Experiment 9:
AC circuits

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INTRO TO EXPERIMENTAL PHYS-LAB
1493/1494/2699

Introduction

- ***Last week (RC circuit):***
  - **Constant** Voltage power source (constant over time)
- ***This week:***
  - A new component: the **inductor**
  - **Alternating Current (AC) circuits**
    - ➢ Time dependent voltage source
  - Leads to:
    - ➢ **Time dependent currents** (alternating currents)
    - ➢ **Phase shifts in voltage and currents** in components with respect to one another
    - ➢ **Resonance**

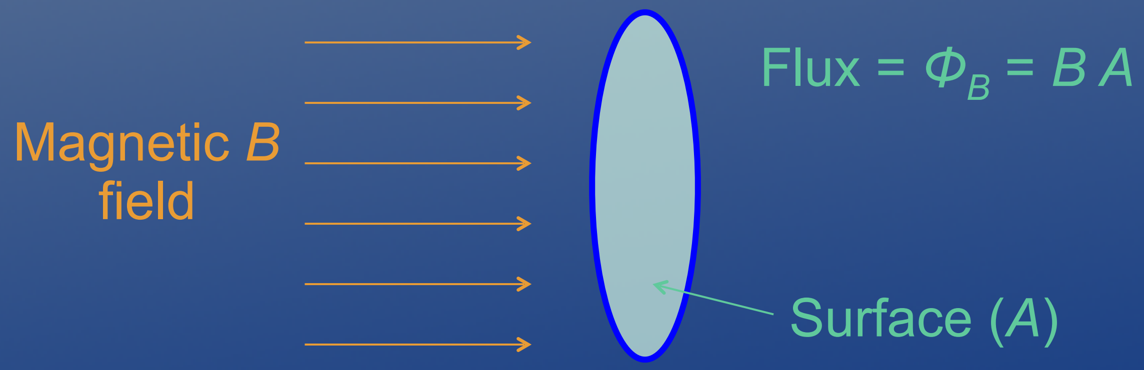
Varying electromagnetic fields

- The 2nd and 4th Maxwell equations in vacuum and with no sources read:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

- Varying magnetic field generates electric field and viceversa!
- We therefore expect from a changing *B* field to give rise to a current or potential difference
- This is summarized by **Faraday's law**: $\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$



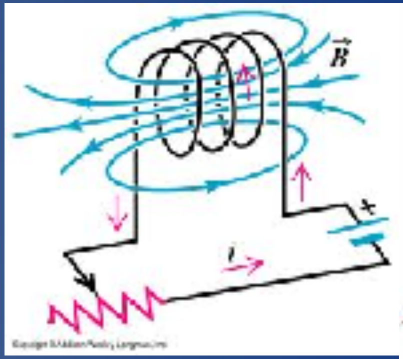
- **Change in magnetic flux creates an e.m.f.**
- Magnetic flux can be varied in two ways:
  1. Change the B field
  2. Change the surface

