Experiment 8: Capacitance and the Oscilloscope

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INTRO TO EXPERIMENTAL PHYS-LAB
1493/1494/2699

Outline

- ***Capacitance:***
  - Capacitor as a charge storage device
  - Capacitors in combination
  - *RC* circuits: exponential growth and decay
- ***Oscilloscope:***
  - Conversion of analog signals to digital
  - Display and signal operations
- ***Measurements:***
  - Large *RC* charging
  - Large *RC* discharging
  - Small *RC* cycle observed with the oscilloscope

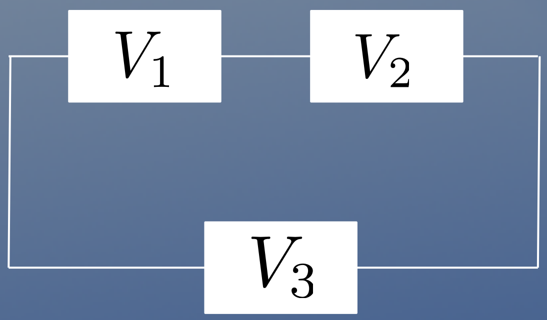
Main components of a circuit

- Even though the behavior of every element of a circuit can be explained on a microscopic electromagnetic basis, it is convenient to <u>introduce a schematic representation of them</u>
- All the microscopic features are embedded in **few macroscopic quantities** (current, resistance, capacitance, ...)
- Most common components are:

1. <u>Battery:</u> it provides a constant potential difference through the circuit. Its macroscopic quantity is the e.m.f. (ε)
2. <u>Resistor:</u> it causes a drop in the voltage due to microscopic collisions between the flowing charges and the atoms of the material or interactions with EM potential. Its macroscopic quantity is the resistance (R)
3. <u>Capacitor:</u> it is composed by two conductors (*e.g.* plates) separated by a non-conducting material. When a battery pumps charges on the plates a potential difference between them is created. Its macroscopic quantity is the capacitance (C)

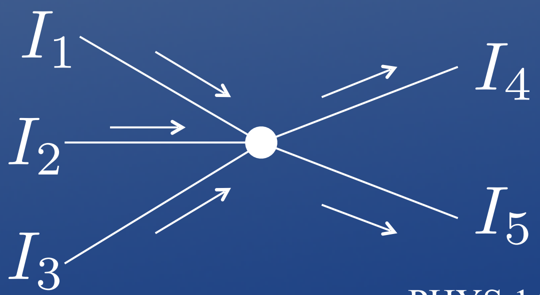
Kirchhoff laws

- The fundamental laws of circuits are the so-called **Kirchhoff's laws**
- 1st law: When considering a closed loop inside a circuit, the total potential difference must be zero



$$V_1 + V_2 + V_3 + \cdots = \sum_{i=1}^{N} V_i = 0$$

- 2nd law: When considering a junction, the sum of the ingoing currents is equal to the sum of the outgoing ones

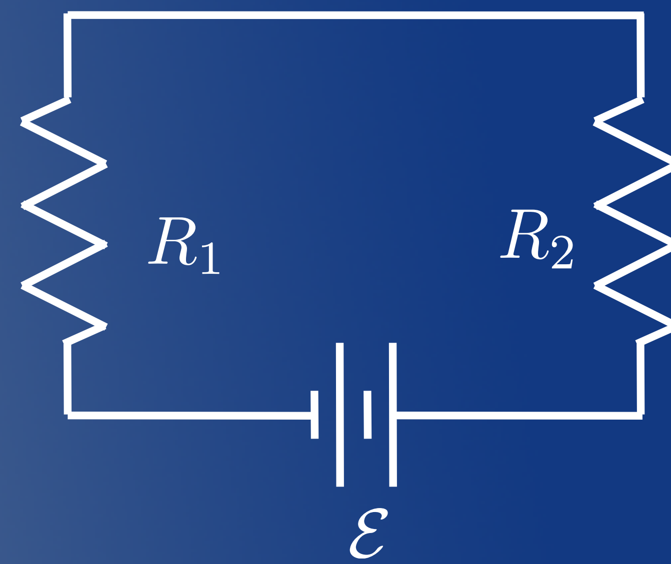


$$I_1 + I_2 + I_3 = I_4 + I_5; \quad \sum_{i=1}^{N} I_{\text{in}} = \sum_{i=1}^{N} I_{\text{out}}$$

A simple circuit

- Battery supplies ***potential difference***
- Current flows from high potential to low
- The voltage drop across a resistor is given by ***Ohm's Law***:

$$\boxed{V = IR}$$

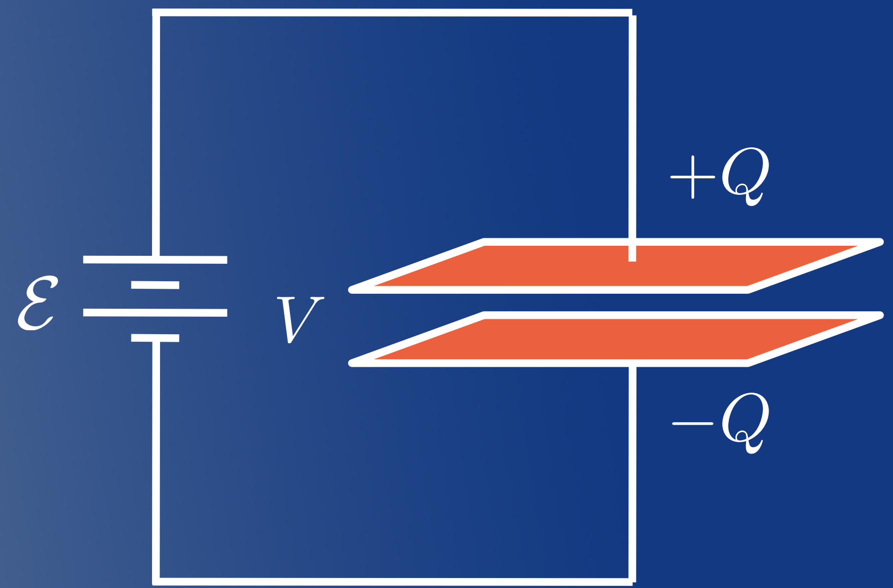


For a series of resistors:

$$V_1 = IR_1; \quad V_2 = IR_2$$
$$\mathcal{E} = V_1 + V_2$$
$$\mathcal{E} = IR_1 + IR_2$$
$$\mathcal{E} = I(R_1 + R_2) = IR_{\text{equiv}}$$

Capacitors

- A capacitor is a device that stores electric charge, and therefore energy.
  - *Examples: camera flashes, computer chips, defibrillators, etc...*
- Example: two conducting plates, separated by a gap, with voltage *V* across them.
- The total charge *Q* that can be stored on the plates is ***proportional*** to the potential generated, *V*.
- Constant of proportionality: a ***geometry-dependent*** quantity called ***capacitance***.

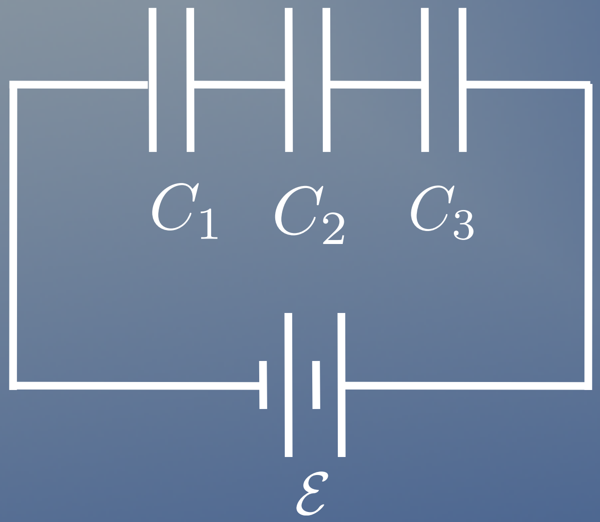


$$\boxed{Q = CV}$$

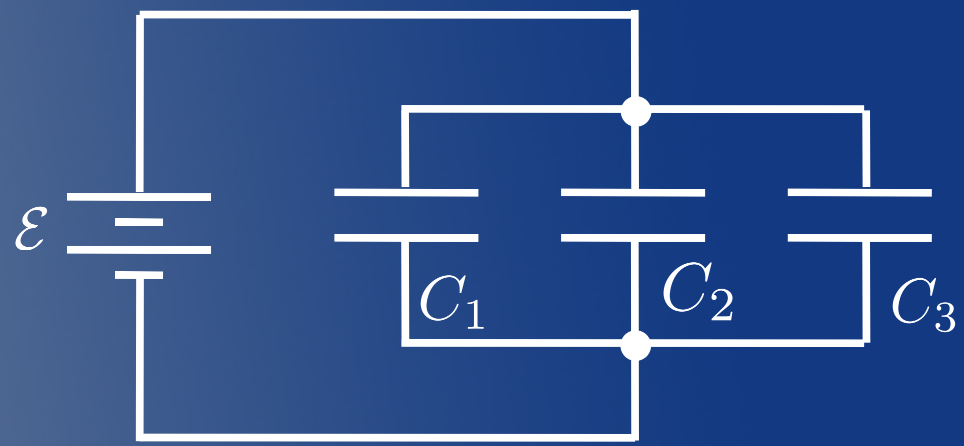
Units of capacitance are Farads (F):
1F = 1C/1V

