# Credit Spreads and Monetary Policy: Technical Appendix*

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## 1 Equilibrium Conditions

This section describes the complete model of credit frictions.[1] The first subsection contains all the non-linear equations and objective welfare function, the second presents the steady state, the third the log-linearized equations, and the fourth presents a detailed description of the parameter values used for the numerical exercises.

### 1.1 Full set of non-linear equilibrium conditions

The objective:

$$\tilde{U}_t = \pi_b \frac{\left(\lambda_t^b\right)^{1-\sigma_b} \bar{C}_t^b}{1-\sigma_b^{-1}} + (1-\pi_b) \frac{\left(\lambda_t^s\right)^{1-\sigma_s} \bar{C}_t^s}{1-\sigma_s^{-1}} - \frac{\psi}{1+\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{Z_t}\right)^{1+\omega_y} \Delta_t \tag{1.1}$$

The equations describing the economy are summarized below:

$$0 = \left(1+i_t^d\right)(1+\omega_t)\,\beta E_t \left[ \left[\delta + (1-\delta)\,\pi_b\right] \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + (1-\delta)(1-\pi_b) \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \right] - \lambda_t^b \tag{1.2}$$

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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[1]For details on the derivations please refer to Cúrdia and Woodford (2009) and its technical appendix.

$$0=\left(1+i_t^d\right)\beta E_t\left[\left(1-\delta\right)\pi_b\frac{\lambda_{t+1}^b}{\Pi_{t+1}}+\left[\delta+\left(1-\delta\right)\left(1-\pi_b\right)\right]\frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right]-\lambda_t^s \tag{1.3}$$

$$0=\Lambda\left(\lambda_t^b,\lambda_t^s\right)\mu^p\left(1+\omega_y\right)\psi\mu_t^w\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right)^{-1}\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y}+\alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}K_{t+1}\right]-K_t \tag{1.4}$$

$$0=\Lambda\left(\lambda_t^b,\lambda_t^s\right)\left(1-\tau_t\right)Y_t+\alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}F_{t+1}\right]-F_t \tag{1.5}$$

$$\begin{aligned}0 \;=\;& \pi_b\left(1-\pi_b\right)B\left(\lambda_t^b,\lambda_t^s,Y_t,\Delta_t;\xi_t\right)-\pi_b b_t^g \\ &+\delta\left[b_{t-1}\left(1+\omega_{t-1}\right)+\pi_b b_{t-1}^g\right]\frac{1+i_{t-1}^d}{\Pi_t}-\left(1+\pi_b\omega_t\right)b_t\end{aligned} \tag{1.6}$$

$$0=\pi_b\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b}+\left(1-\pi_b\right)\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s}+\tilde{\Xi}_t b_t^\eta+G_t-Y_t \tag{1.7}$$

$$0=\alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega_y)}+\left(1-\alpha\right)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}}-\Delta_t \tag{1.8}$$

$$0=\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}-\left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega_y\theta}} \tag{1.9}$$

$$0=1+\chi_t+\eta\tilde{\Xi}_t b_t^{\eta-1}-\left(1+\omega_t\right) \tag{1.10}$$

Auxiliary equations and definitions:

$$\begin{aligned}B\left(\lambda_t^b,\lambda_t^s,Y_t,\Delta_t;\xi_t\right) \;\equiv\;& \bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b}-\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s} \\ &-\left[\left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}}-\left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right]\left(\frac{\tilde{\lambda}_t}{\psi}\right)^{-\frac{1+\nu}{\nu}}\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{Z_t}\right)^{1+\omega_y}\Delta_t\end{aligned} \tag{1.11}$$

$$\Lambda\left(\lambda_t^b,\lambda_t^s\right)\equiv\pi_b\lambda_t^b+\left(1-\pi_b\right)\lambda_t^s \tag{1.12}$$

$$\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right)\equiv\psi\left[\pi_b\left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}}+\left(1-\pi_b\right)\left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right]^{\nu} \tag{1.13}$$

$$\tilde{\Lambda}\left(\lambda_t^b,\lambda_t^s\right)\equiv\psi^{\frac{1}{1+\nu}}\left[\pi_b\psi_b^{-\frac{1}{\nu}}\left(\lambda_t^b\right)^{\frac{1+\nu}{\nu}}+\left(1-\pi_b\right)\psi_s^{-\frac{1}{\nu}}\left(\lambda_t^s\right)^{\frac{1+\nu}{\nu}}\right]^{\frac{\nu}{1+\nu}} \tag{1.14}$$

$$c_t^b=\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b} \tag{1.15}$$

$$c_t^s=\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s} \tag{1.16}$$

$$\omega_y\equiv\phi\left(1+\nu\right)-1 \tag{1.17}$$

## 1.2 Zero inflation steady state

We consider the solution to steady state in which we simply assume zero inflation. We use notation $\bar{x}$ as denoting the steady state value of generic variable $x$, unless otherwise noted.

We set, without loss of generality,

$$\bar{Y} = 1, \tag{1.18}$$

$$\psi = 1. \tag{1.19}$$

We further calibrate the following ratios

$$s_c \equiv \pi_b s_b + (1 - \pi_b) s_s, \tag{1.20}$$

$$\sigma_{bs} \equiv \sigma_b / \sigma_s, \tag{1.21}$$

$$\bar{\sigma} \equiv \pi_b s_b \sigma_b + (1 - \pi_b) s_s \sigma_s, \tag{1.22}$$

$$\bar{\chi} = 0, \tag{1.23}$$

We calibrate $\psi_b$ and $\psi_s$ to imply equal labor for the two types in steady state, implying that

$$\frac{\psi_b}{\psi_s} = \Omega, \tag{1.24}$$

and

$$\psi_s = \psi \left[ \pi_b \Omega^{-\frac{1}{\nu}} + (1 - \pi_b) \right]^{\nu}. \tag{1.25}$$

Further consider the following definitions

$$s_g \equiv \bar{G} / \bar{Y} \tag{1.26}$$

$$s_b \equiv \bar{c}^b / \bar{Y}, \tag{1.27}$$

$$s_s \equiv \bar{c}^s / \bar{Y}, \tag{1.28}$$

$$s_\Xi \equiv \Xi \left( \bar{b} \right) / \bar{Y}. \tag{1.29}$$

For the interest rate we have:

$$1+\bar{r}^d=\beta^{-1}\frac{(\delta+1)+\bar{\omega}\left[\delta+(1-\delta)\pi_b\right]-\sqrt{\left\{(\delta+1)+\bar{\omega}\left[\delta+(1-\delta)\pi_b\right]\right\}^2-4\delta\left(1+\bar{\omega}\right)}}{2\delta\left(1+\bar{\omega}\right)}. \tag{1.30}$$

(Note that if $\bar{\omega}=0$, this reduces to $1+\bar{r}^d=\beta^{-1}$.) We use this steady-state relation to calibrate $\beta$, given assumed values for $\delta$, $\pi_b$, $\bar{\omega}$ and $\bar{r}^d$.

We can also write

$$1+\bar{\imath}^d=1+\bar{r}^d. \tag{1.31}$$

The steady state inflation will determine the steady state price dispersion:

$$\bar{\Delta}=1 \tag{1.32}$$

We assume that the steady state spread is due solely to intermediation costs of the convex type, hence

$$\bar{\tilde{\Xi}}=\frac{\bar{\omega}}{\eta\bar{b}^{\eta-1}},$$

and the fraction of intermediation costs to output is

$$s_{\Xi}=\frac{\bar{\omega}}{\eta}\frac{\bar{b}}{\bar{Y}}. \tag{1.33}$$

Furthermore we can write, from one of the Euler equations:

$$\bar{\lambda}^b=\bar{\Omega}\bar{\lambda}^s, \tag{1.34}$$

where

$$\bar{\Omega}\equiv\frac{1-\left(1+\bar{r}^d\right)\beta\left[\delta+(1-\delta)(1-\pi_b)\right]}{\left(1+\bar{r}^d\right)\beta(1-\delta)\pi_b}. \tag{1.35}$$

This implies that, given $\psi_b$ and $\psi_s$, we get

$$\Lambda\left(\bar{\lambda}^b,\bar{\lambda}^s\right)=\left[\pi_b\bar{\Omega}+(1-\pi_b)\right]\bar{\lambda}^s, \tag{1.36}$$

$$\tilde{\lambda}\left(\bar{\lambda}^b,\bar{\lambda}^s\right)=\left[\pi_b\Omega^{-\frac{1}{\nu}}+(1-\pi_b)\right]^{-\nu}\bar{\lambda}^s, \tag{1.37}$$

$$\tilde{\Lambda}\left(\bar{\lambda}^b,\bar{\lambda}^s\right)=\left(\frac{\pi_b\bar{\Omega}+(1-\pi_b)}{\pi_b\Omega^{-\frac{1}{\nu}}+(1-\pi_b)}\right)^{\frac{\nu}{1+\nu}}\bar{\lambda}^s. \tag{1.38}$$

The inflation equation implies $\bar{F} = \bar{K}$, and using the definitions of $\bar{K}$ and $\bar{F}$ we get,

$$(1-\bar{\tau}) = \mu^p (1+\omega_y) \psi \bar{\mu}^w \tilde{\lambda} \left(\bar{\lambda}^b, \bar{\lambda}^s\right)^{-1} \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}},$$

hence

$$\bar{\lambda}^s = \frac{\psi_s \mu^p (1+\omega_y) \bar{\mu}^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{(1-\bar{\tau})}. \tag{1.39}$$

The resources constraint implies

$$1 - s_c - s_g = s_\Xi. \tag{1.40}$$

which determines $s_g$ given $s_c$ and $s_\Xi$.

