Multiple Equilibria in Open Economy Models with Collateral Constraints: Overborrowing Revisited

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Starting Point

• Open-economy models with collateral constraints typically display a pecuniary externality.
• The externality originates in the fact that the price of pledgable objects is endogenous to the model but exogenous to individual agents.
• A result stressed in the literature is that these economies overborrow, that is, they borrow more than they would if agents internalized the pecuniary externality.
• Open-economy models with collateral constraints are prone to multiple equilibria. This issue has been little explored (indeed avoided) in the related literature.
• Exceptions are Mendoza (2005) and Jeanne and Korinek (2010) who present heuristic analysis of the problem.
This Paper

- Characterizes multiple equilibria in models with flow collateral constraints.

- Shows that open economies with flow collateral constraints have equilibria featuring self-fulfilling financial crises.

- The possibility of multiple equilibria causes the economy to under-borrow in equilibrium.

- **Intuition:** Agents are aware that the economy is prone to self-fulfilling crises, so they engage in extra precautionary savings, driving down the aggregate level of debt.
A Model with a Flow Collateral Constraint

Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

subject to

$$c_t = \left[ ac_t^{T^{1-1/\xi}} + (1 - a)c_t^{N^{1-1/\xi}} \right]^{1/(1-1/\xi)}$$

$$c_t^T + p_t c_t^N + d_t = y^T + p_t y^N + \frac{d_{t+1}}{1 + r}$$

$$d_{t+1} \leq \kappa(y^T + p_t y^N)$$

where $c_t =$ consumption; $c_t^T, c_t^N =$ consumption of tradables, nontradables; $d_t =$ debt due in $t$; $d_{t+1} =$ debt assumed in $t$ and due in $t + 1$; $y^T, y^N =$ endowments of tradables, nontradables; $p_t =$ relative price of nontradables; $r =$ interest rate.
Observations

(1) The last equation is the flow collateral constraint (CC). It says that the amount of debt issued in period $t$, $d_{t+1}$, cannot exceed a fraction $\kappa$ of income, $y^T + p_t y^N$.

(2) From the individual agent’s point of view, the CC is well behaved: the larger is $d_{t+1}$, the closer he gets to hitting the collateral constraint. This is because he takes as exogenous all of the objects on the RHS of the collateral constraint (in particular $p_t$).

(3) Also, from the perspective of the individual agent, the collateral constraint defines a convex set of feasible debt levels: if $d'$ and $d''$ satisfy the collateral constraint, then so does the debt level $\alpha d' + (1 - \alpha) d''$, for any $\alpha \in [0, 1]$.

(4) As we will see shortly, (2) and (3) do not hold from an aggregate perspective.
Three Equilibrium Conditions of Interest

\[ d_{t+1} \leq \kappa (y^T + p_t y^N) \]

\[ p_t = \frac{1 - a}{a} \left( \frac{c_t^T}{y^N} \right)^{1/\xi} \]

\[ c_t^T + d_t = y^T + \frac{d_{t+1}}{1 + r} \]

These three conditions give rise to the following equilibrium collateral constraint

\[ d_{t+1} \leq \kappa \left[ y^T + \left( \frac{1-a}{a} \right) \left( y^T + \frac{d_{t+1}}{1+r} - d_t \right)^{1/\xi} y^N^{1-1/\xi} \right] \]
Observations

(1) $d_{t+1}$ appears on both sides of the equilibrium collateral constraint.

(2) Because $\xi > 0$, the equilibrium value of collateral increases with the level of debt, giving rise to the possibility that the higher is $d_{t+1}$ the less tight is the collateral constraint.

(3) Moreover, collateral (i.e., the RHS of the collateral constraint) is in general nonlinear in $d_{t+1}$, giving rise to the possibility that the set of debt levels that satisfy the equilibrium collateral constraint is nonconvex, that is, if $d'$ and $d''$ satisfy the equilibrium collateral constraint, then $\alpha d' + (1 - \alpha)d''$ may not for some $\alpha \in (0, 1)$. 
Some Simplifying Assumptions

(1) $\sigma = 1/\xi = 2$.

(2) $\beta(1 + r) = 1$.

(3) $a = 0.5$.

(4) $y^N = 1$. A normalization.

The equilibrium collateral constraint then becomes

$$d_{t+1} \leq \kappa \left[ y^T + \left( y^T + \frac{d_{t+1}}{1+r} - d_t \right)^2 \right]$$
The Steady-State Collateral Constraint

\[ d \leq \kappa \left[ y^T + \left( y^T - r d / (1 + r) \right)^2 \right] \]
Observations

(1) the expression under the power of 2 is steady state consumption, \( y^T - rd/(1 + r) \).