Combination of capacitors

- *Capacitors in series:*

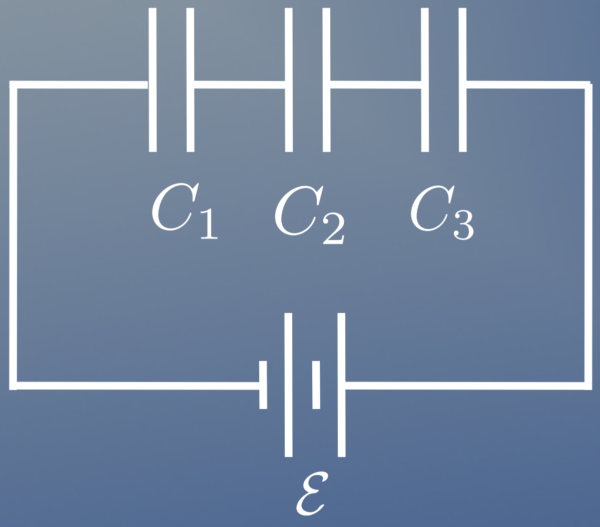


Capacitors in parallel:



Combination of capacitors

- ***Capacitors in series:***



$$(1)\quad \mathcal{E} = V_1 + V_2 + V_3$$

Voltages add

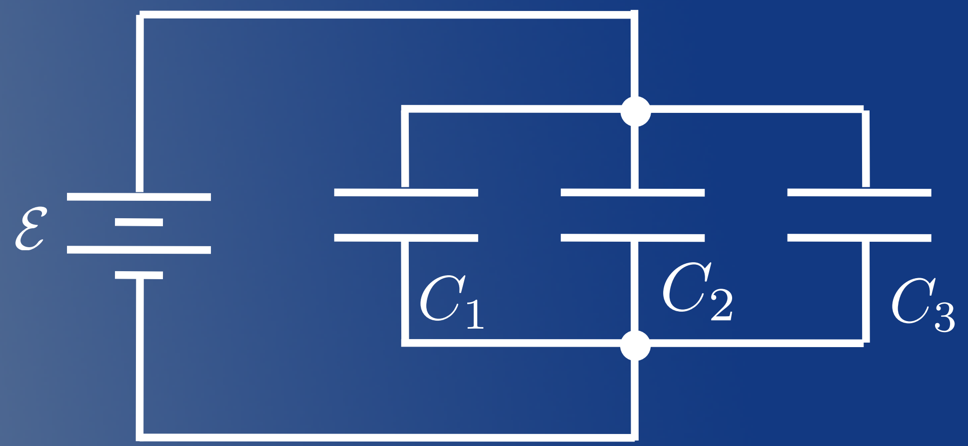
$$(2)\quad Q_1 = Q_2 = Q_3 = Q$$

Charges are the same

$$\frac{Q}{C_{\text{eq}}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

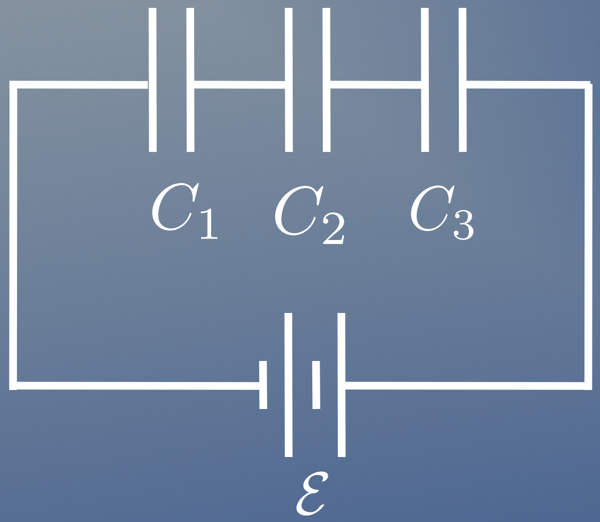
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel:



Combination of capacitors

- ***Capacitors in series:***



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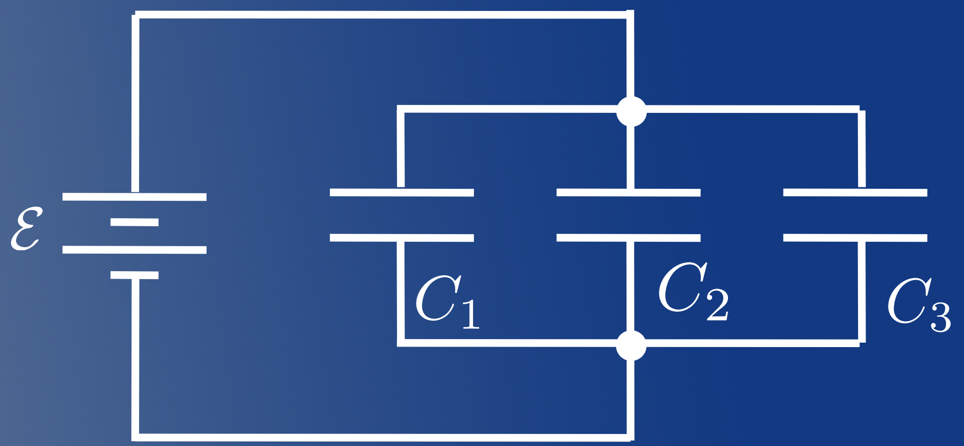
Capacitors in parallel:



$$(1)\quad \mathcal{E} = V_1 = V_2 = V_3$$

Voltages are the same

$$(2)\quad Q = Q_1 + Q_2 + Q_3$$

Charges add

$$CV = C_1V_1 + C_2V_2 + C_3V_3$$
$$C_{\text{eq}} = C_1 + C_2 + C_3$$

Resistor-capacitor *(RC)* combinations

- When ***resistors*** and ***capacitors*** are used together in circuits, interesting things start to happen.

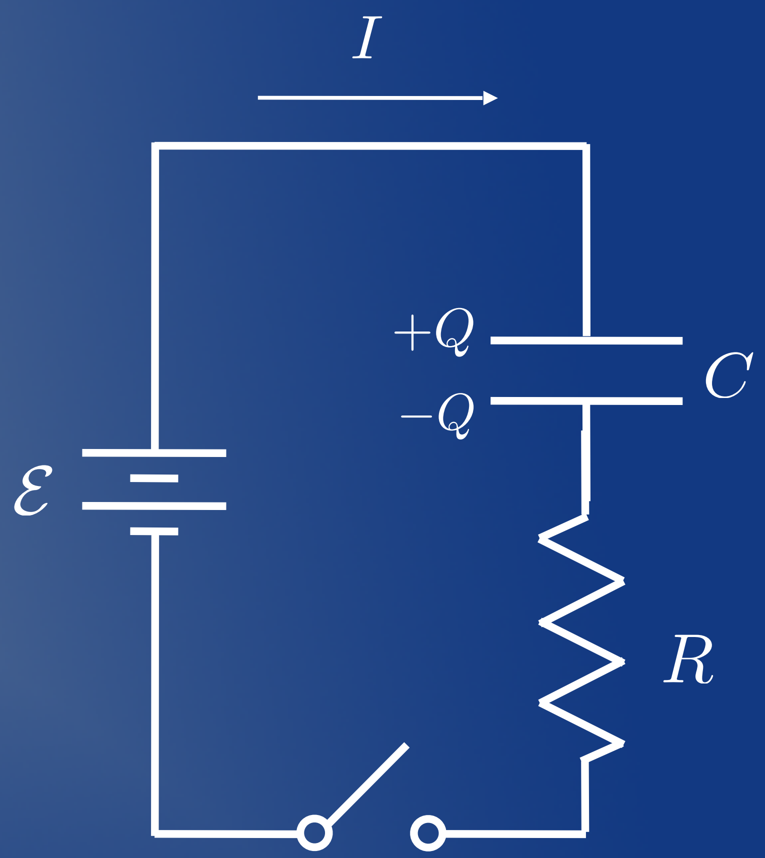
- A resistor will draw current from a battery; a capacitor will store the current's flowing charge.

