Optimal Stabilization Policy When Wages and Prices are Sticky: The Case of a Distorted Steady State*

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In a seminal paper, Chris Erceg, Dale Henderson, and Andy Levin (2000) analyzed the consequences for optimal monetary policy of the stickiness of both wages and prices. A key contribution of their paper was the demonstration that the expected utility of the representative household in their model could be approximated by an objective with three terms, involving measures of the variability of wage inflation, price inflation and the output gap respectively. While wage-inflation stabilization has not commonly been included among the assumed objectives of monetary policy in studies that lack welfare-theoretic foundations, Erceg *et al.* showed that in the context of their model (with Calvo-style staggering of both wage- and price-setting decisions), such an objective is appropriate in the case that wages as well as prices are sticky. This is because variability of the rate of growth of nominal wages implies misalignment of wages that are adjusted at different times, and hence inefficient utilization of different types of labor. They showed furthermore that the existence of this additional stabilization objective implies that a policy aimed solely at inflation stabilization (a strict inflation target) is not generally optimal, and may be quite undesirable. Instead, their numerical analysis suggests that one can do quite well by targeting an appropriately chosen weighted average of wage and price inflation, with a greater relative weight on wage inflation the greater the relative stickiness of wages.

Here we reexamine the issues raised by Erceg *et al.* in a slightly more general setting. Following Rotemberg and Woodford (1997), Erceg *et al.* assume the existence of an output subsidy in order to eliminate the distortion resulting from the market power of the suppliers of differentiated goods, and a similar employment subsidy to eliminate the distortion resulting from the market power of the suppliers of differentiated forms of labor.[1] As a result, the equilibrium allocation of resources would be optimal in their model, in the case that both wages and prices were fully flexible. This is an important simplification, for it implies, even in the model with sticky wages and prices, that the *steady state* level of output under a policy that maintains stable prices is efficient, and hence that (to first order) an increase in the average level of output would neither raise nor lower welfare. Hence in a quadratic approximation to expected utility, obtained as a Taylor series expansion around the allocation associated with this steady state, there is no linear term in the expected level of output. This allows Erceg *et al.* to obtain a purely quadratic loss function,

[1]In fact, as we show below, there is no need for two distinct subsidies to achieve the result that they seek. The presence of a linear term in the quadratic approximation to utility depends only on the overall index Φ of the degree of inefficiency of steady-state output, introduced below.

just as Rotemberg and Woodford (1997) do in the case that only prices are sticky. Hence they obtain a welfare measure that can be evaluated, to second order in the amplitude of the exogenous disturbances, using only an approximate solution for the equilibrium resulting from a given policy rule that is accurate to *first* order, *i.e.*, a log-linear approximation to the model structural relations.

While this feature of their results makes the analysis much more tractable in the case that they consider, the assumption of output and employment subsidies (rather than positive tax rates on sales, payrolls, and wage income) is clearly unrealistic. Furthermore, there is reason to fear that such an analysis may miss an important aspect of the welfare consequences of stabilization policy. As Henderson and Kim (2003), among others, have stressed, in exact models of optimal wage- and price-setting one typically finds that stabilization policy affects the *average* levels of equilibrium output and employment, and not simply their variability. In the welfare analysis of Erceg *et al.*, such effects may be neglected, because a change in the average level of output that is only of second order in the amplitude of the disturbances has no second-order effect on welfare; but this result depends on the fact that (owing to the assumed subsidies) the steady-state level of output is optimal. Under more realistic assumptions, the steady-state level of output would be judged to be inefficiently low, owing to tax distortions as well as market power in both the goods and labor markets; but this would mean that a second-order effect of stabilization policy on average output would make a second-order contribution to welfare, that might be as important (even in the case of arbitrarily small disturbances) as the second-order welfare effects of stabilization policy considered by Erceg *et al.*

Here we show how the analysis of Erceg *et al.* can be extended to take account of such effects, and hence to allow a correct welfare analysis (to second-order accuracy) even in the presence of substantial steady-state distortions. One approach to dealing with such effects that has recently become popular involves solving for equilibrium under alternative policy rules to second-order accuracy, using a second-order Taylor series expansion of the model structural relations. Here we show instead that, even in the case of a distorted steady state, it is possible to obtain a purely quadratic loss function, similar to the one obtained by Erceg *et al.*, which can then be evaluated to second-order accuracy using only log-linear approximations to both the policy rule and the model structural relations. This requires that we substitute out the linear terms in the Taylor series expansion for expected utility in terms of purely quadratic terms, using the method employed by Benigno and Woodford (2004) in the case of an economy with staggered price-setting but flexible wages. (Essentially, the effects of stabilization policy on the average level of output are used to replace a welfare measure that involves the average level of output by one that is purely quadratic.) In this way, we are able to show that results similar to those of Erceg *et al.* continue to obtain in the case of a distorted steady state, though the size of the steady-state distortions matters for one's quantitative conclusions regarding the nature of optimal policy.

We generalize the analysis of Erceg *et al.* other respects as well. Erceg *et al.* consider only policies with the property that in the absence of exogenous disturbances, the equilibrium will correspond to the efficient steady state. (This means policies under which both wages and prices will be constant, in the absence of exogenous disturbances.) This allows them to obtain an approximate welfare measure that involves only the *variances* of macroeconomic variables. We drop this assumption, and so obtain an approximate welfare measure that also allows one to compare policies under which the average inflation rate is not exactly zero. It turns out that in the kind of model considered here, optimal policy *does* involve a zero average inflation rate; but this result can be derived from our evaluation of alternative rules using the quadratic loss function, rather than having to be assumed from the start.[2] Finally, Erceg *et al.* restrict attention to time-invariant policy rules, and evaluate unconditional expected utility in the stationary equilibrium associated with such a rule. We show instead how it is possible to evaluate discounted expected utility conditional upon some initial state, though we propose a criterion for optimality ("optimality from a timeless perspective") under which optimal policy can be shown (rather than being assumed) to be time-invariant.

[2]The conclusion is not an obvious one, in the case that the steady state with zero inflation is no longer assumed to involve an efficient level of output, since the model is one in which the average inflation rate affects the average level of output.

1 Monetary Stabilization Policy: Welfare-Theoretic Foundations

Here we describe our assumptions about the economic environment and pose the optimization problem that a monetary stabilization policy is intended to solve. The approximation method that we use to characterize the solution to this problem is then presented in the following section. Further details of the derivation of the structural equations of our model of nominal price and wage rigidities can be found in Erceg et al. (2000) and Woodford (2003, chapter 3).

1.1 Objective and Constraints

In our model, there is a continuum of measure one of households. Household of type j seeks to maximize

$$U_{t_0}^j \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\tilde{u}(C_t^j; \xi_t) - v(h_t(j); \xi_t) \right], \tag{1.1}$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta_p - 1}{\theta_p}} di \right]^{\frac{\theta_p}{\theta_p - 1}}, \tag{1.2}$$

with an elasticity of substitution equal to $\theta_p > 1$, and $h_t(j)$ is the quantity supplied of labor which is specific to household of type j.

There is a continuum of measure one of differentiated goods and each household consumes all the goods. The objective of policy is to maximize the sum of the utilities of the households at time t_0. We will assume risk-sharing among the households in a way that they will face the same budget constraint and make the same consumption choices even if they have different wages. It follows that the objective of policy is to maximize U_{t_0} defined as

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\tilde{u}(C_t; \xi_t) - \int_0^1 v(h_t(j); \xi_t) dj \right]. \tag{1.3}$$

To simplify the algebraic form of our results we shall restrict attention to the case of isoelastic functional forms,

$$\tilde{u}(C_t; \xi_t) \equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \bar{C}_t^{\tilde{\sigma}^{-1}}}{1 - \tilde{\sigma}^{-1}}, \tag{1.4}$$

$$v(h_t;\xi_t) \equiv \frac{\lambda}{1+\nu} h_t^{1+\nu} \bar{H}_t^{-\nu}, \tag{1.5}$$

where $\tilde{\sigma}, \nu > 0$, and $\{\bar{C}_t, \bar{H}_t\}$ are bounded exogenous disturbance processes. (We use the notation ξ_t to refer to the complete vector of exogenous disturbances, including $\bar{C}_t$ and $\bar{H}_t$.) We assume that the labor used to produce each good is a CES aggregate of the continuum of individual types of labor supplied by the households defined by

$$H_t(i) \equiv \left[\int_0^1 h_t(j)^{\frac{\theta_w - 1}{\theta_w}} dj \right]^{\frac{\theta_w}{\theta_w - 1}}$$

for some elasticity of substitution $\theta_w > 1$. Here $h_t(j)$ is the labor of type j that is hired. Each differentiated type of labor is supplied in a monopolistically-competitive market. It follows that the demand for labor of type j on the part of wage-taking firms is given by

$$h_t(j) = H_t \left(\frac{w_t(j)}{W_t} \right)^{-\theta_w}, \tag{1.6}$$

where $w_t(j)$ is the nominal wage demanded for labor of type j and W_t is the Dixit-Stiglitz wage index

$$W_t \equiv \left[\int_0^1 w_t(j)^{1-\theta_w} dj \right]^{\frac{1}{1-\theta_w}}, \tag{1.7}$$

and H_t is defined as

$$H_t \equiv \int_0^1 H_t(i) di.$$

We assume a common technology for the production of all goods

$$y_t(i) = A_t f(H_t(i)) = A_t H_t(i)^{1/\phi},$$

where A_t is an exogenously varying technology factor, and $\phi > 1$. We first note that we can write

$$\int_0^1 v(h_t(j);\xi_t) dj = \frac{\lambda}{1+\nu} H_t^{1+\nu} \Delta_{w,t} \bar{H}_t^{-\nu}, \tag{1.8}$$

where

$$\Delta_{w,t} = \int_0^1 \left(\frac{w_t(j)}{W_t} \right)^{-\theta_w(1+\nu)} dj \geq 1 \tag{1.9}$$

is a measure of wage dispersion at date t. Moreover

$$H_t = \int_0^1 H_t(i) di = Y_t^{\phi} A_t^{-\phi} \Delta_{p,t}, \tag{1.10}$$

where

$$\Delta_{p,t} \equiv \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta_p(1+\omega_p)} di \geq 1 \tag{1.11}$$

is a measure of price dispersion at date t, in which P_t is the Dixit-Stiglitz price index

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta_p} di \right]^{\frac{1}{1-\theta_p}}, \tag{1.12}$$

and $\omega_p \equiv \phi - 1$ Using (1.8), (1.10) and the identity

$$Y_t = C_t + G_t$$

to substitute for C_t, where G_t is exogenous government demand for the composite good, we can write the utility flow in the form $U(Y_t, \Delta_{p,t}, \Delta_{w,t}; \xi_t)$, where the vector ξ_t now includes the exogenous disturbances G_t and A_t as well as the preference shocks.[3] Hence we can write our objective (1.3) as

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(Y_t, \Delta_{p,t}, \Delta_{w,t}; \xi_t). \tag{1.13}$$

We assume that the wage for each type of labor is set by the monopoly supplier of that type, who stand ready to supply as many hours of work as turn out to be demanded at that wage. We assume that wage setters fix the wages in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983). We let $0 \leq \alpha_w < 1$ be the fraction of wages that remain unchanged in any period. A supplier that changes its wages in period t chooses its new wage $w_t(j)$ to maximize

$$E_t \left\{ \sum_{T=t}^{\infty} (\alpha_w \beta)^{T-t} [\Lambda_T w_t(j) h_T(w_t(j)) - v(h_T(w_t(j)); \xi_t) \right\}, \tag{1.14}$$

where Λ_T is the representative household's marginal utility of nominal income in period T and the dependence of labor demand $h_T(j)$ upon the wage is given by (1.6),

[3] The government is assumed to need to obtain an exogenously given quantity of the Dixit-Stiglitz aggregate each period, and to obtain this in a cost-minimizing fashion. Hence the government allocates its purchases across the suppliers of differentiated goods in the same proportion as do households, and the index of aggregate demand Y_t is the same function of the individual quantities $\{y_t(i)\}$ as C_t is of the individual quantities consumed $\{c_t(i)\}$, defined in (1.2).

and α_w^{T-t} is the probability that a wage chosen in period t will not have been revised by period T.

Each of the wage suppliers that revise their wages in period t choose the same new wage w_t^*, that maximizes (1.14). Note that supplier j's objective function is a concave function of the quantity of working hours supplied $h_t(j)$, since revenues are proportional to $h_t^{\frac{\theta_w-1}{\theta_w}}(j)$ and hence concave in $h_t(j)$, while costs are convex in $h_t(j)$. Moreover, since $h_t(j)$ is proportional to $w_t(j)^{-\theta_w}$, the objective function is also concave in $w_t(j)^{-\theta_w}$. The first-order condition for the optimal choice of the wage $w_t(j)$ is the same as the one with respect to $w_t(j)^{-\theta_w}$; hence the first-order condition with respect to $w_t(i)$,

$$E_t\left\{\sum_{T=t}^{\infty}\alpha_w{}^{T-t}Q_{t,T}H_T W_T^{\theta_w}\left[w_t^* - \mu_w\frac{v_h(h_T(w_t^*);\xi_t)}{\tilde{u}_c(Y_T-G_T;\xi_T)}P_T\right]\right\}=0,$$

where

$$\mu_w \equiv \frac{\theta_w}{\theta_w-1},$$

is both necessary and sufficient for an optimum. In the above expression, $Q_{t,T}$ is the stochastic discount factor by which financial markets discount random nominal income in period T to determine the nominal value of a claim to such income in period t. In equilibrium, this discount factor is given by

$$Q_{t,T}=\beta^{T-t}\frac{\tilde{u}_c(C_T;\xi_T)}{\tilde{u}_c(C_t;\xi_t)}\frac{P_t}{P_T}.$$

Under our assumed isoelastic functional forms, the optimal choice has a closed-form solution

$$\frac{w_t^*}{W_t}=\left(\frac{K_{w,t}}{F_{w,t}}\right)^{\frac{1}{1+\nu\theta_w}}, \tag{1.15}$$

where $K_{w,t}$ and $F_{w,t}$ are functions of current aggregate output Y_t, the real wage W_t/P_t, the index of price dispersion $\Delta_{p,t}$, the current exogenous state ξ_t, and the expected future evolution of wage inflation, output, real wage, price dispersion and disturbances, defined by

$$F_{w,t}\equiv E_t\sum_{T=t}^{\infty}(\alpha_w\beta)^{T-t}u_y(Y_T;\xi_T)Y_T^{\phi}A_T^{-\phi}\Delta_{p,T}\frac{W_T}{P_T}\left(\frac{W_T}{W_t}\right)^{\theta_w-1}, \tag{1.16}$$

$$K_{w,t}\equiv E_t\sum_{T=t}^{\infty}(\alpha_w\beta)^{T-t}\mu_w v_h(Y_T^{\phi};\xi_T)Y_T^{\phi}A_T^{-\phi(1+\nu)}\Delta_{p,T}^{1+\nu}\left(\frac{W_T}{W_t}\right)^{\theta_w(1+\nu)}, \tag{1.17}$$

where we have used the definition

$$u(Y;\xi) \equiv \tilde{u}(Y-G;\xi).$$

The wage index then evolves according to a law of motion

$$W_t = \left[(1-\alpha_w)w_t^{*1-\theta_w} + \alpha_w W_{t-1}^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}}, \tag{1.18}$$

as a consequence of (1.7). Substitution of (1.15) into (1.18) implies that equilibrium wage inflation in any period is given by

$$\frac{1-\alpha_w \Pi_{w,t}^{\theta_w-1}}{1-\alpha_w} = \left(\frac{F_{w,t}}{K_{w,t}}\right)^{\frac{\theta_w-1}{1+\nu\theta_w}}, \tag{1.19}$$

where $\Pi_{w,t} \equiv W_t/W_{t-1}$. This defines a short-run aggregate supply relation between wage inflation and output, real wage and the index of price dispersion, given the current disturbances ξ_t, and expectations regarding future wage inflation, output, real wage, the index of price dispersion and disturbances.

We can also use (1.18) to derive a law of motion of the form

$$\Delta_{w,t} = h_w(\Delta_{w,t-1}, \Pi_{w,t}) \tag{1.20}$$

for the dispersion measure defined in (1.9), where

$$h_w(\Delta_w, \Pi_w) \equiv \alpha_w \Delta_w \Pi_w^{\theta_w(1+\nu)} + (1-\alpha_w)\left(\frac{1-\alpha_w \Pi_w^{\theta_w-1}}{1-\alpha_w}\right)^{-\frac{\theta_w(1+\nu)}{1-\theta_w}}.$$

The producers for each differentiated good fix the prices of their goods in monetary units for a random interval of time. We let $0 \leq \alpha_p < 1$ be the fraction of prices that remain unchanged in any period. A supplier that changes its price in period t chooses its new price $p_t(i)$ to maximize

$$E_t\left\{\sum_{T=t}^{\infty} \alpha_p^{T-t} Q_{t,T} \Pi(p_t(i), P_T; W_T, Y_T, \xi_T)\right\}$$

where the function

$$\Pi(p(i), P; W, Y, \xi) \equiv (1-\tau)p(i)Y(p(i)/P)^{-\theta_p} - W \cdot f^{-1}(Y(p(i)/P)^{-\theta_p}/A) \tag{1.21}$$

indicates the after-tax nominal profits of a supplier with price p when the aggregate price index is equal to P and aggregate demand is equal to Y. Here τ_t is the proportional tax on sales revenues in period t; we treat $\{\tau_t\}$ as an exogenous disturbance process, taken as given by the monetary policymaker. We assume that τ_t fluctuates over a small interval around a non-zero steady-state level $\bar{\tau}$; this is a further reason for inefficiency of the steady-state level of output, in addition to the market power of the suppliers of differentiated goods.[4] The disturbances τ_t and A_t are also included as elements of the vector of exogenous disturbances ξ_t.