The debt equation is

$$\left[1 + \pi_b \bar{\omega} - \delta (1+\bar{\omega}) \left(1+\bar{r}^d\right)\right] \frac{\bar{b}}{\bar{Y}} = \pi_b (1-\pi_b)(s_b - s_s) - \pi_b \frac{\bar{b}^g}{\bar{Y}} \left[1 - \delta \left(1+\bar{r}^d\right)\right],$$

implying that

$$\frac{\bar{b}}{\bar{Y}} = \frac{\pi_b (1-\pi_b)(s_b - s_s) - \pi_b \frac{\bar{b}^g}{\bar{Y}} \left[1 - \delta \left(1+\bar{r}^d\right)\right]}{1 + \pi_b \bar{\omega} - \delta (1+\bar{\omega})(1+\bar{r}^d)}. \tag{1.41}$$

Given that we calibrate $\bar{b}/\bar{Y}$ we can use this equation to determine $s_b - s_s$,

$$s_b - s_s = \frac{\left[1 + \pi_b \bar{\omega} - \delta (1+\bar{\omega}) \left(1+\bar{r}^d\right)\right] \frac{\bar{b}}{\bar{Y}} + \pi_b \frac{\bar{b}^g}{\bar{Y}} \left[1 - \delta \left(1+\bar{r}^d\right)\right]}{\pi_b (1-\pi_b)}. \tag{1.42}$$

Given our calibration of $s_c$ we can then write

$$s_s = s_c - \pi_b (s_b - s_s), \tag{1.43}$$

and

$$s_b = s_c + (1-\pi_b)(s_b - s_s). \tag{1.44}$$

Finally,

$$\bar{C}^b = s_b \left(\bar{\lambda}^b\right)^{\sigma_b}, \tag{1.45}$$

$$\bar{C}^s = s_b \left(\bar{\lambda}^s\right)^{\sigma_s}, \tag{1.46}$$

$$\bar{K} = \bar{F} = \frac{\bar{\Lambda}(1-\bar{\tau})}{1-\alpha\beta}. \tag{1.47}$$

## 1.3 Log-linear equilibrium conditions

In this section we present all the log-linear relations of the model, in which we linearize around the zero inflation steady state. We start by simply presenting the equilibrium conditions in log-linear form without any simplifications, so as to exactly match the set of non-linear equations. Then we proceed to present a simplified set of equations and the exact definitions of the natural rate of output and the natural interest rate used in the policy rules considered.

**Full system**

The full system of log-linear equation is given by:

$$\hat{\lambda}_t^b = \hat{\imath}_t^d + \hat{\omega}_t - E_t\pi_{t+1} + \chi_b E_t\hat{\lambda}_{t+1}^b + (1-\chi_b)\, E_t\hat{\lambda}_{t+1}^s, \tag{1.48}$$

$$\hat{\lambda}_t^s = \hat{\imath}_t^d - E_t\pi_{t+1} + (1-\chi_s)\, E_t\hat{\lambda}_{t+1}^b + \chi_s E_t\hat{\lambda}_{t+1}^s, \tag{1.49}$$

$$\begin{aligned} \hat{K}_t &= (1-\alpha\beta)\left[\hat{\Lambda}_t - \widehat{\bar{\lambda}}_t + \hat{\mu}_t^w - \nu\bar{h}_t + (1+\omega_y)\left(\hat{Y}_t - z_t\right)\right] \\ &\quad + \alpha\beta E_t\left[\theta\,(1+\omega_y)\,\pi_{t+1} + \hat{K}_{t+1}\right], \end{aligned} \tag{1.50}$$

$$\hat{F}_t = (1-\alpha\beta)\left[\hat{\Lambda}_t - \hat{\tau}_t + \hat{Y}_t\right] + \alpha\beta E_t\left[(\theta-1)\,\pi_{t+1} + \hat{F}_{t+1}\right], \tag{1.51}$$

$$0 = \Lambda\left(\lambda_t^b, \lambda_t^s\right)(1-\tau_t)\, Y_t + \alpha\beta E_t\left[\Pi_{t+1}^{\theta-1} F_{t+1}\right] - F_t \tag{1.52}$$

$$\begin{aligned} (1+\pi_b\bar{\omega})\,\hat{b}_t &= \pi_b(1-\pi_b)\frac{\bar{Y}}{\bar{b}}\hat{B}_t - \pi_b(1+\bar{\omega})\,\hat{\omega}_t \\ &\quad + \delta\left(1+\bar{r}^d\right)\left[(1+\bar{\omega}) + \pi_b\frac{\bar{b}^g}{\bar{b}}\right]\left(\hat{\imath}_{t-1}^d - \pi_t\right) \\ &\quad + \delta\left(1+\bar{r}^d\right)(1+\bar{\omega})\left(\hat{b}_{t-1} + \hat{\omega}_{t-1}\right) \\ &\quad - \pi_b\frac{\bar{Y}}{\bar{b}}\left[\hat{b}_t^g - \delta\left(1+\bar{r}^d\right)\hat{b}_{t-1}^g\right], \end{aligned} \tag{1.53}$$

$$\hat{Y}_t = \pi_b s_b\left(\bar{c}_t^b - \sigma_b\hat{\lambda}_t^b\right) + (1-\pi_b)\, s_s\left(\bar{c}_t^s - \sigma_s\hat{\lambda}_t^s\right) + \widehat{\bar{\Xi}}_t + \eta s_\Xi\hat{b}_t + \hat{G}_t, \tag{1.54}$$

$$\hat{\Delta}_t = \alpha\hat{\Delta}_{t-1}, \tag{1.55}$$

$$\pi_t = \frac{1-\alpha}{\alpha}\frac{1}{1+\omega_y\theta}\left(\hat{K}_t - \hat{F}_t\right), \tag{1.56}$$

$$0 = 1 + \chi_t + \eta \tilde{\Xi}_t b_t^{\eta-1} - (1 + \omega_t) \tag{1.57}$$

$$\hat{\omega}_t = \frac{1}{1+\bar{\omega}} \hat{\chi}_t + \frac{\eta \bar{\Xi} \bar{b}^{\eta-1}}{1+\bar{\omega}} \left( \frac{\zeta_{\Xi}}{\bar{\bar{\Xi}}} \hat{\tilde{\Xi}}_t + (\eta - 1)\, \hat{b}_t \right). \tag{1.58}$$

Auxiliary equations:

$$\hat{B}_t = s_b \left( \bar{c}_t^b - \sigma_b \hat{\lambda}_t^b \right) - s_s \left( \bar{c}_t^s - \sigma_s \hat{\lambda}_t^s \right) - \psi \bar{\tilde{\lambda}}^{-1} \bar{\mu}^w \bar{H}^{-\nu} \left( \frac{\bar{Y}}{\bar{\bar{Z}}} \right)^{1+\omega_y} \bar{Y}^{-1} \frac{1}{\nu} \left( \hat{\lambda}_t^b - \hat{\lambda}_t^s \right) \tag{1.59}$$

$$\hat{\tilde{\lambda}}_t = \pi_b \hat{\lambda}_t^b + (1 - \pi_b)\, \hat{\lambda}_t^s, \tag{1.60}$$

$$\hat{\Lambda}_t = \pi_b \frac{\bar{\lambda}^b}{\bar{\lambda}} \hat{\lambda}_t^b + (1 - \pi_b)\, \frac{\bar{\lambda}^s}{\bar{\lambda}} \hat{\lambda}_t^s, \tag{1.61}$$

$$\hat{\tilde{\Lambda}}_t = \pi_b \frac{\psi_b^{-\frac{1}{\nu}} \left( \bar{\lambda}^b \right)^{\frac{1+\nu}{\nu}}}{\psi^{-\frac{1}{\nu}} \bar{\tilde{\Lambda}}^{\frac{1+\nu}{\nu}}} \hat{\lambda}_t^b + (1 - \pi_b)\, \frac{\psi_s^{-\frac{1}{\nu}} \left( \bar{\lambda}^s \right)^{\frac{1+\nu}{\nu}}}{\psi^{-\frac{1}{\nu}} \bar{\tilde{\Lambda}}^{\frac{1+\nu}{\nu}}} \hat{\lambda}_t^s, \tag{1.62}$$

$$\hat{c}_t^b = \bar{c}_t^b - \sigma_b \hat{\lambda}_t^b, \tag{1.63}$$

$$\hat{c}_t^s = \bar{c}_t^s - \sigma_s \hat{\lambda}_t^s. \tag{1.64}$$

The exogenous variables all follow an AR(1) process as follows:

$$\xi_t = \rho_{\xi} \xi_{t-1} + \varepsilon_t \tag{1.65}$$

In the above equations we consider the following definitions

$$\hat{\imath}_t^{\tau} \equiv \ln \left( (1 + i_t^{\tau}) \,/\, (1 + \bar{\imath}_t^{\tau}) \right), \tag{1.66}$$

$$\hat{\omega}_t \equiv \ln \left( (1 + \omega_t) \,/\, (1 + \bar{\omega}) \right), \tag{1.67}$$

$$\pi_t \equiv \ln \Pi_t, \tag{1.68}$$

$$\bar{\lambda}_t^{\tau} \equiv \ln \left( \lambda_t^{\tau} / \bar{\lambda}^{\tau} \right), \tag{1.69}$$

$$\hat{Y}_t \equiv \ln \left( Y_t / \bar{Y} \right), \tag{1.70}$$

$$\hat{F}_t \equiv \ln \left( F_t / \bar{F} \right), \tag{1.71}$$

$$\hat{K}_t \equiv \ln \left( K_t / \bar{K} \right), \tag{1.72}$$

$$\hat{b}_t \equiv \ln \left( b_t / \bar{b} \right), \tag{1.73}$$

$$\bar{h}_t \equiv \ln\left(\bar{H}_t/\bar{H}\right), \tag{1.74}$$

$$z_t \equiv \ln\left(Z_t/\bar{Z}\right), \tag{1.75}$$

$$\hat{\tau}_t \equiv -\log\left((1-\tau_t)/(1-\bar{\tau})\right), \tag{1.76}$$

$$\hat{b}^g_t \equiv \left(b^g_t - \bar{b}\right)/\bar{Y}, \tag{1.77}$$

$$\bar{c}^\tau_t \equiv \ln\left(\bar{C}^\tau_t/\bar{C}^\tau\right), \tag{1.78}$$

$$\hat{\mu}^w_t \equiv \ln\left(\mu^w_t/\bar{\mu}^w\right), \tag{1.79}$$

$$\hat{G}_t \equiv \left(G_t - \bar{G}\right)/\bar{Y}, \tag{1.80}$$

$$\hat{\tilde{\Xi}}_t \equiv \frac{\bar{b}^\eta}{\bar{Y}}\left(\tilde{\Xi}_t - \bar{\tilde{\Xi}}\right), \tag{1.81}$$

$$\hat{\chi}_t \equiv \chi_t, \tag{1.82}$$

and

$$\chi_\tau \equiv \beta\left(1+\bar{r}^\tau\right)\left[\delta + (1-\delta)\,\pi_\tau\right]. \tag{1.83}$$