- Recall: voltage expression for a resistor is given by Ohm's Law: $V = IR$, where $I = dQ/dt$

- Voltage expression for capacitor: $Q = CV$. Put these two together (a series *RC* circuit), and you get **exponentially decreasing current flow**

Charging a capacitor with a battery

- Put a capacitor in series with a battery.
- Capacitor is initially uncharged. Close the switch. By **Kirchoff's 1st law**:



Provides potential difference

Positive charges of current lose energy when getting close to positive charges on plate

Resistors always cause a loss of potential

$$+\mathcal{E} - \frac{Q}{C} - IR = 0$$

$$\mathcal{E} = R\frac{dQ}{dt} + \frac{Q}{C}$$

Charging a capacitor with a battery

$$\mathcal{E} = R\frac{dQ}{dt} + \frac{Q}{C}$$

$$\frac{dQ}{dt} = -\frac{Q}{RC} + \frac{\mathcal{E}}{R}$$

Charging a capacitor with a battery

$$\mathcal{E} = R\frac{dQ}{dt} + \frac{Q}{C}$$

$$\frac{dQ}{dt} = \frac{A}{\tau}e^{-t/\tau}$$

$$\frac{dQ}{dt} = -\frac{Q}{RC} + \frac{\mathcal{E}}{R}$$

- Need *dQ/dt* proportional to *Q* $\longrightarrow$ exponential function

- Note the extra constant, so lets try:

$$Q(t) = A\left(1 - e^{-t/\tau}\right)$$

Charging a capacitor with a battery

$$\mathcal{E} = R\frac{dQ}{dt} + \frac{Q}{C}$$

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- Need *dQ/dt* proportional to $Q \longrightarrow$ exponential function

- Note the extra constant, so lets try:

$$Q(t) = A\left(1 - e^{-t/\tau}\right)$$

$$\frac{dQ}{dt} = \frac{A}{\tau}e^{-t/\tau}$$

Substitute into differential equation:

$$\frac{A}{\tau}e^{-t/\tau} = -\frac{A}{RC}\left(1 - e^{-t/\tau}\right) + \frac{\mathcal{E}}{R}$$

$$\frac{A}{\tau}e^{-t/\tau} = \frac{A}{RC}e^{-t/\tau} + \left(\frac{\mathcal{E}}{R} - \frac{A}{RC}\right)$$

Compare left and right sides to determine A and τ:

$$\left(\frac{\mathcal{E}}{R} - \frac{A}{RC}\right) = 0 \Rightarrow A = \mathcal{E}C = Q_0$$

$$\frac{A}{\tau} = \frac{A}{RC} \Rightarrow \boxed{\tau = RC}$$

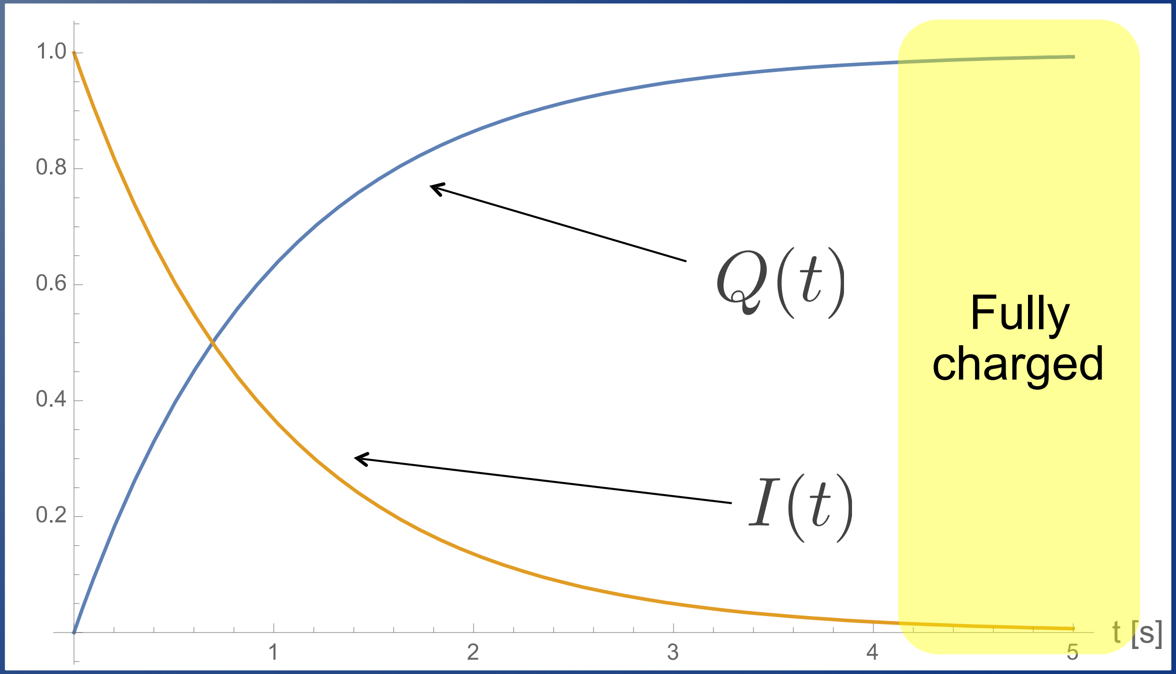
Charging a capacitor with a battery

- So, as we derived, the charge stored on the capacitor as a function of time is:

$$Q(t) = Q_0 \left(1 - e^{-t/RC}\right)$$

- The current flowing through the circuit is instead:

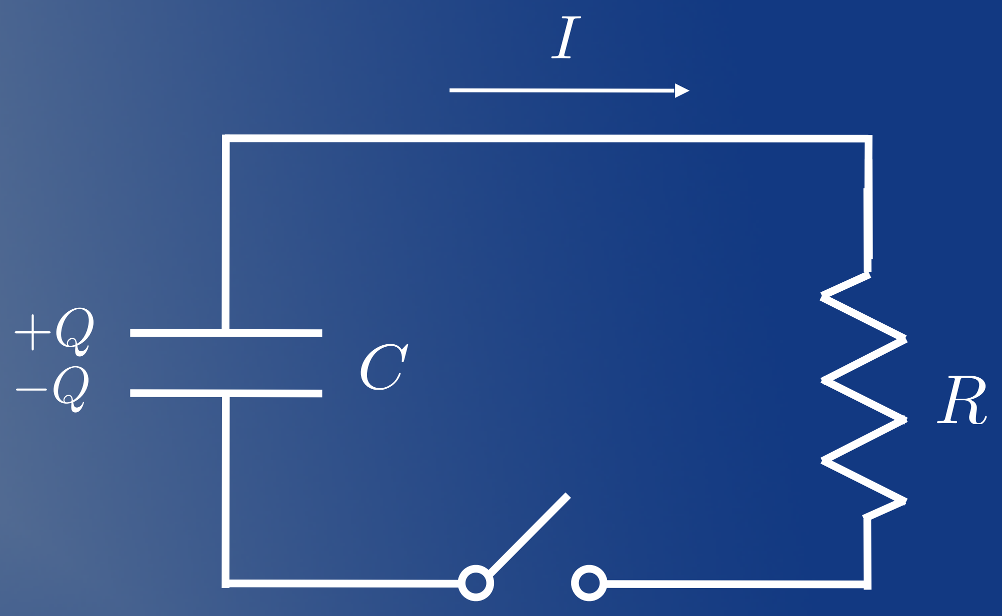
$$I(t) = \frac{dQ}{dt} = -\frac{\mathcal{E}}{R} e^{-t/\tau}$$



- Current decreases exponentially because capacitor is fully charged
- The constant τ is the typical time scale for charging/discharging the system

Discharging a charged capacitor

- Put a *charged capacitor* in series with a resistor.



