynamics are numerically the same under the SGU and EHL formulations. Notably, the Ramsey policy calls for stabilizing price inflation. The optimal standard deviation of inflation is 0.6 percent at an annualized rate.

Our findings that Ramsey dynamics are numerically the same under the SGU and EHL variants of Calvo-type wage stickiness lead us to conjecture that the welfare function of the Ramsey planner must be quite similar under both formulations. We confirm this conjecture by deriving analytically the welfare criterion of the Ramsey planner under SGU and EHL wage stickiness in the much simpler economic environment of Erceg et al. (2000). In particular, we show that up to second order the unconditional expectations of the period utility function are the same under the SGU and EHL variants as the subjective discount factor approaches unity.

The remainder of the paper is organized in six sections. Section 2 presents the SGU and EHL variants of nominal wage stickiness. Section 3 establishes that available estimates of linear wage Phillips curves cannot discriminate between the SGU and EHL models. It also derives the mapping from the coefficient of the Phillips curve to the wage stickiness parameters in the two models. Section 4 demonstrates that Ramsey dynamics are the same under SGU- and EHL-type wage stickiness. Section 6 characterizes analytically second-order accurate welfare functions. Section 7 concludes.

2 Modeling Calvo-Type Wage Stickiness

In this section, we derive the SGU and EHL variants of Calvo-style wage stickiness. We develop in detail the conditions describing equilibrium in the labor market, as this is the dimension along which the two approaches differ. The remaining blocks of the macroeconomic model into which we embed the SGU and EHL wage stickiness mechanisms are shared by these two formulations and are those presented in Schmitt-Grohé and Uribe (2006b). The

Schmitt-Grohé and Uribe (2006b) model is a medium-scale economy featuring a number of nominal and real rigidities. In addition to nominal wage stickiness, the model allows for sticky prices, money demands by households and firms, monopolistic competition in product and labor markets, habit formation, investment adjustment costs, and variable capacity utilization. Contrary to a common practice in the related literature, we do not allow for subsidies in product and labor markets to undo the distortions stemming from imperfect competition in those markets. In this model economy, business cycles are driven by stochastic variations in the growth rate of total factor productivity, investment-specific technological progress, and government spending.

2.1 SGU Wage Stickiness

The economy is assumed to be populated by a large representative family with a continuum of members. Consumption and hours worked are identical across family members. The household's preferences are defined over per capita consumption, c_t, and per capita labor effort, h_t, and are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0, 1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, and strictly concave. The parameter $b > 0$ introduces habit persistence. To facilitate comparison with the EHL model, we will at times appeal to the following specific functional form of the utility index, which is separable in consumption and leisure, logarithmic in habit-adjusted consumption, and iso elastic in labor:

$$U(c_t - bc_{t-1}, h_t) = \ln(c_t - bc_{t-1}) - \frac{h_t^{1+\xi}}{1+\xi}. \tag{2}$$

Firms hire labor from a continuum of labor markets of measure 1 indexed by $j \in [0, 1]$. In each labor market j, wages are set by a monopolistically competitive union, which faces a demand for labor given by $\left(W_t^j / W_t\right)^{-\tilde{\eta}} h_t^d$. Here W_t^j denotes the nominal wage charged by the union in labor market j at time t, $W_t \equiv \left[\int_0^1 {W_t^j}^{1-\tilde{\eta}} dj\right]^{1/(1-\tilde{\eta})}$ is an index of nominal wages prevailing in the economy, and h_t^d is a measure of aggregate labor demand by firms. A formal derivation of this labor demand function is presented in Schmitt-Grohé and Uribe (2006b). In each particular labor market, the union takes W_t and h_t^d as exogenous. The case in which the union takes aggregate labor variables as endogenous can be interpreted as an environment with highly centralized labor unions. Higher-level labor organizations play an important role in some European and Latin American countries, but are less prominent in the United States. Given the wage it charges, the union is assumed to supply enough labor, h_t^j, to satisfy demand. That is, $h_t^j = \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d$, where $w_t^j \equiv W_t^j/P_t$ and $w_t \equiv W_t/P_t$. The total number of hours allocated to the different labor markets/unions must satisfy the resource constraint $h_t = \int_0^1 h_t^j dj$. Combining these two restrictions yields

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj. \tag{3}$$

Households are assumed to have access to a complete set of nominal state-contingent assets. Specifically, each period $t \geq 0$, consumers can purchase any desired state-contingent nominal payment X_{t+1} in period $t+1$ at the dollar cost $E_t r_{t,t+1} X_{t+1}$. The variable $r_{t,t+1}$ denotes a stochastic nominal discount factor between periods t and $t+1$. The household's period-by-period budget constraint is given by:

$$E_t r_{t,t+1} x_{t+1} + c_t = \frac{x_t}{\pi_t} + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj. \tag{4}$$

The variable $x_t \equiv X_t/P_{t-1}$ denotes the real payoff in period t of nominal state-contingent assets purchased in period $t-1$. The variable $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of consumer-price inflation.

We introduce nominal wage stickiness by assuming that each period in a fraction $\tilde{\alpha} \in [0,1)$ of randomly chosen labor markets the nominal wage cannot be reoptimized. In these labor markets nominal wages are indexed to past price inflation, denoted π_{t-1}, and long-run real wage growth, denoted μ_{z^*}. The household chooses processes for c_t, h_t, x_{t+1}, and w_t^j so as to maximize the utility function (1) subject to (3) and (4), the wage stickiness friction, and a no-Ponzi-game constraint, taking as given the processes w_t, h_t^d, $r_{t,t+1}$, and π_t, and the initial condition x_0. The household's optimal plan must satisfy constraints (3) and (4). In addition, letting $\beta^t \lambda_t w_t/\tilde{\mu}_t$ and $\beta^t \lambda_t$ denote Lagrange multipliers associated with constraints (3) and (4), respectively, the Lagrangian associated with the household's optimization problem is

$$\begin{aligned}\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t - bc_{t-1}, h_t) + \lambda_t \left[h_t^d \int_0^1 w_t^i \left(\frac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di - c_t - r_{t,t+1} x_{t+1} + \frac{x_t}{\pi_t} \right] \right. \\ & \left. + \frac{\lambda_t w_t}{\tilde{\mu}_t} \left[h_t - h_t^d \int_0^1 \left(\frac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di \right] \right\}.\end{aligned}$$