**Simplified log-linear system of equilibrium conditions**

We can write the required equations as

$$\hat{\imath}^{avg}_t = \hat{\imath}^d_t + \pi_b\hat{\omega}_t, \tag{1.84}$$

$$\hat{\Omega}_t = \hat{\omega}_t + \hat{\delta}E_t\hat{\Omega}_{t+1}, \tag{1.85}$$

$$\begin{aligned}\hat{Y}_t \;=\;& E_t\hat{Y}_{t+1} - \bar{\sigma}\left(\hat{\imath}^{avg}_t - E_t\pi_{t+1}\right) - E_t\Delta g_{t+1} - E_t\Delta\hat{\Xi}_{t+1} \\ & -\bar{\sigma}s_\Omega\hat{\Omega}_t + \bar{\sigma}\left(s_\Omega + \psi_\Omega\right)E_t\hat{\Omega}_{t+1},\end{aligned} \tag{1.86}$$

$$\begin{aligned}\pi_t \;=\;& \beta E_t\pi_{t+1} + u_t + \kappa\left(\hat{Y}_t - \left(\omega_y + \bar{\sigma}^{-1}\right)^{-1}\left[\bar{\sigma}^{-1}g_t + \nu\bar{h}_t + (1+\omega_y)\,z_t\right]\right) \\ & -\xi\bar{\sigma}^{-1}\hat{\Xi}_t + \xi\left(s_\Omega + \pi_b - \gamma_b\right)\hat{\Omega}_t,\end{aligned} \tag{1.87}$$

$$\hat{\omega}_t = \omega_b\hat{b}_t + \omega_\chi\hat{\chi}_t + \omega_\Xi\hat{\tilde{\Xi}}_t, \tag{1.88}$$

$$\begin{aligned}\hat{b}_t &= \varrho_r\left(\hat{\imath}^d_{t-1}-\pi_t\right)+\varrho_Y\hat{Y}_t+\varrho_\Omega\hat{\Omega}_t+\varrho_\omega\hat{\omega}_t+\varrho_b\left(\hat{b}_{t-1}+\hat{\omega}_{t-1}\right) \\ &\quad +\varrho_\xi\left[\pi_b\left(1-\pi_b\right)s_c\bar{c}_t-s_\Omega\bar{\sigma}^{-1}\left(g_t+\hat{\Xi}_t\right)\right] \\ &\quad -\pi_b\varrho_\xi\left[\hat{b}^g_t-\delta\left(1+\bar{r}^d\right)\hat{b}^g_{t-1}\right],\end{aligned} \tag{1.89}$$

with

$$s_c\bar{c}_t=\pi_b s_b\bar{c}^b_t+\left(1-\pi_b\right)s_s\bar{c}^s_t, \tag{1.90}$$

$$g_t=s_c\bar{c}_t+\hat{G}_t, \tag{1.91}$$

$$u_t\equiv\xi\left(\hat{\mu}^w_t+\hat{\tau}_t\right), \tag{1.92}$$

$$\Delta g_t\equiv g_t-g_{t-1}, \tag{1.93}$$

$$\Delta\hat{\hat{\Xi}}_t\equiv\hat{\hat{\Xi}}_t-\hat{\hat{\Xi}}_{t-1}, \tag{1.94}$$

$$\xi_t=\rho\xi_{t-1}+\varepsilon_t. \tag{1.95}$$

and

$$\hat{\delta}\equiv\chi_b+\chi_s-1, \tag{1.96}$$

$$\psi_\Omega\equiv\pi_b\left(1-\chi_b\right)-\left(1-\pi_b\right)\left(1-\chi_s\right), \tag{1.97}$$

$$s_\Omega\equiv\pi_b\left(1-\pi_b\right)\frac{s_b\sigma_b-s_s\sigma_s}{\bar{\sigma}}, \tag{1.98}$$

$$\xi\equiv\frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega_y\theta}, \tag{1.99}$$

$$\gamma_b\equiv\pi_b\left(\frac{\psi}{\psi_b}\frac{\bar{\lambda}^b}{\bar{\bar{\lambda}}}\right)^{\frac{1}{\nu}}, \tag{1.100}$$

$$\kappa\equiv\xi\left(\omega_y+\bar{\sigma}^{-1}\right), \tag{1.101}$$

$$\omega_b\equiv\frac{\eta\left(\eta-1\right)s_\Xi\frac{\bar{Y}}{\bar{b}}}{1+\bar{\omega}}, \tag{1.102}$$

$$\omega_\chi\equiv\frac{1}{1+\bar{\omega}}, \tag{1.103}$$

$$\omega_\Xi\equiv\frac{\eta}{1+\bar{\omega}}\frac{\bar{Y}}{\bar{b}}, \tag{1.104}$$

$$\tilde{B}_\Omega\equiv\psi\bar{\bar{\lambda}}^{-1}\bar{\mu}^w\bar{H}^{-\nu}\left(\frac{\bar{Y}}{\bar{Z}}\right)^{1+\omega_y}\bar{Y}^{-1}\frac{1}{\nu}\pi_b\left(1-\pi_b\right), \tag{1.105}$$

$$B_\Omega \equiv s_\Omega \pi_b - s_b \sigma_b \pi_b (1-\pi_b) - \tilde{B}_\Omega, \tag{1.106}$$

$$B_\lambda \equiv -s_\Omega, \tag{1.107}$$

$$\varrho_r \equiv \frac{\delta\left(1+\bar{r}^d\right)}{1+\pi_b\bar{\omega}}\left[(1+\bar{\omega})+\pi_b\frac{\bar{b}^g}{\bar{b}}\right], \tag{1.108}$$

$$\varrho_Y \equiv \varrho_\xi s_\Omega \bar{\sigma}^{-1}, \tag{1.109}$$

$$\varrho_\Omega \equiv \varrho_\xi \left(B_\Omega + s_\Omega^2\right), \tag{1.110}$$

$$\varrho_\omega \equiv -\frac{\pi_b(1+\bar{\omega})}{1+\pi_b\bar{\omega}}, \tag{1.111}$$

$$\varrho_\xi \equiv \frac{1}{1+\pi_b\bar{\omega}}\frac{\bar{Y}}{\bar{b}}, \tag{1.112}$$

$$\varrho_b \equiv \frac{\delta\left(1+\bar{r}^d\right)(1+\bar{\omega})}{1+\pi_b\bar{\omega}}. \tag{1.113}$$

**Natural rates**

In the policy rules considered, unless otherwise noted, it is assumed that the interest rate responds to the output gap, $\hat{Y}_t - \hat{Y}_t^n$, and the natural rate of interest, $\hat{r}_t^n$. It is important to notice that in order to be transparent about the role of the response to the financial variables we exclude from this definition any changes in the financial intermediation frictions, implying that neither the natural rate of output nor the natural interest rate respond to changes in the financial frictions. Similarly the natural rates do not respond to shocks to the markup, $\mu_t^w$, or the tax rate, $\tau_t$. Therefore we consider these two variables as solving the flexible price equilibrium of this economy when the intermediation frictions remain at their steady state levels. This means that the natural rate of output is given by

$$\hat{Y}_t^n = \left(\omega_y + \bar{\sigma}^{-1}\right)^{-1}\left[\bar{\sigma}^{-1}g_t + \nu\bar{h}_t + (1+\omega_y)\,z_t\right], \tag{1.114}$$

and the natural interest rate is defined as the rate at which the Euler equation is satisfied when output is at its natural level,

$$\hat{r}_t^n = \bar{\sigma}^{-1}\left(E_t\hat{Y}_{t+1}^n - \hat{Y}_t^n - E_t\Delta g_{t+1}\right). \tag{1.115}$$

# 2 Robustness analysis

In this section we analyze the sensitivity of the main results to different alternative parameter values. Generally speaking the parameters chosen are fairly conventional and used throughout the literature, as discussed in the main text. Here we look at three parameters that are particular to our model. The first is the degree of convexity of the financial intermediaries technology, $\eta$. In the main text we set it to 5 but when discussing the optimal response to the financial shocks we consider two additional values for this parameter: 1 and 50. Setting it to 1 implies that the technology is linear and in particular, movements in the level of credit do not induce any changes to the spread. Setting it to 50 is interesting because it reflects a case with significantly more convex technology.

The second set of parameters of interest are the interest rate sensitivities of the demand of borrowers and savers, $\sigma_b$ and $\sigma_s$. In the text we mention that we choose these to imply an average sensitivity of total demand equal to that in Rotemberg and Woodford (1997), which implies $\bar{\sigma} = 6.25$; and to also imply a ratio $\sigma_b/\sigma_s = 5$. The reason to choose this ratio, is simply to generate a negative response of credit to a monetary policy shock as suggested in VAR analysis.[2] However it might be argued that setting such a big ratio might be unreasonable. Here we consider the alternative with $\sigma_b/\sigma_s = 2$. This implies that we get $\sigma_b = 11.456$ and $\sigma_s = 5.728$ for this ratio.

In Figures 1 and 2 we show how the choice of $\eta$ and $\sigma_b/\sigma_s$ affects the response of the economy to a monetary policy shock. In the first figure we see that for all of the values of $\eta$ we get a contraction in real level of credit, $b_t$. Changing the value of $\eta$ does not change in any substantial way the response of output and inflation but it does affect noticeably the response of credit and the spread. In the case of linear intermediation technology ($\eta = 1$) the contraction in the level of credit does not induce any change in the spread, while in the convex case (the other two levels) it does imply that the spread falls. With a more convex technology the fall in the spread is more significant, hence reducing the cost of borrowing, which leads to a mitigated fall of credit. Because the spread falls and borrowers demand is higher, the equilibrium level of the policy rate is somewhat higher than in the linear technology case.

In the second figure, with the lower ratio of interest rate sensitivity, we see that the impact response of credit to the monetary policy shock is still slightly negative but that is immediately reversed in the following periods, in which there is a persistent increase of the

[2] For example see Lown and Morgan (2002).