(3) The LR collateral constraint achieves a minimum when steady-state consumption is 0, that is, at the natural debt limit.

(4) This means that:
(a) for all relevant values of debt (i.e., all values below the natural debt limit), the LR collateral constraint is well behaved, that is, the larger is debt the tighter it gets.
(b) The LR collateral constraint binds at \( \tilde{d} \), and is violated at any level of debt larger than \( \tilde{d} \). No steady state equilibrium is possible to the right of \( \tilde{d} \).
(c) For any initial debt \( d_0 < \tilde{d} \), the constant debt path \( d_t = d_0 \) for all \( t \) satisfies the resource constraint with a positive and constant level of consumption, \( c_t = c_0 > 0 \) for all \( t \), and ensures that the collateral constraint never binds. Is this an equilibrium? It only remains to show that the Euler equations holds.
The Euler Equation in the Steady-State Equilibrium

When the collateral constraint does not bind, the Euler equation is

\[
\left( \frac{c_{t+1}}{c_t^T} \right)^2 = \beta(1 + r)
\]

Recalling the assumption \( \beta(1 + r) = 1 \), we have that consumption must be constant over time, which is precisely what happens in the steady state. So the Euler equation is satisfied.

We have established that if \( d_0 \leq \tilde{d} \), then \( d_t = d_0 \) and \( c_{t+1}^T = c_t^T \) for all \( t \geq 0 \) is an equilibrium.
The Equilibrium Collateral Constraint and Multiple Equilibria

\[ d \leq \kappa \left[ y^T + \frac{d}{1+r} - d_0 \right]^2 \]
Observations

(1) In the figure, \( d_0 < \bar{d} \). Thus, from the previous analysis we have that point \( A \) is an equilibrium featuring \( d_t = d_0 \) for all \( t \geq 0 \).

(2) The slope of the short-run (SR) CC is proportional to \( c^T_0 \). So an equilibrium must be on an upward sloping range of the SR CC. Thus, point \( C \) is not an equilibrium.

(3) Consider point \( B \). There, consumption is positive in period 0, because the SR CC is upward sloping. Also, at \( B \) the SR CC holds with equality (binding), since it is on the 45-degree line. But we must still check (a) that at \( B \) the Euler equation holds in period 0, and (b) that all equilibrium conditions are satisfied after period 1.

We wish to consider an equilibrium in which the economy jumps from \( A \) to \( B \) in period 0 and then stays at \( B \) forever thereafter.

Note first that deleveraging from \( A \) to \( B \) and then staying at \( B \) forever requires that \( c^T_0 < c^T_1 \). Knowing this makes it easy to that at \( B \) the Euler equation holds in period 0, as we show next.
The Euler Equation at Point B

\[
\left( \frac{c_T^1}{c_T^0} \right)^2 = \frac{1}{1 - \mu_0(1 + r)}
\]

where \( \mu_0 \) is the Lagrange multiplier on the collateral constraint in period 0.
Are Points Between $A$ and $B$ Equilibria?

Take another look at the figure

At any such point: (a) The collateral constrain is satisfied (indeed it is not binding); (b) the resource constraint is satisfied with positive consumption.

However, because at any such point $c_T^0 < c_T^1$, the Euler equation is satisfied only if $\mu_t > 0$. But this violates the slackness condition, which requires $\mu_0 = 0$, because the collateral constraint doesn’t bind in period 0.
A Unique Equilibrium

\[ \kappa \left[ y^T + \left( \frac{1-a}{a} \right) (y^T - \frac{rd}{1+r}) \right] \]

\[ \kappa \left[ y^T + \left( \frac{1-a}{a} \right) (y^T + \frac{d}{1+r} - d_0) \right] \]
Three Equilibria

Welfare ranking: $A > B > C$
Underborrowing

The competitive equilibrium at point $A$ is the first-best allocation.

Therefore, if agents coordinate on equilibrium $A$, there is neither overborrowing nor underborrowing.

But if agents coordinate on equilibria $B$ or $C$, the economy suffers underborrowing.
Implementation Through Capital Control Policy

Suppose that the government imposes a proportional tax on debt, $\tau_t$. The budget constraint of the household becomes

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + \frac{1 - \tau_t}{1 + r} d_{t+1}.$$

(Capital control taxes can be rebated lump-sum or through income-based transfers.)
Implementation Versus Ramsey-Optimal Taxation

The Ramsey optimal tax rate in this economy is $\tau_t = 0$ at all times. This is because the unregulated competitive equilibrium $A$ delivers the first-best allocation.