Each of the suppliers that revise their prices in period t choose the same new price p_t^*, that maximizes (1.21). Note that supplier i's profits are a concave function of the quantity sold $y_t(i)$, since revenues are proportional to $y_t^{\frac{\theta_p-1}{\theta_p}}(i)$ and hence concave in $y_t(i)$, while costs are convex in $y_t(i)$. Moreover, since $y_t(i)$ is proportional to $p_t(i)^{-\theta_p}$, the profit function is also concave in $p_t(i)^{-\theta_p}$. The first-order condition for the optimal choice of the price $p_t(i)$ is the same as the one with respect to $p_t(i)^{-\theta_p}$; hence the first-order condition with respect to $p_t(i)$,

$$E_t\left\{\sum_{T=t}^{\infty}\alpha_p^{T-t}Q_{t,T}\Pi_1(p_t(i),P_T;W_T,Y_T;\xi_T)\right\}=0,$$

is both necessary and sufficient for an optimum. The equilibrium choice p_t^* (which is the same for all the firms that adjust their prices at time t) is the solution to the above equation.

Under our assumed isoelastic functional forms, the optimal choice has a closed-form solution

$$\frac{p_t^*}{P_t}=\left(\frac{K_{p,t}}{F_{p,t}}\right)^{\frac{1}{1+\omega_p\theta_p}}, \tag{1.22}$$

where $F_{p,t}$ and $K_{p,t}$ are functions of current aggregate output Y_t, the current exogenous state ξ_t, and the expected future evolution of inflation, output, real wages and disturbances, defined by

$$F_{p,t}\equiv E_t\sum_{T=t}^{\infty}(\alpha_p\beta)^{T-t}(1-\tau_T)u_y(Y_T;\xi_T)Y_T\left(\frac{P_T}{P_t}\right)^{\theta_p-1}, \tag{1.23}$$

$$K_{p,t}\equiv E_t\sum_{T=t}^{\infty}(\alpha_p\beta)^{T-t}u_y(Y_T;\xi_T)\phi\mu_p\frac{W_T}{P_T}\left(\frac{Y_T}{A_T}\right)^{\phi}\left(\frac{P_T}{P_t}\right)^{\theta_p(1+\omega_p)}, \tag{1.24}$$

[4] Other types of distorting taxes would have similar consequences, since it is the overall size of the steady-state inefficiency wedge that is of greatest importance for our analysis, as we show below. To economize on notation, we assume that the only distorting tax is of this particular kind.

in which expressions

$$\mu_p \equiv \frac{\theta_p}{\theta_p - 1}. \tag{1.25}$$

The price index then evolves according to a law of motion

$$P_t = \left[(1-\alpha_p)p_t^{*1-\theta_p} + \alpha_p P_{t-1}^{1-\theta_p}\right]^{\frac{1}{1-\theta_p}}, \tag{1.26}$$

as a consequence of (1.12). Substitution of (1.22) into (1.26) implies that equilibrium inflation in any period is given by

$$\frac{1-\alpha_p \Pi_{p,t}^{\theta_p - 1}}{1-\alpha_p} = \left(\frac{F_{p,t}}{K_{p,t}}\right)^{\frac{\theta_p - 1}{1+\omega_p\theta_p}}, \tag{1.27}$$

where $\Pi_{p,t} \equiv P_t/P_{t-1}$. This defines a short-run aggregate supply relation between inflation and both output and real wage, given the current disturbances ξ_t, and expectations regarding future inflation, output, real wage and disturbances. We can also use (1.26) to derive a law of motion of the form

$$\Delta_{p,t} = h_p(\Delta_{p,t-1}, \Pi_{p,t}) \tag{1.28}$$

for the dispersion measure defined in (1.11), where

$$h(\Delta_p, \Pi_p) \equiv \alpha_p \Delta_p \Pi_p^{\theta_p(1+\omega_p)} + (1-\alpha_p)\left(\frac{1-\alpha_p \Pi_p^{\theta_p - 1}}{1-\alpha_p}\right)^{-\frac{\theta_p(1+\omega_p)}{1-\theta_p}}.$$

Equations (1.20) and (1.28) are the sources in our model of welfare losses from price and wage inflation or deflation. Finally we note that price and wage inflation rates are related to the real wages as

$$w_{R,t} = w_{R,t-1}\frac{\Pi_{w,t}}{\Pi_{p,t}}, \tag{1.29}$$

where $w_{R,t} \equiv W_t/P_t$.

We assume the existence of a lump-sum source of government revenue (in addition to the proportional tax τ on sales revenues), and assume that the fiscal authority ensures intertemporal government solvency regardless of what monetary policy may be chosen by the monetary authority. This allows us to abstract from the fiscal consequences of alternative monetary policies in our consideration of optimal monetary stabilization policy, as in Erceg *et al.* (2000) and much of the literature on monetary policy rules.

Finally, we follow Erceg *et al.* in abstracting from any monetary frictions that would account for a demand for central-bank liabilities that earn a substandard rate of return; we nonetheless assume that the central bank can control the riskless short-term nominal interest rate i_t, as discussed in Woodford (2003, chapter 2). We also assume that the zero lower bound on nominal interest rates never binds under the optimal policies considered below,[5] so that we need not introduce any additional constraint on the possible paths of output and prices associated with a need for the chosen evolution of prices to be consistent with a non-negative nominal interest rate. We also note that the ability of the central bank to control i_t in each period gives it one degree of freedom each period (in each possible state of the world) with which to determine equilibrium outcomes. Considering (1.20), (1.28) and (1.29) and because of the existence of the aggregate-supply relations (1.19), (1.27) as necessary constraints on the joint evolution of price, wage inflation rates and output, there is exactly one degree of freedom to be determined each period, in order to determine particular stochastic processes $\{\Pi_{w,t}, \Pi_{p,t}, Y_t\}$ from among the set of possible rational-expectations equilibria. Hence we shall suppose that the monetary authority can choose from among the possible processes $\{\Pi_{w,t}, \Pi_{p,t}, Y_t\}$ that constitute rational-expectations equilibria, and consider which equilibrium it is optimal to bring about; the detail that policy is implemented through the control of a short-term nominal interest rate will not actually matter to our calculations.

1.2 Optimal Policy from a "Timeless Perspective"

Under the standard (Ramsey) approach to the characterization of an optimal policy commitment, one chooses among state-contingent paths $\{\Pi_{p,t}, \Pi_{w,t}, Y_t, w_{R,t}, \Delta_{p,t}, \Delta_{w,t}, F_{p,t}, K_{p,t}, F_{w,t}, K_{w,t}\}$ from some initial date t_0 onward that satisfy (1.16), (1.17), (1.19), (1.20), (1.23), (1.24), (1.27), (1.28) and (1.29) for each $t \geq t_0$, given initial price and wage dispersions Δ_{p,t_0-1}, Δ_{w,t_0-1} and initial real wage w_{R,t_0-1}, so as to maximize (1.13). Such a t_0-optimal plan requires commitment, insofar as the corresponding $t-$optimal plan for some later date t, given the initial conditions $\Delta_{p,t-1}$, $\Delta_{w,t-1}$

[5] This can be shown to be true in the case of small enough disturbances, given that the nominal interest rate is equal to $\bar{r} = \beta^{-1} - 1 > 0$ under the optimal policy in the absence of disturbances.

and $w_{R,t-1}$ obtaining at that date, will not involve a continuation of the t_0-optimal plan. This failure of time consistency occurs because the constraints on what can be achieved at date t_0, consistent with the existence of a rational-expectations equilibrium, depend on the expected paths of the above set of variables at later dates; but in the absence of a prior commitment, a planner would have no motive at those later dates to choose a policy consistent with the anticipations that it was desirable to create at date t_0.

However, the degree of advance commitment that is necessary to bring about an optimal equilibrium is of only a limited sort. Paralleling the analysis of Benigno and Woodford (2004), it can be shown that the Ramsey problem can be decomposed in two stages of which the second is fully recursive and of the same form of the Ramsey problem itself except for an additional constraint on a particular set of variables. In our case this set X_t is given by $X_t \equiv (F_{p,t}, K_{p,t}, F_{w,t}, K_{w,t})$.

Our aim here is to characterize policy that solves this constrained optimization problem in which one chooses among state-contingent paths $\{x_t,\, X_t\}$, with $x_t \equiv \{\Pi_{p,t},$ $\Pi_{w,t}, Y_t, w_{R,t},\, \Delta_{p,t}, \Delta_{w,t}\}$ from some initial date t_0 onward that satisfy (1.16), (1.17), (1.19), (1.20), (1.23), (1.24), (1.27), (1.28) and (1.29) for each $t \geq t_0$, given initial price and wage dispersions Δ_{p,t_0-1}, Δ_{w,t_0-1}, real wage w_{R,t_0-1} and an initial condition on the set of variables X_{t_0}, so as to maximize (1.13). Because of the recursive form of this problem, it is possible for a commitment to a time-invariant policy rule from date t onward to implement an equilibrium that solves the problem, for some specification of the initial commitments X_t. A time-invariant policy rule with this property is said by Woodford (2003, chapter 7) to be "optimal from a timeless perspective."[6] Such a rule is one that a policymaker that solves a traditional Ramsey problem would be willing to commit to *eventually* follow, though the solution to the Ramsey problem involves different behavior initially, as there is no need to internalize the effects of prior anticipation of the policy adopted for period t_0. One might also argue that it is desirable to commit to follow such a rule immediately, even though such a policy would not solve the (unconstrained) Ramsey problem, as a way of demonstrating one's willingness to accept constraints that one wishes the public to believe that one will accept in the future.

[6]See also Woodford (1999) and Giannoni and Woodford (2002).

2 A Linear-Quadratic Approximate Problem

In fact, we shall here characterize the solution to this problem (and similarly, derive optimal time-invariant policy rules) only for initial conditions near certain steady-state values, allowing us to use local approximations in characterizing optimal policy. We establish that these steady-state values have the property that if one starts from initial conditions close enough to the steady state, and exogenous disturbances thereafter are small enough, the optimal policy subject to the initial commitments remains forever near the steady state. Hence our local characterization describes the *long run* character of Ramsey policy, in the event that disturbances are small enough.[7] Of greater interest here, it describes policy that is optimal from a timeless perspective in the event of small disturbances.

We first must show the existence of a steady state, *i.e.*, of an optimal policy (under appropriate initial conditions) that involves constant values of all variables. To this end we consider the purely deterministic case, in which the exogenous disturbances $\bar{C}_t$,G_t,$\bar{H}_t$,A_t, τ_t each take constant values $\bar{C}, \bar{H}, \bar{A}, \bar{\tau} > 0$, $\bar{G} \geq 0$ for all $t \geq t_0$. We wish to find initial degree of price and wage dispersions Δ_{p,t_0-1}, Δ_{w,t_0-1}, an initial real wage W_{t_0-1}/P_{t_0-1}, and initial commitments $X_{t_0} = \bar{X}$ such that the solution to the optimal problem involves a constant policy $x_t = \bar{x}$, $X_{t+1} = \bar{X}$ each period, in which $\bar{\Delta}_p$, $\bar{\Delta}_w$ and $\bar{\omega}_R$ are equal to the initial values for these variables. We show in the appendix that the first-order conditions for this problem admit a steady-state solution of this form, and we verify below that (when our parameters satisfy certain bounds) the second-order conditions for a local optimum are also satisfied.

We show that $\bar{\Pi}_p = \bar{\Pi}_w = 1$(zero price and wage inflation), and correspondingly that $\bar{\Delta}_p = \bar{\Delta}_w = 1$(zero price and wage dispersion). We may furthermore assume without loss of generality that the constant values of $\bar{C}$ and $\bar{H}$ are chosen so that in the optimal steady state, $C_t = \bar{C}$ and $H_t = \bar{H}$ each period.[8]

We next wish to characterize the optimal responses to small perturbations of

[7] See Benigno and Woodford (2004) for further discussion. In the simpler model treated there, it is shown explicitly that Ramsey policy converges asymptotically to the steady state of the constrained problem, so that the solution to the LQ approximate problem approximates the response to small shocks under the Ramsey policy, at dates long enough after t_0. A similar result could be established here using similar reasoning.

[8] Note that we may assign arbitrary positive values to $\bar{C}, \bar{H}$ without changing the nature of the implied preferences, as long as the value of λ is appropriately adjusted.

the initial conditions and small fluctuations in the disturbance processes around the above values. To do this, we compute a linear-quadratic approximate problem, the solution to which represents a linear approximation to the solution to the policy problem defined above. An important advantage of this approach is that it allows direct comparison of our results with those obtained in other analyses of optimal monetary stabilization policy. Other advantages are that it makes it straightforward to verify whether second-order conditions hold that imply that a solution to our first-order conditions will represent at least a local optimum, and that it provides us with a welfare measure with which to rank alternative sub-optimal policies, in addition to allowing computation of the optimal policy.

2.1 A Quadratic Approximate Welfare Measure

We begin by computing a Taylor-series approximation to our welfare measure (1.13), expanding around the steady-state allocation defined above, in which $y_t(i) = \bar{Y}$ and $h_t(j) = \bar{H}$ for each good and variety of labor at all times and $\xi_t = 0$ at all times.[9] As a second-order (logarithmic) approximation to this measure, we obtain[10]

$$\begin{aligned} U_{t_0} &= \bar{Y}\bar{u}_c \cdot E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Phi \hat{Y}_t - \frac{1}{2} u_{yy} \hat{Y}_t^2 + \hat{Y}_t u_{y\xi} \xi_t - u_{\Delta_p} \hat{\Delta}_{p,t} - u_{\Delta_w} \hat{\Delta}_{w,t} \\ &+ \text{ t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \tag{2.30}$$

where $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, $\hat{\Delta}_{p,t} \equiv \log \Delta_{p,t}$ and $\hat{\Delta}_{w,t} \equiv \log \Delta_{w,t}$ measure deviations of aggregate output, price and wage dispersion measured from their steady-state levels, the term "t.i.p." collects terms that are independent of policy (constants and functions of exogenous disturbances) and hence irrelevant for ranking alternative policies, and $||\xi||$ is a bound on the amplitude of our perturbations of the steady state.[11] Here

[9] Here the elements of ξ_t are assumed to be $\bar{c}_t \equiv \log(\bar{C}_t/\bar{C}), \bar{h}_t \equiv \log(\bar{H}_t/\bar{H}), a_t \equiv \log(A_t/\bar{A})$, $\hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$, and $\hat{\tau}_t \equiv (\tau_t - \bar{\tau})/\bar{\tau}$, so that a value of zero for this vector corresponds to the steady-state values of all disturbances. The perturbation $\hat{G}_t$ is not defined to be logarithmic so that we do not have to assume positive steady-state value for this variable.

[10] See the appendix for details. Our calculations here follow closely those of Woodford (2003, chapter 6) and Benigno and Woodford (2004).

[11] Specifically, we use the notation $\mathcal{O}(||\xi||^k)$ as shorthand for $\mathcal{O}(||\xi, \hat{\Delta}_{p,t_0-1}^{1/2}, \hat{\Delta}_{w,t_0-1}^{1/2}, \hat{X}_{t_0}||^k)$, where in each case hats refer to log deviations from the steady-state values of the various parameters of the policy problem. We treat $\hat{\Delta}_{p,t_0}^{1/2}$, $\hat{\Delta}_{w,t_0}^{1/2}$ as expansion parameters, rather than $\hat{\Delta}_{p,t_0}, \hat{\Delta}_{w,t_0}$

the coefficient

$$\Phi \equiv 1 - \frac{\theta_w - 1}{\theta_w}\frac{\theta_p - 1}{\theta_p}(1-\bar{\tau}) < 1$$

measures the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor, and hence the inefficiency of the steady-state output level $\bar{Y}$. The coefficients $u_{yy}, u_{y\xi}$, u_{Δ_p} and u_{Δ_w} are defined in the appendix.

In addition, we can take a second-order approximation to equations (1.20) and (1.28) and integrate them to obtain

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\hat{\Delta}_{w,t} = \frac{\alpha_w}{(1-\alpha_w)(1-\alpha_w\beta)}\theta_w(1+\nu)(1+\nu\theta_w)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\frac{\pi_{w,t}^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \tag{2.31}$$

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\hat{\Delta}_{p,t} = \frac{\alpha_p}{(1-\alpha_p)(1-\alpha_p\beta)}\theta_p(1+\omega_p)(1+\omega_p\theta_p)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\frac{\pi_{p,t}^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \tag{2.32}$$

where $\pi_{p,t} \equiv \ln P_t/P_{t-1}$ and $\pi_{w,t} \equiv \ln W_t/W_{t-1}$. Substituting (2.31) and (2.32) into (2.30), we can then approximate our welfare measure by

$$\begin{aligned} U_{t_0} &= \bar{Y}\bar{u}_c \cdot E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\Phi\hat{Y}_t - \frac{1}{2}u_{yy}\hat{Y}_t^2 + \hat{Y}_t u_{y\xi}\xi_t - \frac{1}{2}u_{\pi_p}\pi_{p,t}^2 - \frac{1}{2}u_{\pi_w}\pi_{w,t}^2] \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \tag{2.33}$$

for certain coefficients $u_{\pi_w}, u_{\pi_p} > 0$ defined in the appendix. Note that we can now write our stabilization objective purely in terms of the evolution of the aggregate variables $\{\hat{Y}_t, \pi_{w,t},\, \pi_{p,t}\}$ and the exogenous disturbances.