Table 1: Optimal value of the spread-adjustment coefficient $\phi_\omega$ in policy rule (2.3), in the case of financial disturbances of either of two types, as in Table 2 of the main text, but with $\sigma_b/\sigma_s = 2$.

| | $\eta = 1$ | | $\eta = 5$ | | $\eta = 50$ | |
|---|---|---|---|---|---|---|
| $\phi^*_\omega$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\chi_t$ | $\tilde{\Xi}_t$ |
| $\rho = 0.00$ | 1.35 | 0.89 | 0.70 | 0.54 | 0.70 | 0.58 |
| $\rho = 0.50$ | 1.21 | 1.05 | 0.68 | 0.64 | 0.68 | 0.64 |
| $\rho = 0.90$ | 0.32 | 0.37 | 0.58 | 0.59 | 0.65 | 0.65 |
| $\rho = 0.99$ | -1.56 | -1.48 | 0.27 | 0.35 | 0.58 | 0.61 |

level of credit. In response to the increased level of credit, the spread increases and that gives an incentive for the borrowers to reduce borrowing, hence the degree of increase in credit with high level of $\eta$ is much more mitigated than for the linear case. As before, the response of output and inflation to the monetary policy shock is not changed much, actually it is essentially the same across different ratios of interest rate sensitivity.

Next we consider the implications of the alternative parameter configurations for the main results in the paper.

## 2.1 Spread-Adjusted Taylor Rules

We first consider a policy rule as in equation (2.3) of the text. We start with a discussion for the case of the financial disturbances and then will consider the non-financial ones.

### 2.1.1 Responses to Financial Disturbances

In the main text we discuss the optimal response of the policy rate to the spread in the case of financial shocks. In particular we show in Table 2 of the main text how the optimal response depends on different degrees of persistency and on different values for $\eta$. In Table 1 we replicate those results but for the case with $\sigma_b/\sigma_s = 2$. The important conclusion to take from this table is that this alternative ratio of interest rate sensitivities does not change the main results of the table. The main difference is that this lower degree of heterogeneity between borrowers and savers seems to make the optimal response to the spread somewhat less sensitive to the persistence of the shocks. However, it is still the case that the optimal response is significantly affected by the persistence, especially for very persistent levels and for lower degrees of convexity.

Table 2: Optimal value of the spread-adjustment coefficient $\phi_\omega$ in policy rule (2.3), in the case of non-financial disturbances, as in Table 3 of the main text, but with $\eta = 50$ and $\sigma_b/\sigma_s = 5$.

| $\phi^*_\omega$ | $\bar{C}^b_t$ | $\bar{C}^s_t$ | $G_t$ | $b^g_t$ | $Z_t, \bar{H}_t$ | $\mu^w_t$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|
| $\rho = 0.00$ | 0.64 | 1.07 | 1.54 | 0.62 | 1.08 | 5.77* | 5.43 |
| $\rho = 0.50$ | 0.59 | 0.71 | 2.22 | 0.71 | 1.35 | 5.77* | 5.77* |
| $\rho = 0.90$ | 0.15 | 0.16 | 0.29 | 0.74 | 0.20 | 5.77* | 5.77* |
| $\rho = 0.99$ | -1.39 | -1.39 | -1.37 | 0.64 | -1.39 | 5.77* | 5.77* |

* higher number leads to indeterminacy

### 2.1.2 Responses to Non-Financial Disturbances

When we consider non-financial disturbances it is important to notice that it is pointless to discuss the case with linear intermediation technology ($\eta = 1$) because in that case the spread is unchanged for these shocks. Therefore, for the baseline ratio of $\sigma_b/\sigma_s$ we can show alternative optimized coefficients for the case with very high degree of convexity, $\eta = 50$, shown in Table 2. One interesting result is that with this degree of convexity there are no shocks that would imply a negative $\phi_\omega$ – with $\eta = 5$ we find negative coefficients for $\bar{C}^b_t$, $\bar{C}^s_t$, $G_t$ and $Z_t$. In general the coefficients are higher than in the baseline, except for the two distortionary shocks, $\tau_t$ and $\mu^w_t$. For these two disturbances increasing the convexity implies that the optimal coefficients are restricted by indeterminacy considerations – the welfare increases with the coefficient up to the edge of the determinacy region. For this level of convexity we observe that the optimal response to the spread is less sensitive to the persistence of shocks to the level of government debt level but is more sensitive to the persistence of shocks to private expenditure of both types.

Consider now the same analysis but for the case with lower ratio of interest rate sensitivity of the two expenditures, with $\sigma_b/\sigma_s = 2$. Table 3 shows two panels replicating the same table with the optimized coefficients $\phi_\omega$ for different non-financial disturbances but with the lower ratio and for two levels of convexity. (Notice that, as mentioned before it does not make sense to consider the case of linear for non-financial shocks.)

Overall we can conclude that reducing the difference in interest rate sensitivity reduces to some extent the degree to which the optimal response to the spread depends on the persistence of the shocks, except for the productivity shock, in which and intermediate level of persistence, with $\rho = 0.5$, now implies a very strong response to the spread. However, we still conclude that for shocks that do not affect the size of the distortions in the economy

Table 3: Optimal value of the spread-adjustment coefficient $\phi_\omega$ in policy rule (2.3), in the case of non-financial disturbances, as in Table 3 of the main text, but $\sigma_b/\sigma_s = 2$. We show two panels, one with $\eta = 5$ and the other with $\eta = 50$

| $\phi^*_\omega$ | $\bar{C}^b_t$ | $\bar{C}^s_t$ | $G_t$ | $b^g_t$ | $Z_t, \bar{H}_t$ | $\mu^w_t$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|
| Medium convexity ($\eta = 5$) | | | | | | | |
| $\rho = 0.00$ | 0.39 | 0.64 | 0.64 | 0.53 | 0.46 | 47.39 | 42.90 |
| $\rho = 0.50$ | 0.37 | 0.40 | 5.34 | 0.63 | 2.32 | 29.38 | 26.80 |
| $\rho = 0.90$ | 0.16 | 0.16 | 0.30 | 0.59 | 0.20 | 25.14 | 25.12 |
| $\rho = 0.99$ | -0.63 | -0.63 | -0.62 | 0.35 | -0.63 | 44.89 | 44.91 |
| High convexity ($\eta = 50$) | | | | | | | |
| $\rho = 0.00$ | 0.61 | 0.88 | 0.92 | 0.57 | 0.68 | 5.70 | 4.83 |
| $\rho = 0.50$ | 0.57 | 0.64 | 2.32 | 0.63 | 1.39 | 5.87 | 5.26 |
| $\rho = 0.90$ | 0.30 | 0.30 | 0.62 | 0.65 | 0.40 | 6.75* | 6.75* |
| $\rho = 0.99$ | -0.60 | -0.60 | -0.55 | 0.60 | -0.59 | 6.75* | 6.75* |

* higher number leads to indeterminacy

(shocks to expenditure and shocks to the productivity and labor supply) the optimal response to the spread is very sensitive to the persistence of the shocks. In the case of the supply side distortionary disturbances (tax rate and wage markup) we get even higher response to the spread in the case of medium level of convexity ($\eta = 5$), now well above 20, which would imply a coefficient $\phi_\omega$ far from optimal in the case of other disturbances.

### 2.1.3 Welfare Tradeoffs

In the text we show in Table 4 the welfare changes from responding to the spread compared to the case without any response, measured in steady state consumption equivalent for both types of agents. In Table 4 we present the same analysis but considering two alternative levels of convexity: linear technology, $\eta = 1$, and very convex technology, $\eta = 50$, under the baseline ratio of interest rate sensitivity,$\sigma_b/\sigma_s = 5$.

The first result worth mentioning is that in the case of linear technology the spread only changes in the case of financial shocks and for these the conclusion is that it is welfare improving to have some response to the spread but only a mild one: in the case of baseline persistence $\phi_\omega = 0.25$. In particular we can see that increasing further to $\phi_\omega = 0.5$ will reduce the welfare in both cases, but mostly so in the case of the shock to the default rate. If however we consider the very persistent case then even a mild response to the spread is welfare reducing.

In the case of high convexity, shown in panel B, we get the interesting result that for the

Table 4: Welfare consequences of increasing $\phi_\omega$, for different disturbances, as in Table 4 of the main text, but for different levels of convexity, with $\sigma_b/\sigma_s = 5$.

**Panel A: Linear intermediation, $\eta = 1$**

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ |
|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | |
| $\phi_\omega = 0.25$ | 9.363 | 14.068 |
| $\phi_\omega = 0.50$ | 0.822 | 5.788 |
| $\phi_\omega = 0.75$ | -25.621 | -24.842 |
| $\phi_\omega = 1.00$ | -69.969 | -77.821 |
| High persistence ($\rho = 0.99$) | | |
| $\phi_\omega = 0.25$ | -67.736 | -70.692 |
| $\phi_\omega = 0.50$ | -142.091 | -148.506 |
| $\phi_\omega = 0.75$ | -223.064 | -233.442 |
| $\phi_\omega = 1.00$ | -310.657 | -325.499 |

**Panel B: Convex intermediation, $\eta = 50$**

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\bar{C}_t^b$ | $\bar{C}_t^s$ | $G_t$ | $b_t^g$ | $Z_t, \bar{H}_t$ | $\mu_t^w$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 14.584 | 13.138 | 0.212 | 0.522 | 0.576 | 13.059 | 0.053 | 17.806 | 17.165 |
| $\phi_\omega = 0.50$ | 24.344 | 21.364 | -2.367 | -3.295 | 0.286 | 21.189 | -0.069 | 35.623 | 34.338 |
| $\phi_\omega = 0.75$ | 28.805 | 24.197 | -7.900 | -11.703 | -0.924 | 23.910 | -0.376 | 53.437 | 51.510 |
| $\phi_\omega = 1.00$ | 27.447 | 21.110 | -16.556 | -24.964 | -3.105 | 20.694 | -0.880 | 71.235 | 68.666 |
| High persistence ($\rho = 0.99$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 14.258 | 13.556 | -4.493 | -10.061 | -2.297 | 13.173 | -0.499 | 8.535 | 8.500 |
| $\phi_\omega = 0.50$ | 22.560 | 21.021 | -9.750 | -21.832 | -4.990 | 20.237 | -1.082 | 17.011 | 16.941 |
| $\phi_\omega = 0.75$ | 24.417 | 21.898 | -15.778 | -35.325 | -8.083 | 20.696 | -1.752 | 25.428 | 25.323 |
| $\phi_\omega = 1.00$ | 19.298 | 15.643 | -22.582 | -50.555 | -11.580 | 14.008 | -2.508 | 33.785 | 33.646 |

baseline persistence, with $\rho = 0.9$, having a mild response to the spread, with $\phi_\omega = 0.25$, is always welfare improving, regardless of the shock hitting the economy, as long as all of them share this same degree of persistence. This is no longer true in the case of very high persistence of the shocks, in which case we get results very similar to those presented in the main text.

