Notes on Dynamic Efficiency Wage Models

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These notes discuss how one might develop computationally (and to the extent possible, analytically) tractable general equilibrium models of the real determinants of employment and economic activity, with the aim of better understanding the impact of aggregate fluctuations on the labor market. The approach illustrated here differs from the dominant approach in recent applied general equilibrium analysis of aggregate fluctuations (i.e., "real business cycle theory") in two respects. First, the labor market is not modeled as clearing; unemployment is taken to be a real phenomenon (rather than an artifact of confused procedures on the part of government statistical offices), and the models are intended to explain the effects of various aggregate shocks on the unemployment rate. Second, the predictions of the models regarding the behavior of real wages are another central concern, as opposed to focussing solely upon the predicted fluctuations in quantities (and their ratios, such as average labor productivity). It is recognized that in the presence of multi-period contracts between firms and their employees, the relation between current average wage payments and the shadow price of labor that matters for current employment or participation decisions can be quite indirect (see section 3); but rather than concluding that data on wages are therefore irrelevant for testing the theory, the considerations that determine equilibrium wages are to be explicitly modeled.

Equilibrium unemployment is modeled as resulting from incentive problems in the labor market, in the fashion of "efficiency wage" theory (on which generally, see Calvo, 1979; Akerlof and Yellen, 1986; Katz, 1986; Blanchard and Fischer, 1989, sec. 9.4; Weiss, 1991; and Picard, 1993). I specifically focus upon the role of high real wages in providing an incentive for effort supply, as in the "shirking" model of Shapiro and Stiglitz (1984). Much of the efficiency wage literature has been partial equilibrium in character, and content with a static analysis of the determinants of a steady-state level of unemployment. My task here is to integrate the important insights of this literature with methods of dynamic general equilibrium analysis, in order to allow a complete analysis of the consequences of shocks for unemployment and wages.

The central issues that the models are intended to address are (1) the extent to which efficiency wage models can rationalize the relative "rigidity" of real wages observed over the business cycle, despite sizeable variations in employment, and (2) the extent to which such models are consistent with the observed relative trendlessness of the unemployment rate over the long run, despite trend growth in real wages. The first issue is central to importance of efficiency wages for macroeconomics, but has been little addressed, other than informally, in the literature, and remains a point of contention (see Katz, 1986, and subsequent discussion). The issue becomes an especially delicate one once the models are

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adapted to be able to address the second issue as well; for evidently, the short- and long-run effects of increases in labor demand must be different. An explicitly dynamic, general equilibrium analysis is therefore called for.

Several previous studies provide important starting points for this work. Danthine and Donaldson (1990) and Phelps (1994) incorporate efficiency wage considerations into complete dynamic general equilibrium models, but assume ad hoc and essentially static relations that determine effort supply as a function of an employee's wage and current labor market conditions, after the fashion of Solow (1979) and Akerlof (1982). I show here that explicit attention to the microeconomic foundations of the incentive effects of wages on effort can result in a more complex dynamic specification, with important consequences for the analysis of the short-run response to shocks. \(^1\) Shapiro and Stiglitz (1984) provide explicit microfoundations for the incentive effect, but characterize only an equilibrium steady state. Kimball (1993) analyzes equilibrium dynamics in the Shapiro-Stiglitz model, but abstracts from a number of issues that I intend to emphasize, such as wealth effects, changes in real interest rates, and the existence of advance commitments regarding wages. (His approach is discussed further below.) Wealth effects and interest rate changes are similarly ignored in the dynamic efficiency wage model of Mortensen (1989). This last model, like that of Phelps, also abstracts from firm-level diminishing returns to employment and from the effects of capital accumulation on labor demand.

Picard (1993, chap. 7) and Danthine and Donaldson (1994) provide complete dynamic general equilibrium analyses and are also quite careful in developing the microeconomic foundations of the incentive effects of wages. However, these authors render tractable the general equilibrium analysis by assuming overlapping generations of workers who live for only two consecutive "periods". This makes it difficult to "calibrate" the models for a numerical analysis of what the effects of particular shocks should be, unless one is to understand the "periods" to be quite long, in which case the model cannot be used to analyze dynamics at business cycle frequencies. Here, instead, I develop models in which all households are treated as infinite-lived, as the dynamics with which I am concerned are expected to largely work themselves out over time scales that are short relative to people's lifetimes.

1. Wealth Effects in an Efficiency Wage Model

An important limitation of the Shapiro-Stiglitz model is its failure to allow for any effect of household wealth in equilibrium wage determination. This omission is a consequence of a functional form assumption on preferences made for the sake of tractability. \(^2\) Yet there are a number of reasons to suspect that wealth effects have important consequences for unemployment dynamics. First of all, the Shapiro-Stiglitz model implies that secular growth in labor productivity should result in a secular decline in the unemployment rate,

\(^{1}\) See Woodford (1994) for criticism of the Phelps (1994) analysis along these lines.

\(^{2}\) The assumption of a constant marginal utility of income is also standard in the literature on labor market search, so that wealth effects are similarly abstracted from, even in general equilibrium models of aggregate fluctuations, such as Diamond (1982) and Mortensen (1990).
as the upward shift in the labor demand curve \((LD)\) over time causes the equilibrium point to shift up the efficiency wage schedule \((EW)\), as shown in Figure 1. ³ Yet this has not been observed, despite substantial growth in real wages over the past 150 years; instead, the unemployment rate has been essentially trendless (Layard, Nickell, and Jackman, 1991, chap. 1). This undesirable feature of the Shapiro-Stiglitz model results from the fact that the quantity of wages assumed to contribute as much to utility as does shirking does not change with growth of income over time. The result can be avoided, even with a constant disutility of effort, if one abandons the Shapiro-Stiglitz assumption of a constant marginal utility of income, as I propose. If the marginal utility of income falls as income rises, the reservation wage rises over time, and with it the wage required to deter shirking. ⁴

³ On the vertical axis, \(w\) represents the real wage, while on the horizontal axis, \(N\) represents employment; \(L\) is the size of the labor force. The \(EW\) schedule indicates the efficiency wage that each firm will choose to pay, as a function of the overall current level of employment \(N\); this is the "no shirking condition" of Shapiro and Stiglitz. In a growth model that incorporates this model of the labor market, the curve \(LD\) shifts up over time, due both to technical progress and to capital accumulation. As a result, equilibrium unemployment falls, asymptotically approaching zero.