However, announcing the policy $\tau_t = 0$ for all $t$ does not guarantee that the Ramsey optimal equilibrium will emerge. Indeed, this tax policy also supports the deleveraging equilibrium $B$.

What capital control policy can induce the Ramsey-optimal equilibrium? Consider capital-control policy rules of the form

$$\tau_t = \tau(d_{t+1}, d_t)$$

satisfying $\tau(d, d) = 0$ and $\tau' < 0$. This policy is consistent with the Ramsey equilibrium, since under the Ramsey equilibrium $d_{t+1} = d_t$ for all $t \geq 0$. 


Implementation (continued)

**Off-Equilibrium Threat:** The government sets $\tau_t = 0$ if the economy stays at point $A$, but threatens to tax capital outflows if the economy enters in panic and deleverages to either point $B$ or $C$.

**Result:** If the capital outflow tax is strong enough, then point $B$ and $C$ are ruled out, point $A$ is the unique equilibrium, and capital controls are never imposed.
Implementation (continued)

Under this tax-policy rule, the Euler equation in period 0 becomes:

\[
\left( \frac{c^T_1}{c^T_0} \right)^2 = \frac{1}{1 - \tau(d_1, d_0) - (1 + r)\mu_0}
\]

(1) In the intended (Ramsey) equilibrium, \(c^T_1/c^T_0 = 1\), \(d_1 = d_0\), and \(\mu_0 = 0\), so the Euler equation holds and \(\tau(d_1, d_0) = 0\).

(2) In the unintended equilibrium (point \(B\)), \(c^T_1/c^T_0 > 0\), and \(d_1 < d_0 < 0\). Make \(\tau(d_1, d_0)\) so large that \(\mu_0\) has to be negative for the Euler equation to hold. Since \(\mu_0\) must be nonnegative, this capital-control policy rules out the unintended equilibrium.

**Result:** To rule out undesired self-fulfilling crises, the Ramsey planner commits to impose sufficiently large capital controls in case of speculative capital outflows.
Quantitative Analysis

Time unit: one quarter.
The economy is driven by endowment and interest-rate shocks. Estimate a bivariate AR(1) process for \((y_t, r_t)\) using Argentine data over the period 1983:Q1 to 2001:Q4.

\[\kappa = 1.2 \quad (\Rightarrow \text{upper limit on debt} = 30 \text{ percent of annual output}).\]
\[r = 0.0316\]
\[\beta = 0.9635\]
\[\sigma = 1/\xi = 2\]
\[a = 0.26\]
\[y^N = 1.\]

Discretization: 501 points for \(d\), 21 for \(y^T\), and 11 for \(r\).
Numerical Algorithm

• The numerical solution must take a stance on how to handle the possibility of indeterminacy.

• Failing to address this issue may result in nonconvergence.

• In searching for an equilibrium we favor ones with a binding constraint, as follows:

• if for the current state \((y_t^T, r_t, d_t)\) there are one or two values of \(d_{t+1}\) for which all equilibrium conditions are satisfied and the collateral constraint is binding, pick the smaller debt value.
Ramsey Problem

\[ v(y^T, r, d) = \max_{c^T, d'} \left\{ U(A(c^T, y^N)) + \beta \mathbb{E} \left[ v(y^{T'}, r', d') \mid y^T, r \right] \right\} \]

subject to

\[ c^T + d = y^T + \frac{d'}{1 + r} \]

\[ d' \leq \kappa \left[ y^T + \frac{1 - a}{a} \left( \frac{c^T}{y^N} \right)^{\frac{1}{\xi}} y^N \right] \]

Note. Although the constraints of this control problem may not represent a convex set in tradable consumption and debt, the fact that the Ramsey allocation is the result of a utility maximization problem, implies that its solution is generically unique.
The collateral constraint severely limits the country's ability to borrow. More importantly, it highly compresses the debt distribution (i.e., lowers the unconditional std. dev.).
The collateral constraint seldom binds.
The debt distribution under Ramsey-optimal policy lies to the right of those associated with the unregulated equilibria.
Conclusion

• This paper shows that open economies with flow collateral constraints display multiple equilibria.

• In particular, they are prone to self-fulfilling crises in which deleveraging and Fisherian deflations take place in the absence of changes in fundamentals.

• The competitive equilibrium can display underborrowing, as agents save to protect themselves against expectations-driven financial crises.

• Averting self-fulfilling crises requires the threat of taxing non-fundamental bursts of capital outflows.