Experiment 2: Projectile motion and conservation of energy

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INTRO TO EXPERIMENTAL PHYS-LAB
1494/2699

Overview

- **The physics behind the experiment:**
  - Quick review of conservation of energy
  - Quick review of projectile motion
- **The experiment:**
  - Set up and preliminary measurement of friction
  - Theoretical prediction
  - Measurement procedure
- **Analysis:**
  - Distribution of the landing points
  - Propagation of errors on predicted position
  - Comparison theory vs. experiment

Conservation of energy

- One of the most essential principles of physics is that (neglecting relativistic effects) in a closed system **energy** is always **conserved**
- Energy can have different forms but if we are good enough and we can keep track of all of them, the total energy of the system stays the same
- The energy associated to a body in motion is the *kinetic energy* and it usually receives two contributions:

$$E_{\mathrm{kin}} = E_{\mathrm{transl}} + E_{\mathrm{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

E_{transl}: Energy of the moving center of mass

E_{rot}: Energy of rotation around the center of mass

Conservation of energy

- The energy associated with the “ability of a body to move in the future” is, instead, the *potential energy*
- There are several kinds of potential energy (gravitational, electric, elastic, …)
- In our case we only care about the potential energy associated with the presence of a gravitational field. The expression for this energy near the surface of the Earth is:

$$E_{\text{pot}}(r) = -G_N \frac{M_E m}{r} \simeq mgh + \text{const}$$

- Kinetic and gravitational potential energies are the only two kinds of energy of interest for this experiment

Conservation of energy

- In absence of any kind of *dissipation*, the total amount of mechanical energy (kinetic + potential) is constant through the whole motion
- If, instead, we allow for some mechanical energy to be lost (for example becoming thermal energy due to friction) then the equation for conservation of energy reads:

$$E_{\mathrm{kin}}^{\mathrm{in}} + E_{\mathrm{pot}}^{\mathrm{in}} = E_{\mathrm{kin}}^{\mathrm{fin}} + E_{\mathrm{pot}}^{\mathrm{fin}} + W_f$$

- This is just saying that the total initial energy goes partially into final energy but also into some work done by the frictional force

Projectile motion

- If a point-like object is shot with a certain initial velocity v_0 in a system with vertical downward gravitational acceleration, g, the kinematical equations for the position of the body are:

$$x(t) = x_0 + v_{0,x} t$$

$$y(t) = y_0 + v_{0,y} t - \frac{1}{2} g t^2$$

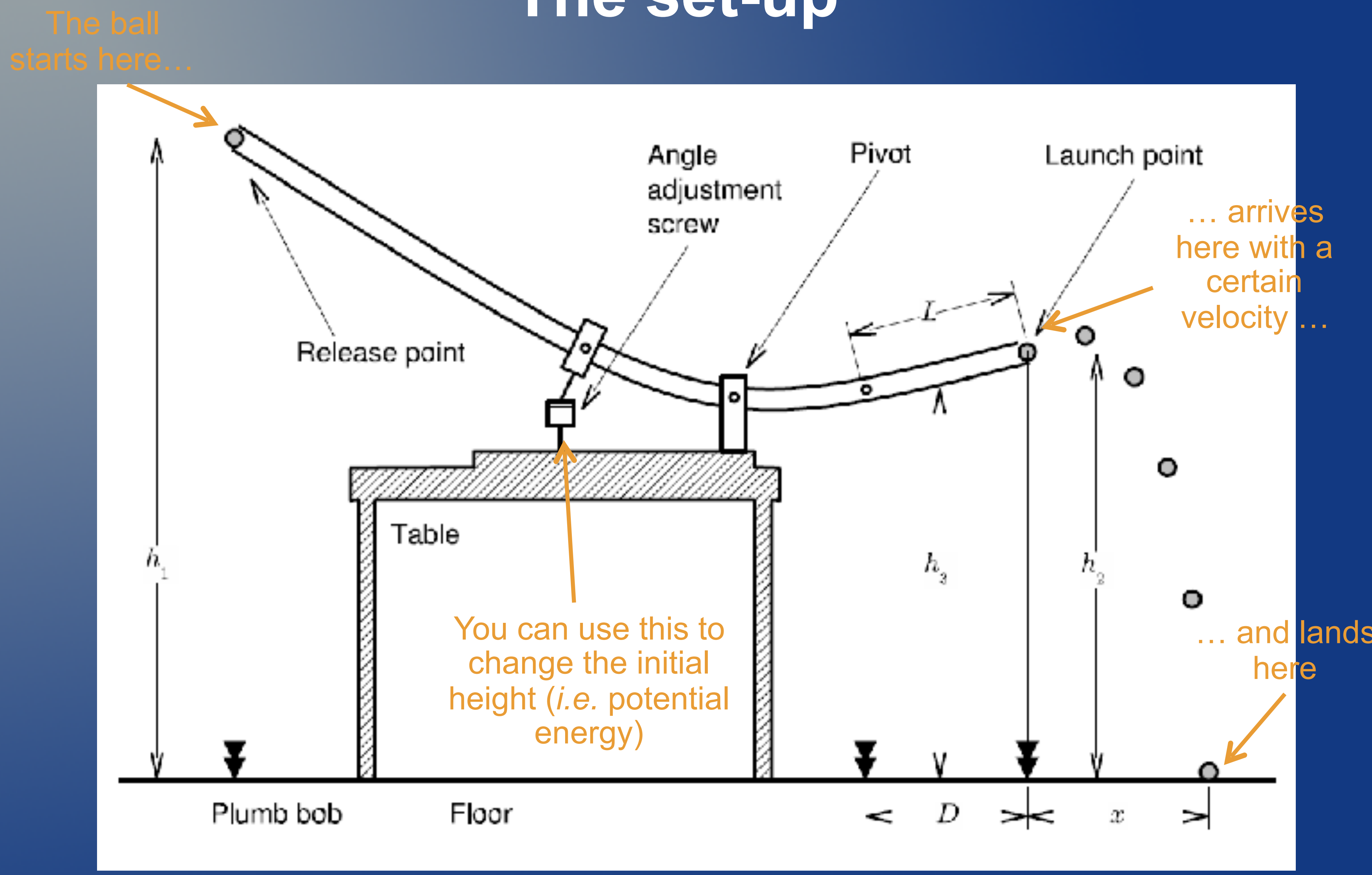
- No acceleration along the horizontal direction ⟶ just motion with constant velocity
- Constant acceleration along the vertical direction ⟶ parabolic motion
- **Piece of cake!**