The first-order conditions with respect to h_t and w_t^i, in that order, are given by

$$-U_h(c_t - bc_{t-1}, h_t) = \frac{\lambda_t w_t}{\tilde{\mu}_t} \tag{5}$$

and

$$w_t^i = \begin{cases} \tilde{w}_t & \text{if } w_t^i \text{ is set optimally in } t \\ w_{t-1}^i \mu_{z^*} \pi_{t-1}/\pi_t & \text{otherwise} \end{cases},$$

where $\tilde{w}_t$ denotes the real wage prevailing in the $1-\tilde{\alpha}$ labor markets in which the union can set wages optimally in period t. Because the labor demand curve faced by the union is identical across all labor markets where the wage rate is optimized in period t, and because the cost of supplying labor is the same for all markets, one can assume that wage rates, $\tilde{w}_t$, are identical across all labor markets updating wages in a given period. In any labor market i in which the union could not reoptimize the wage rate in period t, the real wage is given by $w_{t-1}^i \mu_{z^*} \pi_{t-1}/\pi_t$, as nominal wages are fully indexed to past price inflation and long-run productivity growth.

It remains to derive the optimality condition with respect to the wage rate in those markets where the wage rate is set optimally. To this end, it is of use to track the evolution of real wages in a particular labor market that last reoptimized in period t. In general, s periods after the last wage reoptimization in a particular labor market, the real wage prevailing in that market is given by $\tilde{w}_t \prod_{k=1}^{s} \left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right)$. And similarly, s periods after the last wage reoptimization in a particular labor market, the labor demand in that market is given by $\left(\frac{\tilde{w}_t}{w_{t+s}} \prod_{k=1}^{s} \left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right)\right)^{-\tilde{\eta}} h_{t+s}^d$. The part of the household's Lagrangian that is relevant for optimal wage setting is given by

$$\mathcal{L}^w = E_t \sum_{s=0}^{\infty} (\tilde{\alpha}\beta)^s \lambda_{t+s} \left(\frac{\prod_{k=1}^{s}\left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right)}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \left[\tilde{w}_t^{1-\tilde{\eta}} \prod_{k=1}^{s} \left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right) - \tilde{w}_t^{-\tilde{\eta}} \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right].$$

Using equation (5) to eliminate $\tilde{\mu}_{t+s}$, the first-order condition with respect to $\tilde{w}_t$ is given by

$$E_t \sum_{s=0}^{\infty} (\tilde{\alpha}\beta)^s \lambda_{t+s} \left(\frac{\tilde{w}_t \prod_{k=1}^{s}\left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right)}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \left[\frac{(\tilde{\eta}-1)}{\tilde{\eta}} \tilde{w}_t \prod_{k=1}^{s} \left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right) - \frac{-U_h(t+s)}{\lambda_{t+s}}\right] = 0. \tag{6}$$

This expression states that in labor markets in which the wage rate is reoptimized in period t, the real wage is set so as to equate the union's future expected average marginal revenue to the average marginal cost of supplying labor. The union's marginal revenue s periods after its last wage reoptimization is given by $\frac{\tilde{\eta}-1}{\tilde{\eta}} \tilde{w}_t \prod_{k=1}^{s} \left(\frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right)$. Here, $\tilde{\eta}/(\tilde{\eta}-1)$ represents the markup of wages over marginal cost of labor that would prevail in the absence of wage stickiness. In turn, the marginal cost of supplying labor is given by the marginal rate of substitution between consumption and leisure, or $\frac{-U_h(t+s)}{\lambda_{t+s}} = \frac{w_{t+s}}{\tilde{\mu}_{t+s}}$. The variable $\tilde{\mu}_t$ is a wedge between the disutility of labor and the average real wage prevailing in the economy. Thus, $\tilde{\mu}_t$ can be interpreted as the average markup that unions impose on the labor market. The weights used to compute the average difference between marginal revenue and marginal cost are decreasing in time and increasing in the amount of labor supplied to the market.

We wish to write the wage-setting equation in recursive form. To this end, define

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t E_t \sum_{s=0}^{\infty}(\beta\tilde{\alpha})^s \lambda_{t+s}\left(\frac{w_{t+s}}{\tilde{w}_t}\right)^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\mu_{z^*}\pi_{t+k-1}}\right)^{\tilde{\eta}-1}$$

and

$$f_t^2 = -\tilde{w}_t^{-\tilde{\eta}} E_t \sum_{s=0}^{\infty}(\beta\tilde{\alpha})^s w_{t+s}^{\tilde{\eta}} h_{t+s}^d U_h(c_{t+s} - bc_{t+s-1}, h_{t+s}) \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\mu_{z^*}\pi_{t+k-1}}\right)^{\tilde{\eta}}.$$

One can then express f_t^1 and f_t^2 recursively as

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t\lambda_t\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{\mu_{z^*}\pi_t}\right)^{\tilde{\eta}-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1, \tag{7}$$

$$f_t^2 = -U_h(c_t - bc_{t-1}, h_t)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\tilde{w}_{t+1}\pi_{t+1}}{\mu_{z^*}\tilde{w}_t\pi_t}\right)^{\tilde{\eta}} f_{t+1}^2. \tag{8}$$

With these definitions at hand, the first-order condition of the household's problem with respect to $\tilde{w}_t$ collapses to

$$f_t^1 = f_t^2. \tag{9}$$

Aggregation in the Labor Market

Recall that the demand function for labor of type j is given by $h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d$. Taking into account that at any point in time the nominal wage rate in all labor markets in which wages are set optimally in period t is identical and equal to $\tilde{W}_t$, it follows that labor demand in period t in every labor market in which wages were reoptimized in period t, which we denote by ${}_t h_t$, is given by ${}_t h_t = \left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} h_t^d$. This expression, together with the facts that each period a fraction $1-\tilde{\alpha}$ of unions reoptimizes nominal wages and that $h_t = \int_{-\infty}^{t} {}_s h_t ds$, where ${}_s h_t$ denotes the demand for labor in period t in every labor market where the wage rate was last reoptimized in period s, implies that

$$h_t = (1-\tilde{\alpha})h_t^d \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} (\mu_{z^*} \pi_{t+k-s-1})}{W_t} \right)^{-\tilde{\eta}}.$$