⁴ Phelps (1994a) obtains an upward shift in \(EW\) in proportion to the growth in labor productivity by assuming that employees supply effort as an increasing function, given labor market conditions, not of \(w\), but rather of \(w/y\), where \(y\) represents non-wage income per capita (idenfied in his model with per capita profits). This is in spirit of the proposal here, but has less explicit microeconomic foundations (as is discussed further in Woodford, 1994).
Second, if the LD schedule is interpreted as the marginal productivity of labor schedule derived from an aggregate production function, the Shapiro-Stiglitz model implies that a change in the level of government purchases should have no effect on employment or output in the short run (and that government purchases should be contractionary in the medium run, due to crowding out of private investment, that eventually lowers labor productivity). This is because neither the LD nor the EW schedule is affected by government purchases. But this too depends upon the assumption of a constant marginal utility of income. For the effects of changes in the marginal utility of income upon labor supply play a critical role in the neoclassical theory of the effects of fiscal policy (Barro, 1989). This is also potentially an important channel for the effects of fiscal policy in an efficiency wage model, and indeed is emphasized in the account of Picard (1993, chap. 6).

Finally, the assumption of a constant marginal utility of income implies that households have no desire to insure against the income risk associated with employment risk. This is surely an unfortunate simplification if one intends to use the model for welfare analysis of unemployment policies. It also eliminates the possibility of endogenizing the level of unemployment benefits (as is done for example in Danthine, Donaldson, and Mehra, 1992, a topic discussed further in the next section).

I therefore propose to generalize the preferences assumed by Shapiro and Stiglitz, by assuming that utility each period for household i is given by \( u^i_t = u(c^i_t) - d e^i_t \), where \( c^i_t \) is consumption at date t, \( e^i_t \) is work effort (that takes a value 0 or 1) in period t, and \( d > 0 \) is the disutility of effort. The length of the workweek is assumed to be fixed, and each household each period is either employed for a full shift or is unemployed for the entire period. Households are infinitely-lived, and seek to maximize

\[
E\{\sum_{t=0}^{\infty} \beta^t u^i_t\},
\]

where \( 0 < \beta < 1 \) is a constant discount factor. In order to obtain a balanced growth path with a constant unemployment rate, it is necessary to assume \( u(c) = \log c \), so that the marginal utility of income falls in exactly inverse proportion to the growth of wages and of income and consumption per capita. This contrasts with the Shapiro-Stiglitz assumption of \( u(c) = c \).

An immediate difficulty in the case of concave utility is that households will have different reservation wages if their wealths differ, so that not only aggregate wealth but the distribution of wealth will matter in general for labor market equilibrium. And in a model with unemployment (unlike, say, the neoclassical growth model with endogenous labor supply), together with frictionless financial markets (unlike the model of the next section), households necessarily differ in equilibrium in their employment histories, and hence in general will differ in their current wealth. One way of dealing with this is to assume the existence of perfect insurance against employment risk, as in the model of a

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5 This is also an important reason for the popularity of log utility in the real business cycle literature, where the trendlessness of hours worked per capita despite secular growth in output per capita is cited as a desideratum in model calibration (e.g., King, Plosser and Rebelo, 1988).
competitive labor market with indivisible labor of Hansen (1985) and Rogerson (1988). The idea is to construct an equilibrium in which the wealths of all households remain identical, despite their differing employment histories. In the present case there is an additional complication, not present in the Hansen-Rogerson model, that one must consider the effect of unemployment insurance upon the incentive to shirk. However, with the kind of imperfect monitoring assumed by Shapiro and Stiglitz (i.e., a shirking worker is caught with a probability \( p < 1 \), but a non-shirker is never thought by mistake to have shirked), it is reasonable to assume that the private insurance does not compensate employees who are fired because of being caught shirking (a risk that no one faces in equilibrium, as all employed workers choose \( e = 1 \)), but only those who are not chosen for employment when jobs are rationed.

Suppose, for simplicity, that firms hire workers for a single period at a time, with an independent random drawing of those who will be employed each period, so that past employment history has no effect upon one's probability of employment in the current period. Workers are promised a wage for the period's work at the time of being hired, that may be conditional upon whether they are caught shirking during the period. (One may suppose that the wages are paid at the end of the period.) The wage cannot be conditioned upon the worker's actual effort level \( e_t \), as this is not observed by the employer; it must be non-negative in any event, as the worker is assumed to have the option of walking away from the job without any of his other assets being seized; and it is assumed that workers not caught shirking cannot be rewarded through differential future treatment in the labor market, due to the assumption of single-period contracting.  

It is easily shown that the equilibrium contract involves payment of no wage in the event that the worker is caught shirking (as this provides the greatest possible incentive for effort supply, and does not effect the employee's level of expected utility in equilibrium). Thus the equilibrium contract in period \( t \) is fully described by the real wage, \( w_t \), paid to a worker who is not caught shirking, that may of course be contingent upon the aggregate state. After the terms of employment that will be offered in period \( t \) are announced by the firms, workers declare themselves willing to accept employment on those terms or not, and those seeking employment may insure one another against the risk of failing to be employed. If \( \pi_t \) is the probability that a job-seeker is employed, then as employment risk is fully diversifiable (assuming here a continuum of identical households), it will be possible for a job-seeker to obtain a payment of \( \pi_t b \) contingent upon not being employed, in exchange for a promise to pay \( (1 - \pi_t) b \) if he is, for an arbitrary quantity of insurance \( b \).