Next we consider Table 5 in which we present the same analysis but considering a lower ratio of interest rate sensitivity, $\sigma_b/\sigma_s = 2$. In the case of linear technology the results are very similar to those with $\sigma_b/\sigma_s = 5$. In the case of baseline convexity, with $\eta = 5$, we get again the result discussed in the previous table that with baseline persistency of $\rho = 0.9$ for all shocks welfare is increased by having a mild response to the spread, with $\phi_\omega = 0.25$, but

Table 5: Welfare consequences of increasing $\phi_\omega$, for different disturbances, as in Table 4 of the main text, but for different levels of convexity, with $\sigma_b/\sigma_s = 2$.

**Panel A: Linear intermediation, $\eta = 1$**

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ |
|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | |
| $\phi_\omega = 0.25$ | 24.856 | 41.762 |
| $\phi_\omega = 0.50$ | 18.613 | 41.631 |
| $\phi_\omega = 0.75$ | -18.728 | -0.393 |
| $\phi_\omega = 1.00$ | -87.168 | -84.311 |
| High persistence ($\rho = 0.99$) | | |
| $\phi_\omega = 0.25$ | -83.733 | -87.999 |
| $\phi_\omega = 0.50$ | -179.870 | -189.735 |
| $\phi_\omega = 0.75$ | -288.411 | -305.208 |
| $\phi_\omega = 1.00$ | -409.355 | -434.419 |

**Panel B: Convex intermediation, $\eta = 5$**

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\bar{C}_t^b$ | $\bar{C}_t^s$ | $G_t$ | $b_t^g$ | $Z_t, \bar{H}_t$ | $\mu_t^w$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 35.416 | 39.692 | 0.218 | 0.337 | 0.090 | 41.085 | 0.005 | 5.374 | 5.281 |
| $\phi_\omega = 0.50$ | 51.342 | 58.162 | -1.100 | -1.652 | 0.054 | 60.244 | -0.007 | 10.718 | 10.533 |
| $\phi_\omega = 0.75$ | 47.590 | 55.202 | -3.968 | -5.989 | -0.111 | 57.264 | -0.034 | 16.031 | 15.755 |
| $\phi_\omega = 1.00$ | 23.968 | 30.605 | -8.399 | -12.691 | -0.404 | 31.931 | -0.078 | 21.313 | 20.946 |
| High persistence ($\rho = 0.99$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 15.797 | 28.872 | -2.611 | -4.106 | -0.342 | 29.479 | -0.048 | 3.866 | 3.857 |
| $\phi_\omega = 0.50$ | 3.972 | 25.341 | -6.091 | -9.575 | -0.800 | 25.561 | -0.112 | 7.716 | 7.699 |
| $\phi_\omega = 0.75$ | -35.727 | -10.892 | -10.440 | -16.408 | -1.375 | -12.058 | -0.192 | 11.552 | 11.526 |
| $\phi_\omega = 1.00$ | -103.555 | -80.128 | -15.662 | -24.610 | -2.066 | -83.682 | -0.288 | 15.372 | 15.338 |

**Panel C: Convex intermediation, $\eta = 50$**

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\bar{C}_t^b$ | $\bar{C}_t^s$ | $G_t$ | $b_t^g$ | $Z_t, \bar{H}_t$ | $\mu_t^w$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 17.587 | 17.867 | 2.644 | 2.503 | 0.479 | 17.932 | 0.037 | 9.552 | 9.045 |
| $\phi_\omega = 0.50$ | 26.849 | 27.285 | 1.524 | 1.475 | 0.715 | 27.387 | 0.041 | 18.953 | 17.945 |
| $\phi_\omega = 0.75$ | 27.802 | 28.268 | -3.442 | -3.151 | 0.705 | 28.383 | 0.010 | 28.204 | 26.699 |
| $\phi_\omega = 1.00$ | 20.456 | 20.827 | -12.346 | -11.451 | 0.445 | 20.937 | -0.055 | 37.308 | 35.311 |
| High persistence ($\rho = 0.99$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 17.967 | 19.564 | -2.933 | -4.340 | -0.356 | 19.448 | -0.052 | 4.227 | 4.196 |
| $\phi_\omega = 0.50$ | 25.933 | 28.883 | -6.885 | -10.182 | -0.844 | 28.621 | -0.123 | 8.426 | 8.364 |
| $\phi_\omega = 0.75$ | 24.049 | 28.111 | -11.858 | -17.532 | -1.464 | 27.677 | -0.211 | 12.596 | 12.503 |
| $\phi_\omega = 1.00$ | 12.468 | 17.404 | -17.858 | -26.395 | -2.217 | 16.774 | -0.319 | 16.738 | 16.616 |

Table 6: Optimal value of the credit level-adjustment coefficient $\phi_b$ in policy rule (2.4), in the case of financial disturbances, as in Table 5 of the main text, but with $\sigma_b/\sigma_s = 2$.

| | $\eta = 1$ | | $\eta = 5$ | | $\eta = 50$ | |
|---|---|---|---|---|---|---|
| $\phi_b^*$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\chi_t$ | $\tilde{\Xi}_t$ |
| $\rho = 0.00$ | 0.012 | 0.013 | 0.157 | 0.156 | 0.894 | 0.831 |
| $\rho = 0.50$ | 0.005 | 0.005 | 0.070 | 0.072 | 0.336 | 0.341 |
| $\rho = 0.90$ | 0.000 | 0.000 | 0.021 | 0.023 | 0.044 | 0.048 |
| $\rho = 0.99$ | -0.004 | -0.004 | -0.001 | 0.001 | 0.001 | 0.003 |

above that response we would need to tradeoff the different shocks. Furthermore it is still the case the if different shocks have different persistencies then even the mild response to the spread might not be optimal.

Considering now the case with high convexity, $\eta = 50$, and low ratio of interest rate sensitivity, we get the result that for the baseline persistence of $\rho = 0.9$, a medium response to the spread of $\phi_\omega = 0.5$ is better than no response, but depending on the shock the smaller response with $\phi_\omega = 0.25$ might be better. As before high persistence reduces the benefits from responding to the spread.

## 2.2 Responding to Variations in Aggregate Credit

In the main text we consider adjusting the interest rate rule to respond to the level of aggregate credit, $b_t$, instead of the spread. Here we show similar results, but for the case with a low ratio of the interest rate sensitivity of the expenditure of the two types, with $\sigma_b/\sigma_s = 2$, shown in Table 6. This table shows that the results are extremely similar to those shown in Table 5 of the main text. The optimal coefficient is very small in most cases and in some it can even be negative.

# 3 Taylor Rules With No Natural Rate Adjustments

The main text presents several alternative Taylor rules, all of which consider that the interest rate responds to the natural rate of interest and to deviations of output from its natural level, as described in the baseline rule,

$$i_t^d = r_t^n + \phi_\pi \pi_t + \phi_y \left(\hat{Y}_t - \hat{Y}_t^n\right). \tag{3.1}$$

In this section we shall consider a different version of the interest rate rule, in which the interest rate does not respond to the natural interest rate and it responds to deviations of output from steady state,

$$i_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t. \tag{3.2}$$

This shall be labeled as the "basic" Taylor rule, due to its simplicity.[3] This rule has the advantage of being much simpler to implement than the rule implied by (3.1), in the sense that there is no need to evaluate what is the natural interest rate nor the natural level of output at each period. However, this policy rule is usually farther way from optimal than the one presented in the main text. That is true in the standard New-Keynesian model and it is also true in the current model. The rest of this section considers adjustments to the basic Taylor rule, (3.1), by incorporating an interest rate response to the level of spreads or a response to the level of credit.

Let us consider generalizations of (3.1) of the form

$$i_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t - \phi_\omega \hat{\omega}_t, \tag{3.3}$$

for some coefficient $0 \leq \phi_\omega \leq 1$. Like the rule (2.3) in the main text, these rules reflect the idea that the funds rate should be lowered when credit spreads increase, so as to prevent the increase in spreads from "effectively tightening monetary conditions" in the absence of any justification from inflation or high output relative to potential, except that now we consider a rule without any response to the natural interest rate or the natural rate of output.

It is important to notice that this rule is no different than the one discussed previously for shocks that affect the economy through changes in the size of the distortions, because the definition of the natural rate used precisely ignores those effects. This means that the optimal response to the spread for shocks to the financial frictions is exactly the same, hence we can get back to Table 2 in the main text for those shocks. The same applies to the optimal response to the spread in the case of shocks to government debt, the tax rate, or the wage markup, all of which affect the economy by changing the degree of distortions in the economy, hence are ignored in the definition of the natural variables.[4] Therefore in Table 7

[3] The units quoted here are the ones used by Taylor, in which the inflation rate and interest rates are annualized rates. If instead these are quarterly rates, as in the model equations expounded here, the value of $\phi_y$ is instead $0.5/4 = 0.125$.

[4] Notice in particular that in the absence of credit frictions the level of government debt is irrelevant for the determination of output and inflation and the only way it matters in the current model is precisely by influencing the degree of credit frictions.