Let $\tilde{s}_t \equiv (1-\tilde{\alpha}) \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} (\mu_{z^*} \pi_{t+k-s-1})}{W_t} \right)^{-\tilde{\eta}}$ be a measure of the degree of wage dispersion across different types of labor. Then the above expression can be written as

$$h_t = \tilde{s}_t h_t^d. \tag{10}$$

In turn, the state variable $\tilde{s}_t$ evolves over time according to

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t} \right)^{-\tilde{\eta}} \left(\frac{\pi_t}{(\mu_{z^*} \pi_{t-1})} \right)^{\tilde{\eta}} \tilde{s}_{t-1}. \tag{11}$$

We note that because all job varieties are ex-ante identical, any wage dispersion (i.e., $\tilde{s}_t > 1$) is inefficient. This is reflected in the fact that $\tilde{s}_t$ is bounded below by 1 (see Schmitt-Grohé and Uribe, 2006b, for a proof). Inefficient wage dispersion introduces a wedge that makes the number of hours supplied to the market, h_t, larger than the number of productive units of labor input, h_t^d. In an environment without long-run wage dispersion, up to first order the dead-weight loss created by wage dispersion is nil even in the short run. Formally, a first-order approximation of the law of motion of $\tilde{s}_t$ yields a univariate autoregressive process of the form $\hat{\tilde{s}}_t = \tilde{\alpha} \hat{\tilde{s}}_{t-1}$. This situation emerges, for example, when nominal wages are fully indexed to long-run productivity growth and lagged price inflation, as is the case in the present model. When wages are fully flexible, $\tilde{\alpha} = 0$, wage dispersion disappears, and thus $\tilde{s}_t$ equals 1.

It follows from our definition of the wage index, $W_t \equiv \left[\int_0^1 {W_t^j}^{1-\tilde{\eta}} dj \right]^{1/(1-\tilde{\eta})}$, that the real wage rate w_t evolves over time according to the expression

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha}) \tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{\mu_{z^*} \pi_{t-1}}{\pi_t} \right)^{1-\tilde{\eta}}. \tag{12}$$

Equations (7)-(12) describe equilibrium in the labor market in the SGU model.

The Wage Phillips Curve in the SGU Model

Because most available estimates of the degree of wage stickiness are based on linearized versions of the wage Phillips curve, it is of use to compare this relationship in the SGU and EHL setups. Here we derive a log-linear approximation to the wage Phillips curve in the

SGU model. To facilitate comparison with the linearized wage Phillips curve that obtains in the EHL model (to be derived in the next section), we assume that the period utility function takes the specific form given in equation (2). Let $\pi_t^W \equiv W_t/W_{t-1}$ denote wage inflation in period t. Then log-linearizing equations (7)-(12) around a steady state yields

$$\hat{\pi}_t^W - \hat{\pi}_{t-1} = \beta E_t(\hat{\pi}_{t+1}^W - \hat{\pi}_t) + \gamma \left[\xi \hat{h}_t^d - \hat{w}_t - \hat{\lambda}_t\right], \tag{13}$$

where

$$\gamma = \frac{(1-\tilde{\alpha}\beta)(1-\tilde{\alpha})}{\tilde{\alpha}}. \tag{14}$$

A hat on a variable denotes its log-deviation from the deterministic steady state. Our economy displays long-run stochastic growth in wages and the marginal utility of consumption stemming from variations in the growth rate of neutral and investment specific factor productivity (see Schmitt-Grohé and Uribe, 2006b). For this reason, the variables w_t and λ_t display a stochastic trend in equilibrium. For these two variables, a hat denotes deviations of a stationarity-inducing transformation of the respective variable. The inflation rate π_t appears in the wage Phillips curve because of our maintained assumption that wages are indexed to past inflation.

Existing empirical studies (e.g., Altig et al., 2005 and Levin et al. 2006) estimate the linear wage Phillips curve given above. Such estimates do not provide directly a measure of the degree of wage stickiness $\tilde{\alpha}$. Instead, they deliver an estimate of the coefficient γ on the wage markup. In the SGU model, γ is related to $\tilde{\alpha}$ by equation (14). In the next section, we show that the EHL model implies a wage Phillips curve identical to the one given in (13). The only difference between the linear Phillips curve in the SGU and EHL models lies in the mapping between γ and $\tilde{\alpha}$.

2.2 EHL Wage Stickiness

In this section, we present the household problem under the EHL wage-stickiness assumption. The goal is to derive counterparts to equations (7)-(14). In the EHL model, the economy is assumed to be populated by a large number of differentiated households indexed by j with $j \in [0,1]$. The preferences of household j are defined over per capita consumption, c_t^j, and labor effort, h_t^j, and are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^j - bc_{t-1}^j, h_t^j). \tag{15}$$

To facilitate aggregation, we assume that the period utility index is separable in habit-adjusted consumption and labor effort and isoelastic in labor as in equation (2).