Now since each household faces identical future employment prospects, there is at each date a value function \( v_t(W) \), indicating the maximum attainable present value of the expected utility attainable from period \( t + 1 \) onward, as a function of the household's wealth \( W \) at the end of period \( t \), that is the same for all households (but dependent upon the aggregate state). Under the assumption of log utility, this function can furthermore be shown to be of the form

\[
v_t(W) = \frac{\beta}{1 - \beta} \log(W + H_t) + \xi_t,
\]

where \( \xi_t \) is a random variable capturing the aggregate risk. \( W \) is the wealth and \( H_t \) is the human capital of the household at time \( t \).

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6 The additional incentives that are possible in the case of multi-period employment relations are discussed in section 3.
where \( H_t \) represents the present value of expected future wage income ("human wealth"). The decision problem of household \( i \) in period \( t \) then reduces to the maximization of the expected value of
\[
  u(c_t^i) - de_t^i + v_t(y_t^i - c_t^i)
\]
where \( y_t^i \) is equal to \( A_t^i + w_t - (1 - \pi_t)b_t^i \) if the household chooses to work, is employed, and is not caught shirking; is equal to \( A_t^i - (1 - \pi_t)b_t^i \) if the household chooses to work, is employed, but is caught shirking; is equal to \( A_t^i + \pi_t b_t^i \) if the household chooses to work but is not employed; and is equal to \( A_t^i \) if the household chooses not to seek work. Here \( A_t^i \) denotes the value of the household's assets (non-human wealth) at the beginning of period \( t \), and \( b_t^i \) is the quantity of insurance purchased if the household seeks work.

Given (1), it is clear that optimal consumption (chosen after the wage payment is received) will satisfy
\[
c_t^i = (1 - \beta)(y_t^i + H_t),
\]
and so that expected lifetime utility (counting period \( t \) utility) will equal an expression of the form
\[
\frac{1}{1 - \beta} \log(y_t^i + H_t) - de_t^i + \xi_t^i.
\]

In seeking to maximize this expression, each household will offer itself for employment regardless of the wages promised, since it is always possible to buy no insurance and to plan to supply no effort. It will plan to supply effort (choose \( e = 1 \)) if and only if
\[
\max_b \{ U(H_t + A_t^i, w_t, w_t, b; \pi_t) \} \geq \max_b \{ U(H_t + A_t^i, w_t, 0, b; \pi_t) \} + (1 - \beta)\pi_t d,
\]
where
\[
U(X, w, w', b; \pi) \equiv
\]
\[
(1 - \pi) \log(X + \pi b) + \pi (1 - p) \log(X + w - (1 - \pi)b) + \pi p \log(X + w' - (1 - \pi)b).
\]

When the "no shirking condition" (3) is satisfied, the household does not plan to shirk if employed, in which case one sees that the optimal level of insurance is given by full insurance, \( b_t^i = w_t \). It is shown below that the equilibrium contract satisfies this condition. As a result of the full insurance, households that begin the period with a common level of assets \( A_t^i = A_t \) also end the period with a common level of wealth, independently of whether they are employed, equal to \( \beta(A_t + H_t + \pi_t w_t) \). Households with identical preferences over future consumption streams then choose identical portfolios, and so begin the following period with identical assets \( A_{t+1}^i \) once more. Thus an equilibrium is obtained in which all households have identical wealth at all times.

Each firm in period \( t \) chooses its wage offer \( w_t \) and the number of employees \( N_t \) to hire, so as to maximize its profits, given by
\[
\Pi_t = F(K_t, z_t e_t N_t) - (1 - p + pe_t) w_t N_t,
\]
where \( F \) is a homogeneous degree one production function, \( K_t \) is the capital stock, and the exogenous factor \( z_t \) represents labor-augmenting technical progress, and \( e_t \) represents
the fraction of the firm's employees who choose to supply effort (a predictable consequence of the terms of employment offered). The firm takes as given the overall degree of labor market tightness, summarized by \( \pi_t \), in making its own plans; this matters because it will affect the effort decision of its employees, as indicated in (3).

It is evident that a firm has no reason to offer any contracts with terms such that employees will choose to shirk. Thus the firm chooses its offers to maximize the objective function (4), with \( e_t \) set equal to 1, but subject to the additional constraint (3), which is the same for all households.) This clearly involves setting \( w_t \) at the lowest level that satisfies (3), giving a result of the form

\[
 w_t = \theta(\pi_t)(H_t + A_t), \tag{5}
\]

where \( \theta(\pi) \) is a continuous function, and \( A_t \) is the common level of household assets. Note that this efficiency wage is independent of the number of employees that the firm wishes to hire, and does not depend upon any properties of the production technology (except the assumption that employees who shirk do not increase output).

The variable \( \pi_t \) in (5) depends upon overall labor market conditions through the relation \( \pi_t = N_t / L \), where here \( N_t \) refers to aggregate employment, taken as given by each firm in making its own hiring decision. Substituting this into (5) one obtains an equilibrium relation between the real wage and aggregate employment,

\[
 w_t = \theta(N_t / L)(H_t + A_t). \tag{6}
\]

Given this efficiency wage, each firm chooses a level of employment such that

\[
 w_t = z_t F_2(K_t, z_t N_t). \tag{7}
\]

Note that (6) is a relation like \( EW \) in Figure 1 (though the wage remains finite as \( N \to L \)), while (7) is identical to the relation graphed as \( LD \). However, unlike the \( EW \) relation in the Shapiro-Stiglitz model, (6) is valid not only in steady state, but in each period, regardless of the stochastic process that may be followed by the technology factor \( \{z_t\} \), by government purchases, and so on. Also, the efficiency wage associated with a given level of employment shifts up in proportion to the growth of total wealth \( H_t + A_t \); this is what makes possible a balanced growth path along which unemployment is trendless. (In such a steady state, \( z_t, K_t, w_t, H_t, \) and \( A_t \) all grow at the same rate.)