We note that when $\Phi > 0$, there is a non-zero linear term in (2.33), which means that we cannot expect to evaluate this expression to second order using only an approximate solution for the path of aggregate output that is accurate only to first order. Thus we cannot determine optimal policy, even up to first order, using this approximate objective together with approximations to the structural equations that are accurate only to first order. Erceg et al. (2000) avoid this problem by assuming

because (1.20), (1.28) imply that deviations of the inflation rates from zero of order ϵ only result in deviations in the dispersion measures $\Delta_{p,t}$, $\Delta_{w,t}$ from one of order ϵ^2. We are thus entitled to treat the fluctuations in $\Delta_{p,t}$, $\Delta_{w,t}$ as being only of second order in our bound on the amplitude of disturbances, since if this is true at some initial date it will remain true thereafter.

an output subsidy (*i.e.,* a value $\bar{\tau} < 0$) of the size needed to ensure that $\Phi = 0$. Here we wish to relax this assumption. We show here that an alternative way of dealing with this problem is to use a second-order approximation to the aggregate-supply relations to eliminate the linear terms in the quadratic welfare measure. We show in the appendix that to second order, equations (1.19) and (1.27) can be written as

$$\begin{aligned} V_{j,t} &= \xi_j(c'_{j,x}x_t + c_{j,\xi}\xi_t + \frac{1}{2}x'_t C_{j,xx}x_t - x'_t C_{j,x\xi}\xi_t + \frac{1}{2}c_{j,\pi_p}\pi^2_{p,t} + \frac{1}{2}c_{j,\pi_w}\pi^2_{w,t}) + \beta E_t V_{j,t+1} \\ &\quad + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3), \qquad (2.34) \end{aligned}$$

for $j = p, w$. Here the notation "s.o.t.i.p." indicates terms independent of policy that are entirely of second or higher order, x_t denotes a two-by-one vector whose elements are $\hat{Y}_t$ and $\hat{w}_{R,t} \equiv \log(w_{R,t}/\bar{w}_R)$. We have defined

$$V_{j,t} \equiv \pi_{j,t} + \frac{1}{2}v_{j,\pi}\pi^2_{j,t} + v_{j,z}\pi_{j,t}Z_{j,t},$$

where

$$Z_{j,t} \equiv E_t \sum_{T=t}^{\infty}(\alpha_j\beta)^{T-t}[z_{j,y}\hat{Y}_T + z_{j,r}\hat{w}_{R,T} + z_{j,\pi}\pi_{j,T} + z_{j,\xi}\xi_T];$$

for certain coefficients defined in the appendix. Note that to first order, $V_{j,t} = \pi_{j,t}$, and (2.34) reduces simply to

$$\pi_{j,t} = \xi_j(c'_{j,x}x_t + c_{j,\xi}\xi_t) + \beta E_t\pi_{j,t+1}, \qquad (2.35)$$

for $j = p, w$, which represents two "New Keynesian Phillips curve" relations, for prices and wages respectively, as in Erceg *et al.* (2000).

In the appendix, we sum the two equations in (2.34) and integrate the resulting equation forward to obtain a relation of the form

$$V_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\hat{Y}_t + \frac{1}{2}c_{yy}\hat{Y}_t^2 - \hat{Y}_t c_{y\xi}\xi_t + \frac{1}{2}c_{\pi_p}\pi^2_{p,t} + \frac{1}{2}c_{\pi_w}\pi^2_{w,t}] + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \qquad (2.36)$$

We can then use (2.36) to write the discounted sum of output terms in (2.33) as a function of purely quadratic terms, up to a residual of third order. As shown in the appendix, we can rewrite (2.33) as

$$U_{t_0} = -\Omega E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{q_y}{2}(\hat{Y}_t - \hat{Y}_t^*)^2 + \frac{q_p}{2}\pi^2_{p,t} + \frac{q_w}{2}\pi^2_{w,t}\right\} + T_{t_0} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \qquad (2.37)$$

where [12]

$$\Omega \equiv \bar{Y} u_c > 0,$$

$$q_y \equiv \omega + \sigma^{-1} + \Phi(1-\sigma^{-1}) - \frac{\Phi\sigma^{-1}(s_C^{-1}-1)}{\omega+\sigma^{-1}}, \tag{2.38}$$

$$\hat{Y}_t^* = \omega_1 \hat{Y}_t^n - \omega_2 \hat{G}_t + \omega_3 \hat{\tau}_t, \tag{2.39}$$

$$q_p \equiv \frac{\theta_p}{\xi_p(\omega+\sigma^{-1})}[(\omega+\sigma^{-1}) + \Phi(1-\sigma^{-1})], \tag{2.40}$$

$$q_w \equiv \frac{\theta_w}{\xi_w(\omega+\sigma^{-1})}[(\omega+\sigma^{-1}) + \Phi(1-\sigma^{-1})], \tag{2.41}$$

and

$$\hat{Y}_t^n = \frac{\sigma^{-1} g_t + \omega q_t - \omega_\tau \hat{\tau}_t}{(\omega+\sigma^{-1})}, \tag{2.42}$$

in which expressions

$$\omega_1 = q_y^{-1}[(\omega+\sigma^{-1}) + \Phi(1-\sigma^{-1})],$$

$$\omega_2 = \frac{\Phi s_C^{-1}\sigma^{-1}}{(\omega+\sigma^{-1})^2 + \Phi[(1-\sigma^{-1})(\omega+\sigma^{-1}) - (s_C^{-1}-1)\sigma^{-1}]},$$

$$\omega_3 \equiv \frac{\omega_\tau}{(\omega+\sigma^{-1}) + \Phi[(1-\sigma^{-1}) - (s_C^{-1}-1)\sigma^{-1}(\omega+\sigma^{-1})^{-1}]}.$$

Here $\hat{Y}_t^n$ and $\hat{\omega}_t^n$ represent log-linear approximations to the "natural rate of output and real wage," *i.e.*, the flexible-price equilibrium levels of output and real wages (Woodford, 2003, chap. 3). In terms of this notation, the log-linear aggregate supply relations (2.35) can be written as

$$\pi_{p,t} = \kappa_p[\hat{Y}_t - \hat{Y}_t^n] + \xi_p[\hat{\omega}_{R,t} - \hat{\omega}_t^n] + \beta E_t \pi_{p,t+1}, \tag{2.43}$$

$$\pi_{w,t} = \kappa_w[\hat{Y}_t - \hat{Y}_t^n] - \xi_w[\hat{\omega}_{R,t} - \hat{\omega}_t^n] + \beta E_t \pi_{w,t+1}, \tag{2.44}$$

while

$$\hat{\omega}_{R,t} = \hat{\omega}_{R,t-1} + \pi_{w,t} - \pi_{p,t}, \tag{2.45}$$

where $\kappa_p \equiv \xi_p \omega_p$ and $\kappa_w \equiv \xi_w \nu\phi$. The term $T_{t_0} \equiv \Phi \bar{Y} \bar{u}_c V_{t_0}$ is a transitory component where V_{t_0} is defined in the appendix.

[12]In what follows, the following definitions have been used: $\sigma^{-1} \equiv \tilde{\sigma}^{-1} s_C^{-1}$ with $s_C \equiv \bar{C}/\bar{Y}$; $\omega \equiv \phi\nu + \omega_p$; $\omega q_t \equiv \nu \bar{h}_t + \phi(1+\nu) a_t$; $g_t \equiv \hat{G}_t + s_C \bar{c}_t$; $\omega_\tau \equiv \bar{\tau}/(1-\bar{\tau})$; $\xi_p \equiv (1-\alpha_p\beta)(1-\alpha_p)/[\alpha_p(1+\theta_p\omega_p)]$; $\xi_w \equiv (1-\alpha_w\beta)(1-\alpha_w)/[\alpha_w(1+\theta_w\nu)]$.

Once again, we are interested in characterizing optimal policy from a timeless perspective. We observe from the form of the structural relations (2.34) and the definition of $V_{j,t}$ that the aspects of the expected future evolution of the endogenous variables that affect the feasible set of values for inflation rates, real wage and output in any period t can be summarized (in our second-order approximation to the structural relations) by the expected values of $V_{j,t+1}$, $Z_{j,t+1}$ for $j = p, w$. Hence the only commitments regarding future outcomes that can be of value in improving stabilization outcomes in period t can be summarized by commitments at t regarding the state-contingent values of those two variables in the following period. It follows that we are interested in characterizing optimal policy from any date t_0 onward subject to the constraint that given values for V_{j,t_0}, Z_{j,t_0} for $j = p, w$ be satisfied,[13] in addition to the constraints represented by the structural equations.

But given predetermined values for V_{j,t_0} the value of the transitory component T_{t_0} is predetermined. Hence, over the set of admissible policies, higher values of (2.37) correspond to lower values of

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{q_y}{2} (\hat{Y}_t - \hat{Y}_t^*)^2 + \frac{q_p}{2} \pi_{p,t}^2 + \frac{q_w}{2} \pi_{w,t}^2 \right\}. \tag{2.46}$$

It follows that we may rank admissible policies in terms of the implied value of the discounted quadratic loss function (2.46). Because this loss function is purely quadratic (*i.e.,* lacking linear terms), it is possible to evaluate it to second order using only a first-order approximation to the equilibrium evolution of inflation and output under a given policy. Hence the log-linear approximate structural relations (2.43), (2.44) and (2.45) are sufficiently accurate for our purposes.

Similarly, it suffices that we use log-linear approximations to the variables V_{j,t_0} in describing the initial commitments, which are given by $\hat{V}_{j,t_0} = \pi_{j,t_0}$ for $j = p, w$. Then an optimal policy from a timeless perspective is a policy from date t_0 onward that minimizes the quadratic loss function (2.46) subject to the constraints implied by the linear structural relation (2.43), (2.44) and (2.45) holding in each period $t \geq t_0$ given the initial condition $\hat{\omega}_{R,t_0-1}$and subject also to the constraints that a certain predetermined values for $\hat{V}_{p,t_0}$ and $\hat{V}_{w,t_0}$ be achieved.[14] This last constraint may

[13]Note that a specification of initial values for these four variables corresponds, in our quadratic approximation to the structural equations, to a specification of initial values for the four variables $F_{p,t_0}, K_{p,t_0}, F_{w,t_0}, K_{w,t_0}$ in section 1.

equivalently be expressed as a constraint on the initial inflation rates,

$$\pi_{p,t_0} = \bar{\pi}_{p,t_0} \qquad \pi_{w,t_0} = \bar{\pi}_{w,t_0}. \tag{2.47}$$

2.2 Comparison with Erceg, Henderson and Levin

Thus we obtain a quadratic stabilization objective (2.46) similar to the one derived in Erceg *et al.* (2000) under the assumption that $\Phi = 0$, but now allowing for an arbitrary degree of steady-state distortions. As in the analysis of Erceg *et al.*, the loss function is a sum of three terms, indicating the distortions resulting from variations in the rate of price inflation, the rate of wage inflation, and the output gap, respectively.

There are, however, some noteworthy differences between (2.46) and the loss function of Erceg *et al.*. One is that the loss function of Erceg *et al.* is expressed as a sum of *variances* of the three variables (price inflation, wage inflation, and the output gap), whereas our loss function is linear in the expected values of these variables squared. Our loss function implies (assuming that $q_y, q_p, q_w > 0$, as discussed below) that an increase in the variance of any of the variables, holding constant its mean level, will lower welfare; and indeed our loss function is linear in the variances, holding constant the expected values of the variables. But we find that there are also losses associated with an average rate of price or wage inflation different from zero (in either direction), and similarly with an average output gap different from zero; these effects are neglected by Erceg *et al.* by assumption.[15]

The loss function (2.46) also differs from the one derived by Erceg *et al.* in that it involves expected losses in each of an infinite sequence of periods, with the losses expected in future periods discounted at the rate β^t. The form of loss function derived by Erceg *et al.* is instead obtained, following Rotemberg and Woodford (1997), by evaluating the *unconditional expectation* of the utility of the representative household in the stationary equilibrium implied by one stationary policy rule or another; since

[14]The constraint associated with a predetermined value for Z_{t_0} can be neglected, in a first-order characterization of optimal policy, because the variable Z_t does not appear in the first-order approximation to the aggregate-supply relation.

[15]Erceg *et al.* restrict their attention to policies with the property that in the absence of shocks, the equilibrium obtained will be the optimal steady state. This restriction is innocuous as far as the characterization of optimal stabilization policy is concerned (since the optimal policy belongs to the class considered); but the more general form of loss function provides additional insight into the nature of optimal policy.

the unconditional expectation of the period utility in such an equilibrium is the same each period, one need only consider the unconditional expectation of the utility flow in a single period. The alternative (discounted) welfare measure derived here is instead appropriate if one wishes to characterize optimal policy in the sense described above (what we have called "optimal policy from a timeless perspective"). One advantage of defining the policy problem as we have here is that it allows us to use standard methods for the solution of (discounted) linear-quadratic stochastic control problems to characterize optimal policy.[16]

Apart from these differences in what our loss function measures (and hence in the form in which we report our results), there are also differences in our conclusions that result from the fact that we treat the more general case in which Φ (our measure of the overall severity of steady-state distortions) need not equal zero. First of all, a non-zero value of Φ affects the quantitative magnitudes of the weights q_y, q_p, q_w on the different stabilization objectives. In the case that $\bar{G} = 0$ (there are no steady-state government purchases), each of these weights is proportional to

$$(\omega + \sigma^{-1}) + \Phi(1 - \sigma^{-1}).$$

It then follows that increasing Φ (for given values of the other model parameters) does not change the *relative* weights on alternative stabilization objectives, and hence the relative ranking of alternative equilibria. Even in this case, however, the assumed value of Φ will affect one's conclusion about how much the improvement of stabilization policy matters for welfare; in the case that we judge to be most realistic, in which $\sigma > 1$,[17] a higher value of Φ implies greater welfare gains from stabilization. For example, if we calibrate the parameters ω and σ in accordance with the estimates of Rotemberg and Woodford (1997),[18] then an inefficiency wedge of a more realistic magnitude, $\Phi = 1/3$,[19] would increase the expected losses from any given degree of aggregate volatility by 45 percent, relative to what would be obtained under the assumption that $\Phi = 0$.

[16] For further discussion, see Woodford (2003, chap. 7).

[17] Note that in this model, σ is the intertemporal elasticity of substitution for all private expenditure, and not simply for non-durable consumer expenditure. See Woodford (2003, chaps. 4, 5) for further discussion.

[18] These values are $\omega = .473$ and $\sigma^{-1} = .157$.

[19] This would result, for example, if we assume an elasticity of demand $\theta = 10$, a wage markup of 8 percent, and an average tax rate $\bar{\tau}$ of 20 percent.

In a more realistic parameterization, of course, one should allow for the existence of a positive average level of government purchases, $\bar{G}$, so that $s_C < 1$. In this case, increasing Φ does not increase q_y by as great a factor as the increase in the weights q_p and q_w; hence the relative weight on the output-stabilization objective should be somewhat lower in an economy with a distorted steady state than would be appropriate if $\Phi = 0$. It is not clear, however, how important this qualification is likely to be in practice. Under the calibration just considered, for example, if we assume that $\bar{G}$ is equal to 20 percent of steady-state output, then increasing Φ from 0 to 1/3 will increase q_p and q_w by nearly 45 percent, as just discussed, while it will increase q_y by a factor of only 41 percent. However, the value obtained for the relative weight $q_y/(q_p + q_w)$ under the assumption that $\Phi = 0$ is exaggerated only by slightly more than 2 percent.

Under more extreme assumptions about the share of government purchases in total demand, the mis-estimation of the appropriate relative weight on output stabilization could be much greater. In fact, the correct value of q_y indicated by (2.38) may actually be *negative,* whereas Erceg *et al.* conclude that the relative weight on the output stabilization objective is positive (as we also find, if $\Phi = 0$). This failure of convexity of our welfare-theoretic loss function does not necessarily imply that the second-order conditions for a local welfare maximum fail to hold, or that randomization of policy would be welfare-improving, as discussed in Benigno and Woodford (2004). But such a case would mean that the conclusions of Erceg *et al.* about the degree to which one should be willing to accept greater variability of price and wage inflation for the sake of output-gap stabilization would be quite inaccurate. This will occur, however, only under fairly extreme assumptions. For example, a sufficient condition for q_y to be positive, *regardless* of the magnitude of the steady-state distortions, is that

$$s_G < \frac{(1+\omega)(\omega+\sigma^{-1})}{(1+\omega)(\omega+\sigma^{-1})+\sigma^{-1}}. \tag{2.48}$$

(Here $s_G \equiv \bar{G}/\bar{Y}$ is the steady-state share of government purchases in total demand.) For moderate values of Φ, the value of s_G can be even larger; but even the bound (2.48) is likely to hold. For example, in the case of the Rotemberg-Woodford parameter values, this bound holds as long as government purchases are no more than 85 percent of GDP.

Allowing for $\Phi > 0$ also changes the definition of the target output level $\hat{Y}_t^*$ in the welfare-theoretic loss function (2.46). Contrary to what Erceg *et al.* obtain, $\hat{Y}_t^*$ no longer corresponds in general to the equilibrium level of output under flexible wages and prices, $\hat{Y}_t^n$, as shown by (2.39). We observe that when $\Phi = 0$, $\omega_1 = 1$ and $\omega_2 = 0$, so that (2.39) implies that $\hat{Y}_t^* = \hat{Y}_t^n$, in the absence of fluctuations in the tax rate (also not considered by Erceg *et al.*). If, instead, $\Phi > 0$, and in addition s_G is positive (but less than the upper bound (2.48)), then $\omega_1 > 1$. This means that fluctuations in tastes or technology move $\hat{Y}_t^*$ by *more* than their effect on $\hat{Y}_t^n$.[20] This has the consequence that attempting to stabilize output around trend rather than around the time-varying target level would be an even greater mistake than is indicated by an analysis that assumes that $\Phi = 0$.

Furthermore, when $\Phi > 0$, and s_G satisfies (2.48),[21] $\omega_2 > 0$ in (2.39). Indeed, one can show that

$$\omega_2 > \frac{\sigma^{-1}}{\omega + \sigma^{-1}}(\omega_1 - 1),$$

from which it follows (also using (2.42)) that an increase in government purchases increases $\hat{Y}_t^*$ by *less* than the increase in $\hat{Y}_t^n$. This means that it is not desirable to allow output to increase quite as much in response to an increase in government purchases as would occur under flexible wages and prices.[22]

The fact that the target level of output will move in a somewhat different way than the flexible-wage-and-price equilibrium level of output (or natural rate of output) has consequences for the degree to which stabilization of some combination of wages and prices, without attention to the consequences of policy for aggregate real activity, is likely to provide a good approximation to optimal policy. As a result, some of the more suggestive results of Erceg *et al.* may not be quite so accurate a guide to policy in the case of significant steady-state distortions.