Household j is assumed to be a monopolistic supplier of labor of type j. The household faces a demand for labor given by $\left(W_t^j/W_t\right)^{-\tilde{\eta}} h_t^d$. The household takes W_t and h_t^d as exogenous. Given the wage it charges, household j is assumed to supply enough labor, h_t^j, to satisfy demand. That is,

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d. \tag{16}$$

The household's period-by-period budget constraint is given by:

$$E_t r_{t,t+1} x_{t+1}^j + c_t^j = \frac{x_t^j}{\pi_t} + h_t^d w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}}. \tag{17}$$

Each period t, a random fraction $\tilde{\alpha}$ of households cannot reoptimize their nominal wage. Household j chooses processes for c_t^j, h_t^j, x_{t+1}^j, and w_t^j, so as to maximize the utility function (15) subject to (16) and (17), the wage stickiness friction, and a no-Ponzi-game constraint, taking as given the processes w_t, h_t^d, $r_{t,t+1}$, and π_t, and the initial condition x_0^j. The household's optimal plan must satisfy constraints (16) and (17). In addition, letting $\beta^t \lambda_t^j w_t/\tilde{\mu}_t^j$ and $\beta^t \lambda_t^j$ denote Lagrange multipliers associated with constraints (16) and (17), respectively, the Lagrangian associated with household j's optimization problem is

$$\begin{aligned}
\mathcal{L}^j \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ U(c_t^j - b c_{t-1}^j, h_t^j) + \lambda_t^j \left[w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d - c_t^j - r_{t,t+1} x_{t+1}^j + \frac{x_t^j}{\pi_t} \right] \\
& + \frac{\lambda_t^j w_t}{\tilde{\mu}_t^j} \left[h_t^j - \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d \right] \Bigg\}.
\end{aligned}$$

The first-order conditions with respect to h_t^j and w_t^j, in that order, are given by

$$-U_h(h_t^j) = \frac{\lambda_t^j w_t}{\tilde{\mu}_t^j} \tag{18}$$

and

$$w_t^j = \begin{cases} \tilde{w}_t^j & \text{if } w_t^j \text{ is set optimally in } t \\ w_{t-1}^j \mu_{z^*} \pi_{t-1}/\pi_t & \text{otherwise} \end{cases},$$

where $\tilde{w}_t^j$ denotes the real wage set by household j if it is free to reoptimize this variable in period t. The household's first-order condition with respect to $\tilde{w}_t^j$ is given by

$$0 = E_t \sum_{s=0}^{\infty} (\tilde{\alpha}\beta)^s \lambda_{t+s}^j \left(\frac{\tilde{w}_t^j \prod_{k=1}^{s} \left(\frac{\mu_{z^*} \pi_{t+k-1}}{\pi_{t+k}} \right)}{w_{t+s}} \right)^{-\tilde{\eta}} h_{t+s}^d \left[\frac{(\tilde{\eta}-1)}{\tilde{\eta}} \tilde{w}_t^j \prod_{k=1}^{s} \left(\frac{\mu_{z^*} \pi_{t+k-1}}{\pi_{t+k}} \right) - \frac{-U_h(h_{t+s}^j)}{\lambda_{t+s}^j} \right]. \tag{19}$$

Following EHL (2000), we assume that households can insure against the risk of not being able to reoptimize the nominal wage. This assumption implies that the marginal utility of income, λ_t^j, is indeed the same across all households $j \in [0, 1]$. From this result and the fact that every household faces the same labor demand function, it follows that all households reoptimizing wages in a given period will choose to set the same nominal wage. Therefore, we drop the superscript j from the variables λ_t^j and $\tilde{w}_t^j$. Then, the only difference between the optimality condition (19) and its counterpart in the SGU model, equation (6), is that the argument of the marginal disutility of labor is household-specific in the EHL model (and given by h_{t+s}^j) whereas in the SGU model the argument of the marginal disutility of labor is aggregate per capita labor effort (h_{t+s}). It follows that in the special case of preferences that are linear in hours worked ($\xi = 0$ in equation (2)), the SGU and EHL models deliver identical expressions for the optimality condition with respect to the wage rate. This result is intuitive: if agents are risk-neutral with respect to hours worked, it is irrelevant whether households pool employment uncertainty (as in the SGU model) or not (as in the EHL model).

At this point existing models that use wage stickiness à la EHL (2000) proceed to log-linearizing the optimality condition for wages given in equation (19), yielding the well-known linear wage Phillips curve. Because our eventual goal is to compute Ramsey dynamics and to perform welfare evaluations, we are interested in deriving the true nonlinear wage Phillips curve. To this end, we derive a recursive representation of equilibrium condition (19). We explain this derivation in some detail, as it is a novel feature of the present paper. Let f_t^1 be defined as in equation (7). As before, let the number of hours worked in period $t+s$ by a household who received a wage change signal for the last time in period t be denoted by ${}_t h_{t+s}$. Then define f_t^2 as

$$f_t^2 = -\tilde{w}_t^{-\tilde{\eta}} E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s w_{t+s}^{\tilde{\eta}} h_{t+s}^d U_h({}_t h_{t+s}) \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\mu_{z^*} \pi_{t+k-1}} \right)^{\tilde{\eta}}.$$

With these definitions at hand, the first-order condition with respect to $\tilde{w}_t$, given in equation (19), can be written as $f_t^1 = f_t^2$. To express f_t^2 recursively proceed as follows. Note that

${}_t h_{t+s}$ is given by

$$
\begin{aligned}
{}_t h_{t+s} &= \left(\frac{W^j_{t+s}}{W_{t+s}}\right)^{-\tilde{\eta}} h^d_{t+s} \\
&= \left(\frac{\tilde{w}_t}{w_{t+s}} \prod_{k=1}^{s} \frac{\mu_{z^*}\pi_{t+k-1}}{\pi_{t+k}}\right)^{-\tilde{\eta}} h^d_{t+s}.
\end{aligned}
$$

Clearly, the variable ${}_t h_{t+1+s}$ is related to ${}_{t+1} h_{t+1+s}$ by the expression,

$$
{}_t h_{t+1+s} = \left(\frac{\tilde{w}_t \mu_{z^*}\pi_t}{\tilde{w}_{t+1}\pi_{t+1}}\right)^{-\tilde{\eta}} {}_{t+1} h_{t+1+s}.
$$

Now we resort to the assumption made earlier that U_h is homogenous of degree ξ in h (see equation (2)). Then we have that

$$
U_h({}_t h_{t+1+s}) = \left(\frac{\tilde{w}_t \mu_{z^*}\pi_t}{\tilde{w}_{t+1}\pi_{t+1}}\right)^{-\xi\tilde{\eta}} U_h\left({}_{t+1} h_{t+1+s}\right).
$$

This expression allows us to rewrite f^2_t recursively as follows:

$$
f^2_t = -U_h({}_t h_t)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h^d_t + \tilde{\alpha}\beta E_t \left(\frac{\tilde{w}_{t+1}\pi_{t+1}}{\mu_{z^*}\tilde{w}_t\pi_t}\right)^{\tilde{\eta}(1+\xi)} f^2_{t+1}, \tag{20}
$$

where ${}_t h_t$ is given by

$$
{}_t h_t = \left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} h^d_t. \tag{21}
$$

Labor market equilibrium in the EHL model is given by equation (7), (9), (12), (20), and (21).