The Experiment

Main goals

- In general, we want to test our ability to predict the motion of a projectile using a simple mathematical model
- **Part 1:**
  - Estimating the amount of energy lost because of friction. This is needed to describe the motion realistically
- **Part 2:**
  - Use kinematic equations of motion to predict where the ball will land
  - Verify if the prediction and the observation are in agreement

The set-up


The ball
starts here…
Angle
adjustment
screw
Pivot
Launch point
… arrives
here with a
certain
velocity …
Release point
L
Table
You can use this to
change the initial
height (i.e. potential
energy)
h1
h3
h2
… and lands
here
Plumb bob
Floor
D
x

Theoretical prediction for projectile landing position

Predicting the total range

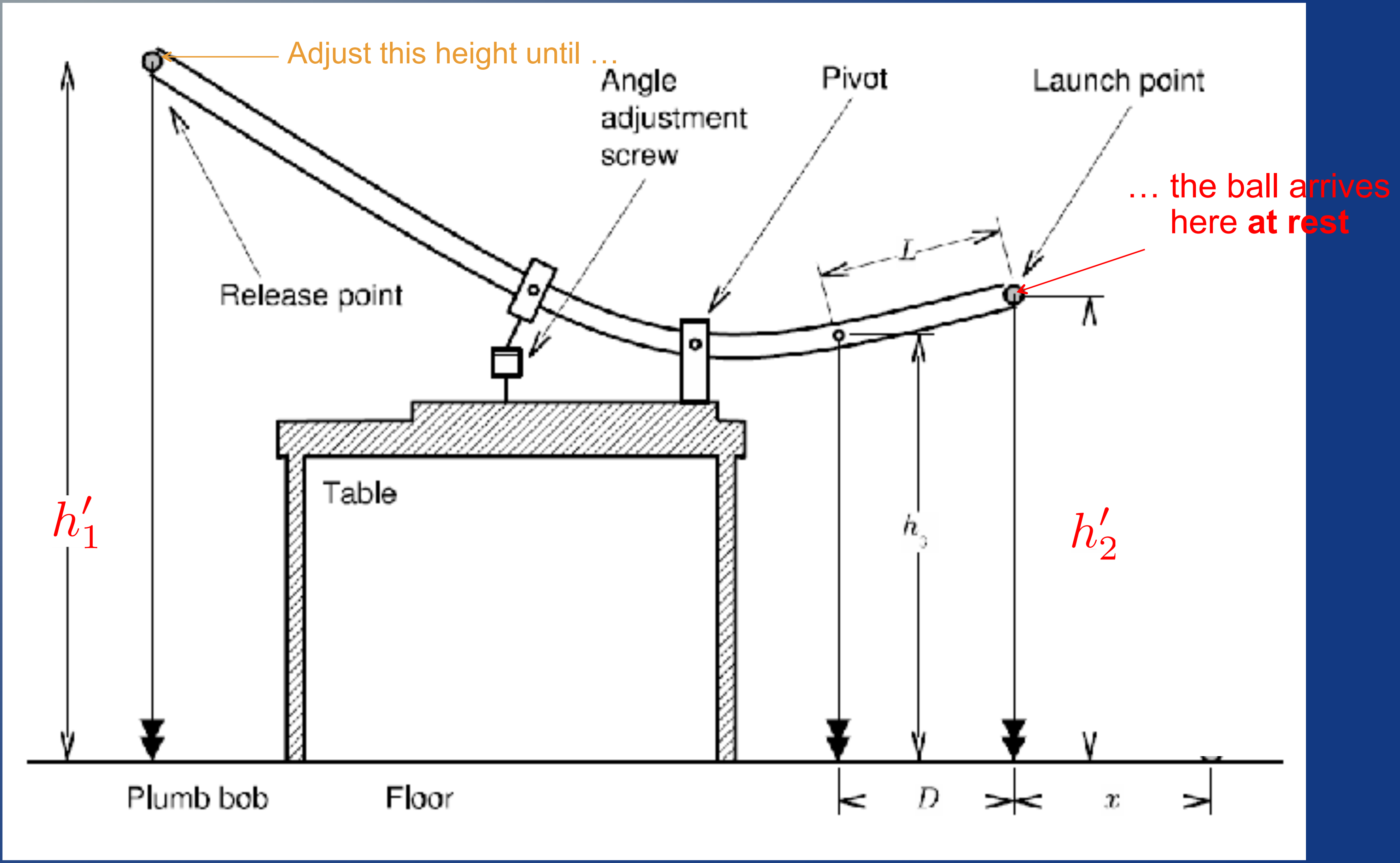
- The calculation needed to estimate the landing point of the ball is rather long and ugly
- The best approach is to split the process in many sub-sections and study each of them separately