- Close switch; charge will ***dissipate*** through the resistor. Again, by Kirchhoff's loop rule

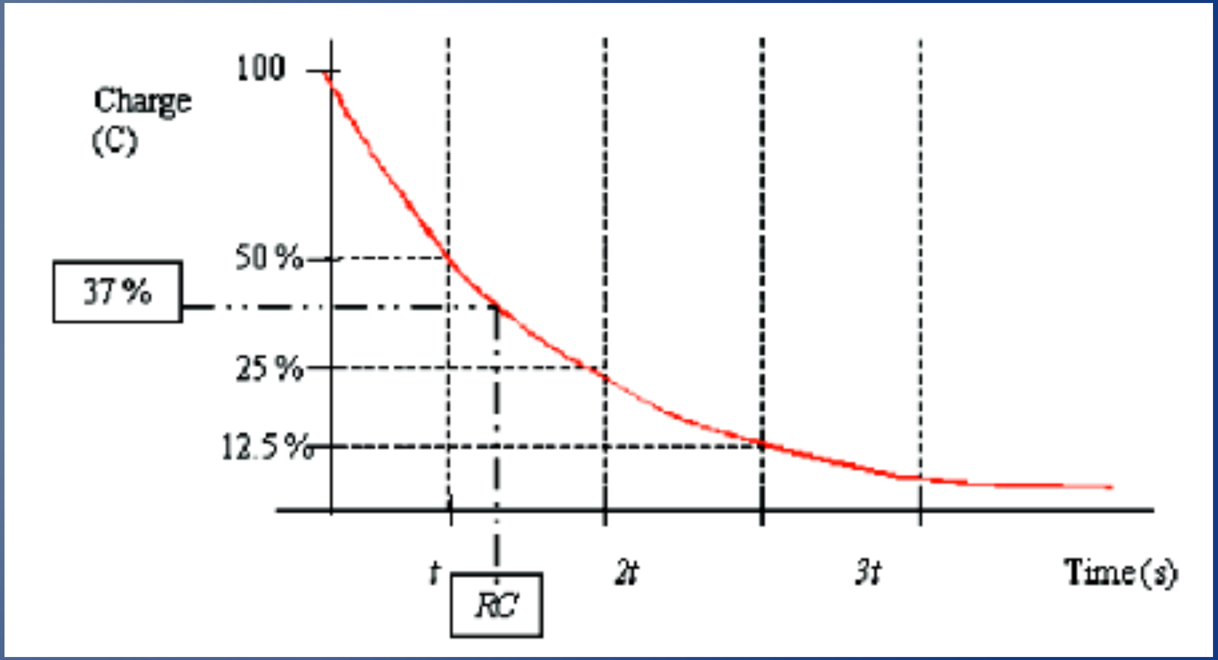
$$
\begin{aligned}
0 &= \frac{Q}{C} + IR \\
&= \frac{Q}{C} + R\frac{dQ}{dt}
\end{aligned}
$$

The calculation is now very similar to the case we just studied…

Discharging a charged capacitor

$$Q(t) = Q_0 e^{-t/RC}$$

$$I(t) = -\frac{\mathcal{E}}{R} e^{-t/RC}$$

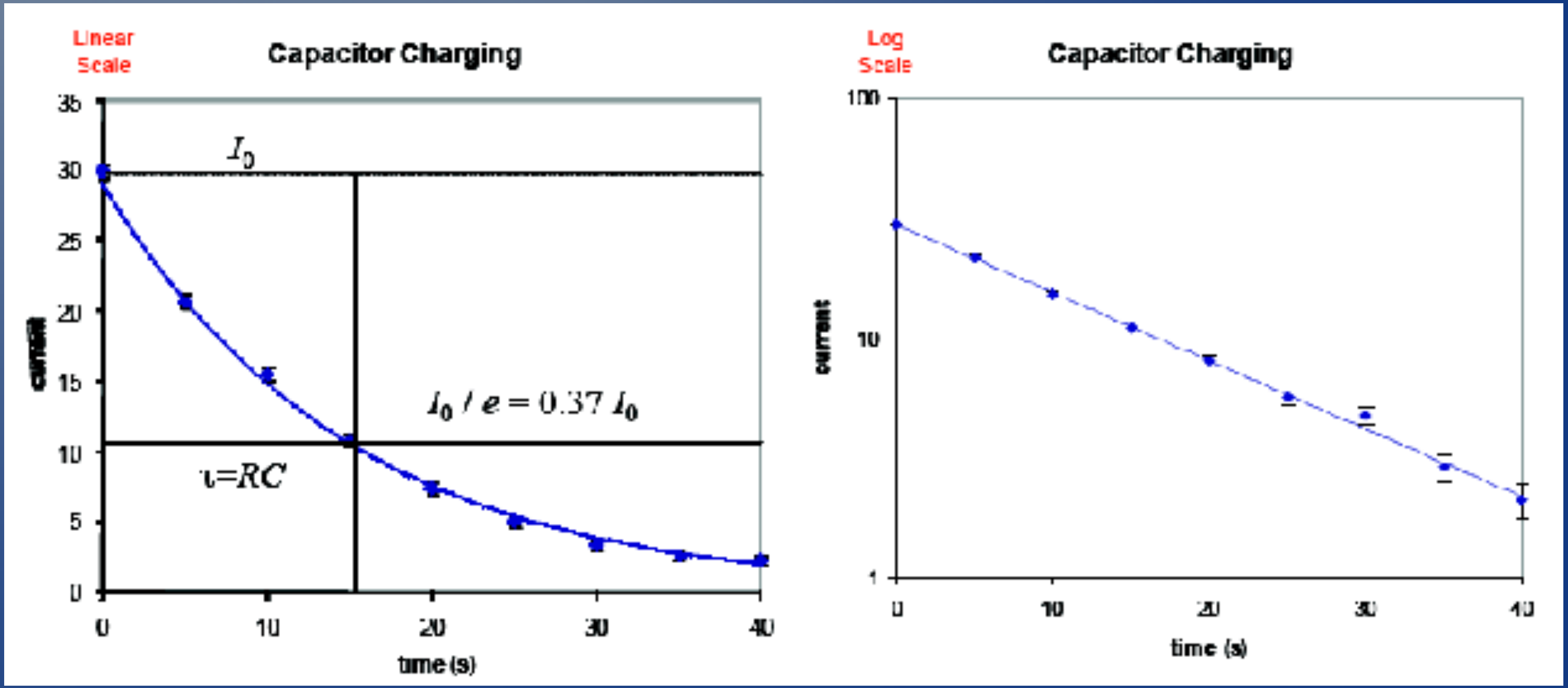


- In this case, the current *I(t)* decreases ***exponentially***, just like the previous charging case. However, in this case, is because the system is "running out" of charges
- Sign of current is ***negative***, indicating that flow is in the opposite direction

A better plot for current

- On a standard plot of *I* vs. *t*, *I(t)* approaches zero exponentially. This is <u>hard to visualize or quantify</u>
- **Better solution:** plot *log(I) vs. t*. In this semi-log plot the current will look like a straight line

<u>Horrible plots…</u>



Linearization of the exponential function

- Why does the exponential function look like a **straight line** on a semi-log plot?

$$
\begin{aligned}
I(t) &= I_0 e^{-t/RC} \\
\ln I(t) &= \ln\left(I_0 e^{-t/RC}\right) \\
&= \ln I_0 + \ln e^{-t/RC} \\
&= \ln I_0 - \frac{1}{RC} t \\
&\equiv b - at
\end{aligned}
$$

- Taking the (natural) logarithm converts the exponential function to a linear function in time *t*.