3 Can Existing Econometric Evidence Distinguish Between the SGU and the EHL Models?

To derive a log-linear approximation to the wage Phillips curve in the EHL model, we continue to assume that the period utility function takes the specific form given in equation (2). Log-linearizing equations (7), (9), (12), (20), and (21) yields equation (13) with

$$
\gamma = \frac{(1-\tilde{\alpha}\beta)(1-\tilde{\alpha})}{\tilde{\alpha}} \frac{1}{1+\tilde{\eta}\xi}. \tag{22}
$$

Existing econometric studies that estimate the degree of wage stickiness using aggregate data are based on linear models. Studies that are limited to estimating the wage Phillips curve deliver an estimate of the coefficient γ in equation (13). Because both the EHL and SGU models give rise to a linearized Phillips curve of the form given in equation (13), estimates of γ are not sufficient to tell the SGU and EHL models apart.

Furthermore, even if one were to estimate the complete set of linearized equilibrium conditions using aggregate data (as, for instance, in Altig et al. Levin et al., and others), the SGU and EHL models would continue to be observationally equivalent. This is because the parameter $\tilde{\alpha}$ appears only in the coefficient γ and because one can show that up to first order, all equations of the general equilibrium model are identical in the SGU and EHL models (except, of course, for the mapping between the coefficient γ and $\tilde{\alpha}$).

Given an estimate of γ, one will draw different conclusions about the size of $\tilde{\alpha}$ depending on whether one assumes that the model displays EHL or SGU-type wage stickiness. Let $\tilde{\alpha}^{SGU}$ denote the degree of wage stickiness that one would infer from given values for γ, ξ, β, and $\tilde{\eta}$ in the SGU model of wage stickiness and similarly let $\tilde{\alpha}^{EHL}$ be the inferred degree of wage stickiness in the EHL model. It is clear from equations (14) and (22) that $\tilde{\alpha}^{SGU}$ and $\tilde{\alpha}^{EHL}$ are linked by the following implicit function:

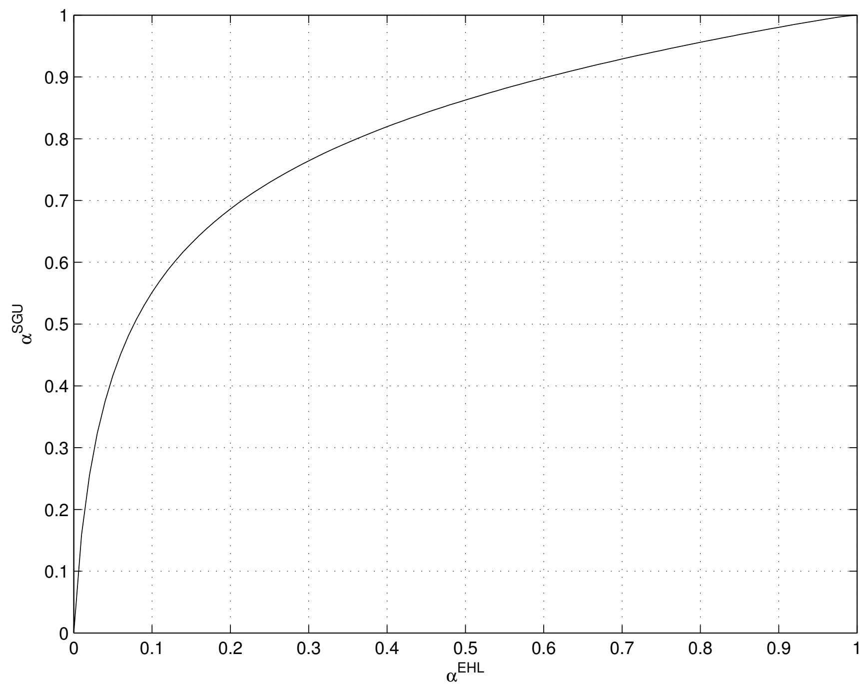
$$\frac{(1-\tilde{\alpha}^{SGU}\beta)(1-\tilde{\alpha}^{SGU})}{\tilde{\alpha}^{SGU}} = \frac{(1-\tilde{\alpha}^{EHL}\beta)(1-\tilde{\alpha}^{EHL})}{\tilde{\alpha}^{EHL}} \frac{1}{1+\tilde{\eta}\xi}. \tag{23}$$

Clearly, for a given value of γ the implied degree of wage stickiness is always higher in the SGU than in the EHL model, or

$$\tilde{\alpha}^{SGU} > \tilde{\alpha}^{EHL}.$$

Figure 1 displays the graph of the implicit function linking $\tilde{\alpha}^{SGU}$ to $\tilde{\alpha}^{EHL}$ given in equation (23). In constructing the graph, we draw from the calibration used in Altig et al. (2005) and set $\beta = 1.03^{-1/4}$, $\xi = 1$, and $\tilde{\eta} = 21$. Consider, for example, a value of 0.69 for $\tilde{\alpha}^{EHL}$, which is the degree of wage stickiness estimated by Altig et al. in their high-markup case under the assumption of EHL-type wage stickiness. The corresponding value for $\tilde{\alpha}^{SGU}$ is 0.9261. Thus, under the Altig et al. estimation of γ the SGU model implies that nominal wages are reoptimized on average every 13 quarters, whereas the EHL model implies that they are reoptimized every 3 quarters.

Figure 1: The Relation Between $\tilde{\alpha}^{SGU}$ and $\tilde{\alpha}^{EHL}$



The figure displays the graph of the implicit function given in equation (23) for $\beta = 1.03^{-1/4}$, $\xi = 1$, and $\tilde{\eta} = 21$.