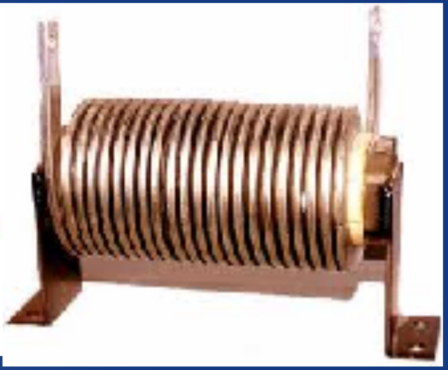
Introducing the inductor

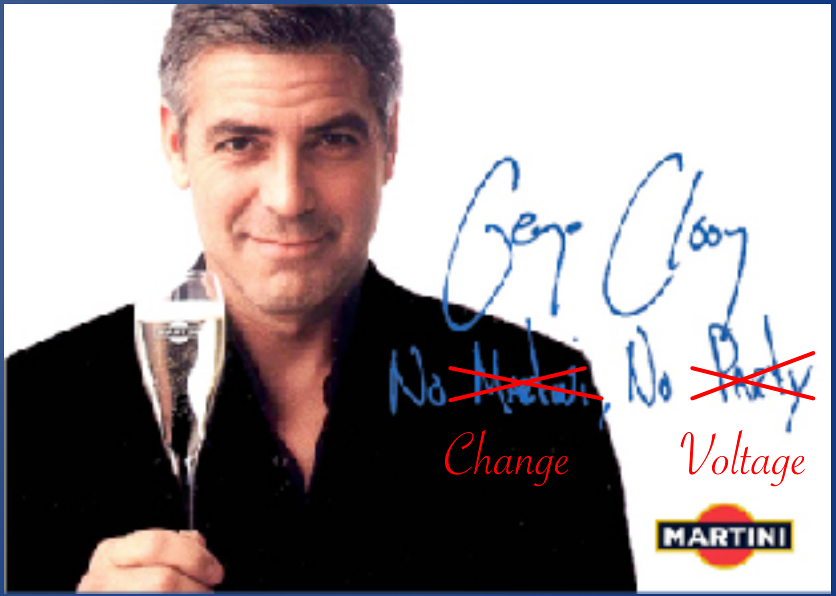
- It ***stores energy*** in form of magnetic fields (analogous to the capacitor)
- From Faraday’s law one deduces the expression for the potential difference at the two ends of the inductor:

$$\boxed{V_L = -L\frac{dI}{dt}}$$

- The inductor is only sensitive to the **change in current**! *No change = no voltage*
- Negative sign indicates that the inductor opposes any change in current (***Lenz's Law***)





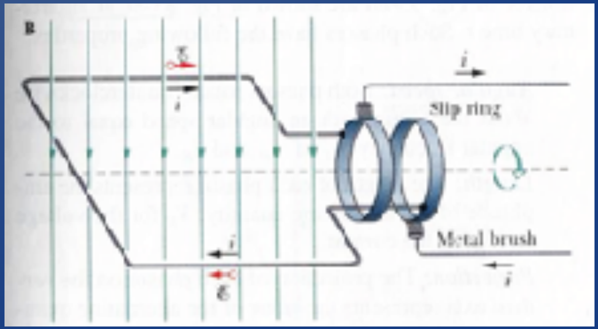


Why AC circuits?

- Sensitive to input **frequency** (*i.e.* function generator frequency)
- Serve as signal frequency filters:
  - High-frequency filters
  - Low-frequency filters
  - Band-pass filters
- Transformers
  - Induction effects - Ability to raise or lower the voltage amplitude.
- Generators and Motors

***e.g.* Speakers**

***e.g.* Radios**

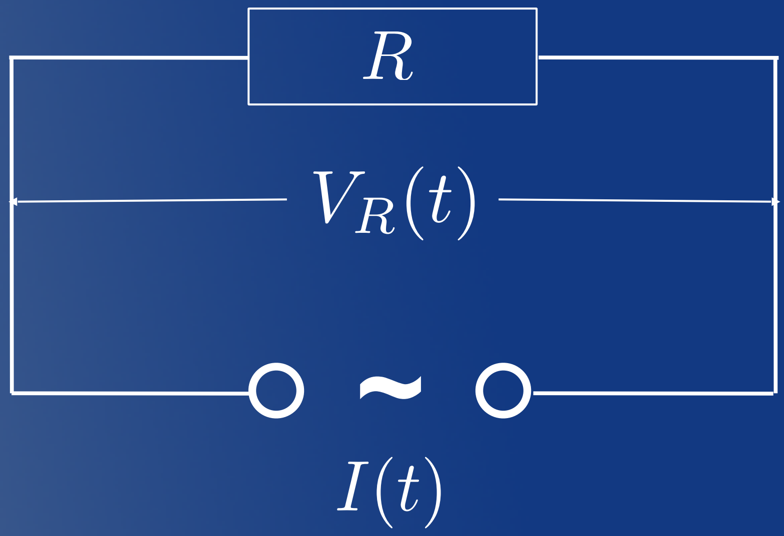


AC circuits: resistors

- AC circuits have an enormous range of applications. Here we cover the most important aspects
- For this lab:
  - Consider only sources that vary ***sinusoidally***:

$$I(t) = I_{\max} \sin(\omega t)$$

- Simple example:
  - Function generator + resistor
- ***<u>Ohm's Law:</u>*** voltage across the resistor is just $V_R = IR$