The Experiment

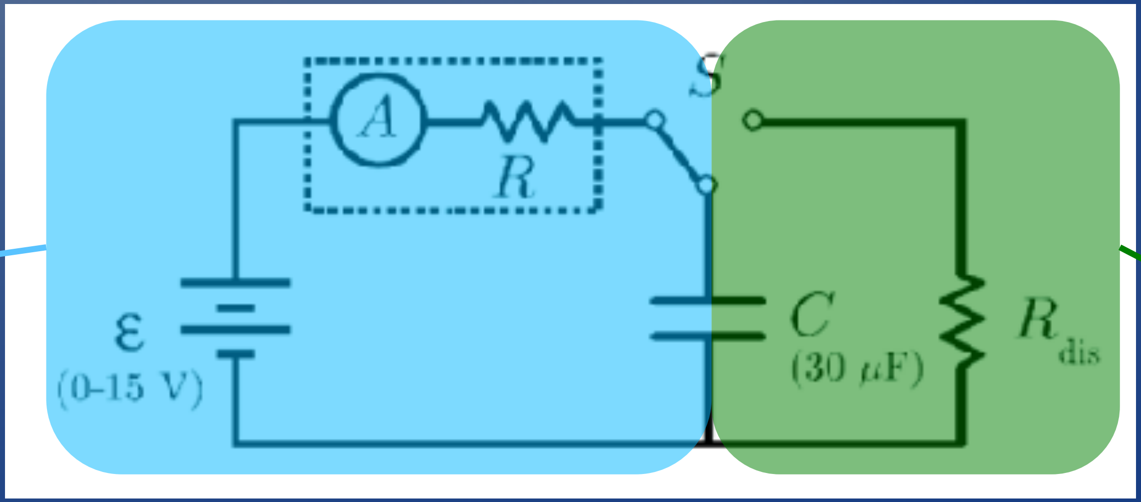
Goals

- In this experiment you will study different *RC* circuits and observe their properties
- Main goals are:
  - Use a timer to measure RC from the charge and discharge curve of a circuit with larger RC (slow)
  - Familiarize with an oscilloscope
  - Use the oscilloscope to study a circuit with small RC (fast)

Measurement of large $\tau = RC$

- In the first part of the lab, you will observe the time dependence of the current in two circuits with large RC values (*i.e.* long charge/discharge typical time).
- **Idea:** charge the capacitor bank (C = 10 μF, 20 μF, 30 μF). Pass the current through the ammeter so we can measure it.
- Ammeter has large unknown internal resistance R that determines the time constant $\tau = RC$.