4 Ramsey Policy in the SGU and EHL Models

The Ramsey optimization problem consists in maximizing a weighted average of lifetime utility of all households in the economy subject to the set of equations defining a competitive equilibrium. Thus far, we have established that for a given monetary regime and up to first order the SGU and EHL models of wage stickiness result in identical equilibrium dynamics provided that in both models $\tilde{\alpha}$ is chosen appropriately. This is because the linearized equilibrium conditions of both models are the same. Furthermore, as we will show shortly, the objective function of the Ramsey planner is the same in the SGU and EHL models. However, it does not follow directly from these results that Ramsey dynamics must be the same in the SGU and EHL models up to first order. The reason is that linear approximations to the equilibrium conditions of the Ramsey problem involve higher than first-order derivatives of the competitive equilibrium conditions. We therefore resort to a numerical analysis to compare Ramsey dynamics under the SGU and EHL models.

4.1 The Ramsey Planner's Objective Function

The SGU economy is populated by a representative household. As a consequence, the Ramsey planner's objective function coincides with that of the representative household. Recalling that $h_t = \tilde{s}_t h_t^d$ (see equation (10)) and imposing the specific functional form for the period utility function given in equation (2), we can write the Ramsey planner's objective function as

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j} - bc_{t+j-1}) - \frac{(\tilde{s}_{t+j} h_{t+j}^d)^{1+\xi}}{1+\xi} \right], \tag{24}$$

with

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{w_{t-1}\mu_{z^*}\pi_{t-1}}{w_t \pi_t} \right)^{-\tilde{\eta}} \tilde{s}_{t-1}.$$

In the EHL economy, households are heterogeneous. In equilibrium consumption is identical across households but labor supply varies cross sectionally. We assume that the Ramsey planner cares about all households equally. Therefore, the planner's objective function is given by

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j} - bc_{t+j-1}) - \int_0^1 \frac{(h_{t+j}^i)^{1+\xi}}{1+\xi} di \right].$$

Recall that because households must satisfy labor demand at the posted wage, we have that $h_t^i = (w_t^i/w_t)^{-\tilde{\eta}} h_t^d$ (see equation (16)). Let $\tilde{s}_t$ be defined by $\tilde{s}_t^{1+\xi} = \int_0^1 (w_t^i/w_t)^{-\tilde{\eta}(1+\xi)} di$. It follows that in the EHL model the Ramsey planner's objective function is given by equation (24), which is identical to its counterpart in the SGU model. However, the evolution of $\tilde{s}_t$ differs in the two models. In effect, taking into account that in the EHL model only a fraction $1 - \tilde{\alpha}$ of households are allowed to reoptimize wages in any given period and that every reoptimizing household charges the same wage, we can express the above expression for $\tilde{s}_t$ recursively as follows:

$$\tilde{s}_t^{1+\xi} = (1 - \tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\eta}(1+\xi)} + \tilde{\alpha} \left(\frac{\mu_{z^*} \pi_{t-1} w_{t-1}}{\pi_t w_t} \right)^{-\tilde{\eta}(1+\xi)} \tilde{s}_{t-1}^{(1+\xi)}.$$

The Ramsey problem in the SGU model consists in maximizing equation (24) subject to the equilibrium conditions given in sections A.1 and A.2 of the appendix. The Ramsey problem in the EHL model consists in maximizing equation (24) subject to the equilibrium conditions given in sections A.1 and A.3 of the appendix.

4.2 The Optimal Degree of Inflation Stabilization

We compute Ramsey dynamics by approximating the Ramsey equilibrium conditions up to first order. We calibrate the SGU and EHL models to the U.S. economy following Schmitt-Grohé and Uribe (2006b). Table 1 presents the values of the deep structural parameters implied by our calibration strategy. The only structural parameter that takes a different value in the SGU and EHL models is the degree of wage stickiness $\tilde{\alpha}$. As discussed earlier, there is a one-to-one relationship between $\tilde{\alpha}^{SGU}$ and $\tilde{\alpha}^{EHL}$ that makes both models consistent with the empirical estimates of the wage Phillips curve. Altig et al. estimate $\tilde{\alpha}^{EHL}$ to be 0.69. The corresponding value for $\tilde{\alpha}^{SGU}$ is 0.9261. We adopt these values in our calibration of $\tilde{\alpha}$ in the EHL and SGU models, respectively.

Table 2 displays the standard deviation, first-order autocorrelation, and correlation with output growth of price inflation, wage inflation, the nominal interest rate and the growth rates of output, consumption, and investment. The table shows that the SGU and EHL models imply identical second moments under the Ramsey policy. This result suggests that differences in the characteristics of optimal monetary policy reported in studies using SGU- or EHL-type wage stickiness must be attributed not to the way wage stickiness is modeled, but rather either to other differences in the theoretical environments employed across studies, or to calibrations of the parameter $\tilde{\alpha}$ that do not satisfy the one-to-one mapping linking these two parameters given by equation (23).

Table 2 shows that under the Ramsey-optimal policy the volatility of inflation is low at 0.6 percentage points at an annual rate. We take this number to suggest that inflation stability should be a central goal of optimal monetary policy. Furthermore, the results shown in the table suggest that in the Ramsey-optimal competitive equilibrium price inflation is