Table 7: Optimal value of the spread-adjustment coefficient $\phi_\omega$, in the case of non-financial disturbances, as in Table 3 of the main text, but for policy rule 3.3.

| $\phi^*_\omega$ | $\bar{C}^b_t$ | $\bar{C}^s_t$ | $G_t$ | $b^g_t$ | $Z_t, \bar{H}_t$ | $\mu^w_t$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|
| $\rho = 0.00$ | -0.47 | 1.52 | 6.64 | 0.60 | 52.25 | 16.41 | 14.12 |
| $\rho = 0.50$ | -0.33 | 0.69 | 4.42 | 0.71 | 12.73 | 12.09 | 11.73 |
| $\rho = 0.90$ | 1.70 | -0.77 | -2.06 | 0.62 | 13.03 | 13.03 | 13.02 |
| $\rho = 0.99$ | 6.96 | -3.98 | -8.61 | 0.16 | 21.75 | 21.77 | 21.78 |

we can see that the optimal coefficients for those shocks are unchanged compared to Table 3 in the text.

The first striking feature in Table 7, compared to Table 3 of the main text, is that the optimal coefficient $\phi_\omega$ for the productivity shock is much bigger, higher than 10 for all levels of persistence. The reason for this difference is that under a Taylor rule that does not respond to the natural rates of output and interest, output and inflation are weaker compared to the optimal, while our baseline Taylor is very close to optimal for this shock, as shown in Figure 3 (shows responses to this shock for four different policies: optimal policy, baseline Taylor rule, the basic Taylor rule described above and another version of the Taylor rule with response to the output gap but not the natural rate of interest, to be discussed in the next section). Furthermore, this shock leads to an increase in the spread which means that by having a very strong negative response to the spread ($\phi_\omega >> 0$) it is possible to mitigate partially the inefficiency of the basic Taylor rule. Even in this case it is still the case that the level of optimal response to the spread is very sensitive to the persistency of the shock, as in our main results.

Regarding the other three shocks for which the baseline and basic Taylor rules are not equivalent, we still get negative coefficients for the shocks to the expenditures of savers and government, in the case of high enough persistence ($\rho \geq 0.9$) and positive for lower levels of persistence but now the sensitivity of the coefficient to the persistence is magnified. To understand this we show in Figure 4 the response to a shock in $\bar{C}^s_t$ for the baseline specification with alternative specifications of the Taylor rule. The basic Taylor rule implies weaker than optimal output and inflation. In this case spreads fall, which means that a negative coefficient will help bringing the response of output and inflation towards the optimal. A similar result applies for the case of the shock to government expenditures.

On the other hand, for the shock to the expenditure of borrowers, the sign of the coefficient is flipped, compared to the results for the baseline Taylor rule. The reason for this can be seen in Figure 5, which shows that, in response to a shock to $\bar{C}^b_t$ with persistence of $\rho = 0.9$,

Table 8: Welfare consequences of increasing $\phi_\omega$, for different disturbances, as in Table 4 of the main text, but for policy rule 3.3.

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\bar{C}_t^b$ | $\bar{C}_t^s$ | $G_t$ | $b_t^g$ | $Z_t, \bar{H}_t$ | $\mu_t^w$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 27.592 | 27.588 | 6.087 | -10.857 | -4.534 | 28.169 | 2.165 | 9.425 | 9.322 |
| $\phi_\omega = 0.50$ | 42.519 | 41.715 | 11.257 | -24.878 | -9.637 | 42.540 | 4.313 | 18.774 | 18.570 |
| $\phi_\omega = 0.75$ | 44.201 | 41.766 | 15.491 | -42.120 | -15.319 | 42.474 | 6.442 | 28.042 | 27.740 |
| $\phi_\omega = 1.00$ | 32.034 | 27.100 | 18.770 | -62.639 | -21.589 | 27.311 | 8.550 | 37.225 | 36.825 |
| High persistence ($\rho = 0.99$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 0.252 | 4.155 | 15.824 | -25.095 | -12.424 | 2.727 | 5.093 | 7.712 | 7.703 |
| $\phi_\omega = 0.50$ | -15.889 | -10.151 | 31.115 | -51.787 | -25.234 | -13.120 | 10.146 | 15.365 | 15.347 |
| $\phi_\omega = 0.75$ | -49.053 | -43.636 | 45.870 | -80.081 | -38.431 | -48.254 | 15.159 | 22.955 | 22.929 |
| $\phi_\omega = 1.00$ | -99.880 | -97.029 | 60.083 | -109.983 | -52.016 | -103.395 | 20.130 | 30.484 | 30.450 |

the basic Taylor rule implies a path for inflation and output much lower to what would be optimal. Because in this case, much like that of the productivity shock, borrowing and spreads increase, then having a negative coefficient on the spread helps getting closer to the optimal policy and thus is welfare improving.

In Table 8 we show the equivalent of Table 4 in the main text but under the basic Taylor rule discussed here. Notice that the welfare changes are exactly the same for the shocks affecting the size of the distortions in the economy, ignored in the definition of the natural rate. An important difference is that now all shocks imply similar size of changes in welfare in response to the spread. The only two shocks that induce a welfare reduction in response to the spread for the baseline persistence are the shocks to the savers and government expenditures. Notice that in Table 3 in addition to these two we also have welfare reductions for the productivity and borrowers expenditure shocks. Similar results apply to the case of more persistent shocks.

We do not consider the case with response to the credit level because in the text we focus on the financial shocks and for those, as discussed previously there is no difference between the basic and baseline Taylor rules.

# 4 Taylor Rules With Output Gap Adjustment

In this section we reevaluate the main results of the paper for the case in which we have an interest rate rule with adjustment to the output gap but no time-varying intercept (no

Table 9: Optimal value of the spread-adjustment coefficient $\phi_\omega$, in the case of non-financial disturbances, as in Table 3 of the main text, but for policy rule 4.2.

| $\phi_\omega^*$ | $\bar{C}_t^b$ | $\bar{C}_t^s$ | $G_t$ | $b_t^g$ | $Z_t, \bar{H}_t$ | $\mu_t^w$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|
| $\rho = 0.00$ | -0.82 | 1.73 | 11.74 | 0.60 | 10.27 | 16.41 | 14.12 |
| $\rho = 0.50$ | -1.00 | 1.13 | 6.96 | 0.71 | 6.54 | 12.09 | 11.73 |
| $\rho = 0.90$ | -1.21 | 0.27 | 1.19 | 0.62 | 1.15 | 13.03 | 13.02 |
| $\rho = 0.99$ | -1.80 | -1.37 | -1.18 | 0.16 | -1.18 | 21.77 | 21.78 |

adjustment to the natural rate of interest),

$$i_t^d = \phi_\pi \pi_t + \phi_y \left( \hat{Y}_t - \hat{Y}_t^n \right), \tag{4.1}$$

which we will be referring to as the Taylor rule with output gap. This policy rule is an interesting case to consider because often, when the Taylor rule is discussed, movements in the natural rate of interest are ignored or it is assumed that the natural rate of interest is constant. Hence it is worth considering the effects of such a rule for the analysis undertaken in the main text.

Let us consider generalizations of (4.1) of the form

$$i_t^d = \phi_\pi \pi_t + \phi_y \left( \hat{Y}_t - \hat{Y}_t^n \right) - \phi_\omega \hat{\omega}_t, \tag{4.2}$$

for some coefficient $0 \leq \phi_\omega \leq 1$. The analysis follows that of the Basic Taylor rule case: there is no reason to reevaluate the cases of disturbances that operate through changes in the size of distortions in the economy ($\chi_t$, $\tilde{\Xi}_t$, $\tau_t$, $\mu_t^w$ and $b_t^g$) because they do not affect the natural variables in any way.

Table 9 shows the equivalent to Table 3 in the main text but for the Taylor rule with output gap, as in 4.2. In the case of the baseline persistence level, $\rho = 0.9$, we get positive coefficients for all disturbances, except the shock to the borrowers expenditure. In the case of the productivity shock, the reason for the change in the sign of the coefficient is that, as shown in Figure 3, the Taylor rule with output gap implies lower output and inflation than the baseline one (even if less suboptimal than the basic Taylor) and because the spread increases, having some positive coefficient helps bringing the response of output closer to the optimal. In the case of a shock to the expenditure of savers (and similarly for the government expenditure) the response of output and inflation are actually too big, as shown in Figure 4, but because the spread falls in this case, then a positive coefficient $\phi_\omega$ is still helpful in

Table 10: Welfare consequences of increasing $\phi_\omega$, for different disturbances, as in Table 4 of the main text, but for policy rule 4.2.

| $\varphi \times 10^5$ | $\chi_t$ | $\tilde{\Xi}_t$ | $\bar{C}_t^b$ | $\bar{C}_t^s$ | $G_t$ | $b_t^g$ | $Z_t, \bar{H}_t$ | $\mu_t^w$ | $\tau_t$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline persistence ($\rho = 0.90$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 27.592 | 27.588 | -5.841 | 1.727 | 1.932 | 28.169 | 0.297 | 9.425 | 9.322 |
| $\phi_\omega = 0.50$ | 42.519 | 41.715 | -12.842 | 0.546 | 3.424 | 42.540 | 0.524 | 18.774 | 18.570 |
| $\phi_\omega = 0.75$ | 44.201 | 41.766 | -21.023 | -3.603 | 4.469 | 42.474 | 0.680 | 28.042 | 27.740 |
| $\phi_\omega = 1.00$ | 32.034 | 27.100 | -30.401 | -10.775 | 5.056 | 27.311 | 0.763 | 37.225 | 36.825 |
| High persistence ($\rho = 0.99$) | | | | | | | | | |
| $\phi_\omega = 0.25$ | 0.252 | 4.155 | -4.738 | -8.976 | -1.759 | 2.727 | -0.366 | 7.712 | 7.703 |
| $\phi_\omega = 0.50$ | -15.889 | -10.151 | -10.106 | -19.480 | -3.862 | -13.120 | -0.804 | 15.365 | 15.347 |
| $\phi_\omega = 0.75$ | -49.053 | -43.636 | -16.108 | -31.520 | -6.309 | -48.254 | -1.312 | 22.955 | 22.929 |
| $\phi_\omega = 1.00$ | -99.880 | -97.029 | -22.747 | -45.102 | -9.103 | -103.395 | -1.893 | 30.484 | 30.450 |

preventing the overheating of the economy. As in the main text all of the shocks imply that the optimal responses to the spread are very sensitive to the degree of persistency of the shocks.