- In particular, we will follow the following steps:
  1. Estimate energy lost by friction
  2. Compute the launching velocity
  3. Compute the time it takes for the ball to land
  4. Compute the final range of the motion

Estimating the energy lost

- **Question:** How can we estimate the work (W_f) done by the frictional force?
- If the ball arrives to the very end of the pipe with zero velocity this means that **all the energy has gone into work of friction**
- We can then do:
  1. Adjust the height of the release point until the ball arrives to the launching point (end of the pipe) at rest
  2. Measure the heights of the release and launching point and label them h'_1 and h'_2 respectively
  3. In this case we have $\Delta E_{kin} = 0$ and by conservation of energy:
  $$\Delta E_{\mathrm{kin}} + \Delta E_{\mathrm{pot}} - W_f = \Delta E_{\mathrm{pot}} - W_f = 0$$
  4. The work done by the friction is then: $W_f = mg(h'_1 - h'_2) \equiv mg\Delta h'$

Estimating the energy lost



Calculating initial velocity

- ***Increase height*** *such that*: $h_1 - h_2 \geq 2(h'_1 - h'_2)$
- **Potential energy:** The total difference in potential energy between the release and the launch point is just:

$$\Delta E_{\text{pot}} = mg(h_1 - h_2) \equiv mg\Delta h$$

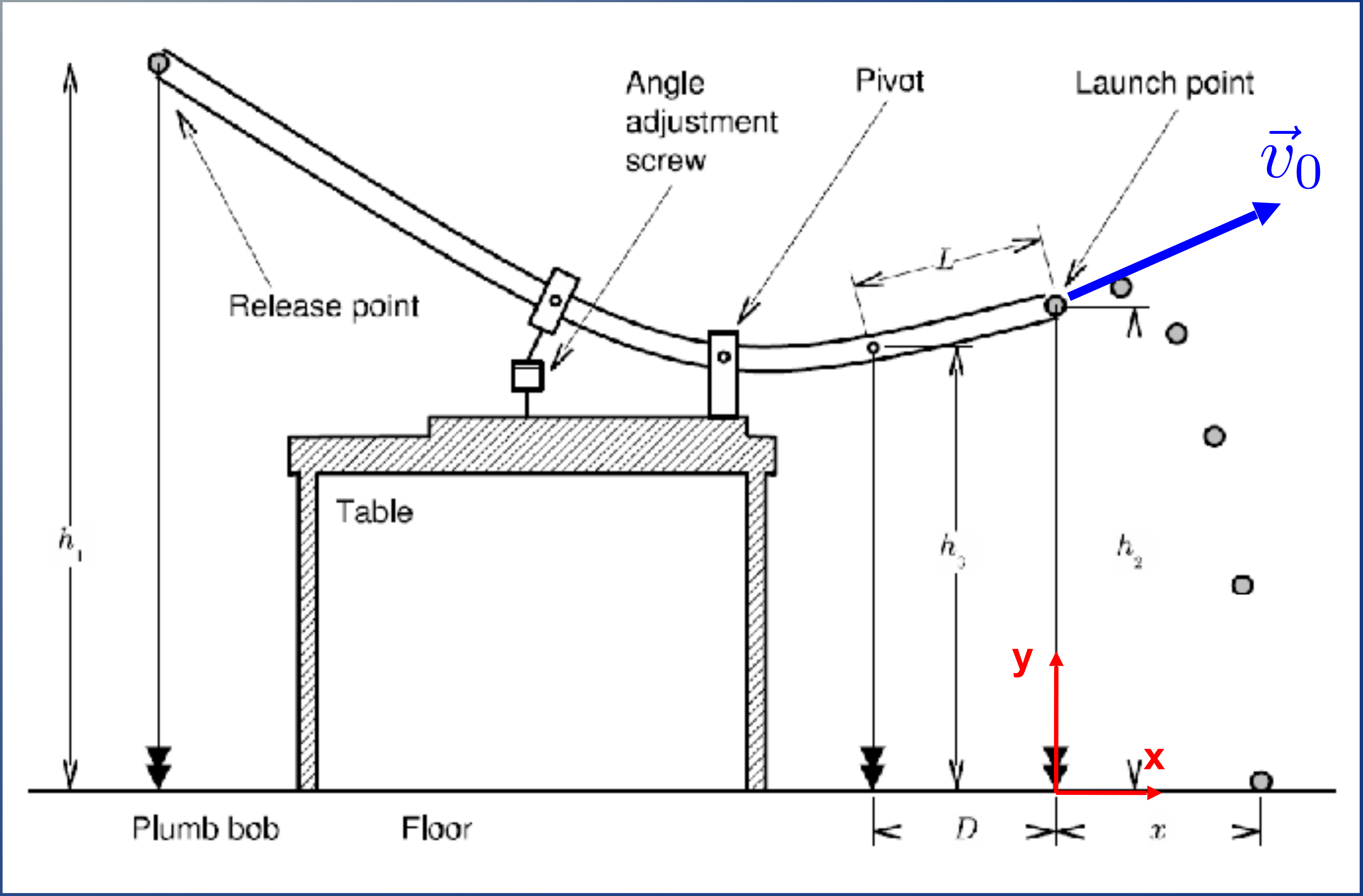
- **Kinetic energy:** The ball is assumed to be rolling without slipping and we can use the momentum of inertia of a solid sphere ($I = 2mR^2/5$)
- The total kinetic energy at the end of the pipe is then:

$$\Delta E_{\text{kin}} = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I(v_0/R)^2 = \frac{7}{10}mv_0^2$$

For pure rolling $\omega = v / R$

- We can then find the **launching velocity** imposing conservation of energy (including W_f !)

Projectile motion



Projectile motion

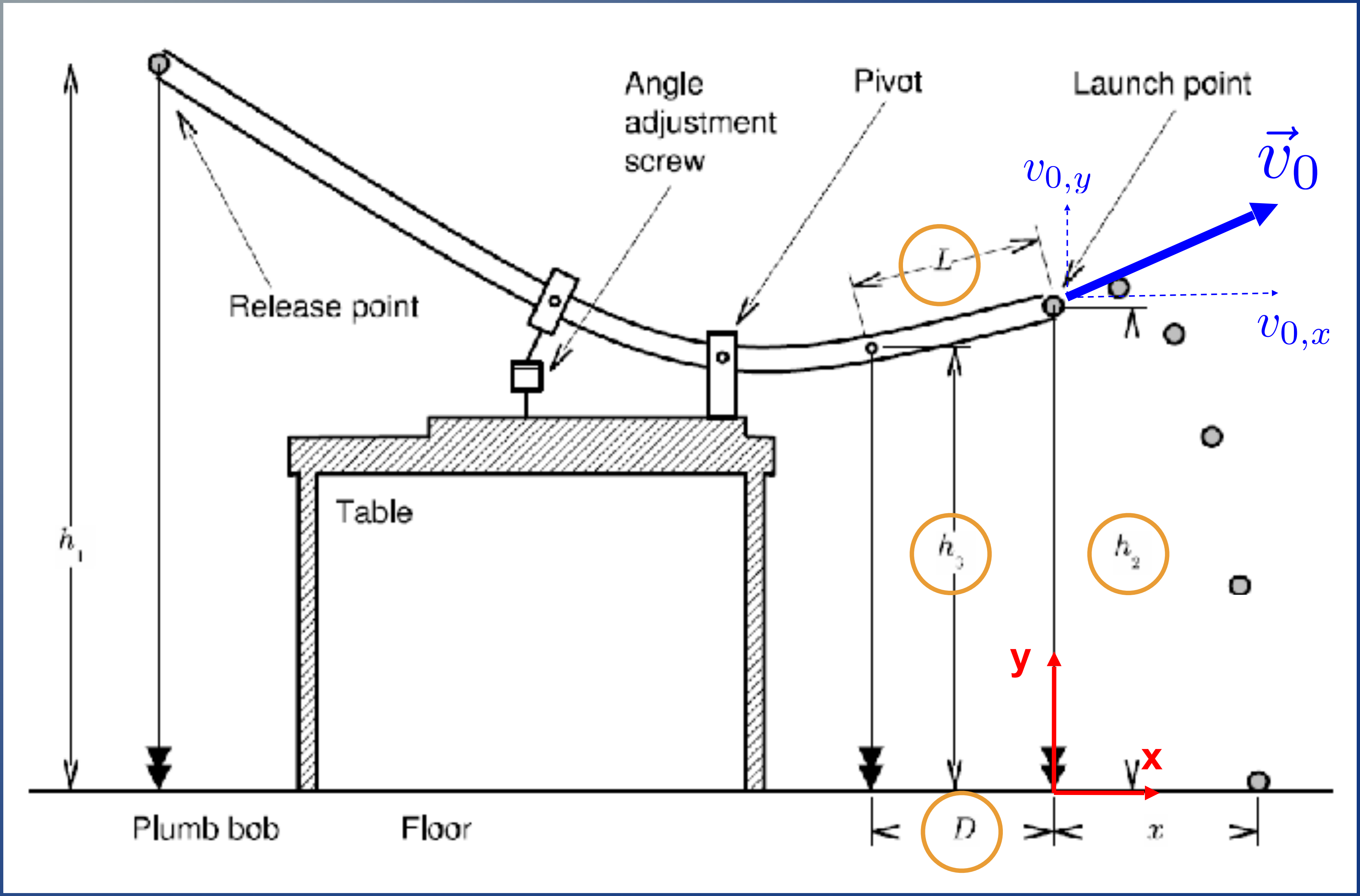
- Now that we used conservation of energy to find the magnitude of the velocity, v_0, we can find its components:

$$\vec{v}_0 = v_{0,x}\hat{i} + v_{0,y}\hat{j}$$

$$v_{0,x} = v_0 \cos\theta$$

$$v_{0,y} = v_0 \sin\theta$$

Projectile motion



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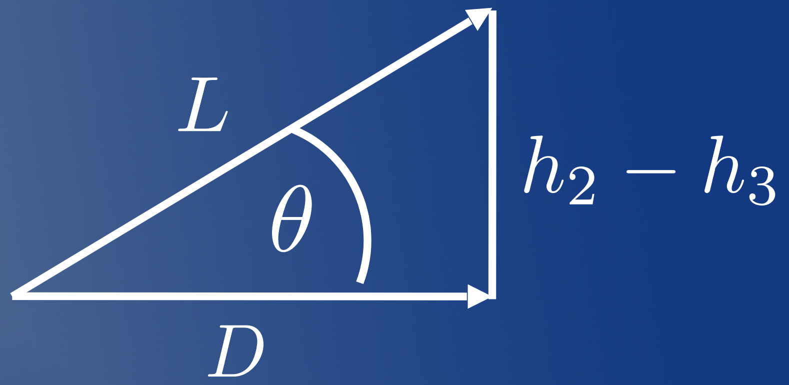


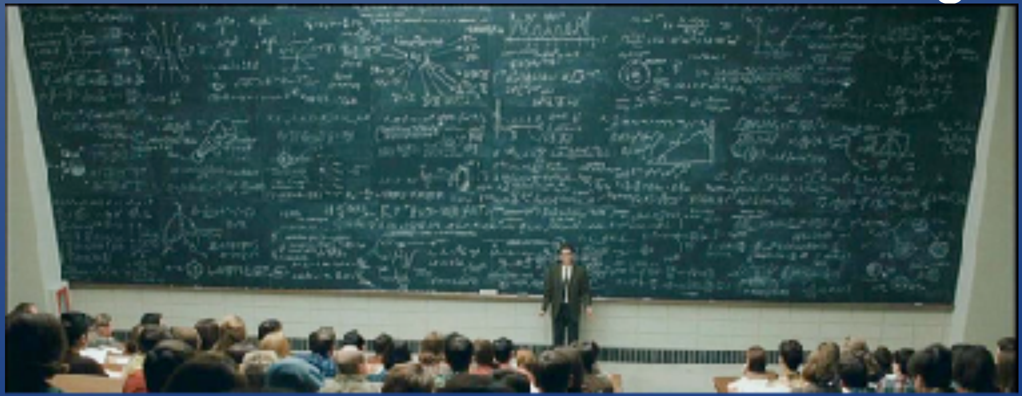
- Therefore we can compute the components without actually finding θ. They are

$$v_{0,x} = v_0(D/L)$$

$$v_{0,y} = v_0(h_2 - h_3)/L$$

Projectile motion

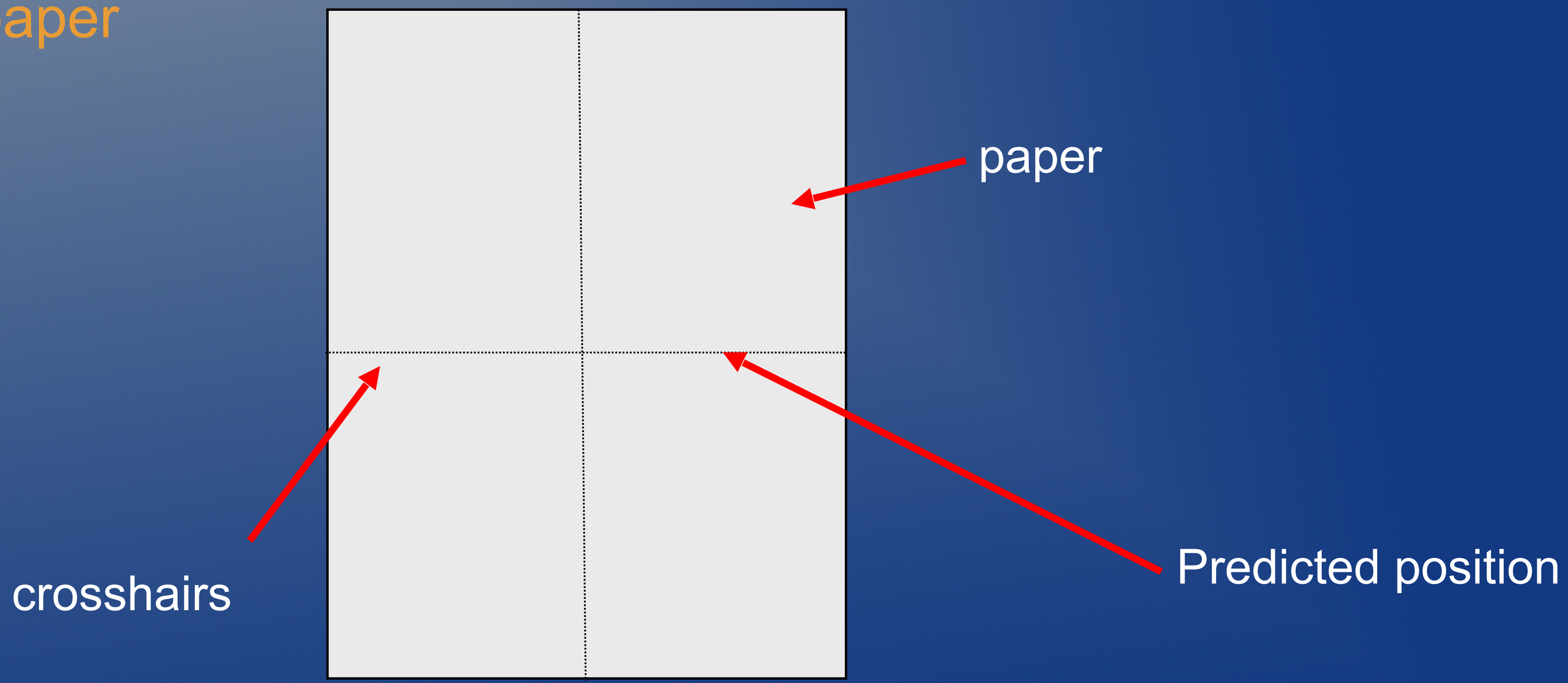
- Now that we know how the components of the initial velocity look like we have everything we need to find the range of the parabolic motion
- I will leave this derivation to you! **You will have to derive it before arriving to lab and the derivation must be included in the lab report**
- **TIP:** It is much simpler to compute the following quantities step by step: landing time $\longrightarrow$ total distance traveled
- Don't plug everything into the final expression! It will get long and ugly and it will be much easier to make algebra mistakes