Table 1: Structural Parameters

Parameter	Value	Description
β	$1.03^{1/4}$	Subjective discount factor (quarterly)
θ	0.36	Share of capital in value added
ψ	0.5317	Fixed cost parameter
δ	0.025	Depreciation rate (quarterly)
ν	0.6011	Fraction of wage bill subject to a CIA constraint
η	6	Price-elasticity of demand for a specific good variety
$\tilde{\eta}$	21	Wage-elasticity of demand for a specific labor variety
α	0.8	Fraction of firms not setting prices optimally each quarter
$\tilde{\alpha}^{SGU}$	0.9261	Fraction of labor markets not setting wages optimally in SGU model
$\tilde{\alpha}^{EHL}$	0.69	Fraction of labor markets not setting wages optimally in EHL model
b	0.69	Degree of habit persistence
ϕ_1	0.0459	Transaction cost parameter
ϕ_2	0.1257	Transaction cost parameter
ξ	1	Preference parameter
κ	2.79	Parameter governing investment adjustment costs
γ_1	0.0412	Parameter of capacity-utilization cost function
γ_2	0.0601	Parameter of capacity-utilization cost function
χ	0	Degree of price indexation
μ_Υ	1.0042	Quarterly growth rate of investment-specific technological change
σ_{μ_Υ}	0.0031	Std. dev. of the innovation to the investment-specific technology shock
ρ_{μ_Υ}	0.20	Serial correlation of the log of the investment-specific technology shock
μ_z	1.00213	Quarterly growth rate of neutral technology shock
σ_{μ_z}	0.0007	Std. dev. of the innovation to the neutral technology shock
ρ_{μ_z}	0.89	Serial correlation of the log of the neutral technology shock
$\bar{g}$	0.4549	Steady-state value of government consumption (quarterly)
σ_{ϵ^g}	0.008	Std. dev. of the innovation to log of gov. consumption
ρ_g	0.9	Serial correlation of the log of government spending

Note. All parameter values are as in Schmitt-Grohé and Uribe (2006b) except for those assigned to ψ and $\bar{g}$, which change because the number of hours worked in the steady state in both models is different due to different specifications for the period utility function.

Table 2: Second Moments under Ramsey Optimal Stabilization Policy

Variable	SGU Model	EHL Model
	Standard Deviation	
Nominal Interest Rate	0.5	0.5
Price Inflation	0.6	0.6
Wage Inflation	0.9	0.9
Output Growth	0.9	0.9
Consumption Growth	0.5	0.5
Investment Growth	1.8	1.8
	Serial Correlation	
Nominal Interest Rate	0.8	0.8
Price Inflation	0.9	0.9
Wage Inflation	0.4	0.4
Output Growth	0.5	0.5
Consumption Growth	0.9	0.9
Investment Growth	0.7	0.7
	Correlation with Output Growth	
Nominal Interest Rate	-0.3	-0.3
Price Inflation	-0.5	-0.5
Wage Inflation	0.2	0.2
Output Growth	1	1
Consumption Growth	0.4	0.4
Investment Growth	0.6	0.6
	Welfare	
Unconditional (EV_0)	-183.1090	-183.0913
Conditional (V_0)	-183.0456	-183.0508

Standard deviations are measured in percentage points per year. Conditional welfare is computed under the assumption that the initial state is the deterministic Ramsey steady state. In computing (conditional and unconditional) welfare levels the welfare criterion was appropriately transformed to induce stationarity.

somewhat smoother than wage inflation. Finally, a remarkable feature of monetary policy is that a significant degree of inflation stability is brought about with little volatility in the policy instrument. In effect, the Ramsey-optimal standard deviation of the nominal interest rate is only 0.5 percentage points at an annualized rate.

5 Optimal Operational Interest-Rate Rules

To be added.

6 Second-Order Accurate Welfare Functions

The facts that the SGU and EHL wage-stickiness formulations deliver identical Ramsey-optimal dynamics up to first order, identical optimal operational rules, and identical equilibrium conditions up to first order suggest that the welfare function associated with both models must be similar up to second order. This impression is supported by the welfare levels reported at the bottom of table 2. There, we show that up to second order the Ramsey policy induces an unconditional level of welfare of -183.1090 under the SGU specification and of -183.0913 under the EHL specification.

In this section, we establish analytically the similarity between the welfare criteria in the SGU and EHL models. To this end, we consider a much simpler economic environment than the one studied thus far. In particular, we consider the sticky-price sticky-wage model of Erceg et al. (2000). This model features no habit formation, no capital accumulation, no money, no growth, and no variable capacity utilization. Furthermore, the Erceg et al. model assumes the existence of subsidies to production and labor supply aimed at neutralizing the distortions arising from the presence of monopolistic competition in product and factor markets.

We find that up to second order, the difference in the unconditional expectation of the period utility function under SGU and EHL wage stickiness, which we denote by $E[U_t^{SGU} - U_t^{EHL}]$, is given by:[1]

$$E[U_t^{SGU} - U_t^{EHL}] = \left[\frac{1-\beta\tilde{\alpha}^{SGU}}{1-\tilde{\alpha}^{SGU}} - \frac{1-\beta\tilde{\alpha}^{EHL}}{1-\tilde{\alpha}^{EHL}}\right]\frac{hU_h(h)\tilde{\eta}}{2\gamma}\mathrm{Var}(\ln(\pi^W)).$$

To obtain this expression, we use the fact that up to second order both the SGU and EHL models give rise to the same volatility of wage inflation, $\mathrm{Var}(\ln(\pi^W))$. Also, the parameter

[1] The derivation of this expression is available from the authors upon request.

γ is the wage-markup coefficient in the linearized wage Phillips curve. Given an estimate of the wage Phillips curve, γ takes the same value under SGU and EHL wage stickiness.

It is clear from the above expression that the unconditional expectation of the difference in the period utility functions under the SGU and EHL wage-stickiness specifications vanishes as the discount factor approaches unity. Formally,[2]

$$\lim_{\beta \to 1} E[U_t^{SGU} - U_t^{EHL}] = 0.$$

Clearly, because, as shown earlier, $\alpha^{SGU} > \alpha^{EHL}$, and recalling that $U_h < 0$, it follows that welfare under the SGU formulation is always slightly smaller than under the EHL specification, or $EU_t^{SGU} < EU_t^{EHL}$. This result appears to hold for the more complex model studied earlier in the paper, as suggested by table 2.

7 Conclusion

To be added.

[2]Of course, this statement is true only insofar as $U_h h \text{Var}(\ln(\pi^W))$ remains bounded as the discount factor approaches unity.

Appendix

This appendix lists the complete set of equilibrium conditions in the SGU and EHL models. Suitable stationarity-inducing transformations were applied to variables containing a trend in equilibrium. A derivation of the equilibrium conditions common to both models can be found in Schmitt-Grohé and Uribe (2006b). In the equations below, the parameter $\tilde{\chi}$ measures the degree of nominal wage indexation to lagged price inflation and long-run productivity growth. To obtain the case of full indexation analyzed in the main body of the paper, set $\tilde{\chi} = 1$.