This provides an equilibrium model of unemployment that can easily be integrated into a complete intertemporal general equilibrium framework, by adjoining standard equations for capital accumulation and asset pricing in a model where, insofar as consumption, saving and portfolio decisions are concerned, all households behave identically. In fact, the equations of the model turn out to be similar to those of a neoclassical growth model with endogenous labor supply. In particular, one can show that in the limiting case of \( \beta \) approaching 1 (i.e., the case in which households are very patient, relative to the length of the "period" between successive labor markets), in the efficiency wage model

\[
(1 - \beta)^{-1} \theta(\pi) \to d/p,
\]
a constant function. Thus in this limit, the efficiency wage schedule becomes completely horizontal, up to the point of full employment. In this case, (6) has exactly the form of the neoclassical labor supply curve, for an economy made up of identical infinite-lived households, each of which determines each period a quantity of consumption $c_i^t$ to divide among its members, and a fraction $n_i^t$ of members of the household who will work, so as to maximize expected discounted utility as above, with a single-period utility function of the form

$$u_i^t = \log c_i^t - \nu n_i^t,$$

where $\nu > 0$ indicates each member's disutility of work. In such an economy, competitive labor supply implies a wage relation of the form (6), where now

$$\theta(\pi) \equiv \frac{(1 - \beta)\nu}{1 - (1 - \beta)\nu\pi},$$

and $\pi_t = N_t/L$ is a choice variable for each household. Here too $(1 - \beta)^{-1}\theta(\pi)$ approaches a positive constant, $\nu$, in the high-patience limit.

Thus, in this limit, the efficiency wage model makes identical predictions to those of a neoclassical growth model with a linear disutility of work – which also imples, to those of the Hansen (1985) model with indivisible labor. The efficiency wage model is thus able to predict significant variations in equilibrium unemployment in response to shocks; indeed, in this limiting case, it predicts as much variation in employment, relative to the variation in real wages as does the neoclassical growth model in the preference specification that is most favorable toward employment fluctuations, i.e., the case of no increasing disutility of work. This indicates that the pessimism of Danthine and Donaldson (1990) as to the helpfulness of an efficiency wage model in accounting for the relative variability of employment and wages (or productivity) is not justified.

It might be felt that if the model's predictions are the same as those of the Hansen model, then allowing for efficiency wages brings no increase in the ability of theory to account for the variability of employment, relative to what a simpler model could achieve. However, it may be argued that this reinterpretation of the structural equations of the Hansen model makes them more empirically plausible. The near-insensitivity of the real wage to the current level of employment, given current household wealth, is here obtained without having to postulate preferences under which households have a high intertemporal elasticity of substitution of leisure – as in the representative household model with a linear disutility of work, a theoretical possibility that does not fit the evidence of microeconomic studies of intertemporal labor supply (see, e.g., Card, 1994). (In the model proposed here, households do not vary at all the length of their desired workweek in response to interest rate changes or expected future growth of real wages.) Similarly, the "indivisibility" of labor that is relied upon in the Hansen-Rogerson account is better motivated; employers prefer to hire a single worker for the entire period, rather than hiring several part-time, because of the greater incentive that the single worker has not to shirk. And finally, the highly elastic "wage curve" is obtained without one's having to postulate that the typical employed worker is indifferent between participating in the work force and dropping out, at the level of real wages currently received.
The alternative account given here also implies a different welfare evaluation of policies that change the equilibrium level of unemployment. Since in this model, unlike the Hansen model, the marginal utility product of labor exceeds the disutility of working in equilibrium, it follows that an employment subsidy raises the welfare of the representative household, if it can be financed in a way that does not create too large of other distortions. (This contrasts with the result for the competitive model, where such a subsidy reduces welfare even if financed through lump-sum taxation.) Thus the availability of the alternative interpretation of the wage-unemployment relation is of no small importance for policy analysis, even if the predicted character of fluctuations in wages and unemployment is essentially the same.

2. Consequences of Credit-Constrained Workers

The model of the previous section still fails (as does the Hansen (1985) competitive model) to explain the relative acyclicalty of real wages, in the following respect. The model implies (in the limiting case of patient households) that the efficiency wage is essentially independent of the unemployment rate, given the current wealth level $A_t + H_t$. But it also implies that the real wage should be procyclical to the extent that aggregate wealth is, and given (2), one can infer the procyclicalty of this latter variable from the observed procyclicalty of consumption spending (technically, non-durable consumption). The model thus implies that the real wage should be as pro-cyclical as is aggregate consumption. But this is not observed, and this problem has been much discussed as a defect of representative-household models of competitive labor supply (e.g., Barro and King, 1984; Mankiw, Rotemberg and Summers, 1985).

A possible solution to this difficulty is to suppose that the marginal utilities of income of households are not all equal, and thus not all to be inferred from aggregate consumption. This requires that we abandon the convenient assumption of insurance against employment risk, used in the previous section. Another case in which general equilibrium analysis is tractable, without assuming a constant marginal utility of income, is that in which workers have no access whatsoever to asset or credit markets of any kind, so that they consume their wage income immediately. This assumption, common in the efficiency wage and implicit contract literatures alike, has the consequence that all employed workers have the same level of wealth in a given period, in the absence of any need for employment insurance, and so without the counter-factual consequence that unemployed workers have the same level of consumption as do the employed. 7 It is also known that allowing for a fraction of the population to be constrained to consume its current income improves the fit of Euler equations for aggregate consumption (e.g., Hall and Mishkin, 1982; Campbell and Mankiw, 1989, 1990; Beaudry and Van Wincoop, 1992), and can help to explain the “equity premium puzzle” (e.g., Mankiw, 1986). Thus the use of a similar assumption to explain the failure of the relation between wages and the labor input implied by a representative household (or full insurance) model has some appeal.