We have shown that the policy objective (2.46) can be expressed solely as a function of the evolution of the inflation rates and the welfare-relevant output gap

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^*.$$

It is useful to write the linear constraints implied by our model's structural equations in terms of the welfare-relevant output gap as well. The aggregate-supply relations

[20] Under the parameter values considered above, for example, one would obtain $\omega_1 = 1.02$.

[21] In fact, it suffices for this conclusion (and for those of the previous paragraph) that s_G be small enough for q_y to be positive.

[22] For example, under the parameter values considered above, an increase in government purchases equal to one percent of steady-state output would increase $\hat{Y}_t^n$ by 0.25 percent, while it would increase $\hat{Y}_t^*$ by only 0.14 percent.

(2.43) and (2.44) can alternatively be expressed as

$$\pi_{p,t} = \kappa_p x_t + \xi_p[\hat{\omega}_{R,t} - \hat{\omega}_t^n] + \beta E_t \pi_{p,t+1} + u_{p,t}, \tag{2.49}$$

$$\pi_{w,t} = \kappa_w x_t - \xi_w[\hat{\omega}_{R,t} - \hat{\omega}_t^n] + \beta E_t \pi_{w,t+1} + u_{w,t} \tag{2.50}$$

where $u_{j,t}$, for $j = p, w$ are composite "cost-push" terms. In terms of our previous notation for the exogenous disturbances in the model, these are given by

$$\begin{aligned} u_{j,t} &\equiv \kappa_j(\hat{Y}_t^* - \hat{Y}_t^n) \\ &= \kappa_j(\omega_1 - 1)\hat{Y}_t^n - \kappa_j\omega_2\hat{G}_t + \kappa_j\omega_3\hat{\tau}_t. \end{aligned}$$

The presence of these "cost-push" terms[23] (not present in the aggregate-supply relations of Erceg *et al.*) implies a tension between the goals of wage and price stabilization, on the one hand, and output-gap stabilization (in the welfare-relevant sense) on the other. In the case that $\Phi = 0$, then $\omega_1 = 1, \omega_2 = 0$, and $u_{p,t} = u_{w,t} = 0$, except if there are fluctuations in the tax rate. If, instead, $\Phi > 0$, then there are other reasons for the cost-push terms to be non-zero. As we have just discussed, in the case of greatest interest, fluctuations in preferences or technology that raise the natural rate of output will result in positive cost-push terms in both (2.49) and (**??**), while increases in government purchases will result in negative cost-push terms in both equations.

This makes it even more difficult for all three stabilization goals to be simultaneously achieved than is indicated by the analysis of Erceg *et al.* For example, Erceg *et al.* conclude that if either wages or prices are completely flexible (so that the welfare-theoretic weight on one of the stabilization objectives is zero), then it should be possible to fully achieve both of the remaining stabilization objectives by completely stabilizing wage inflation (if only wages are sticky) or price inflation (if only prices are sticky). In the presence of cost-push terms, this ceases to be the case.

[23]Here we adopt the terminology of Clarida *et al.* (1999) for the case of a model with sticky prices only. A more desirable terminology might be "inefficient supply shocks," as we are interested in disturbances to the aggregate-supply relations that are not due to changes in the efficient level of output. There are variety of reasons for non-zero terms of this kind to appear, which need not correspond to the specific sorts of disturbances traditionally associated with the "cost-push" terminology. And it is equally important to recognize that not all disturbances that affect the cost of supplying output represent "cost-push" shocks in the sense in which we use the term here, since such disturbances usually imply a change in the efficient level of output.

Even when prices are fully flexible, the presence of the cost-push terms implies that complete stabilization of wage inflation will not imply complete stabilization of the welfare-relevant output gap, or vice versa.[24]

Erceg *et al.* find, on the basis of numerical analysis of a calibrated model, that a simple policy rule that stabilizes an index of wages and prices provides a close approximation to optimal policy, if the relative weight on wages as opposed to prices in this index is appropriately chosen.[25] However, this result most likely depends on their having made assumptions under which there are no cost-push terms. For example, it is easy to see why the result is true, if there are no cost-push terms, in the case just discussed in which only wages are sticky. (In that case, the appropriate index to target involves nominal wages only.) But when cost-push terms are present, as is almost inevitably the case if the steady state is distorted, optimal policy no longer corresponds to stabilization of the nominal wage; instead, the nominal wage should be a function of the history of the cost-push terms.[26] On the other hand, the optimal evolution of the real wage (and hence, of goods prices) should depend on the evolution of the natural real wage ω_t^n as well. In general, real disturbances will affect the natural real wage in a different way than they affect the cost-push terms, and so one cannot expect there to be any linear combination of wages and prices that will be constant in an optimal equilibrium. Since in this case, the optimal simple rule is *fully* optimal when there are no cost-push terms, but can be far from optimal when Φ is far from zero, one suspects that the same is true when both wages and prices are sticky.[27]

Erceg *et al.* also find that another class of simple policy rules, in which a weighted average of price inflation and the output gap is stabilized, also provides a good approximation to optimal policy when the weights are appropriately chosen. But here again, it is likely that the result depends on the absence of cost-push terms (in the case of substantial stickiness of wages). Let us once more consider the simple case of perfectly flexible prices but sticky wages. In this case, optimal policy requires complete

[24]The corresponding result in the case of an economy in which only prices are sticky is established by Benigno and Woodford (2004).

[25]See also Woodford (2003, chap. 6) for results in the same vein.

[26]A method that can be used to characterize the way in which the wage should depend on the history of disturbances is discussed in the next section.

[27]The same conclusion is supported by a consideration of the case of "equally sticky" wages and prices in section 3.1 below.

stabilization of the nominal wage when there are no cost-push shocks. Flexibility of prices means that the pricing relation (2.49) reduces to

$$\hat{\omega}_{R,t} - \hat{\omega}_t^n + \omega_p x_t = 0,$$

if there is no cost-push term. At the same time, (2.50) implies that in the optimal equilibrium, since wage inflation is always zero,

$$\kappa_w x_t - \xi_w[\hat{\omega}_{R,t} - \hat{\omega}_t^n] = 0,$$

if there is no cost-push term. Together, these two relations imply that $x_t = 0$ in the optimal equilibrium, which is a limiting case of the class of simple rules.

On the other hand, if $\Phi > 0$, cost-push terms are present in both (2.49) and (2.50). It is no longer optimal to fully stabilize wage inflation, exactly because this will no longer imply complete stability of the output gap; instead, the optimal evolution of both the nominal wage and the output gap will be a function of the history of cost-push disturbances. At the same time, the optimal evolution of the real wage (and hence, of goods prices), will depend on the evolution of the natural real wage as well. Once again, there will in general be no linear combination of price inflation and the output gap for which these different sorts of dependence on the history of real disturbances will happen to cancel. And since the family of simple rules ceases to include an optimal rule even in this special case, it is likely that it ceases to include any rule that is so close to being optimal as Erceg *et al.* report, in the case that both wages and prices are sticky.

3 Optimal Stabilization Policy

We now use our linear-quadratic approximate policy problem to characterize optimal policy in the event of small enough disturbances. We begin by noting that the first-order conditions associated with an LQ problem of this kind characterize an optimum only in the case that certain second-order conditions are satisfied as well. However, it follows from our results in the previous section that the weights $q_p, q_w > 0$. Hence the loss function (2.46) is convex (and the second-order conditions are necessarily satisfied) as long as $q_y > 0$ as well.[28] A sufficient condition for this, in turn, is that

[28]This condition is sufficient but not necessary. See further discussion in Benigno and Woodford (2004).

the share of government purchases in total demand satisfy (2.48). As long as this bound is satisfied, the solution to the first-order conditions will represent an optimum of the LQ problem. This means that in the even of small enough disturbances, this same solution will represent a linear approximation to a policy that represents at least a *local* welfare optimum in the exact model.

3.1 The Case of "Equally Sticky" Wages and Prices

As stressed by Erceg et al. (2000), it is not in general possible to fully stabilize all the target variables in the loss function (2.46). However, in the absence of cost-push shocks, optimal policy still corresponds to complete stabilization of an appropriately defined index of wages and prices, in at least one special case. Suppose that $\theta_w \phi^{-1} = \theta_p$ and that $\kappa_p = \kappa_w = \kappa$. (We can think of this special case as one in which wages and prices are "equally sticky".) In this case the loss function (2.46) can be written as

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{q_y}{2} {x_t}^2 + \frac{q_\pi}{2} \bar{\pi}_t^2 + \frac{q_p}{2}(1-\gamma)(\hat{\omega}_{R,t} - \hat{\omega}_{R,t-1})^2 \right\} \tag{3.51}$$

where $\bar{\pi}_t \equiv \gamma \pi_{p,t} + (1-\gamma)\pi_{w,t}$ is a weighted average of the price and wage inflation rates, with weight $0 < \gamma < 1$ determined by $\gamma \equiv \omega_p/(\omega + \sigma^{-1})$, and where $q_\pi \equiv q_p/\gamma$. Under these conditions, by subtracting (2.50) from (2.49) and using (2.45), we obtain a difference equation for the evolution of the real wage, from which it follows that the real wage is independent of policy. Moreover, by taking a weighted average of (2.50) and (2.49), we obtain

$$\bar{\pi}_t = \kappa x_t + \beta E_t \bar{\pi}_{t+1} + u_t, \tag{3.52}$$

where $u_t \equiv \gamma u_{p,t} + (1-\gamma) u_{w,t}$.

In the case that $\Phi = 0$ and there are no variations in the tax rate, as assumed by Erceg *et al.*, there are no cost-push terms, and $u_t = 0$ at all times in (3.52). It then follows that complete stabilization of $\bar{\pi}_t$ implies complete stabilization of x_t as well. Since the real wage evolves independently of policy in this case, it is then obvious that (3.51) attains its lowest possible value under such a policy. Hence it is optimal to completely stabilize a weighted average of price inflation and wage inflation.

However, even when wages and prices are "equally sticky," this result fails to obtain in the case of a distorted steady state.[29] When $\Phi > 0$, real disturbances of

[29] It would also fail if there are variations in tax rates or in market power that would give rise to

any sort will generally result in a non-zero cost-push term u_t in (3.52), as discussed in the previous section. Complete stabilization of $\bar{\pi}_t$ continues to be possible, but in this case requires fluctuations in x_t, and it will be preferable to allow some degree of variation in $\bar{\pi}_t$ for the sake of greater stability of the output gap.

Since real wages are independent of policy, to characterize the optimal tradeoff one can simply consider the processes $\{x_t, \bar{\pi}_t\}$ that maximize (3.51) under the constraint (3.52) for each $t \geq t_0$, given an initial commitment for the value of $\bar{\pi}_{t_0}$. One observes that the form of this problem is the same — and that the solution is therefore the same (in the case of a given $\{u_t\}$ process and given values of q_π and q_y) — as in the $\Phi = 0$ case treated in Woodford (2003, chap. 7).[30] We recall here some of the main results presented there, which directly apply to the present case as well.

The first-order conditions for the optimization problem just stated are of the form

$$q_\pi \bar{\pi}_t + \varphi_t - \varphi_{t-1} = 0, \tag{3.53}$$

$$q_y x_t - \kappa \varphi_t = 0, \tag{3.54}$$

for each $t \geq t_0$, where φ_t is the Lagrange multiplier associated with the constraint (3.52) in period t. Bounded processes $\{\bar{\pi}_t, x_t, \varphi_t\}$ that satisfy (3.52) and (3.53) – (3.54) for each $t \geq t_0$ and are consistent with the initial condition (2.47) represent an optimum. Using (3.53) to eliminate $\bar{\pi}_t$ and (3.54) to eliminate x_t,[31] (3.52) becomes an equation for the evolution of the multiplier

$$\beta q_y E_t \varphi_{t+1} - [(1+\beta) q_y + \kappa^2 q_\pi] \varphi_t + q_y \varphi_{t-1} = q_\pi q_y u_t. \tag{3.55}$$

The initial condition (2.47) can similarly be expressed as a constraint on the path of the multipliers

$$\varphi_{t_0} - \varphi_{t_0 - 1} = -q_\pi \bar{\pi}_{t_0}. \tag{3.56}$$

An optimum can then be described by a bounded process $\{\varphi_t\}$ for all dates $t \geq t_0 - 1$ that satisfies (3.55) for each $t \geq t_0$ and is also consistent with (3.56).

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cost-push terms even in the case that $\Phi = 0$.

[30] See also Clarida, Gali and Gertler (1999) for analysis of an LQ problem of this form.

[31] Here we assume that both $q_\pi, q_y \neq 0$. Note that if either q_π or q_y happens to equal zero, optimal policy is easily characterized: it consists simply of the complete stabilization of the variable with the non-zero weight in the loss function.

Equation (3.55) has a unique bounded solution consistent with (3.56) if and only if the characteristic equation

$$\beta\mu^2 - \left[1 + \beta + \frac{\kappa^2 q_\pi}{q_y}\right]\mu + 1 = 0 \tag{3.57}$$

has exactly one root such that $|\mu| < 1$. This requires that the characteristic equation have real roots, exactly one of which lies in the interval between -1 and 1; this in turn is true if and only if $q_\pi \neq 0$ and

$$\frac{q_y}{q_\pi} > -\frac{\kappa^2}{2(1+\beta)}. \tag{3.58}$$

Note that (3.58) is necessarily satisfied if (2.48) holds, since in that case $q_\pi, q_y > 0$.

A characterization of the optimal equilibrium is then obtained by solving (3.53) and (3.54) for $\bar{\pi}_t$ and x_t respectively, where the multiplier process $\{\varphi_t\}$ is specified recursively by the relation[32]

$$\varphi_t = \mu\varphi_{t-1} - q_\pi \sum_{j=0}^{\infty} \beta^j \mu^{j+1} E_t u_{t+j}. \tag{3.59}$$

Here μ is the root of (3.57) that satisfies $-1 < \mu < 1$, and the initial value φ_{t_0-1} is chosen so that the solution is consistent with the precommitted value for $\bar{\pi}_{t_0}$.

We note that even in the special case that wages and prices are "equally sticky," optimal policy will not involve complete stabilization of any weighted average of wages and prices. Instead, the optimal evolution of $\bar{\pi}_t$ will depend on the history of cost-push disturbances. The optimal evolution of any other index of wages and prices will depend both on this and the exogenous determinants of real wages, and since different real disturbances will affect u_t and the real wage $\omega_{R,t}$ in different ways, there will not generally be any index of wages and prices that will remain constant in the optimal equilibrium.

[32]Details of this derivation are given in the appendix.

3.2 The General Case

More generally, to derive the optimal policy we can write the Lagrangian as

$$
\begin{aligned}
\mathcal{L}_{t_0} \;=\; & E_{t_0}\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{q_y}{2}x_t^2+\frac{q_p}{2}\pi_{p,t}^2+\frac{q_w}{2}\pi_{w,t}^2+\varphi_{p,t}(\pi_{p,t}-\kappa_p x_t-\xi_p\hat{\omega}_{R,t}-\beta\pi_{p,t+1}) \\
& +\varphi_{w,t}(\pi_{w,t}-\kappa_w x_t+\xi_w\hat{\omega}_{R,t}-\beta\pi_{w,t+1})+\varphi_{r,t}(\hat{\omega}_{R,t}-\hat{\omega}_{R,t-1}-\pi_{w,t}+\pi_{p,t})+ \\
& +\varphi_{1,t_0-1}\pi_{p,t_0}+\varphi_{2,t_0-1}\pi_{w,t_0}\}.
\end{aligned}
$$

The first-order conditions obtained by differentiation are then

$$q_y x_t-\kappa_p\varphi_{p,t}-\kappa_w\varphi_{w,t}=0; \tag{3.60}$$

$$q_p\pi_{p,t}+\varphi_{p,t}-\varphi_{p,t-1}+\varphi_{r,t}=0; \tag{3.61}$$

$$q_w\pi_{w,t}+\varphi_{w,t}-\varphi_{w,t-1}-\varphi_{r,t}=0; \tag{3.62}$$

$$\xi_p\varphi_{p,t}-\xi_w\varphi_{w,t}-\varphi_{r,t}+\beta E_t\varphi_{r,t+1}=0, \tag{3.63}$$

for each $t\geq t_0$. The first-order conditions (3.60) to (3.63) together with the structural equations (2.50), (2.49) and (2.45) need to be solve for the optimal path of the lagrange multipliers $\{\varphi_{p,t},\,\varphi_{w,t},\,\varphi_{r,t}\}$ and the variables $\{x_t,\,\pi_{p,t},\,\pi_{w,t},\,w_{R,t}\}$ given the initial conditions (2.47). We note that the initial conditions can similarly be expressed as a constraint on the path of the multipliers

$$\varphi_{p,t_0}-\varphi_{p,t_0-1}+\varphi_{r,t_0}=-q_p\bar{\pi}_{p,t_0},$$

$$\varphi_{w,t_0}-\varphi_{w,t_0-1}-\varphi_{r,t_0}=-q_w\bar{\pi}_{w,t_0}.$$

We show in the appendix that we can express the above conditions as a linear system of the form

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_4 \end{bmatrix} E_t\begin{pmatrix} z_{1,t+1} \\ z_{2,t}\end{pmatrix}=\begin{bmatrix} B_1 & B_2 \\ B_3 & B_4\end{bmatrix}\begin{pmatrix} z_{1,t} \\ z_{2,t-1}\end{pmatrix}+\begin{bmatrix} C_1 \\ C_2\end{bmatrix}\upsilon_t,$$

for matrices defined in the appendix, where

$$z_{1,t}'\equiv[\varphi_{p,t}\;\;\varphi_{w,t}\;\;\varphi_{r,t}],$$

$$z_{2,t-1}'\equiv[\hat{\omega}_{R,t-1}\;\varphi_{p,t-1}\;\;\varphi_{w,t-1}]$$

and

$$\upsilon_t' \equiv [\hat{\omega}_t^n \quad u_{p,t} \quad u_{w,t}].$$

The determinacy of the equilibrium depends on the roots of the characteristic equation associated with the system (A.67)

$$\det(B - \mu A) = 0.$$

Rational-expectations equilibrium is determinate if the number of roots μ_i such that $|\mu_i| < 1$ is exactly equal to the number of predetermined variables, which in this case is three. Under this condition, we show in the appendix that the unique non-explosive solution is of the form

$$z_{1,t} = -(VA)_1^{-1}(VA)_2 z_{2,t-1} - (VA)_1^{-1} E_t \sum_{j=0}^{\infty} \Phi^{-(j+1)} VC\upsilon_{t+j}, \tag{3.64}$$

$$z_{2,t} = A_4^{-1}(B_4 - B_3(VA)_1^{-1}(VA)_2) z_{2,t-1} + A_4^{-1} C_2 \upsilon_t - A_4^{-1} B_3 (VA)_1^{-1} E_t \sum_{j=0}^{\infty} \Phi^{-(j+1)} VC\upsilon_{t+j}, \tag{3.65}$$

for matrices again defined in the appendix. Using (3.60) to (3.63) and (3.64), (3.65) we can obtain the optimal paths of the variables $\{x_t, \pi_{p,t}, \pi_{w,t}, w_{R,t}\}$.