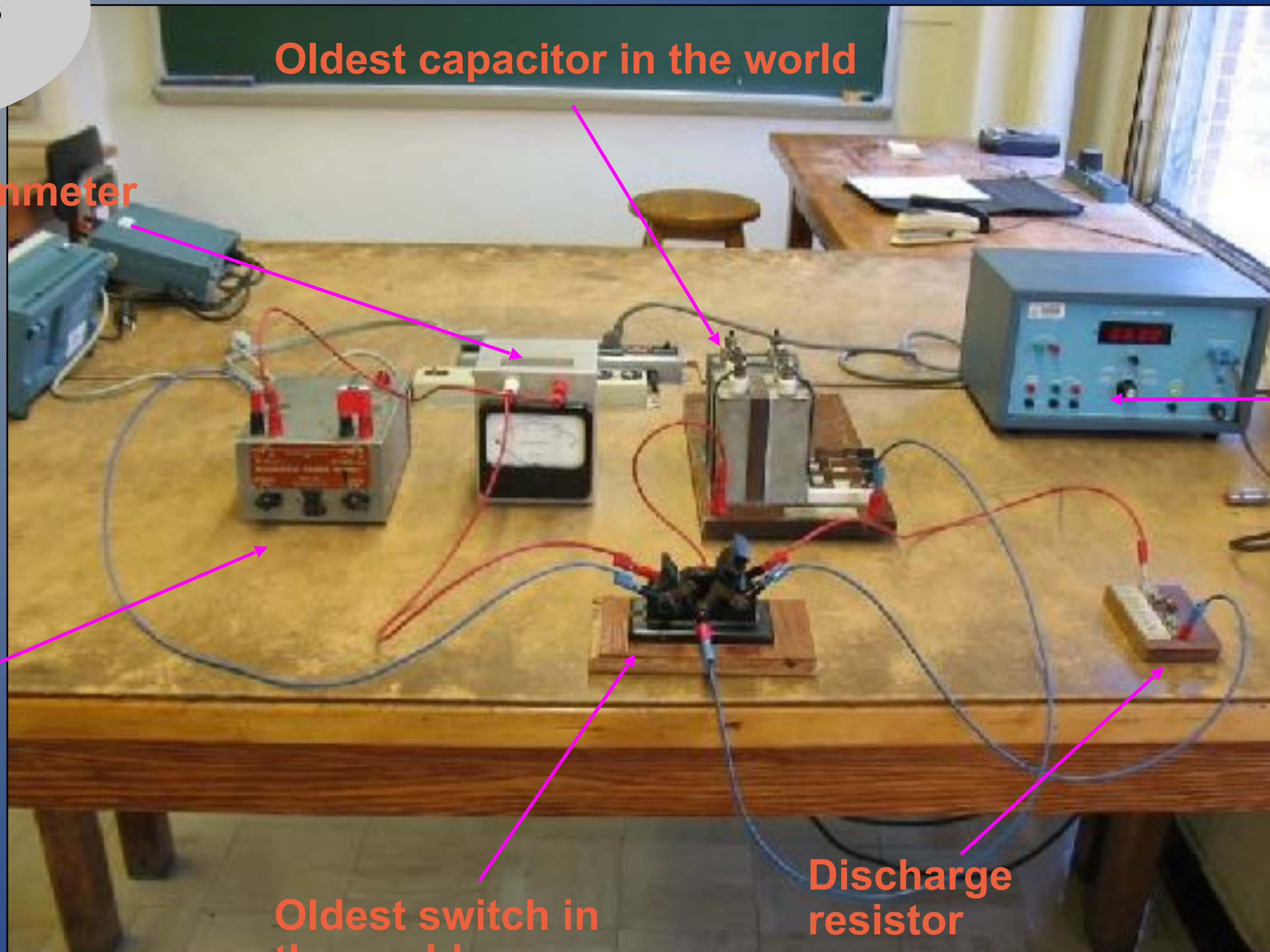
RC circuit of interest



This is only used to discharge

Large charge setup

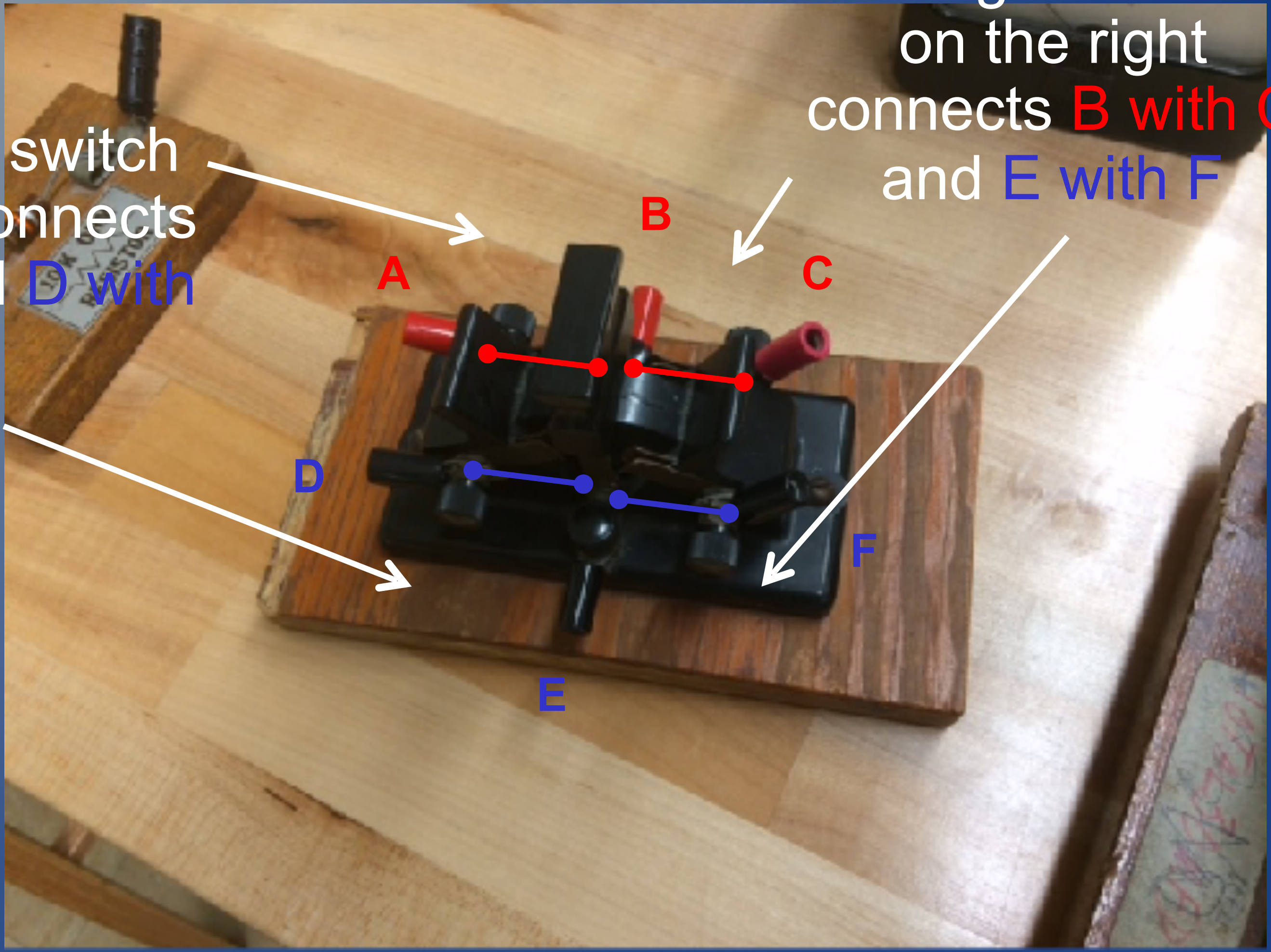
Back in my days, this was high tech…



How to use the oldest switch in the world

Closing the switch on the left connects A with B and D with E

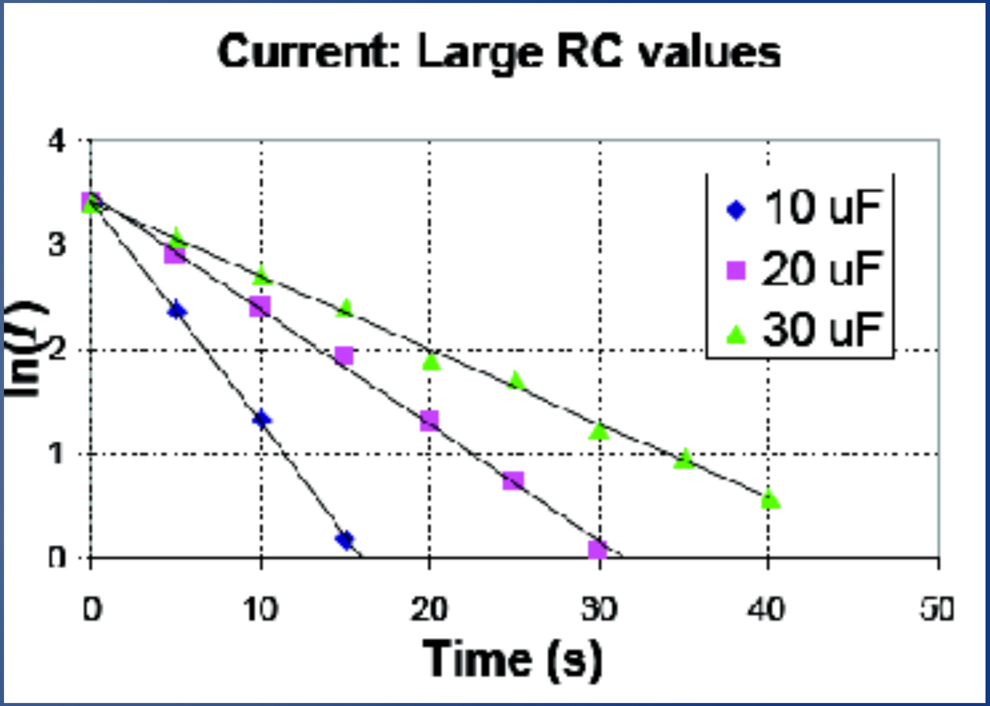
Closing the switch on the right connects B with C and E with F



Measurement of large $\tau = RC$

- For three settings of the capacitor bank C = 10μF, 20μF, 30μF, observe *I(t)*.
  - There is a timer you can use to read off *e.g.* 1s intervals
- Calculate ln(*I(t)*) and perform a linear fit of the logarithm vs. time.
- Get three values for the time constant τ, then get three values for internal resistance *R*.
- Calculate mean *R* and the standard error on the mean. Compare your results obtained with charging/discharging.

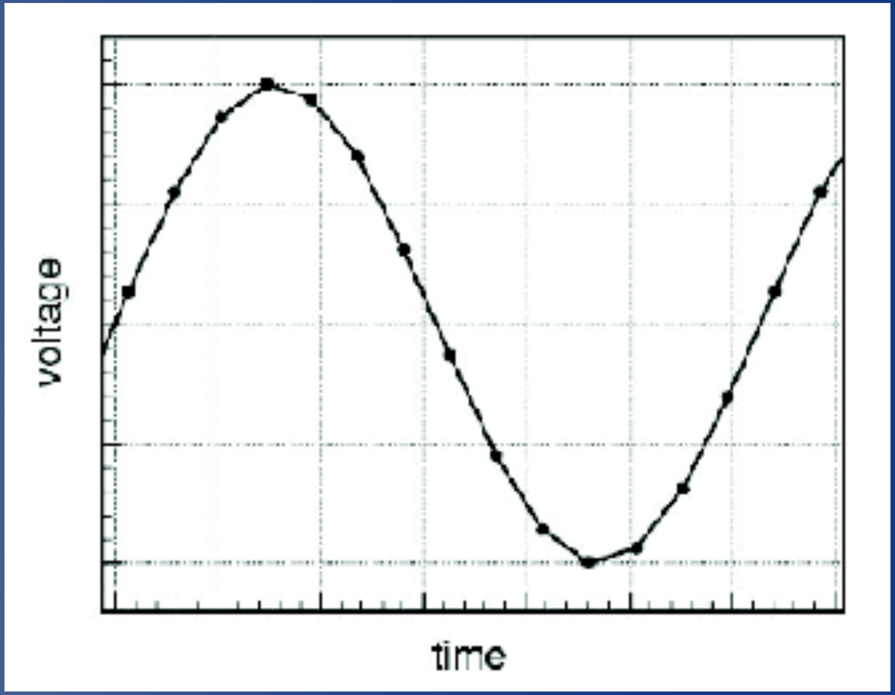
Watch out! Horrible plot…



What your plot of log current vs. time might look like

Oscilloscope

- The oscilloscope allows us to visualize signals that vary rapidly with time. Very handy!
- **Idea:** scope converts voltage waveforms from analog to digital, then displays the signal on an LCD screen.
- What you need to do:
  - Learn how to display voltage signals from function generator on the scope.
  - Learn how to use the scope's measurement tools.



A digital oscilloscope samples input voltage signal at regular intervals using an analog-to-digital converter (ADC)

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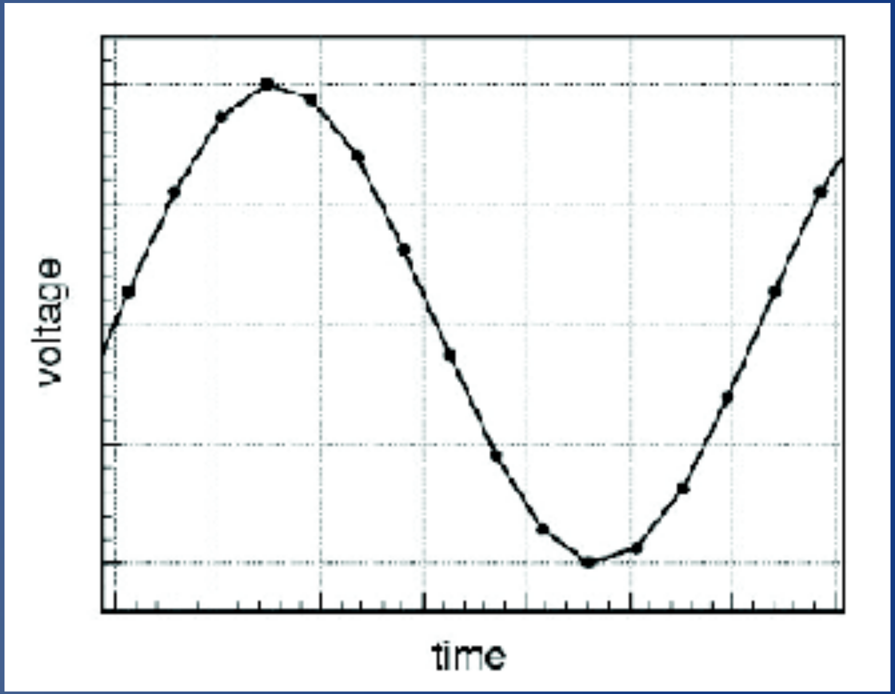


A digital oscilloscope samples input voltage signal at regular intervals using an analog-to-digital converter (ADC)

Question for you: why is this plot worth several points less on the report?