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7 It is not actually necessary to assume that workers are unable to save. It suffices that they be unable to borrow against future wage income, and that their rate of time preference be higher than that of the asset holders. See Woodford (1988).
Given that workers hold no assets, it is necessary to specify a source of non-wage income for the unemployed, so that the utility of this state is defined. I will therefore assume a real level of unemployment benefits $b_t > 0$ in period $t$; this is for the moment treated as exogenous. (It could alternatively represent the productivity of an activity other than employment.) An employed worker therefore has consumption level $c_t^e = w_t$, while an unemployed worker has $c_t^u = b_t$. One-period labor contracts are assumed as in the previous section, and worker preferences, the production technology, and the monitoring technology are all as before. As before, the optimal contract involves paying a wage only to workers not caught shirking. An employed worker then chooses to supply effort if and only if

$$u(w_t) - d \geq pu(b_t) + (1 - p)u(w_t).$$

Each firm accordingly pays the lowest wage that satisfies this "no-shirking constraint" - which wage is the same for all workers - so that in equilibrium (again assuming log utility for the workers)

$$\log w_t = \log b_t + d/p.$$  \hspace{1cm} (8)

Given this efficiency wage, labor demand is again determined by (7). Equations (7) and (8) together determine employment and the real wage, given the capital stock, the state of technology, and the level of unemployment benefits.

Embedding this model of the labor market into a general equilibrium framework requires the introduction of a second type of household, the asset holders, who own the capital stock. For simplicity we may assume that they do not work, and simply consume out of their wealth. Suppose that these households seek to maximize

$$E\{\sum_{t=0}^{\infty} \gamma^t \log c_t^a\},$$

where $c_t^a$ denotes the consumption of a representative asset holder, and $0 < \gamma < 1$ is a discount factor. (One may wish to assume $\beta < \gamma$ to explain why workers hold no assets.) Then one obtains as above

$$c_t^a = (1 - \gamma)(A_t - T_t),$$  \hspace{1cm} (9)

$$K_{t+1} = \gamma A_t + (1 - \gamma)T_t,$$  \hspace{1cm} (10)

where $A_t$ denotes household assets at the beginning of the period (after date $t$ tax liabilities), and $T_t$ denotes the present value of future tax liabilities, from date $t + 1$ onward (understood to be lump-sum). The saving equation (10) describes the evolution of the aggregate capital stock, since in equilibrium the aggregate savings of the asset holders correspond to exactly the value of the aggregate capital stock. The lump-sum tax levied upon

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8 Here I assume for simplicity that the unemployment benefits are financed exclusively through taxation of the asset holders, rather than the employed workers. Results are qualitatively similar in the case of a tax on employed workers, as long as the tax rate does not vary cyclically in order to allow balanced-budget financing of the unemployment benefits. The latter possibility can result in multiple equilibria, as in Blanchard and Summers (1987).
asset holders each period is assumed to be the amount needed to finance the unemployment benefits, namely \( b_t(L - N_t) \). Beginning-of-period assets are then given by

\[
A_t = F(K_t, z_t N_t) - w_t N_t + (1 - \delta) K_t - b_t(L - N_t),
\]

where \( \delta \) is the rate of depreciation of the capital stock. The present value of tax liabilities is given by

\[
T_t = q_t^{-1} \sum_{j=1}^{\infty} E_t[q_{t+j} b_{t+j}(L - N_{t+j})],
\]

where present values are defined using the intertemporal marginal rates of substitution of the representative asset holder,

\[
q_t = \gamma^t [c_t^a]^{-1}.
\]

Equations (7)–(13) then comprise a complete system of equations to determine the evolution of the variables \( \{w_t, N_t, K_t, c_t^a, A_t, T_t, q_t\} \).

It will be of interest to study the responses of this system to a variety of shocks. The system can easily be analyzed using the linearization techniques illustrated in King, Plosser and Rebelo (1988) or in [3]. It is clear, however, that real wages can easily be less procyclical than is aggregate consumption. Consider, for example, the short-run effects of an increase in the technology factor \( z_t \), assuming no immediate change in \( b_t \) and given the predetermined capital stock \( K_t \). Total consumption is the sum of consumption by asset holders, by employed workers, and by unemployed workers, or

\[
C_t = c_t^a + w_t N_t + b_t(L - N_t).
\]

Since neither \( w_t \) nor \( b_t \) changes, and \( w_t > b_t \) by (8), the increase in \( N_t \) increases total consumption by workers. Given that the real wage does not change, returns to capital necessarily increase; and the tax burden falls, so \( A_t \) increases. Thus in general \( c_t^a \) increases as well. Thus aggregate consumption and employment are both procyclical, while real wages do not change at all. The explanation, of course, is that the consumption of each employed worker does not change, even though aggregate consumption increases.

Such a model easily generates real wage rigidity over the business cycle (if unemployment benefits \( b_t \) are assumed to be acyclical). On the other hand, in order to account also for the trendlessness of unemployment, one needs for \( b_t \) to have the same trend growth rate as \( z_t \). One might simply postulate that the productivity of time in the household sector grows at the same trend rate as productivity at work, as is common in growth models with "household production" (e.g., Greenwood, Rogerson, and Wright, 1994). Probably more appealing is to explicitly model the political determination of the level of unemployment benefits, as in Danthine, Donaldson, and Mehra (1992). This plausibly leads to a decision to keep the marginal utility of income of the unemployed, on average, in a certain proportion to that of the employed; this will be taken up in future work.

The assumption of no borrowing or lending by workers is obviously extreme. One way of relaxing this, with little loss in tractability, is to assume that a fraction \( \phi \) of workers are credit-constrained, as above, while the remaining \( 1 - \phi \) not only trade in financial markets,
but insure one another against employment risk as in section 1. (There need not in this case be any category of non-working asset holders.) Then wealth will differ across the two types of households, but all households of a given type will have identical wealth, and so identical no-shirking conditions. If one supposes that the two types of households supply labor of two distinct kinds, then there can be a single real wage in each market. Evidence on the fraction of total consumption that represents spending by credit-constrained households (from Euler-equation studies of the kind mentioned above) could then be used to calibrate \( \phi \).