3.3 Optimal Targeting Rules

Finally, following Giannoni and Woodford (2003), we can use the first-order conditions to eliminate the three Lagrange multipliers, obtaining a target criterion of the form

$$(\kappa_w - \kappa_p)\pi_t^{asym} + (\xi_p + \xi_w) q_t + (\kappa_w - \kappa_p)\{E_t[\beta q_{t+1} - q_t] - E_{t-1}[\beta q_t - q_{t-1}]\} = 0, \tag{3.66}$$

where

$$\pi_t^{asym} \equiv q_p \xi_p \pi_{p,t} - q_w \xi_w \pi_{w,t}$$

is a measure of the asymmetry between price and wage inflation,

$$\pi_t^{sym} \equiv \frac{q_p \xi_p \pi_{p,t} + q_w \xi_w \pi_{w,t}}{q_p \xi_p + q_w \xi_w}$$

is a average of the rates of price and wage inflation, and

$$q_t \equiv (q_p \xi_p + q_w \xi_w) \left[\pi_t^{sym} + \frac{q_y}{q_p \xi_p + q_w \xi_w} (x_t - x_{t-1}) \right].$$

This criterion holds at all times in the optimal equilibrium, and a commitment to use monetary policy to ensure that it holds ensures that the only non-explosive rational-expectations equilibrium consistent with the policy will be the optimal one. In the special case analyzed above in which $\kappa_w = \kappa_p = \kappa > 0$, the optimal target criterion reduces to $q_t = 0$, or

$$\pi_t^{sym} + \frac{q_y}{q_p \xi_p + q_w \xi_w}(x_t - x_{t-1}) = 0. \tag{3.67}$$

This again allows us to consider the degree to which simple policy rules of either of the two kinds discussed by Erceg *et al.* are likely to provide close approximations to optimal policy in the general case. In the special case that wages and prices are "equally sticky", there *is* a linear combination of wage inflation, price inflation and the output gap that it would be optimal to stabilize, given by the optimal target criterion (3.67). However, all three target variables enter with non-zero weights in this criterion, and because real disturbances should influence these three variables in two distinct ways in the optimal equilibrium (both through their effects on the cost-push terms and through their effects on the natural real wage), as discussed in the previous section, it will not generally be possible to closely approximate any one of them by a linear combination of the other two (*except* for the relation implied by this target criterion itself). Hence one should not expect optimal policy to be well-characterized by a rule that stabilizes any linear combination of wage inflation and price inflation alone, or by a rule that stabilizes a linear combination of price inflation and the output gap alone. In the more general case, optimal policy cannot even be characterized by a static relation between all three variables; but there is even less reason to believe that a good approximation to optimal policy can be obtained without reference to all three variables.

4 Conclusion

We have shown how to extend the analysis of Erceg *et al.* to treat the case in which the steady-state equilibrium level of output under a policy that maintains zero inflation is suboptimal, due to tax distortions and market power, and in which, as a consequence, the effects of stabilization policy on the average level of output are important for the welfare evaluation of such policies. Even in this case, it is possible to approximate the expected utility of the representative household by a purely quadratic objective, so that welfare can be evaluated, to second-order accuracy, using only a first-order accurate solution for the equilibrium implied by a given policy rule.

As in the case of an efficient steady state treated by Erceg *et al.,* the welfare-theoretic loss function can be expressed as a sum of three quadratic terms, indicating the distortions due to non-zero levels of wage inflation, price inflation and an appropriately defined output gap, respectively. The inefficiency of the steady state does not change the general form of the loss function, but it does have quantitative implications for both the weights on each of the three stabilization objectives, and for the definition of the target level of output, deviations from which define the welfare-relevant output gap. An important consequence of a distorted steady state is that except under extremely special circumstances, one cannot expect real disturbances to move the target level of output and the natural rate of output (the equilibrium output level in the case of flexible wages and prices) to the same extent. This means that almost any kind of real disturbances will create a tension between the objectives of stabilizing the welfare-relevant output gap on the one hand and stabilizing wage and price inflation and the other. As a result, it is likely that neither of the kinds of simple rules considered by Erceg *et al.* — rules that stabilize a weighted average of wage and price inflation with no reference to the output gap, and rules that stabilize a weighted average of price inflation and the output gap with no reference to wage inflation — will come as close to approximating fully optimal policy in an economy with a distorted steady state as in the numerical examples that they consider.

Nonetheless, the most important of the conclusions of Erceg *et al.* remain valid. The stickiness of wages implies that variations in the rate of wage inflation are as closely related to distortions that monetary policy should seek to mitigate as are variations in price inflation, and as a consequence, a strict (goods-price) inflation target will not be optimal. Indeed, we have shown that in the more general model considered here, optimal policy can be characterized by a targeting rule, but the optimal target criterion generally involves the projected paths of price inflation, wage inflation, *and* the output gap. The welfare gains from adoption of a more sophisticated form of inflation target may be substantial; and our analysis suggests that they may be even larger when one takes account of the likely degree of distortion of the steady-state level of output in a realistic model.

A Appendix

A.1 The deterministic steady state

Here we show the existence of a steady state, *i.e.,* of an optimal policy (under appropriate initial conditions) of the recursive policy problem that involves constant values of all variables. We consider a deterministic problem in which the exogenous disturbances $\bar{C}_t$, G_t, $\bar{H}_t$, A_t, τ_t each take constant values $\bar{C}$, $\bar{H}$, $\bar{A}$, $\bar{\tau} > 0$ and $\bar{G} \geq 0$ for all $t \geq t_0$. We wish to find an initial degree of price and wage dispersions Δ_{p,t_0-1}, Δ_{w,t_0-1}, initial real wage $w_{R,t_0-1} \equiv W_{t_0-1}/P_{t_0-1}$ and initial commitments $X_{t_0} = \bar{X}$ such that the recursive (or "stage two") problem involves a constant policy $x_{t_0} = \bar{x}$, $X_{t+1} = \bar{X}$ each period, in which $\bar{\Delta}_p$, $\bar{\Delta}_w$ and $\bar{w}$ are equal to the initial values.

We thus consider the problem of maximizing

$$U_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(Y_t, \Delta_{p,t}, \Delta_{w,t}) \tag{A.1}$$

subject to the constraints

$$K_{p,t} p(\Pi_{p,t})^{\frac{1+\omega_p\theta_p}{\theta_p-1}} = F_{p,t}, \tag{A.2}$$

$$F_{p,t} = (1-\bar{\tau}) u_y(Y_t - \bar{G}) Y_t + \alpha_p \beta \Pi_{p,t+1}^{\theta_p-1} F_{p,t+1}, \tag{A.3}$$

$$K_{p,t} = \phi \mu_p u_y(Y_t - \bar{G}) w_{R,t} Y_t^{\phi} \bar{A}^{-\phi} + \alpha_p \beta \Pi_{p,t+1}^{\theta_p(1+\omega_p)} K_{p,t+1}, \tag{A.4}$$

$$\Delta_{p,t} = \alpha_p \Delta_{p,t-1} \Pi_{p,t}^{\theta_p(1+\omega_p)} + (1-\alpha_p) p(\Pi_{p,t})^{-\frac{\theta_p(1+\omega_p)}{1-\theta_p}}, \tag{A.5}$$

$$K_{w,t} p(\Pi_{w,t})^{\frac{1+\nu\theta_w}{\theta_w-1}} = F_{w,t}, \tag{A.6}$$

$$F_{w,t} = u_y(Y_t - \bar{G}) Y_t^{\phi} \bar{A}^{-\phi} \Delta_{p,t} w_{R,t} + \alpha_w \beta \Pi_{w,t+1}^{\theta_w-1} F_{w,t+1}, \tag{A.7}$$

$$K_{w,t} = \mu_w v_h(Y_t^{\phi}) Y_t^{\phi} \bar{A}^{-\phi(1+\nu)} \Delta_{p,t}^{1+\nu} + \alpha_w \beta \Pi_{w,t+1}^{\theta_w(1+\nu)} K_{w,t+1}, \tag{A.8}$$

$$\Delta_{w,t} = \alpha_w \Delta_{w,t-1} \Pi_{w,t}^{\theta_w(1+\nu)} + (1-\alpha_w) p(\Pi_{w,t})^{-\frac{\theta_w(1+\nu)}{1-\theta_w}}, \tag{A.9}$$

$$w_{R,t} = \frac{\Pi_{w,t}}{\Pi_{p,t}} w_{R,t-1}, \tag{A.10}$$

and given the specified initial conditions $\Delta_{p,t_0-1}, \Delta_{w,t_0-1}$, w_{R,t_0-1}, X_{t_0} where we have defined

$$p(\Pi_{p,t}) \equiv \left(\frac{1-\alpha_p \Pi_{p,t}^{\theta_p-1}}{1-\alpha_p} \right),$$

$$p(\Pi_{w,t}) \equiv \left(\frac{1 - \alpha_w \Pi_{w,t}^{\theta_w - 1}}{1 - \alpha_w} \right).$$

We introduce Lagrange multipliers ϕ_{1t} through ϕ_{9t} corresponding to constraints (A.2) through (A.10) respectively. We also introduce multipliers dated t_0 corresponding to the constraints implied by the initial conditions $X_{t_0} = \bar{X}$; the latter multipliers are normalized in such a way that the first-order conditions take the same form at date t_0 as at all later dates. The first-order conditions of the maximization problem are then the following. The one with respect to Y_t is

$$\begin{aligned} 0 &= U_y(Y_t, \Delta_{p,t}, \Delta_{w,t}) - (1 - \bar{\tau})[u_{yy}(Y_t - \bar{G})Y_t + u_y(Y_t - \bar{G})]\phi_{2,t} \\ &\quad - \phi \mu_p \bar{A}^{-\phi} w_{R,t}[u_{yy}(Y_t - \bar{G})Y_t^{\phi} + \phi Y_t^{\phi-1} u_y(Y_t - \bar{G})]\phi_{3,t} + \\ &\quad - \bar{A}^{-\phi} w_{R,t} \Delta_{p,t}[u_{yy}(Y_t - \bar{G})Y_t^{\phi} + \phi Y_t^{\phi-1} u_y(Y_t - \bar{G})]\phi_{6,t} \\ &\quad - \mu_w \bar{A}^{-\phi(1+\nu)} \Delta_{p,t}^{1+\nu}[\phi v_{hh}(Y_t^{\phi})Y_t^{2\phi-1} + \phi v_h(Y_t^{\phi})Y_t^{\phi-1}]\phi_{7,t} \end{aligned} \tag{A.11}$$

that with respect to $\Delta_{p,t}$ is

$$\begin{aligned} 0 &= U_{\Delta_p}(Y_t, \Delta_{p,t}, \Delta_{w,t}) + \phi_{4t} - \alpha_p \beta \Pi_{p,t+1}^{\theta_p(1+\omega_p)} \phi_{4,t+1} - u_y(Y_t - \bar{G})Y_t^{\phi} \bar{A}^{-\phi} w_{R,t} \phi_{6,t} \\ &\quad - (1+\nu)\mu_w v_h(Y_t^{\phi}) \bar{A}^{-\phi(1+\nu)} Y_t^{\phi} \Delta_{p,t}^{\nu} \phi_{7,t} \end{aligned} \tag{A.12}$$

that with respect to $\Pi_{p,t}$ is

$$\begin{aligned} &\frac{1 + \omega_p \theta_p}{\theta_p - 1} p(\Pi_{p,t})^{\frac{(1+\omega_p\theta_p)}{\theta_p - 1} - 1} p_\pi(\Pi_{p,t}) K_{p,t} \phi_{1,t} - \alpha_p(\theta_p - 1)\Pi_{p,t}^{\theta_p - 2} F_{p,t} \phi_{2,t-1} \\ &- \theta_p(1+\omega_p)\alpha_p \Pi_{p,t}^{\theta_p(1+\omega_p)-1} K_{p,t} \phi_{3,t-1} - \theta_p(1+\omega_p)\alpha_p \Delta_{p,t-1} \Pi_{p,t}^{\theta_p(1+\omega_p)-1} \phi_{4,t} + \\ &- \frac{\theta_p(1+\omega_p)}{\theta_p - 1}(1 - \alpha_p) p(\Pi_{p,t})^{\frac{(1+\omega_p\theta_p)}{\theta_p - 1}} p_\pi(\Pi_{p,t}) \phi_{4,t} + \Pi_{w,t} \Pi_{p,t}^{-2} w_{R,t-1} \phi_{9,t} = 0; \end{aligned} \tag{A.13}$$

that with respect to $F_{p,t}$ is

$$-\phi_{1,t} + \phi_{2,t} - \alpha_p \Pi_{p,t}^{\theta_p - 1} \phi_{2,t-1} = 0; \tag{A.14}$$

that with respect to $K_{p,t}$ is

$$p(\Pi_{p,t})^{\frac{1+\omega_p\theta_p}{\theta_p - 1}} \phi_{1,t} + \phi_{3t} - \alpha_p \Pi_{p,t}^{\theta_p(1+\omega_p)} \phi_{3,t-1} = 0; \tag{A.15}$$

that with respect to $\Delta_{w,t}$ is

$$0 = U_{\Delta_w}(Y_t, \Delta_{p,t}, \Delta_{w,t}) + \phi_{8,t} - \alpha_w \beta \Pi_{w,t+1}^{\theta_w(1+\nu)} \phi_{8,t+1} \tag{A.16}$$

that with respect to $\Pi_{w,t}$ is

$$\begin{aligned}&\frac{1+\nu\theta_w}{\theta_w-1}p(\Pi_{w,t})^{\frac{(1+\nu\theta_w)}{\theta_w-1}-1}p_\pi(\Pi_{w,t})K_{w,t}\phi_{5,t}-\alpha_w(\theta_w-1)\Pi_{w,t}^{\theta_w-2}F_{w,t}\phi_{6,t-1}\\&-\theta_w(1+\nu)\alpha_w\Pi_{w,t}^{\theta_w(1+\nu)-1}K_{w,t}\phi_{7,t-1}-\theta_w(1+\nu)\alpha_w\Delta_{w,t-1}\Pi_{w,t}^{\theta_w(1+\nu)-1}\phi_{8,t}+\\&-\frac{\theta_w(1+\nu)}{\theta_w-1}(1-\alpha_w)p(\Pi_{w,t})^{\frac{(1+\nu\theta_w)}{\theta_w-1}}p_\pi(\Pi_{w,t})\phi_{8,t}-\Pi_{p,t}^{-1}w_{R,t-1}\phi_{9,t}=0; \qquad (A.17)\end{aligned}$$

that with respect to $F_{w,t}$ is

$$-\phi_{5,t}+\phi_{6,t}-\alpha_w\Pi_{w,t}^{\theta_w-1}\phi_{6,t-1}=0; \qquad (A.18)$$

that with respect to $K_{w,t}$ is

$$p(\Pi_{w,t})^{\frac{1+\nu\theta_w}{\theta_w-1}}\phi_{5,t}+\phi_{7,t}-\alpha_w\Pi_{w,t}^{\theta_w(1+\nu)}\phi_{7,t-1}=0; \qquad (A.19)$$

that with respect to $w_{R,t}$ is

$$\begin{aligned}0 &= -\phi\mu_p u_y(Y_t-\bar{G})Y_t^{\phi}\bar{A}^{-\phi}\phi_{3,t}-u_y(Y_t-\bar{G})Y_t^{\phi}\bar{A}^{-\phi}\Delta_{p,t}\phi_{6,t}\\&\quad+\phi_{9,t}-\beta\Pi_{w,t}\Pi_{p,t}^{-1}\phi_{9,t+1} \qquad (A.20)\end{aligned}$$

We search for a solution to these first-order conditions in which $\Pi_{p,t}=\Pi_{w,t}=\bar{\Pi}$, $\Delta_{p,t}=\bar{\Delta}_p$, $\Delta_{w,t}=\bar{\Delta}_w$, $w_{R,t}=\bar{w}_R$, $Y_t=\bar{Y}$ at all times. A steady-state solution of this kind also requires that the Lagrange multipliers take constant values. We furthermore conjecture the existence of a solution in which $\bar{\Pi}=1$, as stated in the text. Note that such a solution implies that $\bar{\Delta}_p=\bar{\Delta}_w=1$, $p(\bar{\Pi}_p)=1$, $p(\bar{\Pi}_w)=1$, $p_\pi(\bar{\Pi}_p)=-(\theta_p-1)\alpha_p/(1-\alpha_p)$, $p_\pi(\bar{\Pi}_w)=-(\theta_w-1)\alpha_w/(1-\alpha_w)$ and $\bar{K}_p=\bar{F}_p$ and $\bar{K}_w=\bar{F}_w$. Using these substitutions, we find that (the steady-state version of) each of the first-order conditions (A.11) – (A.20) is satisfied if the steady-state values satisfy

$$\begin{aligned}0 &= U_y(\bar{Y},1,1)-(1-\bar{\tau})[u_{yy}(\bar{Y}-\bar{G}))\bar{Y}+u_y(\bar{Y}-\bar{G}))]\phi_2+\\&\quad+\phi\mu_w\mu_p\bar{A}^{-\phi(1+\nu)}[\phi v_{hh}(\bar{Y}^{\phi})\bar{Y}^{2\phi-1}+\phi v_h(\bar{Y}^{\phi})\bar{Y}^{\phi-1}]\phi_2\end{aligned}$$

$$\begin{aligned}(1-\alpha_p\beta)\phi_4 &= -U_{\Delta_p}(\bar{Y},1)+u_y(\bar{Y}-\bar{G})\bar{A}^{-\phi}\bar{Y}^{\phi}\bar{w}_R\phi_6\\&\quad-(1+\nu)\mu_w v_h(\bar{Y}^{\phi})\bar{A}^{-\phi(1+\nu)}\bar{Y}^{\phi}\phi_6,\end{aligned}$$

$$\phi_1=(1-\alpha_p)\phi_2,$$

$$\phi_3 = -\phi_2,$$

$$(1-\alpha_w\beta)\phi_8 = -U_{\Delta_w}(\bar{Y},1,1)$$

$$\phi_5 = (1-\alpha_w)\phi_6,$$

$$\phi_7 = -\phi_6$$

$$\phi_6 = \phi\mu_p\phi_2$$

$$\phi_9 = 0$$

These equations can obviously be solved (uniquely) for the steady-state multipliers, given any value $\bar{Y} > 0$ and $\bar{w}_R > 0$.