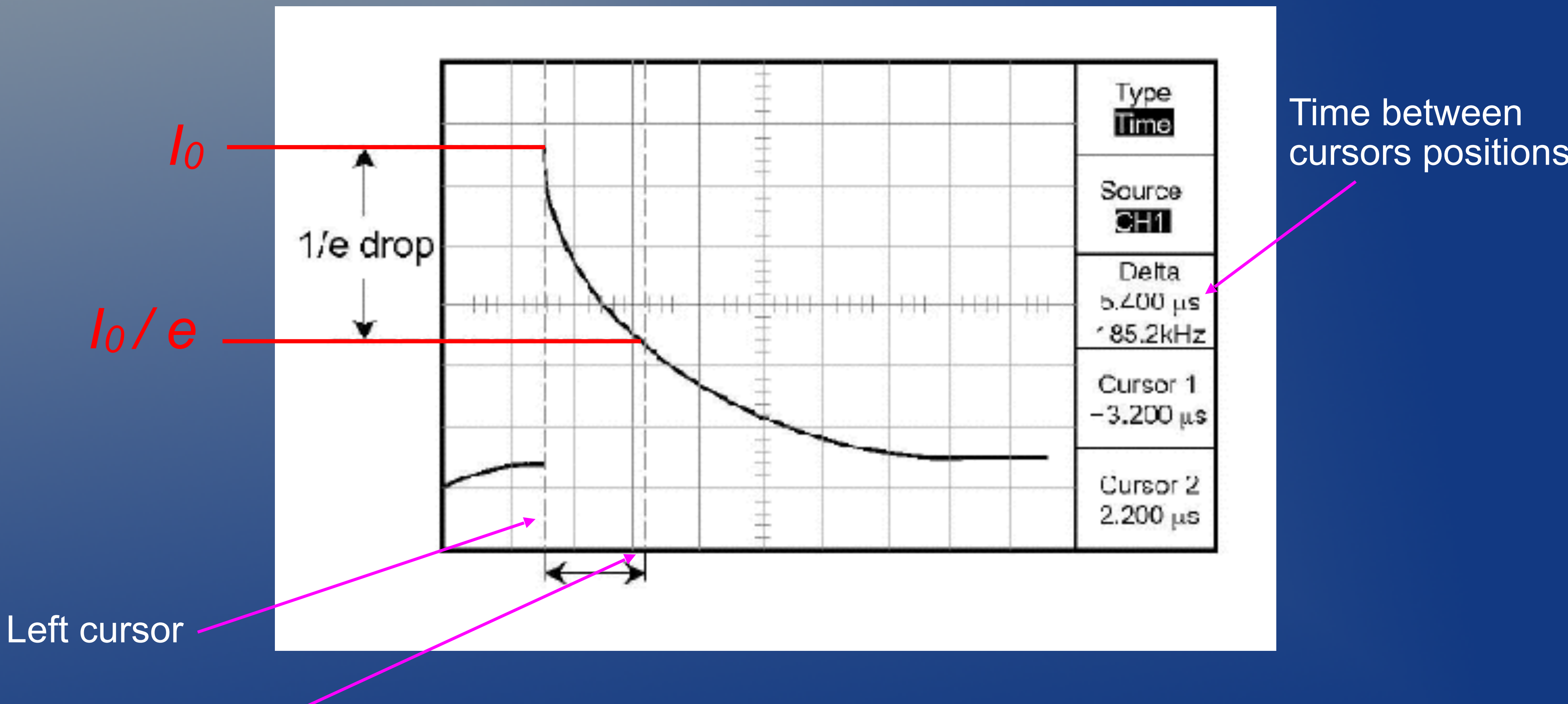
Using the oscilloscope

- *This part of the lab is mostly playing around and getting familiar with the oscilloscope.*

- You need to know the following:
  - Display different signal types (square, sine, triangle waves) from function generator.
  - Make sure you know **how to scale axes**.
  - Using a sine wave input, use the scope's **MEASURE tool** to display the period and amplitude of the signal.
  - Learn how to use the cursors to read a time period or an amplitude directly off the screen.

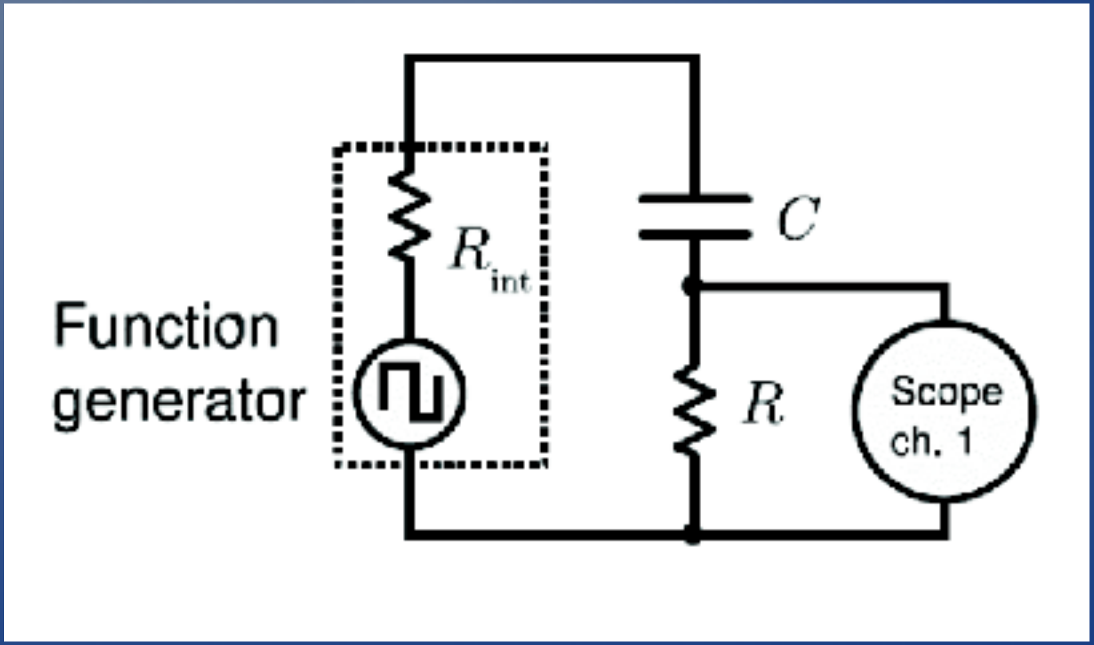
Using the oscilloscope

- The channel 1,2 menu position buttons double as cursors controls. You can position the cursors by hand and find the ***time difference*** between them.



Measurement of short *RC*

- Once you are comfortable with the oscilloscope, set up the circuit below.
- Drive the function generator with a square wave, and then analyze the wave on the scope as it moves through the *RC* circuit.

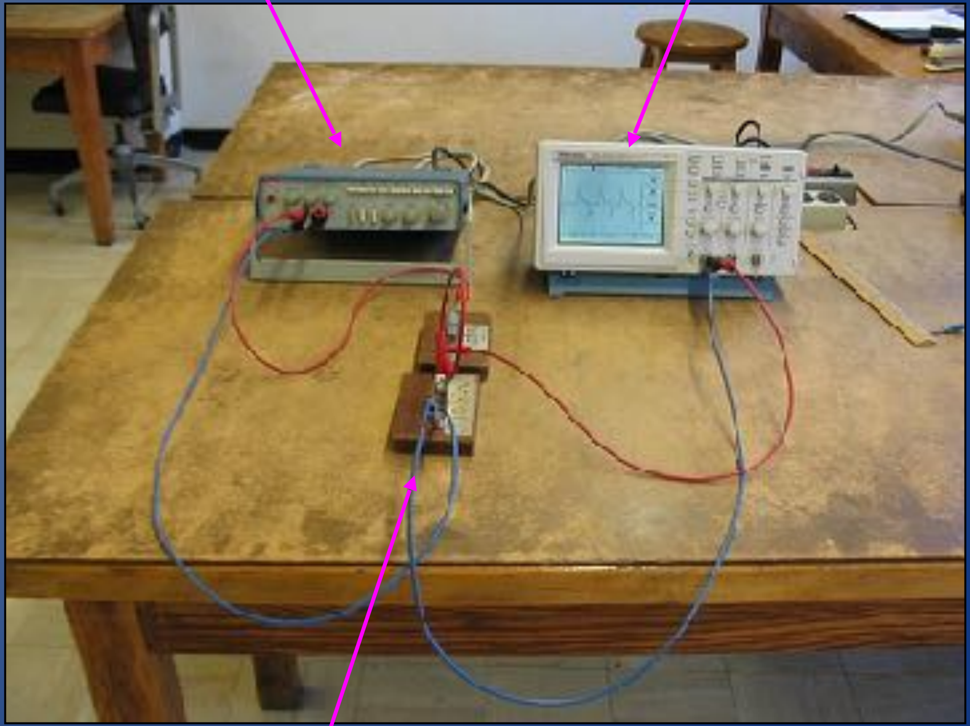


Oscilloscope and generator setup tips

- ***Function generator***
  - Amplitude: 10V peak-peak
  - Frequency: 150Hz
- ***Oscilloscope***
  - Ch.1 coupling: DC
  - BW limit: ON
  - Volts/Div: coarse
  - Probe: 1X
- *R* and *C* used are 10kΩ and 82nF.