In Table 10 we show the table with the welfare impact of alternative responses to the spread for the different disturbances, much like in Table 4 of the main text, but under the Taylor rule with output gap. The main difference, under the baseline persistence ($\rho = 0.90$), is that in this case the only shock for which even a mild response of $\phi_\omega = 0.25$ is welfare reducing is the case of a shock to the borrowers expenditure, which is in accordance to the discussion above. In the case of very high persistence the welfare impact of the responding to the spread is similar to the one shown in Table 4 for the baseline Taylor rule.

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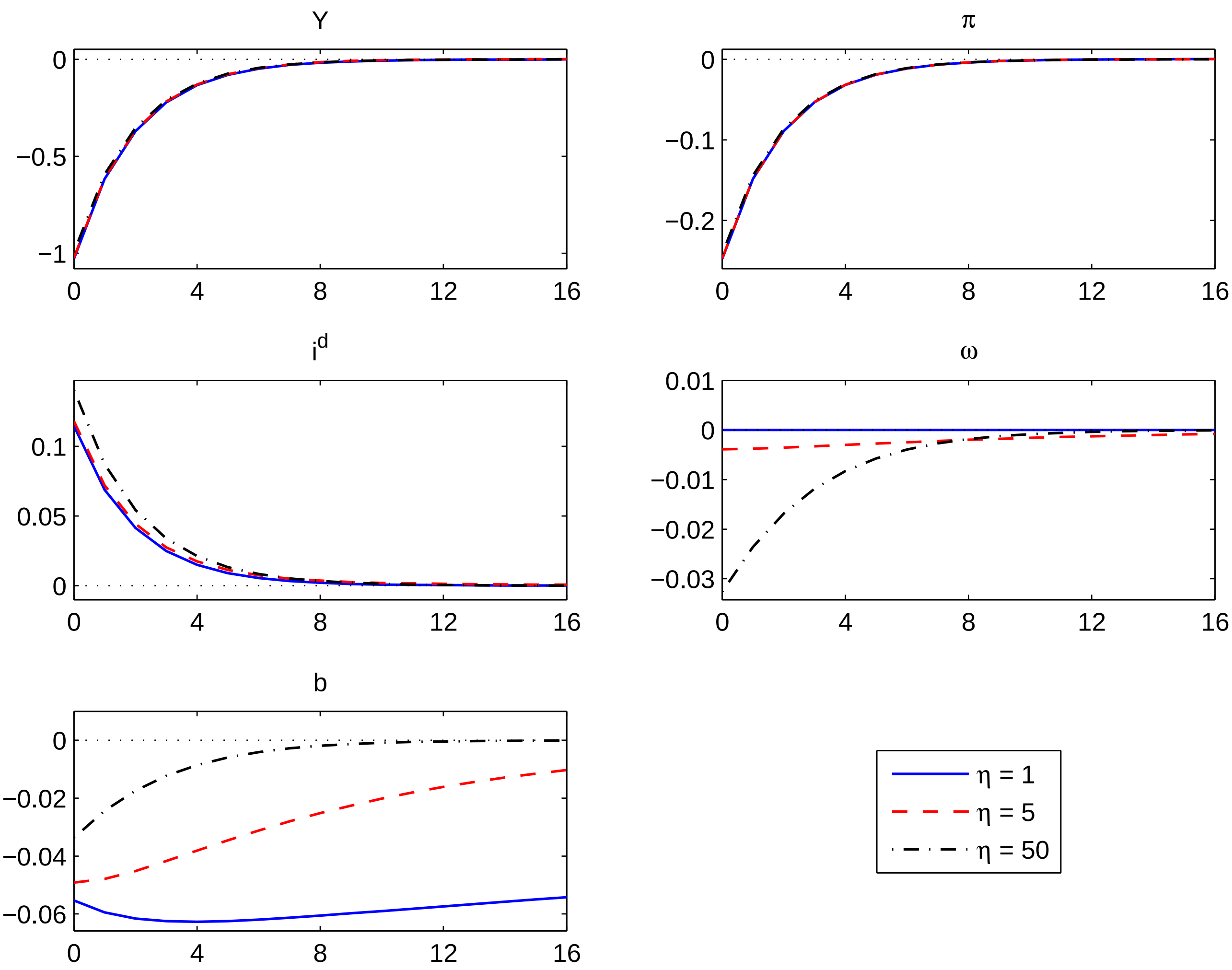


Figure 1: Impulse responses to a 1 percent shock to $\epsilon_t^m$, for different values of $\eta$, under the Taylor rule, for the case with $\sigma_b/\sigma_s = 5$.

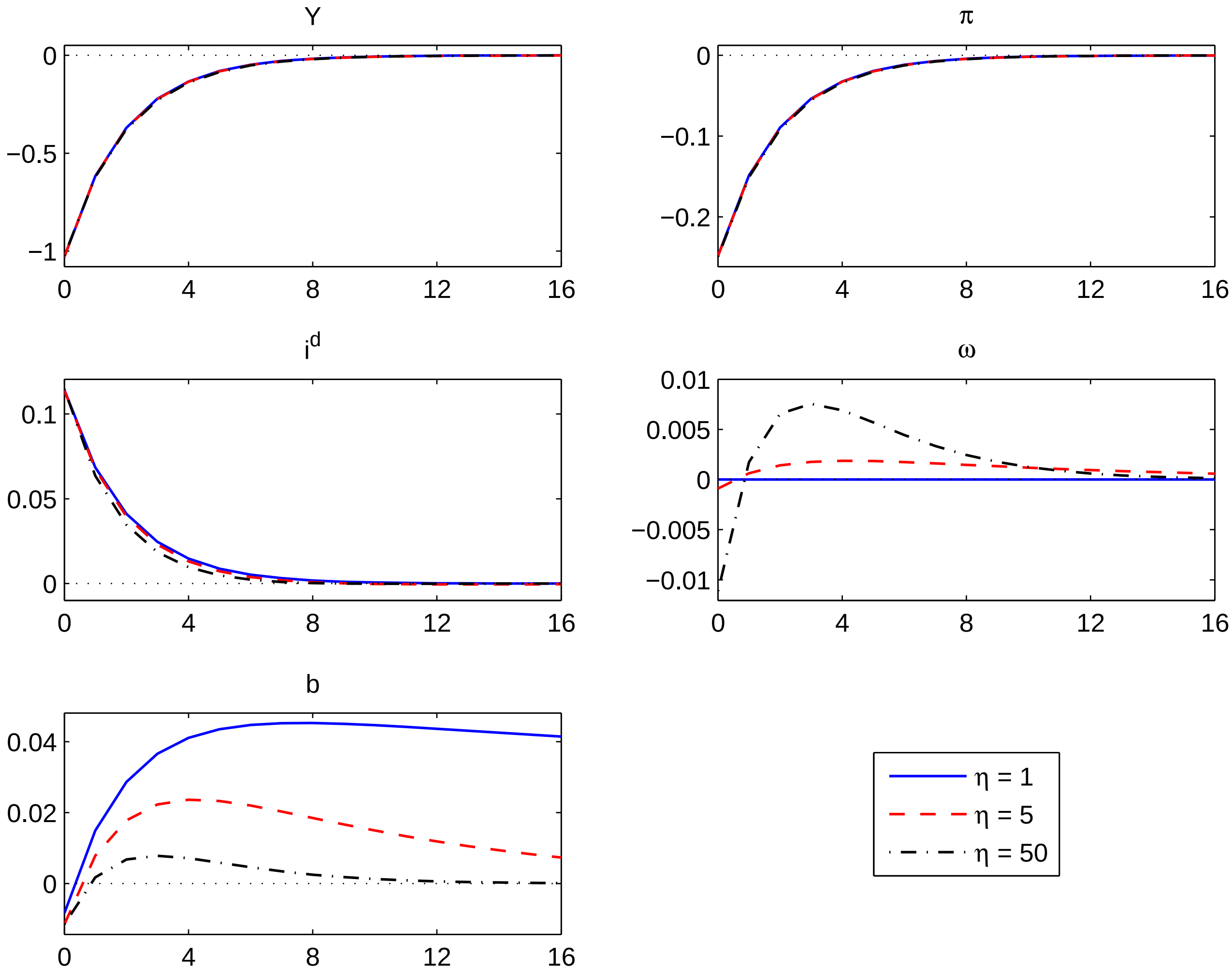


Figure 2: Impulse responses to a 1 percent shock to $\epsilon_t^m$, for different values of $\eta$, under the Taylor rule, for the case with $\sigma_b/\sigma_s = 2$.