Similarly, (the steady-state versions of) the constraints (A.2) – (A.10) are satisfied if

$$(1-\bar{\tau})\bar{Y}^{1-\phi} = \phi\mu_p\bar{w}_R\bar{A}^{-\phi} \tag{A.21}$$

$$u_y(\bar{Y}-\bar{G})\bar{A}^{-\phi}\bar{w}_R = \mu_w v_h(\bar{Y}^{\phi})\bar{A}^{-\phi(1+\nu)}. \tag{A.22}$$

Substituting (A.21) into (A.22) we can obtain

$$\frac{(1-\bar{\tau})}{\phi\mu_p\mu_w}u_y(\bar{Y}-\bar{G})\bar{Y} = v_h\left(\left(\frac{\bar{Y}}{\bar{A}}\right)^{\phi}\right)\left(\frac{\bar{Y}}{\bar{A}}\right)^{\phi},$$

which can be solved for the steady-state value $\bar{Y}$. Then either (A.21) or (A.22) can be solved to obtain the steady-state value $\bar{w}_R$ given $\bar{Y}$.

A.2 A second-order approximation to utility (equations (2.30) and (2.33))

We derive here equations (2.30) and (2.33) in the main text, taking a second-order approximation to (equation (1.13)) following the treatment in Woodford (2003, chapter 6). We start by approximating the expected discounted value of the sum of the utilities of the households (the policy-objective function)

$$U_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[u(Y_t;\xi_t) - \int_0^1 v(h_t(j);\xi_t)dj\right]. \tag{A.23}$$

First we note that

$$\int_0^1 v(h_t(j);\xi_t)dj = \frac{\lambda}{1+\nu}H_t^{1+\nu}\Delta_{w,t}\bar{H}_t^{-\nu} = v(H_t;\xi_t)\Delta_{w,t}$$

where $\Delta_{w,t}$ is the measure of price dispersion defined in the text. We can then write (A.23) as

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[u(Y_t; \xi_t) - v(H_t; \xi_t)\Delta_{w,t} \right]. \tag{A.24}$$

The first term in (A.24) can be approximated using a second-order Taylor expansion around the steady state defined in the previous section as

$$\begin{aligned} u(Y_t; \xi_t) &= \bar{u} + \bar{u}_c \tilde{Y}_t + \bar{u}_\xi \xi_t + \frac{1}{2} \bar{u}_{cc} \tilde{Y}_t^2 + \bar{u}_{c\xi} \xi_t \tilde{Y}_t + \frac{1}{2} \xi_t' \bar{u}_{\xi\xi} \xi_t + \mathcal{O}(||\xi||^3) \\ &= \bar{u} + \bar{Y} \bar{u}_c \cdot (\hat{Y}_t + \frac{1}{2} \hat{Y}_t^2) + \bar{u}_\xi \xi_t + \frac{1}{2} \bar{Y} \bar{u}_{cc} \hat{Y}_t^2 + \\ &\qquad + \bar{Y} \bar{u}_{c\xi} \xi_t \hat{Y}_t + \frac{1}{2} \xi_t' \bar{u}_{\xi\xi} \xi_t + \mathcal{O}(||\xi||^3) \\ &= \bar{Y} u_c \hat{Y}_t + \frac{1}{2} [\bar{Y} \bar{u}_c + \bar{Y}^2 \bar{u}_{cc}] \hat{Y}_t^2 - \bar{Y}^2 \bar{u}_{cc} g_t \hat{Y}_t + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \\ &= \bar{Y} \bar{u}_c \left\{ \hat{Y}_t + \frac{1}{2}(1 - \sigma^{-1}) \hat{Y}_t^2 + \sigma^{-1} g_t \hat{Y}_t \right\} + \\ &\qquad + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \tag{A.25}$$

where a bar denotes the steady-state value for each variable, a tilde denotes the deviation of the variable from its steady-state value (*e.g.*, $\tilde{Y}_t \equiv Y_t - \bar{Y}$), and a hat refers to the log deviation of the variable from its steady-state value (*e.g.*, $\hat{Y}_t \equiv \ln Y_t/\bar{Y}$). We use ξ_t to refer to the entire vector of exogenous shocks,

$$\xi_t' \equiv \left[\begin{array}{cccccc} \hat{G} & g_t & q_t & \hat{\tau}_t & \bar{h}_t & a_t \end{array} \right],$$

in which $\hat{G}_t \equiv (G_t - \bar{G})/\overline{Y}$, $g_t \equiv \hat{G}_t + s_C \bar{c}_t$, $\omega \equiv (\phi - 1) + \phi\nu$, $\omega q_t \equiv \nu \bar{h}_t + \phi(1+\nu) a_t$, $\hat{\tau}_t \equiv (\tau_t - \bar{\tau})/\bar{\tau}$, $\bar{c}_t \equiv \ln \bar{C}_t/\bar{C}$, $a_t \equiv \ln A_t/\bar{A}$, $\bar{h}_t \equiv \ln \bar{H}_t/\bar{H}$. Moreover, we use the definitions $\sigma^{-1} \equiv \tilde{\sigma}^{-1} s_C^{-1}$ with $s_C \equiv \bar{C}/\bar{Y}$. We have used the Taylor expansion

$$Y_t/\bar{Y} = 1 + \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 + \mathcal{O}(||\xi||^3)$$

to get a relation for $\tilde{Y}_t$ in terms of $\hat{Y}_t$. Finally the term "t.i.p." denotes terms that are independent of policy, and may accordingly be suppressed as far as the welfare ranking of alternative policies is concerned.

We may similarly approximate $v(H_t;\xi_t)\Delta_{w,t}$ by

$$\begin{aligned}
v(H_t;\xi_t)\Delta_{w,t} &= \bar{v}+\bar{v}(\Delta_{w,t}-1)+\bar{v}_h(H_t-\bar{H})+\bar{v}_h(\Delta_{w,t}-1)(H_t-\bar{H})+(\Delta_{w,t}-1)\bar{v}_\xi\xi_t+ \\
&\quad +\frac{1}{2}\bar{v}_{hh}(H_t-\bar{H})^2+(H_t-\bar{H})\bar{v}_{h\xi}\xi_t+\frac{1}{2}\xi_t'\bar{v}_{\xi\xi}\xi_t+\mathcal{O}(||\xi||^3) \\
&= \bar{v}(\Delta_{w,t}-1)+\bar{v}_h\bar{H}\left(\hat{H}_t+\frac{1}{2}\hat{H}_t^2\right)+\bar{v}_h(\Delta_{w,t}-1)\bar{H}\hat{H}_t+(\Delta_{w,t}-1)\bar{v}_\xi\xi_t+ \\
&\quad +\frac{1}{2}\bar{v}_{hh}\bar{H}^2\hat{H}_t^2+\bar{H}\hat{H}_t\bar{v}_{h\xi}\xi_t+\text{t.i.p.}+\mathcal{O}(||\xi||^3) \\
&= \bar{v}_h\bar{H}[\frac{\hat{\Delta}_{w,t}}{1+\nu}+\hat{H}_t+\frac{1}{2}(1+\nu)\hat{H}_t^2+\hat{\Delta}_{w,t}\hat{H}_t-\nu\hat{H}_t\bar{h}_t+ \\
&\quad -\frac{\hat{\Delta}_{w,t}}{1+\nu}\nu\bar{h}_t]+\text{t.i.p.}+\mathcal{O}(||\xi||^3).
\end{aligned}$$

We take a second-order expansion of (1.20), obtaining

$$\hat{\Delta}_{w,t}=\alpha_w\hat{\Delta}_{w,t-1}+\frac{\alpha_w}{1-\alpha_w}\theta_w(1+\nu)(1+\nu\theta_w)\frac{\pi_{w,t}^2}{2}+\mathcal{O}(||\xi||^3), \quad \text{(A.26)}$$

from which it follows that $\hat{\Delta}_{w,t}$ is a second-order terms (since the equation can be solved backward from date t_0-1 and written showing $\hat{\Delta}_{w,t}$ as a function of t.i.p. and quadratic terms). We now use (1.10) that in an exact form implies that

$$\hat{H}_t=\phi(\hat{Y}_t-a_t)+\hat{\Delta}_{p,t}$$

We take a second-order expansion of (1.28), obtaining

$$\hat{\Delta}_{p,t}=\alpha_p\hat{\Delta}_{p,t-1}+\frac{\alpha_p}{1-\alpha_p}\theta_p(1+\omega_p)(1+\omega_p\theta_p)\frac{\pi_{p,t}^2}{2}+\mathcal{O}(||\xi||^3), \quad \text{(A.27)}$$

from which it follows that also $\hat{\Delta}_{p,t}$ is a second-order term for the same reasons as above. This implies that

$$\hat{H}_t^2=\phi^2(\hat{Y}_t^2-2a_t\hat{Y}_t)+\mathcal{O}(||\xi||^3)$$

These results in turn allow us to approximate $v(H_t;\xi_t)\Delta_{w,t}$

$$\begin{aligned}
v(H_t;\xi_t)\Delta_{w,t} &= \bar{v}_h\bar{H}\phi\left\{\frac{\hat{\Delta}_{w,t}}{\phi(1+\nu)}+\hat{Y}_t+\frac{\hat{\Delta}_{p,t}}{\phi}+\frac{1}{2}(1+\nu)\phi(\hat{Y}_t^2-2a_t\hat{Y}_t)-\nu\hat{Y}_t\bar{h}_t\right\}+ \\
&\quad +\text{t.i.p.}+\mathcal{O}(||\xi||^3), \\
&= (1-\Phi)\bar{u}_c\bar{Y}\left\{\frac{\hat{\Delta}_{w,t}}{1+\omega}+\hat{Y}_t+\frac{\hat{\Delta}_{p,t}}{\phi}+\frac{1}{2}(1+\omega)\hat{Y}_t^2-\omega q_t\hat{Y}_t\right\}+ \\
&\quad +\text{t.i.p.}+\mathcal{O}(||\xi||^3) \qquad \text{(A.28)}
\end{aligned}$$

where we have used the steady state relation $\bar{v}_h \bar{H}\phi = (1-\Phi)\bar{u}_c\bar{Y}$ where

$$\Phi \equiv 1 - \left(\frac{\theta_p - 1}{\theta_p}\right)\left(\frac{\theta_w - 1}{\theta_w}\right)(1-\bar{\tau}) < 1$$

measures the inefficiency of steady-state output $\bar{Y}$.

Combining (A.25) and (A.28), we finally obtain equation (2.30) in the text,

$$\begin{aligned} U_{t_0} &= \bar{Y}\bar{u}_c \cdot E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Phi\hat{Y}_t - \frac{1}{2}u_{yy}\hat{Y}_t^2 + \hat{Y}_t u_{y\xi}\xi_t - u_{\Delta p}\hat{\Delta}_{p,t} - u_{\Delta_w}\hat{\Delta}_{w,t} \\ &+ \text{ t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \quad \text{(A.29)}$$

where

$$\begin{aligned} u_{yy} &\equiv (\omega + \sigma^{-1}) - \Phi(1+\omega), \\ u_{y\xi}\xi_t &\equiv [\sigma^{-1}g_t + (1-\Phi)\omega q_t], \\ u_{\Delta w} &\equiv \frac{(1-\Phi)}{1+\omega}, \\ u_{\Delta p} &\equiv \frac{(1-\Phi)}{\phi}. \end{aligned}$$

We finally observe that (A.26) and (A.27) can be integrated to obtain

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\hat{\Delta}_{w,t} = \frac{\alpha_w}{(1-\alpha_w)(1-\alpha_w\beta)}\theta_w(1+\nu)(1+\nu\theta_w)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\frac{\pi_{w,t}^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \quad \text{(A.30)}$$

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\hat{\Delta}_{p,t} = \frac{\alpha_p}{(1-\alpha_p)(1-\alpha_p\beta)}\theta_p(1+\omega_p)(1+\omega_p\theta_p)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\,\frac{\pi_{p,t}^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \quad \text{(A.31)}$$

where $\pi_{p,t} \equiv \ln P_t/P_{t-1}$ and $\pi_{w,t} \equiv \ln W_t/W_{t-1}$.

By substituting (A.30) and (A.31) into (A.29), we obtain

$$\begin{aligned} U_{t_0} &= \bar{Y}\bar{u}_c \cdot E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\Phi\hat{Y}_t - \frac{1}{2}u_{yy}\hat{Y}_t^2 + \hat{Y}_t u_{y\xi}\xi_t - \frac{1}{2}u_{\pi_p}\pi_{p,t}^2 - \frac{1}{2}u_{\pi_w}\pi_{w,t}^2] \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned}$$

This coincides with equation (2.33) in the text, where we have further defined

$$\begin{aligned}
\xi_p &\equiv \frac{(1-\alpha_p\beta)(1-\alpha_p)}{\alpha_p(1+\theta_p\omega_p)}, \\
\xi_w &\equiv \frac{(1-\alpha_w\beta)(1-\alpha_w)}{\alpha_w(1+\theta_p\nu)}, \\
u_{\pi_p} &\equiv \frac{\theta_p(1-\Phi)}{\xi_p}, \\
u_{\pi_w} &\equiv \frac{\theta_w(1-\Phi)}{\xi_w\phi}.
\end{aligned}$$

A.3 A second-order approximation to the AS equations (equations (1.19) and (1.27))

Here we need to take approximations of two similar equations of the form

$$\Gamma_{j,t} \equiv \left(\frac{1-\alpha_j\Pi_{j,t}^{\theta_j-1}}{1-\alpha_j}\right)^{-\frac{1+\omega_j\theta_j}{\theta_j-1}} = \left(\frac{F_{j,t}}{K_{j,t}}\right)^{-1}$$

for $j = p,w$. In what follows, $\omega_w = \nu$. We show below that we can do it just once and take care of the difference with some additional notation. We further re-define the variables $F_{j,t}$ and $K_{j,t}$ as

$$\begin{aligned}
F_{j,t} &\equiv E_t\left\{\sum_{T=t}^{\infty}(\alpha_j\beta)^{T-t}f_{t,T}^j\right\}, \\
K_{j,t} &\equiv E_t\left\{\sum_{T=t}^{\infty}(\alpha_j\beta)^{T-t}k_{t,T}^j\right\},
\end{aligned}$$

with

$$f_{t,T}^p \equiv (1-\tau_T)C_T^{-\tilde{\sigma}^{-1}}\bar{C}_T^{\tilde{\sigma}^{-1}}Y_T P_{t,T}^{1-\theta_p}, \tag{A.32}$$

$$k_{t,T}^p \equiv \phi\mu_p C_T^{-\tilde{\sigma}^{-1}}\bar{C}_T^{\tilde{\sigma}^{-1}}w_{R,T}Y_T^{\phi}A_T^{-\phi}P_{t,T}^{-\theta_p(1+\omega_p)}, \tag{A.33}$$

$$f_{t,T}^w \equiv C_T^{-\tilde{\sigma}^{-1}}\bar{C}_T^{\tilde{\sigma}^{-1}}Y_T^{\phi}A_T^{-\phi}\Delta_{p,T}w_{R,T}W_{t,T}^{1-\theta_w} \tag{A.34}$$

$$k_{t,T}^w \equiv \lambda\mu_w Y_T^{\phi(1+\nu)}\bar{H}_t^{-\nu}A_T^{-\phi(1+\nu)}\Delta_{p,T}^{1+\nu}W_{t,T}^{-\theta_w(1+\nu)} \tag{A.35}$$

where we have defined $P_{t,T} \equiv P_t/P_T$, $W_{t,T} \equiv W_t/W_T$. We can then obtain in an exact log-linear form that

$$\hat{\Gamma}_{j,t} + \hat{F}_{j,t} = \hat{K}_{j,t}. \tag{A.36}$$

We take a second-order expansion of $F_{j,t}$ and $K_{j,t}$, obtaining

$$\begin{aligned}\hat{F}_{j,t} + \frac{1}{2}\hat{F}_{j,t}^2 &= (1-\alpha_j\beta)E_t\left\{\sum_{T=t}^{+\infty}(\alpha_j\beta)^{T-t}(\hat{f}_{t,T}^j + \frac{1}{2}(\hat{f}_{t,T}^j)^2)\right\} \\ &\quad +\mathcal{O}(||\xi||^3), \quad (A.37)\\ \hat{K}_{j,t} + \frac{1}{2}\hat{K}_{j,t}^2 &= (1-\alpha_j\beta)E_t\left\{\sum_{T=t}^{+\infty}(\alpha_j\beta)^{T-t}(\hat{k}_{t,T}^j + \frac{1}{2}(\hat{k}_{t,T}^j)^2)\right\} \\ &\quad +\mathcal{O}(||\xi||^3). \quad (A.38)\end{aligned}$$

Plugging (A.37) and (A.38) into (A.36), we obtain

$$\begin{aligned}\hat{\Gamma}_{j,t} &= (1-\alpha_j\beta)E_t\left\{\sum_{T=t}^{+\infty}(\alpha_j\beta)^{T-t}(\hat{k}_{t,T}^j - \hat{f}_{t,T}^j)\right\} + \\ &\quad +\frac{(1-\alpha_j\beta)}{2}E_t\left\{\sum_{T=t}^{+\infty}(\alpha\beta)^{T-t}((\hat{k}_{t,T}^j)^2 - (\hat{f}_{t,T}^j)^2)\right\} + \\ &\quad \frac{1}{2}(\hat{F}_{j,t} - \hat{K}_{j,t})(\hat{F}_{j,t} + \hat{K}_{j,t}) + \mathcal{O}(||\xi||^3). \quad (A.39)\end{aligned}$$

We note that in an exact log-linear form

$$\hat{k}_{t,T}^p - \hat{f}_{t,T}^p = -(1+\omega_p\theta_p)\hat{P}_{t,T} + \hat{w}_{R,T} + \phi(\hat{Y}_T - a_T) - \hat{Y}_T - \hat{S}_T,$$

$$\begin{aligned}\hat{k}_{t,T}^w - \hat{f}_{t,T}^w &= -(1+\nu\theta_w)\hat{W}_{t,T} + \phi\nu\hat{Y}_T - \nu h_T - \phi\nu a_T + \nu\hat{\Delta}_{p,T} - \hat{w}_{R,T} \\ &\quad +\tilde{\sigma}^{-1}(\hat{C}_T - \bar{c}_T),\end{aligned}$$

where $\hat{S}_t \equiv \ln(1-\tau_t)/(1-\bar{\tau})$.