Experimental Details

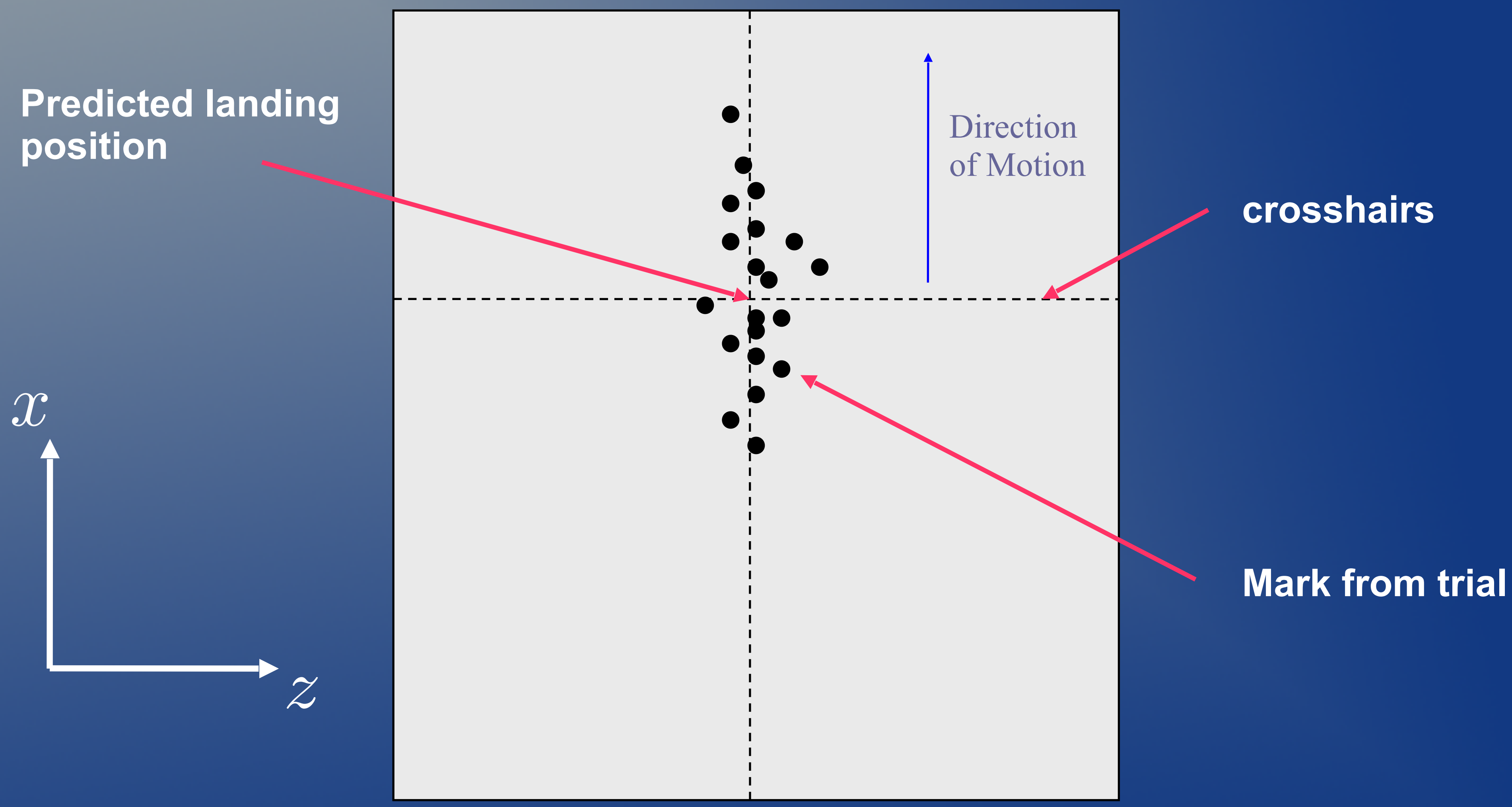
Procedure

- After you determined the energy lost by friction you can change the height of the pipe
- It is ***important*** to make sure that in the new set up $\Delta h \geq 2\Delta h'$ in order not to be affected by fluctuations of W_f
- Now compute the ***predicted landing position*** and mark it on the paper