3. Consequences of Multi-Period Wage Contracting

In a sense, both of the models just sketched predict too much "real wage rigidity." For both predict that the efficiency wage should be relatively insensitive to current labor market conditions; in particular, shocks that have only a transitory effect on labor demand (and so have little effect on \( H_t \) in the first model) are predicted in both models to change equilibrium employment with little effect on the real wage. But many would interpret the considerable evidence for a negative effect of the unemployment rate on the wages of employed workers (e.g., Blanchflower and Oswald, 1994) as evidence that the efficiency wage locus (or more generally, the "wage-setting curve" in the sense of Layard, Nickell and Jackman, 1991, and Lindbeck, 1993) should be upward-sloping, and much of the efficiency wage literature stresses the ability of such models to rationalize such a "wage curve". Certainly the models sketched above do not capture the (Marx-) Shapiro-Stiglitz idea that unemployment exists because it is "a worker discipline device"; the current unemployment rate has no incentive effect, in those models, because employment terminates at the end of the period in any event, and the only loss suffered by a worker caught shirking is loss of the current period's wage.

It is obviously more realistic to allow for employment relations that persist, so that workers who lose their job can expect a lower probability of being employed in the immediate future as a result of this. In this section, I propose to emphasize this new incentive for effort supply exclusively, by assuming that an employer who catches a worker shirking cannot withhold his current period's wage, but can only terminate his employment at the end of the period. (This is, perhaps, more consistent in any event with the idea that the firm cannot seize any assets of the worker.)

This modification has an important consequence for the structure of the theory: now high wages provide no incentive for effort supply, except insofar as they represent a commitment to pay high wages in the future. One can no longer assume that firms commit themselves each period only to the current period's wage. For in that case, a higher current wage would not deter shirking; the current period's wage is collected even if one is caught, and the wage expected in the future (if one is still employed) is whatever it will be optimal for the employer to pay then, independent of the wage paid now. But then an employer has no reason to pay a higher current wage than is required to induce workers to accept employment, which is to say, any positive wage at all. Thus the equilibrium wage will be zero, and as this is foreseeable the case, no employee expects a positive wage in the future either, and all employees shirk. Thus in such an equilibrium, no production is possible.
Kimball (1993) avoids this conclusion, despite his use of a continuous-time framework (as in Shapiro and Stiglitz). For his continuous-time model does not represent the infinitesimal-period-length limit of a model in which the only penalty is termination; rather, it is the limit of a model in which the current period's wage is also withheld. The difference matters, even in the limit of an infinitesimal period length, because effort is a discrete choice variable, and in equilibrium each employed worker is poised at the point of indifference between supplying zero effort and full effort. Thus the loss of the current period's wage still matters, even when the period is extremely short: it is the prospect of losing this small amount, in addition to the future wages, that makes an employee willing to supply effort, and if the current wage were any smaller he would shirk instead. The result is plainly a fragile one. If, instead, effort were a continuous choice variable, and equilibrium involved an interior choice, then variations in the current period wage should in any event make little difference to the effort decision if the period is short; thus in the infinitesimal-period-length limit, an employer should come to have no reason to keep wages high, even if he can withhold the current period wage of a shirker who is caught. Thus the assumption proposed here would seem to be the more attractive one, if one thinks of the "period" as short (i.e., if wages are paid frequently).

As a first approach to the problem, I propose to investigate the consequences of assuming that multi-period contracts must take a special form, that is intended to describe, in a stylized way, observed labor market practices, and to allow construction of an equilibrium that is in the spirit of (if slightly different from) that of Shapiro and Stiglitz, in order to analyze the dynamics of such a model. Specifically, I assume that the only enforceable wage commitment, made prior to the period to which it applies, is a commitment under which the employee's real wage each period is the previous period's real wage multiplied by a factor $\lambda$, intended to represent a conventional expectation regarding the rate of wage increase. The idea is that more complex commitments are not enforceable because their terms cannot be verified by third parties, whereas the wage commitment intended under an agreement of the kind just mentioned can be determined by observing the employee's previous wage (assumed to be a matter of public record). Note that the factor $\lambda$ is not itself subject to negotiation, for intended deviations from the conventional rate of wage

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9 Whether this is also true of the model of Shapiro and Stiglitz is not entirely clear, as they consider only the steady state. The more attractive interpretation, in my view, is as one in which an employer commits himself to a particular real wage for the entire life of the employment relation.

10 That the results of Shapiro and Stiglitz depend upon their assumption of a particular kind of contracting has been often noted (e.g., Carmichael, 1985). The class of contracts that I consider here increase the role of high wages in providing an incentive for effort supply, by excluding second-best wage profiles of the kind considered by Lazear (1981). However, as Akerlof and Katz (1989) show, even in the case of general wage profiles, equilibrium unemployment will still exist, as long as the wage must be non-negative in all periods, and employment fees are disallowed. See also MacLeod and Malcomson (1989), and Picard (1993, chap. 6). A consideration of the consequences of broader classes of contracts will be an aim of the proposed research.

11 I am grateful to Julio Rotemberg for suggesting this interpretation.
increase would again be terms that could not be verified by third parties. Thus the only aspect of such a commitment that is negotiable in the case of a particular employment relation is the initial wage. (This does seem to be a typical contractual form.) It is possible to assume that employers are not required to commit themselves to the conventional rate of wage increase; one might suppose that such commitments are enforceable without making them mandatory. However, if it is the only enforceable type of commitment, then in equilibrium all employees will have such commitments. For an employer that chose not to make such commitments would have no reason to pay a wage higher than the reservation wage in any period (as in each period, the wage has no effect upon the effort decision); but an employee with this expectation has no incentive to supply effort, and an employer therefore no reason to hire him.