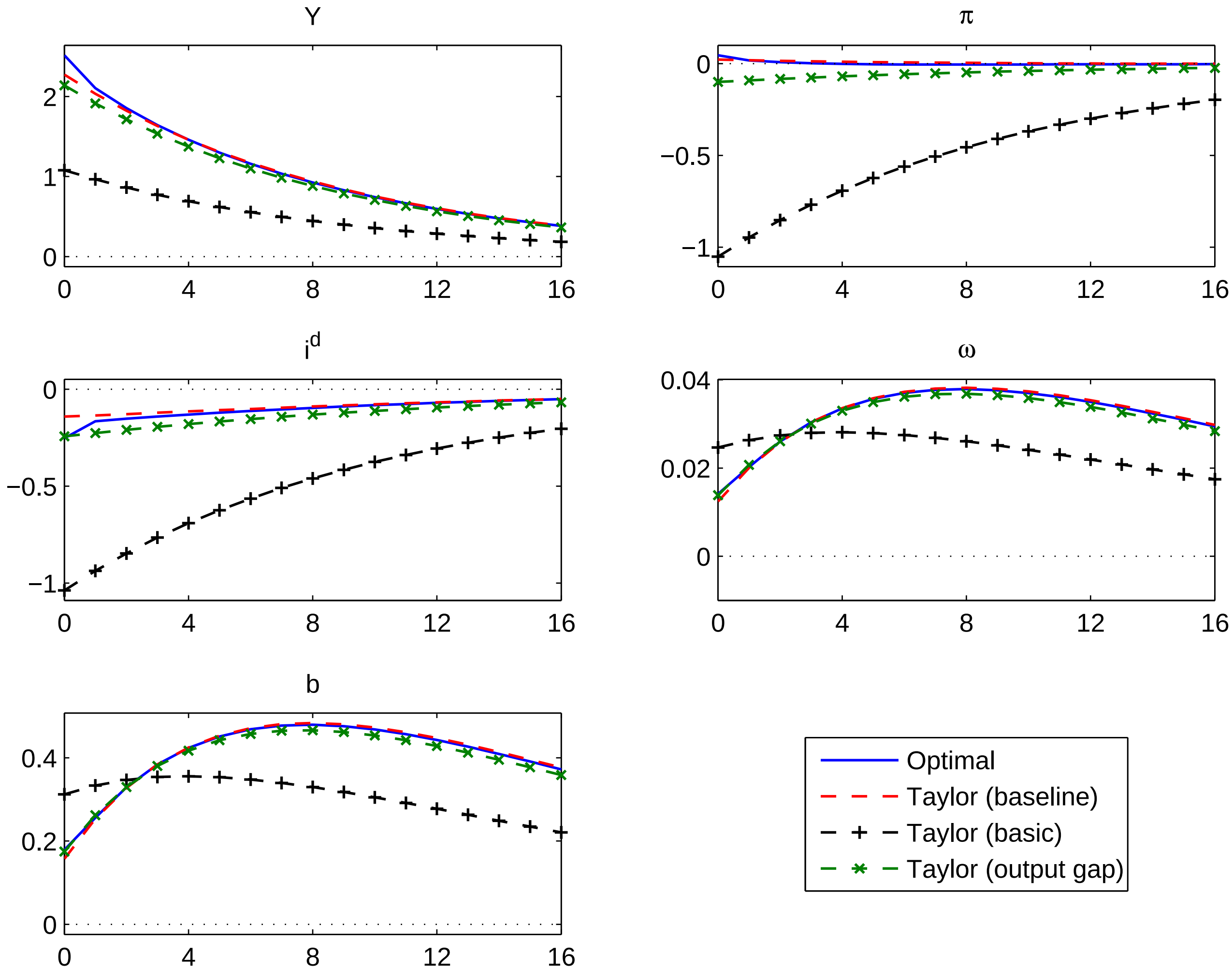


Figure 3: Impulse responses to a 1 percent shock to $Z_t$, for alternative specifications of the Taylor rule, for the case with $\sigma_b/\sigma_s = 5$, $\eta = 5$ and $\rho = 0.9$.

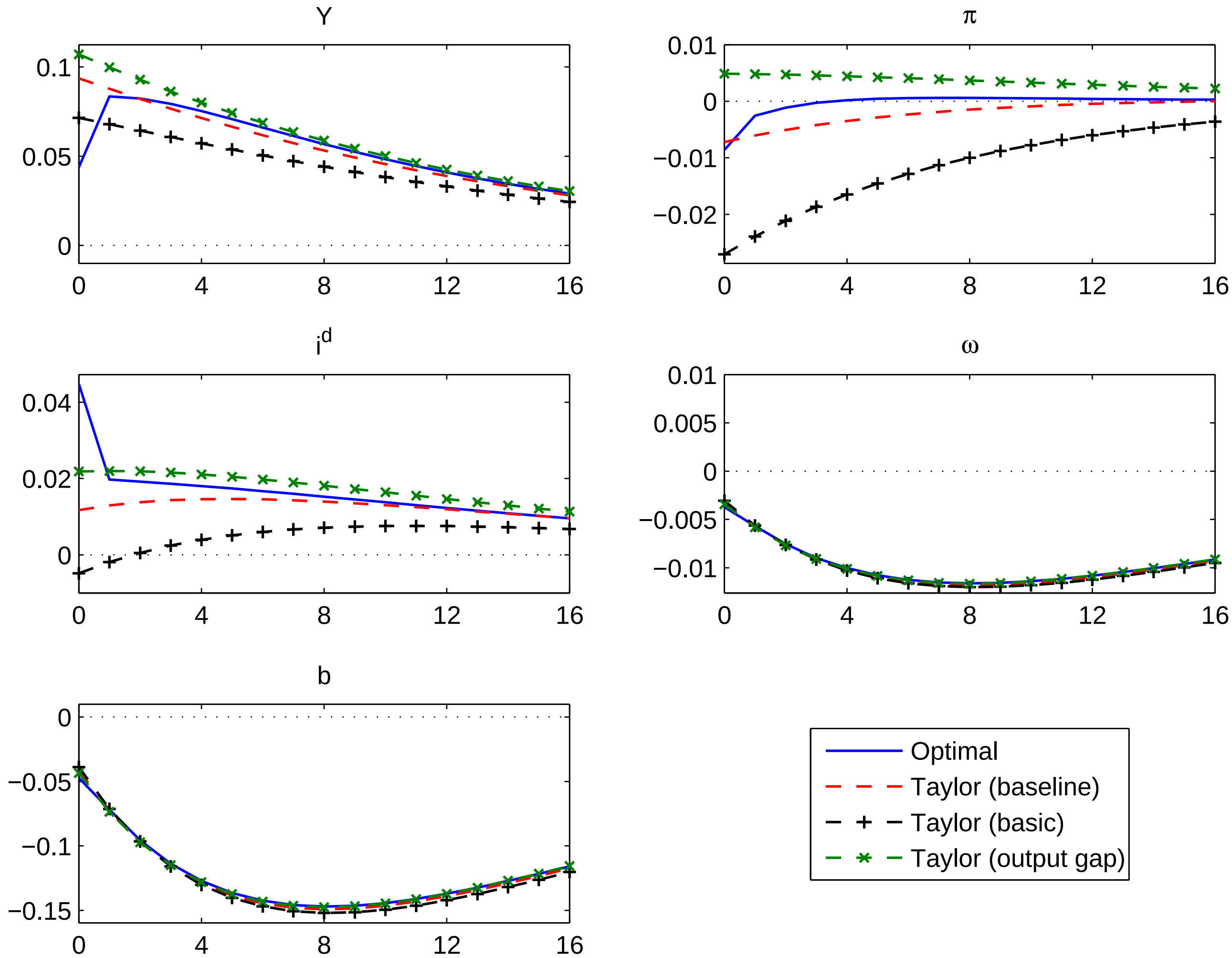


Figure 4: Impulse responses to a 1 percent shock to $\bar{C}_t^s$, for alternative specifications of the Taylor rule, for the case with $\sigma_b/\sigma_s = 5$, $\eta = 5$ and $\rho = 0.9$.

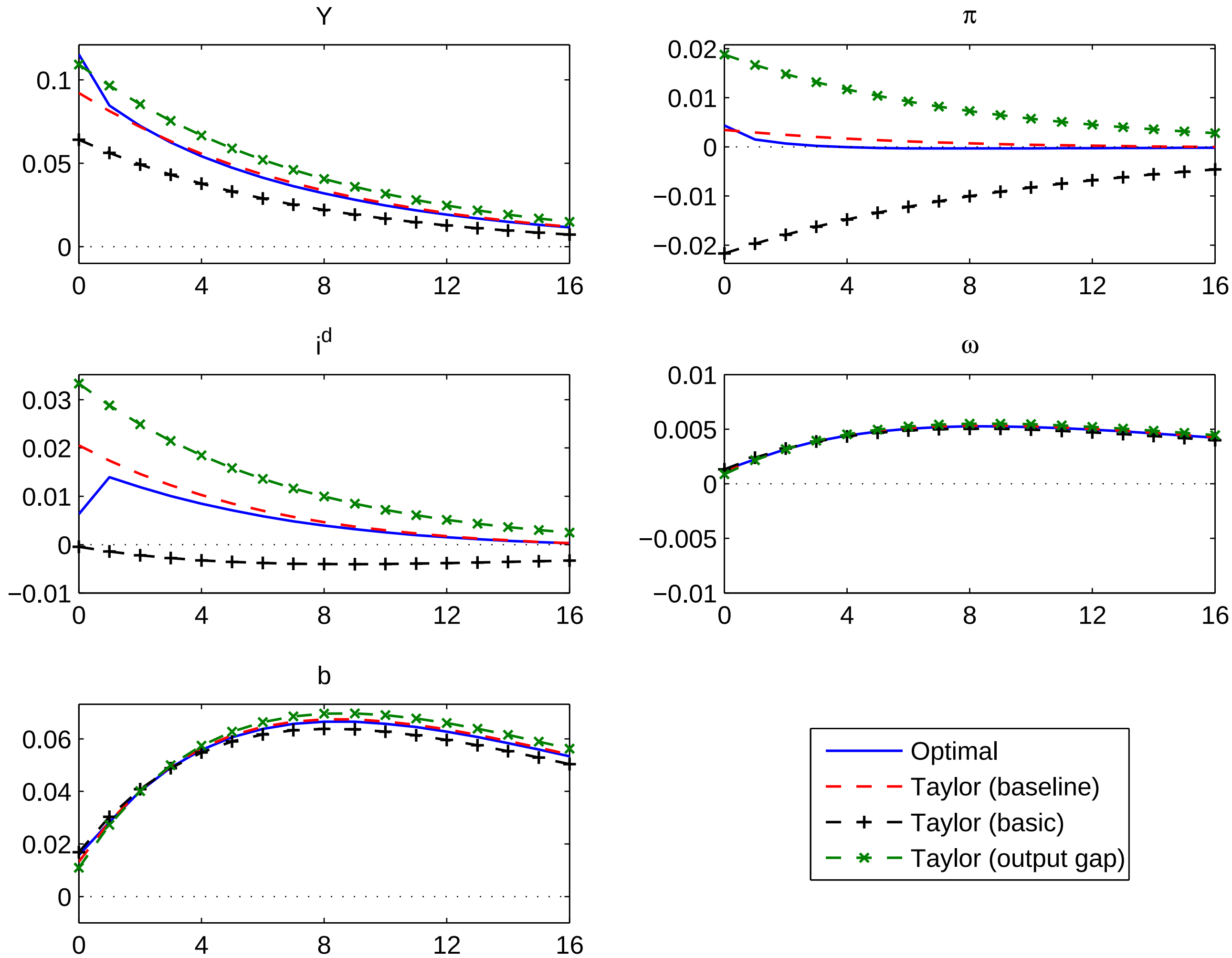


Figure 5: Impulse responses to a 1 percent shock to $\bar{C}_t^b$, for alternative specifications of the Taylor rule, for the case with $\sigma_b/\sigma_s = 5$, $\eta = 5$ and $\rho = 0.9$.