Spread in landing position

- After measuring everything you need for the setup, roll the ball (at least) **20 times**. The distribution of landing points will be:



Data Analysis for landing position measurement

- You can now measure the value of the (x_i , z_i) coordinates of each of the marked points
- Given these 20 points compute:

1. Means:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i; \qquad \bar{z} = \frac{1}{N}\sum_{i=1}^{N} z_i$$

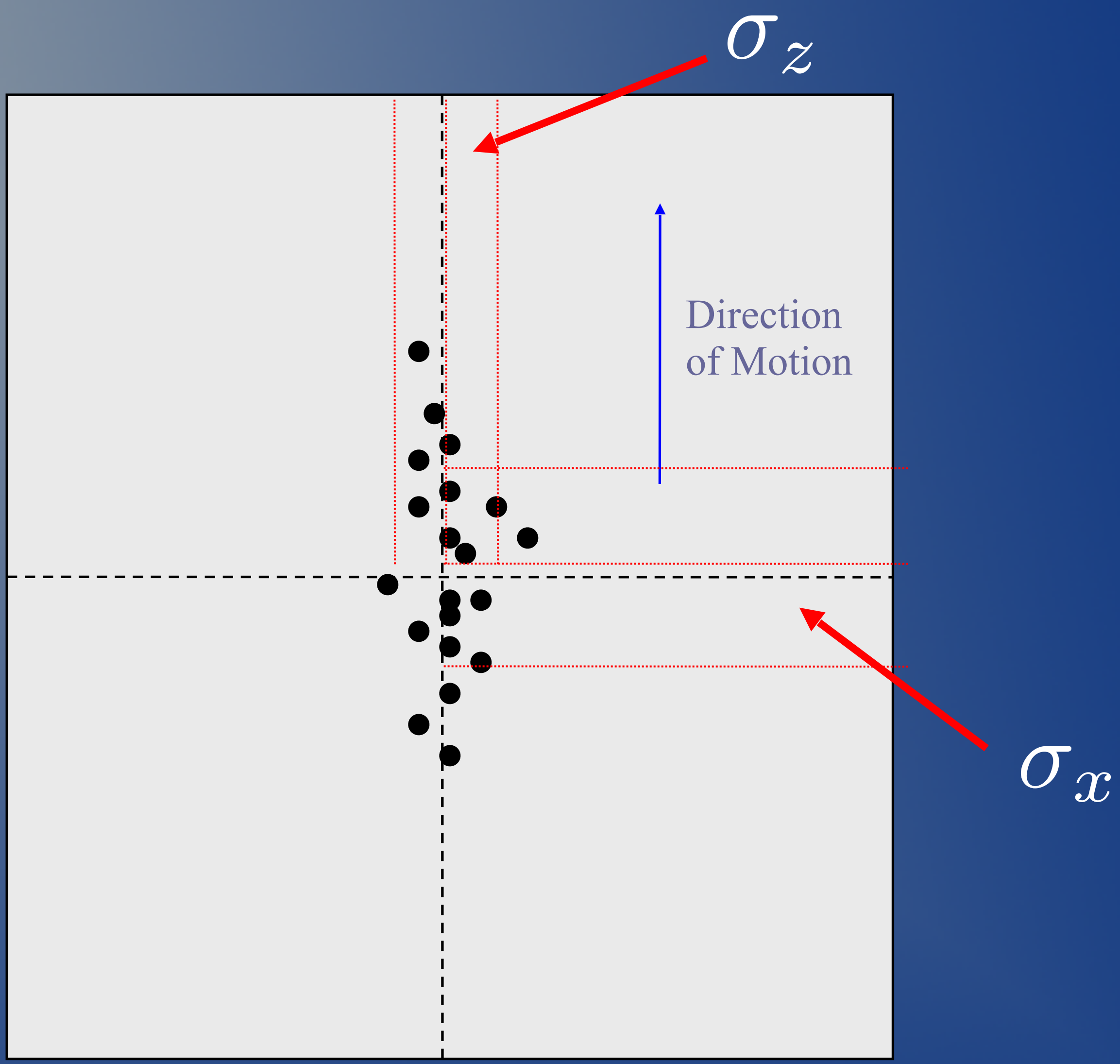
2. Standard deviations:

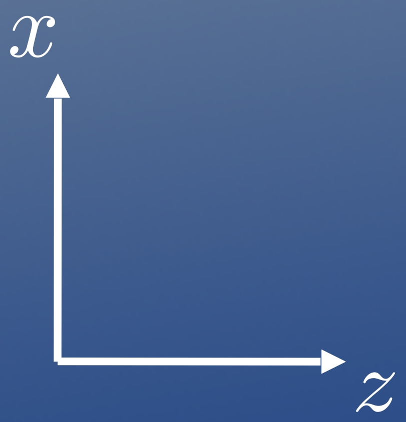
$$s_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}; \qquad s_z = \sqrt{\frac{\sum_{i=1}^{N}(z_i - \bar{z})^2}{N-1}}$$