It is furthermore necessary to assume that the advance commitments place only a lower bound on the wage that an employee may be paid in a given period. Obviously, there is no reason for an employee to insist upon his “right” to receive a lower wage, if his employer wishes to raise his wage (thereby raising the commitment for future wages as well, for the remainder of their relationship). But given this, an employer never has any reason to commit to a wage path higher than the lowest commitment that suffices to deter shirking in the current period. Let that wage in period \( t \) be denoted \( w_t \). I will assume as in the previous section that workers have no access to capital markets, so that the wage commitment that suffices to deter shirking is the same for all, regardless of past employment history.) Then a newly hired worker will receive a wage commitment of \( w_t = w_t \), while a previously employed worker will be given a wage commitment of \( w_t = \max\{\lambda w_{t-1}, w_t\} \). This implies that a worker who has been employed for \( s \) previous periods at his current employer has a current wage commitment (and receives a current wage) of

\[
    w_t = \max_{0 \leq i \leq s} \{\lambda^i w_{t-i}\}. \tag{21}
\]

Equation (21) is consistent with a number of observations about the determinants of individual workers’ wages. It says that individual histories, during a single employment relation, should be characterized by a constant rate of wage increase except for particular occasions upon which a larger raise is received, which permanently raises the baseline to which further increments are added; this is often observed, and indeed is something that other labor contracting theories have also sought to explain (e.g., Harris and Holmstrom, 1982; Beaudry and DiNardo, 1991; MacLeod and Malcolmson, 1993). It implies that wages should differ among workers who are identical in productivity, simply as a consequence of their employment histories, and in particular that a worker’s current wage should depend, not only upon current labor market conditions, but upon conditions during the employee’s entire tenure at his current job – and in particular, upon the tightest conditions observed during this time. The model provides an explanation for the finding of Akerlof, Rose and Yellen (1990) that during business downturns, the wages of existing employees ("stayers") rise relative to those of newly employed workers ("movers"), and similarly for the finding that employees who were hired during boom times continue thereafter to have higher wages (at a given point in time) than workers hired during slumps (their “lock-in effect”).

\[ \text{Note, however, that in my proposed interpretation of these facts, unlike that of Akerlof,} \]
Beaudry and DiNardo (1991) find that the variable
\[
\min_{0 \leq s \leq s} \{u_{t-s}\}
\]
is more important than the current unemployment rate \(u\) in explaining the period \(t\) wages of employees with \(s\) periods of tenure – that is, upon the tightness of the labor market at the tightest time to have occurred during the employee’s tenure. Equation (21) provides a possible interpretation of this finding. Beaudry and DiNardo interpret their finding as evidence for an implicit contract model of the kind first discussed by Harris and Holmstrom (1982). Their interpretation depends upon assuming one-sided commitment, despite a lack of any asymmetric information; the one-sided commitment is better motivated in a model with an incentive problem of the kind proposed here. Furthermore, their interpretation of the role of past unemployment rates in determining an employee’s current wage depends upon an assumption that the reservation wage of the marginal employee at each point in time is a decreasing function of the unemployment rate at that time. But it is not clear why this should be so, in a model like theirs, without job rationing. In the efficiency wage model proposed here, instead, the current unemployment rate is inversely related to the current value of \(w^*\) (though the relation also depends upon expected changes in the two variables, as shown below), because the unemployment rate determines the amount of time that a worker caught shirking expects to be without a job.

I now sketch a model with wage commitments of the kind just described. Preferences and production technology are as in the previous two sections, and for simplicity, workers have no access to capital markets, as in the previous section; but I abstract here from trend growth in the technology factor \(\{z_t\}\). I now assume that a worker employed by a given firm in the period \(t - 1\), and not caught shirking, is employed again by that firm in period \(t\) (and covered by the prior wage commitment), unless an exogenous separation occurs. Separation is an independent event for each employee, with probability \(0 < \eta < 1\), realized at the beginning of period \(t\). (Exogenous separations are required, in the absence of growth, in order to allow an equilibrium with a positive flow of new hires each period; no employee ever chooses to quit prior to such an event, in equilibrium.) Employees for whom such separations occur begin period \(t\) as members of the pool of job seekers, as do any employees who are caught shirking during period \(t - 1\), and workers who were unemployed in period \(t - 1\). (An employer commits not to terminate employment unless the employee is caught shirking or an exogenous separation occurs; employers commit to terminate the employment of any employee caught shirking.) Each member of the pool of job seekers has an equal probability \(\pi_t\) of finding employment in period \(t\), given by

\[
\pi_t = \frac{N_t - (1 - \eta)N_{t-1}}{L - (1 - \eta)N_{t-1}}. \tag{22}
\]
Let $V_t$ represent the expected discounted utility from date $t$ onward, that a worker expects to obtain if he begins period $t$ already employed, and with a prior wage commitment of $w$. This is evaluated after the date $t$ aggregate state has been realized. The function is the same for all workers, because of the lack of access to capital markets. Note that it is a non-decreasing function, strictly increasing for $w > w_t^*$, but constant for all $w \leq w_t^*$, since the employee will receive $w_t^*$ in the latter case regardless of the prior commitment. Let $\tilde{V}_t$ denote the corresponding expected utility for a household that begins period $t$ in the pool of job seekers. Then an employee $i$ in period $t$ will choose to supply effort if and only if

$$E_t[V_{t+1}(\lambda w_t^*)] \geq E_t[\tilde{V}_{t+1}] + \frac{d}{(1-\eta)p\beta}$$

where $w_t^*$ is the employee’s wage commitment in period $t$. The wage required to deter shirking, $w_t^*$, is then the minimum value of $w_t^*$ that satisfies this “no shirking condition”. Thus in equilibrium one must have

$$E_t[V_{t+1}(\lambda w_t^*)] = E_t[\tilde{V}_{t+1}] + \frac{d}{p\beta(1-\eta)}.$$

(23)

And in addition, conditional upon the aggregate state at date $t$, there must be a positive probability that

$$\lambda w_t^* > w_{t+1}^*$$

(24).

This latter condition is necessary for $E_t[V_{t+1}(\lambda w)]$ to be strictly increasing for $w$ in a left neighborhood of $w_t^*$. If it does not hold, the “no shirking condition” would also be satisfied by a lower wage. Thus a deterministic steady state seems not to be possible unless $\lambda > 1$.