Signal generator

Oscilloscope

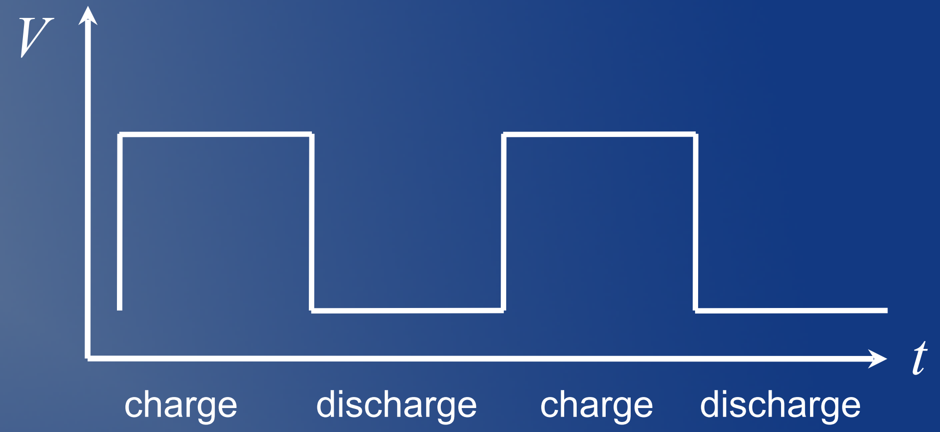


***R*,*C* in series**

Measurement of short *RC*

- When the square wave is "high", the capacitor will ***charge up***.
- When the square wave is "low", the capacitor will ***discharge***.

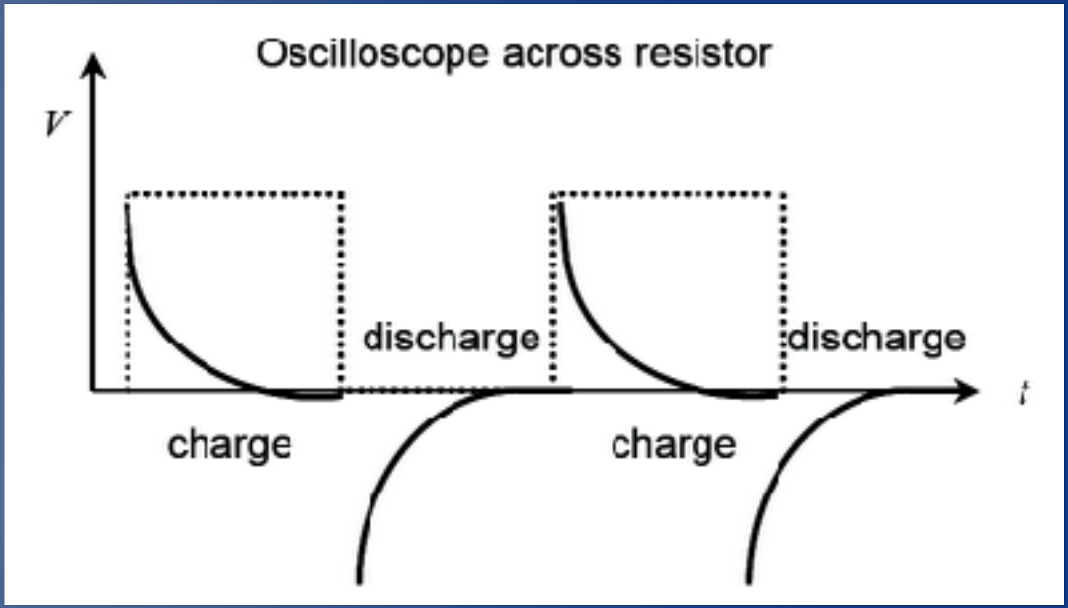
Input to RC circuit from function generator:



Measurement of short *RC*

- Oscilloscope connected across resistor will show ***exponential decay*** to zero for both charging and discharging a cycles.

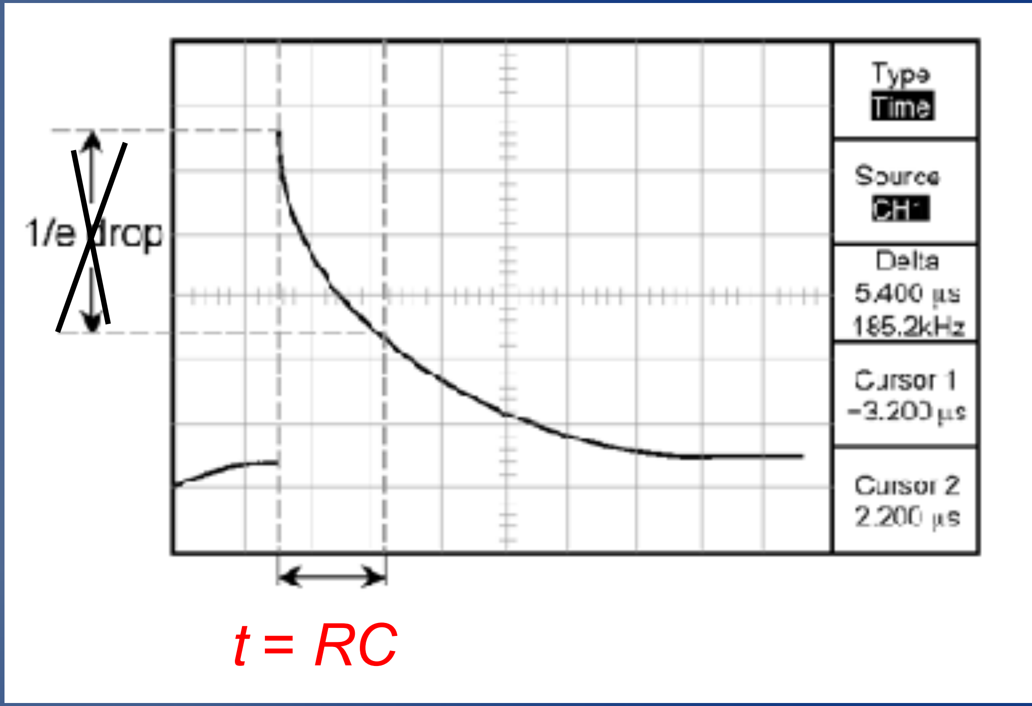
- You can view the rapid charge/discharge cycle on the scope. Use it to measure τ = *RC*.



$$V = IR = I_0 Re^{-t/RC}$$

Measurement of short *RC*

- Measurement: zoom in on a single part of the charge/ discharge cycle.
- Using the cursors, observe the time it takes for the voltage to drop to 37% of its initial value. ***This is the time constant.*** See if it matches the prediction $\tau = RC$
- Try this for several driving frequencies on the scope. **Note:** do not use frequencies much larger or smaller than ***f = 1 /*** τ



Qualitatively: what happens when the ***frequency*** is much smaller than $1/\tau$? When is it much larger? Can you explain what you see?

Tips

- This experiment is one of the easiest and therefore there are just a few tips:

1. The apparati are not dangerous. However, please always be careful when handling the electric components. In particular, when changing set up, always turn the instruments off.
2. The most essential part of this experiment is the set up itself, *i.e.* connecting all the right components to the right spots. Try to follow the pictures on the lab manual as close as possible. Ask your TA to check your links.
3. If you have troubles showing the signals on the oscilloscope, remember that the AUTOSET button is often the solution to everything