3. Standard errors of the mean:

$$\sigma_x = \frac{s_x}{\sqrt{N}}; \qquad \sigma_z = \frac{s_z}{\sqrt{N}}$$

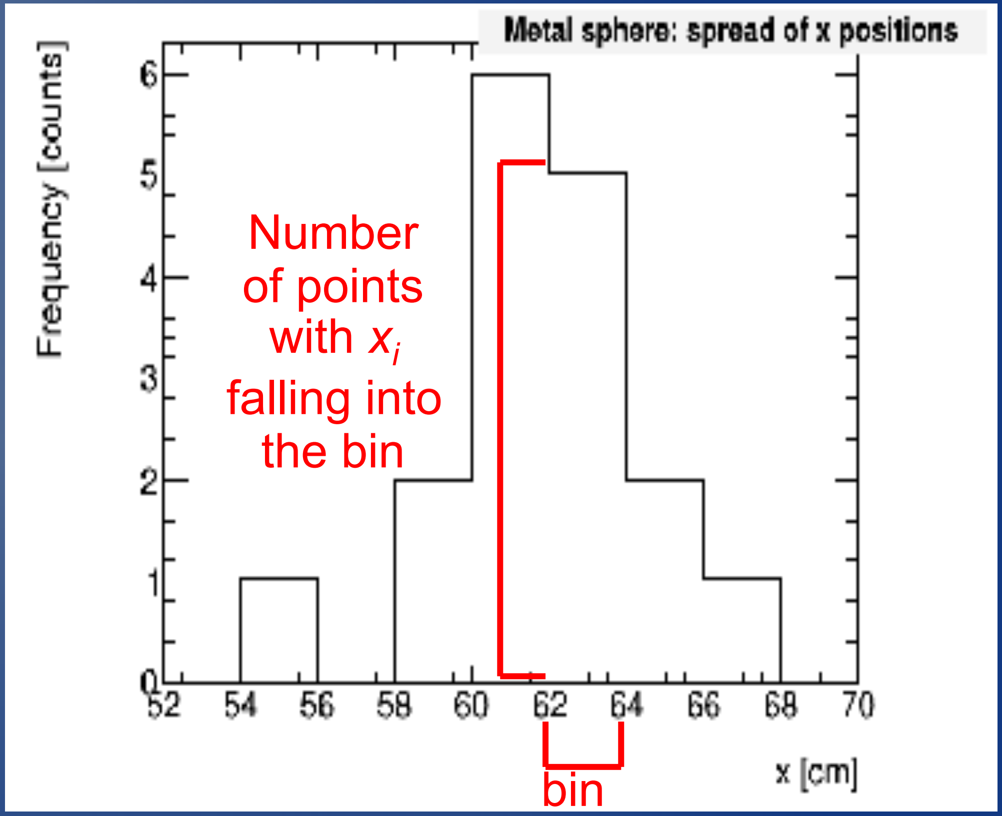
Spread in landing position





Are the data randomly distributed?

- A first thing one can do with the (x_i, z_i) values at hand is to make an histogram of x_i and one of z_i
- Most softwares can choose the binning automatically, if not make a wise choice (neither to large nor too small!)
- **Recall**: 68% of Gaussian data contained within 1σ of the mean
- Check to see that ~ 2/3 of your data points are within 1σ of the mean



Propagating error on the prediction

- If we want to meaningfully compare the measured value of $\bar{x} \pm \sigma_x$ with the predicted one ***we have to find the error on the latter***
- To determine the error on the predicted position (x_p) it is better to split the propagation in two parts:

1. Error on the initial velocity:

$$\sigma_{v_0}^2 = \left(\frac{\partial v_0}{\partial h_1}\right)^2 \sigma_{h_1}^2 + \left(\frac{\partial v_0}{\partial h_2}\right)^2 \sigma_{h_2}^2 + \left(\frac{\partial v_0}{\partial h_1'}\right)^2 \sigma_{h_1'}^2 + \left(\frac{\partial v_0}{\partial h_2'}\right)^2 \sigma_{h_2'}^2$$

2. Error on the predicted *x*: you will find this EXTREMELY long and tedious. The reason is that *x* depends on 6 variables (h_1, h_2, h_3, D, L , v_0)! Also, the dependence involves square roots and other ugly stuff… There is a small trick…

Propagating error on the prediction

- Let us define the following "approximate" position:

$$x_{\text{approx}} = \frac{D\sqrt{2h_2 h_E}}{L}; \quad \text{with:} \quad h_E = \frac{10}{7}(\Delta h - \Delta h')$$

- It turns out that the relative errors of the real and the approximate positions are more or less the same:

$$\frac{\sigma_{x_p}}{x_p} \simeq \frac{\sigma_{x_{\text{approx}}}}{x_{\text{approx}}}$$

- Use this expression to find σ_{x_p}
- **NOTE:** We are using x_{approx} only to find the error on the predicted position since the calculations are much easier. However, to predict the actual value of the landing point you still have to use the full formula for x_p !
- Finally, check if theory and experiment agree within uncertainty

Tips

- Do not forget to derive the equations for the predicted position ***before*** coming to lab! Otherwise you will not have enough time to both take data and do the calculation
- On the lab manual there are some trial values that you can use to check if your formulae are correct
- Do not forget to write down the final formulae on the lab report!

- The experiment is extremely sensitive to the value of W_f, so measure it as carefully as possible!
- Also, watch out of one particular systematic error
- To check if your *Δh'* is the right one do:
  - For a fixed *Δh'* roll the ball 10 times
  - If ~5 times falls and ~5 times comes back then *Δh'* is correct