A.1. Equilibrium Conditions Common to the SGU and EHL Model

$$K_{t+1} = (1-\delta)\frac{K_t}{\mu_{I,t}} + I_t\left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2\right]$$

$$\frac{1}{(C_t - \mu_{z^*,t}^{-1} bC_{t-1})} - b\beta E_t \frac{1}{(\mu_{z^*,t+1}C_{t+1} - bC_t)} = \Lambda_t[1 + \ell(v_t) + v_t\ell'(v_t)]$$

$$\Lambda_t Q_t = E_t \frac{\beta\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}\left[R^k_{t+1}u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right]$$

$$\Lambda_t = \Lambda_t Q_t\left[1 - \frac{\kappa}{2}\left(\frac{\mu_{I,t}I_t}{I_{t-1}} - \mu_I\right)^2 - \left(\frac{\mu_{I,t}I_t}{I_{t-1}}\right)\kappa\left(\frac{\mu_{I,t}I_t}{I_{t-1}} - \mu_I\right)\right]$$
$$+\beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}Q_{t+1}\left(\mu_{I,t+1}\frac{I_{t+1}}{I_t}\right)^2 \kappa\left(\mu_{I,t+1}\frac{I_{t+1}}{I_t} - \mu_I\right)$$

$$v_t^2\ell'(v_t) = 1 - \beta E_t \mu_{\Lambda,t+1}\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1}{\pi_{t+1}}$$

$$R^k_t = a'(u_t)$$

$$f^1_t = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{W}_t\Lambda_t\left(\frac{W_t}{\tilde{W}_t}\right)^{\tilde{\eta}} h^d_t + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\mu_{z^*,t+1}\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{\tilde{\eta}-1}\mu_{\Lambda,t+1}\mu_{z^*,t+1}f^1_{t+1},$$

$$f^1_t = f^2_t$$

$$\Lambda_t = \beta R_t E_t \mu_{\Lambda,t+1}\frac{\Lambda_{t+1}}{\pi_{t+1}}$$

$$Y_t = C_t[1 + \ell(v_t)] + G_t + [I_t + a(u_t)\mu_{I,t}^{-1}K_t]$$

$$X_t^1 = Y_t \mathrm{mc}_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\mu_{\Lambda,t+1}\Lambda_{t+1}}{\Lambda_t}(\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta}\mu_{z^*,t+1}X_{t+1}^1$$

$$X_t^2 = Y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\mu_{\Lambda,t+1}\Lambda_{t+1}}{\Lambda_t}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta}\mu_{z^*,t+1}X_{t+1}^2$$

$$\eta X_t^1 = (\eta - 1)X_t^2$$

$$1 = \alpha \pi_t^{\eta-1}\pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}$$

$$(u_t \mu_{I,t}^{-1} K_t)^{\theta} h_t^{d\,1-\theta} - \psi = \{[1 + \ell(v_t)]C_t + G_t + [I_t + a(u_t)K_t/\mu_{I,t}]\}\, s_t$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}$$

$$\mathrm{mc}_t(1-\theta)(u_t K_t/\mu_{I,t})^{\theta} h_t^{d\,-\theta} = W_t\left[1 + \zeta\frac{R_t - 1}{R_t}\right]$$

$$\mathrm{mc}_t \theta (u_t K_t/\mu_{I,t})^{\theta-1} h_t^{d\,1-\theta} = R_t^k$$

$$W_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{W}_t^{1-\tilde{\eta}} + \tilde{\alpha}(W_{t-1}/\mu_{z^*,t})^{1-\tilde{\eta}}\left(\frac{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}$$

$$\mu_{I,t} = \mu_{\Upsilon,t}\mu_{z^*,t}$$

$$\mu_{\Lambda,t} = \mu_{z^*,t}^{-1}$$

$$\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{\theta}{1-\theta}}\mu_{z,t}$$

$$\mu_{z,t} \equiv \frac{z_t}{z_{t-1}} \quad \text{and} \quad \hat{\mu}_{z,t} \equiv \ln(\mu_{z,t}/\mu_z))$$

$$\hat{\mu}_{z,t} = \rho_{\mu_z}\hat{\mu}_{z,t-1} + \epsilon_{\mu_z,t} \quad \text{with } \epsilon_{\mu_z,t} \sim (0, \sigma_{\mu_z}^2)$$

$$\mu_{\Upsilon,t} \equiv \Upsilon_t/\Upsilon_{t-1} \quad \text{and} \quad \hat{\mu}_{\Upsilon,t} \equiv \ln(\mu_{\Upsilon,t}/\mu_{\Upsilon})$$

$$\hat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon}\hat{\mu}_{\Upsilon,t-1} + \epsilon_{\mu_\Upsilon,t} \quad \text{with } \epsilon_{\mu_\Upsilon,t} \sim (0, \sigma_{\mu_\Upsilon}^2)$$

$$\ln\left(\frac{G_t}{G}\right) = \rho_g \ln\left(\frac{G_{t-1}}{G}\right) + \epsilon_{g,t}$$

A.2. Equilibrium Conditions Specific to the SGU Model

$$f_t^2 = \tilde{s}_t^{\xi}\left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} h_t^{d(1+\xi)} + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\mu_{z^*,t+1}\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{\tilde{\eta}} f_{t+1}^2$$

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{W_{t-1}(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t \mu_{z^*,t} W_t}\right)^{-\tilde{\eta}}\tilde{s}_{t-1}$$

A.3. Equilibrium Conditions Specific to the EHL Model

$$f_t^2 = \left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}(1+\xi)} h_t^{d(1+\xi)} + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}(1+\xi)} \left(\frac{\mu_{z^*,t+1}\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{\tilde{\eta}(1+\xi)} f_{t+1}^2$$

$$\tilde{s}_t^{1+\xi} = (1-\tilde{\alpha})\left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}(1+\xi)} + \tilde{\alpha}\left(\frac{W_{t-1}(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t\mu_{z^*,t}W_t}\right)^{-\tilde{\eta}(1+\xi)} \tilde{s}_{t-1}^{(1+\xi)}$$

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