Furthermore we obtain that

$$\begin{aligned}\hat{k}_{t,T}^p + \hat{f}_{t,T}^p &= (1+\phi)\hat{Y}_T - \phi a_T + (1-2\theta_p-\omega_p\theta_p)\hat{P}_{t,T} + \hat{S}_T - 2\tilde{\sigma}^{-1}(\hat{C}_T - \bar{c}_T) + \hat{w}_{R,T} \\ &= X_{p,T} + (1-2\theta_p-\omega_p\theta_p)\hat{P}_{t,T},\end{aligned}$$

$$\begin{aligned}\hat{k}_{t,T}^w + \hat{f}_{t,T}^w &= \phi(2+\nu)\hat{Y}_T + (2+\nu)\hat{\Delta}_T - \nu\bar{h}_T - \phi(2+\nu)a_T + \hat{w}_{R,T} + \\ &\quad (1-2\theta_w-\nu\theta_w)\hat{W}_{t,T} - \tilde{\sigma}^{-1}(\hat{C}_T - \bar{c}_T) \\ &= X_{w,T} + (1-2\theta_w-\nu\theta_w)\hat{W}_{t,T},\end{aligned}$$

where we have defined

$$X_{p,T} \equiv (1+\phi)\hat{Y}_T - \phi a_T + \hat{S}_T - 2\tilde{\sigma}^{-1}(\hat{C}_T - \bar{c}_T) + \hat{w}_{R,T}.$$

$$X_{w,T} \equiv \phi(2+\nu)\hat{Y}_T + (2+\nu)\hat{\Delta}_T - \nu h_t - \phi(2+\nu)a_t + \hat{w}_{R,T} - \tilde{\sigma}^{-1}(\hat{C}_T - \bar{c}_T).$$

We can then substitute into (A.39) and get

$$\begin{aligned}\frac{1}{(1-\alpha\beta)}\hat{\Gamma}_{j,t} &= -\frac{1}{2}\hat{\Gamma}_{j,t}Z_{j,t} + E_t\sum_{T=t}^{+\infty}(\alpha_j\beta)^{T-t}(\hat{k}^j_{t,T} - \hat{f}^j_{t,T}) + \\ &+\frac{1}{2}E_t\sum_{T=t}^{+\infty}(\alpha_j\beta)^{T-t}[(\hat{k}^j_{t,T} - \hat{f}^j_{t,T})][X_{j,T} + (1-2\theta_j - \omega_j\theta_j)\hat{P}^j_{t,T}] + \\ &+\mathcal{O}(||\xi||^3), \qquad \text{(A.40)}\end{aligned}$$

where we use the definition $\hat{P}^w_{t,T} = \hat{W}_{t,T}$ and

$$Z_{j,t} \equiv E_t\sum_{T=t}^{+\infty}(\alpha_j\beta)^{T-t}[X_{j,T} + (1-2\theta_j - \omega_j\theta_j)\hat{P}^j_{t,T}].$$

By using (A.40), and defining

$$z_{j,T} \equiv \hat{k}^j_{t,T} - \hat{f}^j_{t,T} + (1+\omega_j\theta_j)\hat{P}^j_{t,T},$$

we can write

$$\begin{aligned}\frac{\hat{\Gamma}_{j,t}}{(1-\alpha_j\beta)} &= z_{j,t} + \frac{\alpha_j\beta}{(1-\alpha_j\beta)}E_t(\hat{\Gamma}_{j,t+1} - (1+\omega_j\theta_j)\hat{P}^j_{t,t+1}) - \frac{1}{2}\hat{\Gamma}_{j,t}Z_{j,t} + \frac{1}{2}\alpha_j\beta E_t\hat{\Gamma}_{j,t+1}Z_{j,t+1} + \\ &+\frac{1}{2}z_{j,t}X_{j,t} + \frac{\alpha_j\beta}{2}E_t\{\sum_{T=t+1}^{+\infty}(\alpha_j\beta)^{T-t-1}(1+\omega_j\theta_j)(1-2\theta_j-\omega_j\theta_j)(-\hat{P}^{j2}_{t,t+1} + \\ &-2\hat{P}^j_{t,t+1}\hat{P}^j_{t+1,T}) - (1+\omega_j\theta_j)\hat{P}^j_{t,t+1}X_{j,T} + (1-2\theta_j-\omega_j\theta_j)\hat{P}^j_{t,t+1}z_{j,T}\} + \mathcal{O}(||\xi||^3),\end{aligned}$$

which can be simplified to

$$\begin{aligned}\frac{\hat{\Gamma}_{j,t}}{(1-\alpha_j\beta)} &= z_{j,t} + \alpha_j\beta\frac{1}{(1-\alpha_j\beta)}E_t(\hat{\Gamma}_{j,t+1} - (1+\omega_j\theta_j)\hat{P}^j_{t,t+1}) + \frac{1}{2}z_{j,t}X_{j,t} + \\ &-\frac{1}{2}\hat{\Gamma}_{j,t}Z_{j,t} + \frac{1}{2}\alpha_j\beta E_t\{(\hat{\Gamma}_{j,t+1} - (1+\omega_j\theta_j)\hat{P}^j_{t,t+1})Z_{j,t+1}\} \\ &+\frac{\alpha_j\beta}{2(1-\alpha_j\beta)}(1-2\theta_j-\omega_j\theta_j)E_t\{(\hat{\Gamma}_{j,t+1} - (1+\omega_j\theta_j)\hat{P}^j_{t,t+1})\hat{P}^j_{t,t+1}\} + \\ &+\mathcal{O}(||\xi||^3), \qquad \text{(A.41)}\end{aligned}$$

We next take a second-order expansion of $\hat{\Gamma}_{j,t}$

$$\frac{\hat{\Gamma}_{j,t}}{(1+\omega_j\theta_j)} = \frac{\alpha_j}{1-\alpha_j}\pi_{j,t} - \frac{1-\theta_j}{2}\frac{\alpha_j}{(1-\alpha_j)^2}\pi_{j,t}^2 + \mathcal{O}(||\xi||^3), \tag{A.42}$$

and note that and $\hat{P}^j_{t-1,t} = -\pi_{j,t}$. We can then plug (A.42) into (A.41) obtaining

$$\begin{aligned}\pi_{j,t} \quad = \quad & \frac{1-\theta_{j,t}}{2}\frac{1}{(1-\alpha_j)}\pi_{j,t}^2 + \xi_j z_{j,t} + \beta E_t\pi_{j,t+1} - \frac{1-\theta_j}{2}\frac{\alpha_j\beta}{(1-\alpha_j)}E_t\pi_{j,t+1}^2 \\ & + \frac{1}{2}\xi_j z_{j,t}X_{j,t} - \frac{1}{2}(1-\alpha_j\beta)\pi_{j,t}Z_{j,t} + \frac{\beta}{2}(1-\alpha_j\beta)E_t\{\pi_{j,t+1}Z_{j,t+1}\} \\ & - \frac{\beta}{2}(1-2\theta_j-\omega_j\theta_j)E_t\{\pi_{j,t+1}^2\} + \mathcal{O}(||\xi||^3).\end{aligned} \tag{A.43}$$

By integrating equation (A.43) forward from time t_0 we can finally obtain

$$\begin{aligned}V_{j,t_0} \quad = \quad & \xi_j E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}z_{j,t} + \frac{1}{2}\xi_j E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}z_{j,t}X_{j,t} \\ & + \frac{\theta_j(1+\omega_j)}{2}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_{j,t}^2 + \mathcal{O}(||\xi||^3),\end{aligned} \tag{A.44}$$

where

$$V_{j,t_0} \equiv \pi_{j,t_0} - \frac{1-\theta_j}{2(1-\alpha_j)}\pi_{j,t_0}^2 + \frac{(1-\alpha_j\beta)}{2}\pi_{j,t_0}Z_{j,t_0} + \frac{\theta_j(1+\omega_j)}{2}\pi_{j,t_0}^2$$

and

$$Z_{j,t} = X_{j,t} - \frac{\alpha_j\beta}{1-\alpha_j\beta}(1-2\theta_j-\omega_j\theta_j)E_t\pi_{j,t+1} + \alpha_j\beta E_t Z_{j,t+1}.$$

Finally, we can take a second-order approximation of the relation between output and consumption $Y_t = C_t + G_t$ obtaining

$$\hat{C}_t = s_C^{-1}\hat{Y}_t - s_C^{-1}\hat{G}_t + \frac{s_C^{-1}(1-s_C^{-1})}{2}\hat{Y}_t^2 + s_C^{-2}\hat{Y}_t\hat{G}_t + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3), \tag{A.45}$$

while

$$\hat{S}_t = -\omega_\tau\hat{\tau}_t + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3), \tag{A.46}$$

where $\omega_\tau \equiv \bar{\tau}/(1-\bar{\tau})$. By substituting (A.45) and (A.46) into the definition of $z_{j,t}$ and $Z_{j,t}$ in (A.43), we finally obtain a quadratic approximation to the AS relations.

For the price constraint we obtain.

$$\begin{aligned}
V_{p,t_0} &= \xi_p E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [(\phi - 1)\hat{Y}_t - \phi a_t + \hat{w}_{R,t} + \omega_\tau \hat{\tau}_t] \\
&+ \frac{1}{2} \xi_p E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{[\phi^2 - 1 - 2\sigma^{-1}(\phi - 1)]\hat{Y}_t^2 + 2[\phi - \sigma^{-1}]\hat{w}_{R,t}\hat{Y}_t + \hat{w}_{R,t}^2\} \\
&+ \xi_p E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{[\sigma^{-1} g_t - \phi a_t]\hat{w}_{R,t} + [\phi(\sigma^{-1} - \phi)a_t + \sigma^{-1}(\phi - 1)g_t + (1 - \sigma^{-1})\omega_\tau \hat{\tau}_t]\hat{Y}_t\} \\
&+ \frac{\theta_p(1 + \omega_p)}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \pi_{p,t}^2 + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3), \qquad \text{(A.47)}
\end{aligned}$$

This can be expressed compactly in the form

$$\begin{aligned}
V_{p,t_0} &= E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_p (c'_{p,x} x_t + c_{p,\xi} \xi_t + \frac{1}{2} x'_t C_{p,xx} x_t - x'_t C_{p,x\xi} \xi_t + \frac{1}{2} c_{p,\pi_p} \pi_{p,t}^2) \\
&+ \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3) \qquad \text{(A.48)}
\end{aligned}$$

or as

$$\begin{aligned}
V_{p,t} &= \xi_p (c'_{p,x} x_t + c_{p,\xi} \xi_t + \frac{1}{2} x'_t C_{p,xx} x_t - x'_t C_{p,x\xi} \xi_t + \frac{1}{2} c_{p,\pi_p} \pi_{p,t}^2) + \beta E_t V_{p,t+1} \\
&+ \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3) \qquad \text{(A.49)}
\end{aligned}$$

where we have defined

$$c'_{p,x} \equiv \begin{bmatrix} (\phi - 1) & 1 \end{bmatrix}$$

$$c_{p,\xi}\xi_t \equiv -\phi a_t + \omega_\tau \hat{\tau}_t,$$

$$C_{p,xx} \equiv \begin{bmatrix} \phi^2 - 1 - 2\sigma^{-1}(\phi - 1) & \phi - \sigma^{-1} \\ \phi - \sigma^{-1} & 1 \end{bmatrix}$$

$$C_{p,x\xi} \equiv \begin{bmatrix} 0 & -\sigma^{-1}(\phi - 1) & 0 & -(1 - \sigma^{-1})\omega_\tau & 0 & 0 & -\phi(\sigma^{-1} - \phi) \\ 0 & -\sigma^{-1} & 0 & 0 & 0 & 0 & \phi \end{bmatrix}$$

$$c_{p,\pi} \equiv \frac{\theta_p(1 + \omega_p)}{\xi_p}$$

and

$$V_{p,t} = \pi_{p,t} + \frac{1}{2} v_{\pi_p} \pi_{p,t}^2 + v_{p,z} \pi_{p,t} Z_{p,t},$$

$$Z_{p,t} = z_{p,y} \hat{Y}_t + z_{p,r} \hat{w}_{R,t} + z_{\pi_p} \pi_{p,t} + z_{p,\xi} \xi_t + \alpha_p \beta E_t Z_{p,t+1},$$

in which the coefficients are defined as

$$v_{p,\pi} \equiv \theta_p (1 + \omega_p) - \frac{1 - \theta_p}{(1 - \alpha_p)}, \qquad v_{p,z} \equiv \frac{(1 - \alpha_p \beta)}{2},$$

$$v_{p,k} \equiv \frac{\xi_p \alpha_p}{1 - \alpha_p \beta} (1 - 2\theta_p - \omega_p \theta_p),$$

$$z_{p,y} \equiv (1 + \phi - 2\sigma^{-1}) + v_{p,k}(\omega + \sigma^{-1})$$

$$z_{p,r} \equiv (1 + v_{p,k})$$

$$z_{p,\xi} \xi_t \equiv 2\sigma^{-1} g_t - \phi(1 + v_{p,k}) a_t - \omega_\tau (1 - v_{p,k}) \hat{\tau}_t,$$

$$z_{p,\pi} \equiv -\frac{v_{p,k}}{\xi_p}.$$

Note that in a first-order approximation, (A.49) can be written simply as

$$\pi_{p,t} = \xi_p [(\phi - 1)\hat{Y}_t + \hat{w}_{R,t} + c_{p,\xi} \xi_t] + \beta E_t \pi_{p,t+1}. \qquad \text{(A.50)}$$

We can also write (A.48) as

$$\begin{aligned} V_{t_0} &= E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_p (c'_{p,x} x_t + \frac{1}{2} x'_t C_{p,xx} x_t - x'_t C_{p,x\xi} \xi_t + \frac{1}{2} c_{p,\pi_p} \pi_{p,t}^2) \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \qquad \text{(A.51)} \end{aligned}$$

where the term $c_\xi \xi_t$ is now included in terms independent of policy. (Such terms matter when part of the log-linear constraints, as in the case of (A.50), but not when part of the quadratic objective.)