A case that is especially simple to analyze is that in which (24) holds at all times. Then each employee’s wage is determined solely by the commitment received at the date at which he was originally hired. In this case one can show that

$$V_t(w) - \tilde{V}_t = E_t \left[ \sum_{j=0}^{\infty} [\beta(1-\eta)]^j [u(\lambda^j w) + v_{t+j}] \right],$$

for all $w \geq w_t^*$, where

$$v_t \equiv \pi_t[u(w_t^*) + d/p] + (1 - \pi_t)[u(b) + d].$$

(25)

It then follows from (23) that

$$\sum_{j=0}^{\infty} [\beta(1-\eta)]^j u(\lambda^{j+1} w_t^*) = \sum_{j=0}^{\infty} [\beta(1-\eta)]^j E_t[v_{t+j+1}] + \frac{d}{p\beta(1-\eta)}.$$

The Shapiro-Stiglitz equilibrium can be understood as one in which the firm promises to pay a constant real wage until termination, but this appears to require that the firm binds itself neither to decrease or increase the employee’s wage, though this is plainly an unappealing assumption.
This can be quasi-differenced to yield a relation of the form
\[ w^*_t = E_t[f(w^*_{t+1}, v_{t+1})], \]
which through substitution of (22) and (25) yields a difference equation of the form
\[ w^*_t = E_t[g(w^*_{t+1}; N_t, N_{t+1})], \]  
(26)
to determine the wage \( w^*_t \) paid to new hires, as a function of the current level and expected future path of aggregate employment \( \{N_{t+j}\} \). The evolution over time of \( \{w^*_t\} \) then determines the entire wage distribution at each point in time through (21). Finally, given these stochastic processes for wages, that are independent of the hiring decisions of an individual firm, firms choose employment \( N_t \) to satisfy
\[ z_t F_2(K_t, z_t N_t) = w^*_t + q_t^{-1} \sum_{j=1}^{\infty} E_t[q_{t+j}(1 - \eta)^j(w^*_t - w^*_t)], \]  
(27)
where again the process \( \{q_t\} \) defines present values for the shareholders. (In writing this, I assume that (27) has a solution for which \( N_t > (1 - \eta)N_{t-1} \).) Equations (26) and (27) define a pair of equilibrium relations between employment and real wages, that can be conjoined with the conditions for optimal capital accumulation by the shareholders, as in the previous section, to yield a complete dynamic general equilibrium model.

Equations (26) and (27) play the role of equations (6) and (7) in the model of section 1, or equations (7) and (8) in the model of section 2, but imply that expected future labor market conditions affect the determination of wages and employment in the present. The long-run versions of these equations (i.e., the relations that they imply between the \textit{constant} values \( \{w^*, N\} \) that characterize a deterministic steady state) are similar to those obtained by Shapiro and Stiglitz. The long-run version of (27) is just (7), graphed as \( LD \) in Figure 1. The long-run version of (26), passing to the limit of an infinitesimal period length, is given by
\[ \log w^* = \log b + \frac{r + p}{p} - \frac{g}{\eta + r} + \frac{d}{p}(\frac{L}{L - N}), \]
where \( r \equiv -\log \beta \) is the rate of time preference of workers, and \( g \equiv \log \lambda \) is the growth rate of wages with tenure. The graph of this relation is like the curve \( EW \) in Figure 1; in particular, it becomes vertical as full employment is approached. It follows that a steady state necessarily involves unemployment, regardless of how high labor productivity may be relative to the disutility of effort and the level of unemployment benefits, as in the analysis of Shapiro and Stiglitz. On the other hand, these static relations between wages and employment do not apply in the short run; in particular, contrary to the analysis of Kimball (1993), labor market equilibrium does not at all times lie on the long-run labor demand curve (7).  

\[ ^{14} \] An important consequence of the alternative specification here is that, at least in the case of sufficient substitutability between capital and labor, rational expectations equilibrium is determinate, so that the equilibrium response to shocks can be uniquely determined without resorting to an equilibrium selection rule of the kind needed by Kimball.
This model clearly can account for the relative acyclicality of average real wages, given that the wages of most existing employees will be determined by prior commitments. It predicts that the wages of new hires should be much more procyclical than average wages, and more generally that categories of workers in which employment is more variable should have more procyclical real wages, as is found by Akerlof, Rose and Yellen (1990). (See also Solon, Barsky and Parker, 1994.) In order to account for the trendlessness of unemployment, one again needs for \( b \) to grow with the same trend rate as productivity. However, the credit constraint (and indeed, the absence of insurance against employment risk) are not really needed here to obtain relatively acyclical average real wages; there may be no reason to reject a model that predicts that \( w^*_t \) should be more procyclical than aggregate consumption. Hence an attempt will be made to integrate the model of wealth effects from section 1 with this model. In this case it is expected that even without unemployment benefits, \( w^*_t \) will grow over the long run with aggregate wealth, thus generating a trendless unemployment rate.

It should be of interest to analyze the consequences of the dynamic relations sketched above between wages and employment, for the predicted effects of a variety of shocks to the economy (such as exogenous productivity shocks, energy price changes, and changes in the level of government purchases), and for the predicted cyclical properties of real wages under alternative assumptions about the driving shocks. In the case of an equilibrium involving sufficiently small fluctuations around the steady state (so that the inequalities \( N_t > (1 - \eta)N_{t-1} \) and (24) hold at all times), linearization techniques can again be used to analyze stationary stochastic equilibria. However, it will also be of some interest to analyze the case of larger shocks, in which case wages of employees are not determined solely by conditions at the time of hiring; this will require numerical methods of greater complexity. It is to be expected that in the more general case, the degree of uncertainty about future conditions will be an important determinant of current labor demand, as in the literature on firing costs (e.g., Bentolila and Bertola, 1990).

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15 This explanation of the acyclicality of the average real wage thus depends upon the fact that it is related only indirectly to the shadow price of labor that is actually allocative, due to the existence of long-term contracts, as argued by Hall (1980).
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