For the wage constraint we obtain that

$$\begin{aligned}
V_{w,t_0} &= \xi_w E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[(\phi\nu+\sigma^{-1})\hat{Y}_t-\hat{w}_{R,t}-\phi\nu a_t-\sigma^{-1}g_t-\nu h_t] \\
&+\frac{1}{2}\xi_w E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\{[(\phi(2+\nu)-\sigma^{-1})(\phi\nu+\sigma^{-1}))+\sigma^{-1}(1-s_C^{-1})]\hat{Y}_t^2 \\
&-2[\phi-\sigma^{-1}]\hat{w}_{R,t}\hat{Y}_t-\hat{w}_{R,t}^2\} \\
&+\xi_w E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[-\sigma^{-1}g_t+\phi a_t]\hat{w}_{R,t}+ \\
&+\xi_w E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\sigma^{-1}(\sigma^{-1}-\phi)g_t-\phi\nu(1+\nu)h_t+\sigma^{-1}s_C^{-1}\hat{G}_t]\hat{Y}_t \\
&-\xi_w E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\phi[2\phi\nu+\phi\nu^2+\sigma^{-1}]a_t\hat{Y}_t+\frac{\theta_w(1+\nu)}{2}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_{w,t}^2 \\
&+\frac{\theta_p\nu(1+\omega_p)\xi_w}{\xi_p}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_{p,t}^2+\text{s.o.t.i.p.}+\mathcal{O}(||\xi||^3),
\end{aligned}$$

This can be expressed compactly in the form

$$\begin{aligned}
V_{w,t_0} &= E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_w(c_{w,x}'x_t+c_{w,\xi}\xi_t+\frac{1}{2}x_t'C_{w,xx}x_t-x_t'C_{w,x\xi}\xi_t+\frac{1}{2}c_{w,\pi_w}\pi_{w,t}^2+\frac{1}{2}c_{w,\pi_p}\pi_{p,t}^2) \\
&+\text{s.o.t.i.p.}+\mathcal{O}(||\xi||^3) \qquad \text{(A.52)}
\end{aligned}$$

or as

$$\begin{aligned}
V_{w,t} &= \xi_w(c_{w,x}'x_t+c_{w,\xi}\xi_t+\frac{1}{2}x_t'C_{w,xx}x_t-x_t'C_{w,x\xi}\xi_t+\frac{1}{2}c_{w,\pi_w}\pi_{w,t}^2+\frac{1}{2}c_{w,\pi_p}\pi_{p,t}^2)+\beta E_tV_{w,t+1} \\
&+\text{s.o.t.i.p.}+\mathcal{O}(||\xi||^3) \qquad \text{(A.53)}
\end{aligned}$$

where we have defined

$$c_{w,x}'\equiv\begin{bmatrix}\phi\nu+\sigma^{-1} & -1\end{bmatrix}$$

$$c_{w,\xi}\xi_t\equiv-\phi\nu a_t-\sigma^{-1}g_t-\nu h_t,$$

$$C_{w,xx}\equiv\begin{bmatrix}(\phi(2+\nu)-\sigma^{-1})(\phi\nu+\sigma^{-1})+\sigma^{-1}(1-s_C^{-1}) & -(\phi-\sigma^{-1}) \\ -(\phi-\sigma^{-1}) & -1\end{bmatrix}$$

$$C_{w,x\xi} \equiv \begin{bmatrix} -\sigma^{-1}s_C^{-1} & -\sigma^{-1}(\sigma^{-1}-\phi) & 0 & 0 & \phi\nu(1+\nu) & \phi[2\phi\nu+\phi\nu^2+\sigma^{-1}] \\ 0 & \sigma^{-1} & 0 & 0 & 0 & -\phi \end{bmatrix}$$

$$c_{w,\pi_w} \equiv \frac{\theta_w(1+\nu)}{\xi_w}$$
$$c_{w,\pi_p} \equiv \frac{\theta_p\nu(1+\omega_p)}{\xi_p}$$

and

$$V_{w,t} = \pi_{w,t} + \frac{1}{2}v_{w,\pi}\pi_{w,t}^2 + v_{w,z}\pi_{w,t}Z_{w,t},$$
$$Z_{w,t} = z_{w,y}\hat{Y}_t + z_{w,r}\hat{w}_{R,t} + z_{w,\pi}\pi_{w,t} + z_{w,\xi}\xi_t + \alpha_w\beta E_t Z_{w,t+1},$$

in which the coefficients are defined as

$$v_{w,\pi} \equiv \theta_w(1+\omega_w) - \frac{1-\theta_w}{(1-\alpha_w)}, \qquad v_{w,z} \equiv \frac{(1-\alpha_w\beta)}{2},$$

$$v_{w,k} \equiv \frac{\alpha_w\xi_w}{1-\alpha_w\beta}(1-2\theta_w-\nu\theta_w),$$
$$z_{w,y} \equiv \phi(2+\nu) - \sigma^{-1} + v_{w,k}(\phi\nu+\sigma^{-1})$$
$$z_{w,\xi}\xi_t \equiv \sigma^{-1}(1-v_{w,k})g_t - \nu(1+v_{w,k})\bar{h}_t - [\phi(2+v)+\phi\nu v_{w,k}]a_t,$$
$$z_{w,\pi} \equiv -\frac{v_{w,k}}{\xi_w}.$$

Note that in a first-order approximation, (A.53) can be written as simply

$$\pi_{w,t} = \xi_w[(\phi\nu+\sigma^{-1})\hat{Y}_t - \hat{w}_{R,t} - \phi\nu a_t - \sigma^{-1}g_t - \nu h_t] + \beta E_t\pi_{w,t+1}. \tag{A.54}$$

We can also write (A.52) as

$$\begin{aligned} V_{w,t_0} &= E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_w(c_{w,x}'x_t + \frac{1}{2}x_t'C_{w,xx}x_t - x_t'C_{w,x\xi}\xi_t + \frac{1}{2}c_{w,\pi_w}\pi_{w,t}^2 + \frac{1}{2}c_{w,\pi_p}\pi_{p,t}^2) \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned} \tag{A.55}$$

We can add (A.51) and (A.55) to obtain

$$V_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}(\hat{Y}_t + \frac{1}{2}c_{yy}\hat{Y}_t^2 - \hat{Y}_t c_{y\xi}\xi_t + \frac{1}{2}c_{\pi_w}\pi_{w,t}^2 + \frac{1}{2}c_{\pi_p}\pi_{p,t}^2) + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

which is equation (2.34) in the text, where now

$$c_{yy} \equiv 2 + \omega - \sigma^{-1} + \sigma^{-1}(1 - s_C^{-1})(\omega + \sigma^{-1})^{-1}$$

$$c_{y\xi}\xi \equiv (\omega + \sigma^{-1})^{-1}[-\sigma^{-1}s_C^{-1}\hat{G}_t + (1 - \sigma^{-1})\sigma^{-1}g_t + \omega(1 + \omega)q_t - \omega_\tau(1 - \sigma^{-1})\hat{\tau}_t]$$

$$c_{\pi_w} \equiv \frac{\theta_w(1 + \nu)}{\xi_w(\omega + \sigma^{-1})}$$

$$c_{\pi_p} \equiv \frac{\theta_p(1 + \omega)}{\xi_p(\omega + \sigma^{-1})}$$

and

$$V_t \equiv \frac{V_{w,t}}{\xi_w(\omega + \sigma^{-1})} + \frac{V_{p,t}}{\xi_p(\omega + \sigma^{-1})}$$

A.4 Derivation of equation (2.37)

We can multiply equation (2.36) by $\Phi\bar{Y}\bar{u}_c$ and subtract from (2.30) to obtain

$$\begin{aligned} U_{t_0} &= -\bar{Y}\bar{u}_c E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} q_y \hat{Y}_t^2 - \hat{Y}_t(u_{y\xi}\xi_t + \Phi c_{y\xi}\xi_t) + \frac{1}{2} q_p \pi_{p,t}^2 + \frac{1}{2} q_w \pi_{w,t}^2 \right\} + \\ &\quad + T_{t_0} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned}$$

where

$$\begin{aligned} q_p &\equiv u_{\pi_p} + \Phi c_{\pi_p} \\ &= \frac{\theta_p(1 - \Phi)}{\xi_p} + \Phi \frac{\theta_p(1 + \omega)}{\xi_p(\omega + \sigma^{-1})} \\ &= \frac{\theta_p}{\xi_p(\omega + \sigma^{-1})}[(\omega + \sigma^{-1}) + \Phi(1 - \sigma^{-1})], \end{aligned}$$

$$\begin{aligned} q_w &\equiv u_{\pi_w} + \Phi c_{\pi_w} \\ &= \frac{\theta_w(1 - \Phi)}{\phi\xi_w} + \Phi \frac{\theta_w(1 + \nu)}{\xi_w(\omega + \sigma^{-1})} \\ &= \frac{\theta_w}{\xi_w\phi(\omega + \sigma^{-1})}[(\omega + \sigma^{-1}) + \Phi(1 - \sigma^{-1})], \end{aligned}$$

$$\begin{aligned} q_y &\equiv u_{yy} + \Phi c_{yy} \\ &= (\omega + \sigma^{-1}) - \Phi(1 + \omega) + \Phi(2 + \omega - \sigma^{-1}) + \Phi\sigma^{-1}(1 - s_C^{-1})(\omega + \sigma^{-1})^{-1} \\ &= (\omega + \sigma^{-1}) + \Phi(1 - \sigma^{-1}) + \frac{\Phi\sigma^{-1}(1 - s_C^{-1})}{\omega + \sigma^{-1}}. \end{aligned}$$

This can be rewritten in the form (2.37) given in the text, where

$$\begin{aligned}
\hat{Y}_t^* &\equiv q_y^{-1}[u_{y\xi}\xi_t + \Phi c_{y\xi}\xi_t] \\
&= q_y^{-1}\{\sigma^{-1}g_t + (1-\Phi)\omega q_t + (\omega+\sigma^{-1})^{-1}\Phi[-\sigma^{-1}s_C^{-1}\hat{G}_t + \sigma^{-1}(1-\sigma^{-1})g_t + \omega(1+\omega)q_t \\
&\quad -\omega_\tau(1-\sigma^{-1})\hat{\tau}_t]\} \\
&= \omega_1\hat{Y}_t^n - \omega_2\hat{G}_t + \omega_3\hat{\tau}_t,
\end{aligned}$$

and Ω, $\hat{Y}_t^n$, and the ω_i are defined as in the text.

A.5 Determinacy conditions

Consider the first-order conditions

$$q_y x_t = \kappa_p\varphi_{p,t} + \kappa_w\varphi_{w,t}, \tag{A.56}$$

$$q_p\pi_{p,t} = -(\varphi_{p,t} - \varphi_{p,t-1}) - \varphi_{r,t}, \tag{A.57}$$

$$q_w\pi_{w,t} = -(\varphi_{w,t} - \varphi_{w,t-1}) + \varphi_{r,t}, \tag{A.58}$$

$$\xi_p\varphi_{p,t} - \xi_w\varphi_{w,t} - \varphi_{r,t} + \beta E_t\varphi_{r,t+1} = 0, \tag{A.59}$$

and the structural equations

$$\pi_{p,t} = \kappa_p x_t + \xi_p(\hat{\omega}_{R,t} - \hat{\omega}_t^n) + u_{p,t} + \beta E_t\pi_{p,t+1}, \tag{A.60}$$

$$\pi_{w,t} = \kappa_w x_t - \xi_w(\hat{\omega}_{R,t} - \hat{\omega}_t^n) + u_{w,t} + \beta E_t\pi_{w,t+1}, \tag{A.61}$$

$$\hat{\omega}_{R,t} = \hat{\omega}_{R,t-1} + \pi_{w,t} - \pi_{p,t}. \tag{A.62}$$

We can substitute equations (A.56), (A.57), (A.58), (A.59) and (A.62) into (A.60) to obtain

$$\begin{aligned}
\beta q_w q_y E_t\varphi_{p,t+1} &= [q_w q_y(1+\beta) + q_p q_w\kappa_p^2 + 2q_w q_y\xi_p]\varphi_{p,t} + \\
&\quad -[q_w q_y + q_w q_y\xi_p]\varphi_{p,t-1} + q_p q_y\xi_p\varphi_{w,t-1} + \\
&\quad +[q_w q_p\kappa_w\kappa_p - \xi_w q_y q_w - \xi_p q_y q_p]\varphi_{w,t} - q_p q_w q_y\xi_p\hat{\omega}_t^n + \\
&\quad +q_p q_w q_y u_{p,t} + q_y\xi_p q_p q_w\hat{\omega}_{R,t-1} + q_y\xi_p(q_p+q_w)\varphi_{r,t}
\end{aligned} \tag{A.63}$$

We can substitute equations (A.56), (A.57), (A.58), (A.59) and (A.62) into (A.61) to obtain

$$\begin{aligned}\beta q_p q_y E_t \varphi_{w,t+1} &= [q_p q_y(1+\beta) + q_p q_w \kappa_w^2 + 2q_p q_y \xi_w]\varphi_{w,t} + \\ &-[q_p q_y + q_p q_y \xi_w]\varphi_{w,t-1} + q_w q_y \xi_w \varphi_{p,t-1} + \\ &+[q_p q_w \kappa_w \kappa_p - \xi_p q_y q_p - \xi_w q_y q_w]\varphi_{p,t} + q_p q_w q_y \xi_w \hat{\omega}_t^n \\ &+q_p q_w q_y u_{w,t} - q_y \xi_w q_p q_w \hat{\omega}_{R,t-1} + q_y \xi_w (q_p + q_w)\varphi_{r,t}, \quad \text{(A.64)}\end{aligned}$$

Substitution of (A.57) and (A.58) yields

$$\begin{aligned}q_w q_p \hat{\omega}_{R,t} &= q_w q_p \hat{\omega}_{R,t-1} + q_w(\varphi_{p,t} - \varphi_{p,t-1}) - q_p(\varphi_{w,t} - \varphi_{w,t-1}) \\ &+(q_p + q_w)\varphi_{r,t} \quad \text{(A.65)}\end{aligned}$$

finally (A.59) implies

$$\beta E_t \varphi_{r,t+1} = \varphi_{r,t} + \xi_w \varphi_{w,t} - \xi_p \varphi_{p,t}, \quad \text{(A.66)}$$

We can write the set of the above conditions (A.64), (A.63), (A.65), (A.66) in the following system

$$A E_t z_{t+1} = B z_t + C \upsilon_t \quad \text{(A.67)}$$

where

$$z_t' \equiv [\varphi_{p,t} \;\; \varphi_{w,t} \;\; \varphi_{r,t} \;\; \hat{\omega}_{R,t-1} \;\; \varphi_{p,t-1} \;\; \varphi_{w,t-1}],$$

and

$$\upsilon_t' \equiv [\hat{\omega}_t^n \;\; u_{p,t} \;\; u_{w,t}],$$

and

$$A \equiv \begin{bmatrix} \beta q_w q_y & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta q_p q_y & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & q_w q_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B \equiv \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ -\xi_p & \xi_w & 1 & 0 & 0 & 0 \\ q_w & -q_p & (q_p + q_w) & q_w q_p & -q_w & q_p \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$b_{11} \equiv [q_w q_y (1+\beta) + q_p q_w \kappa_p^2 + 2 q_w q_y \xi_p],$$

$$b_{12} \equiv [q_w q_p \kappa_w \kappa_p - \xi_w q_y q_w - \xi_p q_y q_p],$$

$$b_{13} \equiv q_y \xi_p (q_p + q_w),$$

$$b_{14} \equiv q_y \xi_p q_p q_w,$$

$$b_{15} \equiv -[q_w q_y + q_w q_y \xi_p],$$

$$b_{16} \equiv q_p q_y \xi_p,$$

$$b_{21} \equiv [q_p q_w \kappa_w \kappa_p - \xi_p q_y q_p - \xi_w q_y q_w],$$

$$b_{22} \equiv [q_p q_y (1+\beta) + q_p q_w \kappa_w^2 + 2 q_p q_y \xi_w],$$

$$b_{23} \equiv q_y \xi_w (q_p + q_w),$$

$$b_{24} \equiv -q_y \xi_w q_p q_w,$$

$$b_{25} \equiv q_w q_y \xi_w,$$

$$b_{26} \equiv -[q_p q_y + q_p q_y \xi_w],$$

$$C \equiv \begin{bmatrix} -q_p q_w q_y \xi_p & q_p q_w q_y & 0 \\ q_p q_w q_y \xi_w & 0 & q_p q_w q_y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The determinacy of the equilibrium depends on the roots of the characteristic equation associated with the system (A.67)

$$\det(B - \mu A) = 0.$$

Rational-expectations equilibrium is determinate if the number of roots μ_i such that $|\mu_i| < 1$ is exactly equal to the number of predetermined variables which in our case is three. Under this condition, we can solve the above system in the following way. Consider as V the matrix of the left eigenvector associated with the roots of the characteristic polynomial which are above the unit circle. The matrix V has the property that $VB = \Phi VA$, where Φ is a diagonal matrix that contains the roots μ_i such that $|\mu_i| > 1$. By premultiplying (A.67) by V we obtain

$$E_t k_{t+1} = \Phi k_t + VC\upsilon_t \tag{A.68}$$

where we have defined $k_t \equiv VAz_t$. A unique and stable solution for $\{z_t\}$ can be obtained by

$$z_t = -E_t \sum_{j=0}^{\infty} \Phi^{-(j+1)} VC\upsilon_{t+j}.$$

We can partition VA as $VA = [(VA)_1 \ (VA)_2]$ according to the non-predetermined and predetermined endogenous variables in $z_t = [z_{1,t} \ z_{2,t-1}]$ and we can obtain

$$(VA)_1 z_{1,t} + (VA)_2 z_{2,t} = -E_t \sum_{j=0}^{\infty} \Phi^{-(j+1)} C\upsilon_{t+j},$$

which can be solved (under conditions of invertibility on VA_1) as

$$z_{1,t} = -(VA)_1^{-1}(VA)_2 z_{2,t-1} - (VA)_1^{-1} E_t \sum_{j=0}^{\infty} \Phi^{-(j+1)} VC\upsilon_{t+j} \tag{A.69}$$

We note that we can partition the system (A.67) in the following way

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_4 \end{bmatrix} E_t \begin{pmatrix} z_{1,t+1} \\ z_{2,t} \end{pmatrix} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{pmatrix} z_{1,t} \\ z_{2,t-1} \end{pmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \upsilon_t$$

We can then substitute (A.69) into the lower block of the above system to obtain

$$z_{2,t} = A_4^{-1}(B_4 - B_3(VA)_1^{-1}(VA)_2) z_{2,t-1} + A_4^{-1} C_2 \upsilon_t - A_4^{-1} B_3 (VA)_1^{-1} E_t \sum_{j=0}^{\infty} \Phi^{-(j+1)} VC\upsilon_{t+j} \tag{A.70}$$

Using (A.69) and (A.70) and (A.56), (A.57), (A.58) and (A.62) we can obtain the optimal path for $\{x_t,\ \pi_{p,t},\ \pi_{w,t},\ \hat{\omega}_{R,t}\}$.

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