The voltage across the resistor is then simply:

$$\begin{aligned} V_R(t) &= I(t)R \\ &= I_{\max} R \sin(\omega t) \\ &\equiv V_{\max} \sin(\omega t) \end{aligned}$$

AC circuits: capacitors

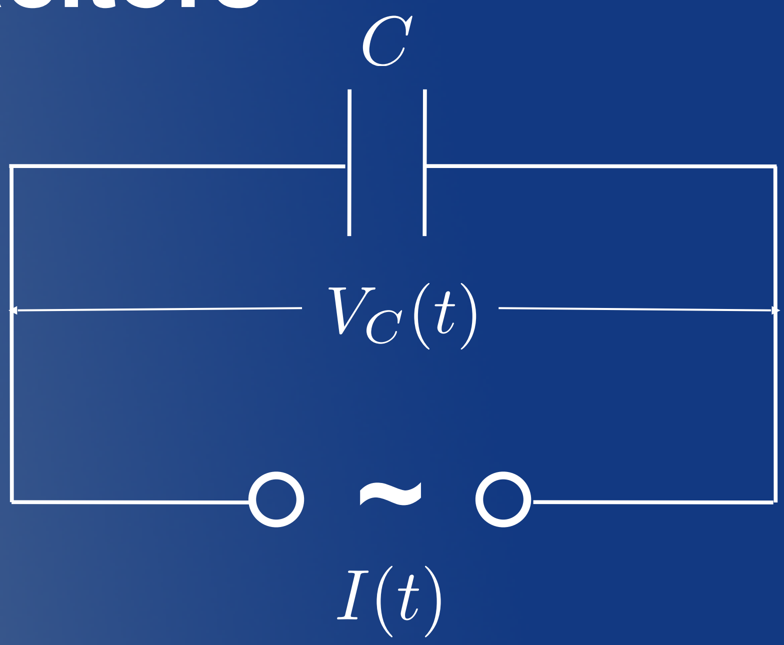
- More interesting case: connect a capacitor to the AC voltage source
- Last time we saw that the voltage across a capacitor is given by:

$$Q = CV_C$$

$$\Downarrow$$

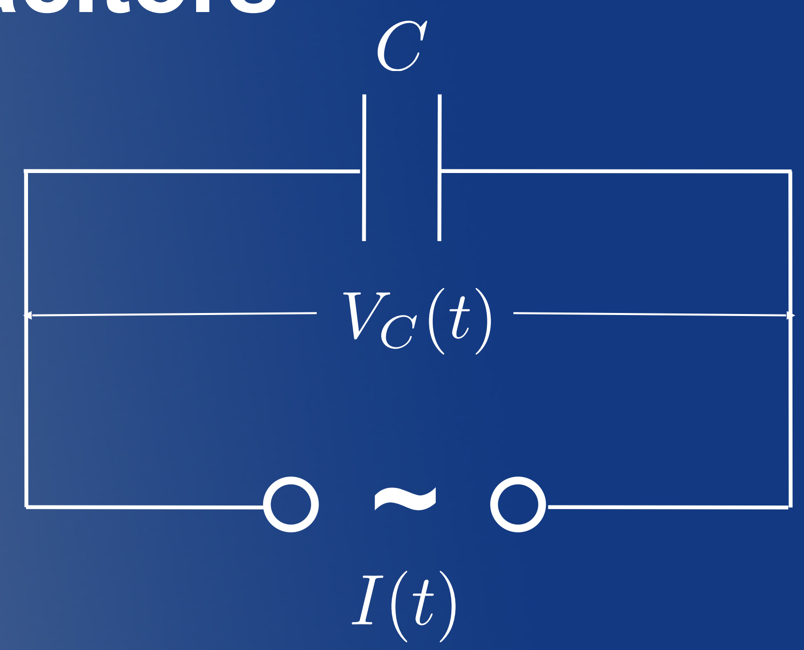
$$\int I(t)dt = CV_C$$

- Therefore, when the current is sinusoidal the voltage is given by:



$$\begin{aligned} V_C(t) &= \frac{1}{C}\int I(t)dt \\ &= \frac{1}{C}\int I_{\max}\sin(\omega t)dt \\ &= -\frac{I_{\max}}{\omega C}\cos(\omega t) \end{aligned}$$

AC circuits: capacitors

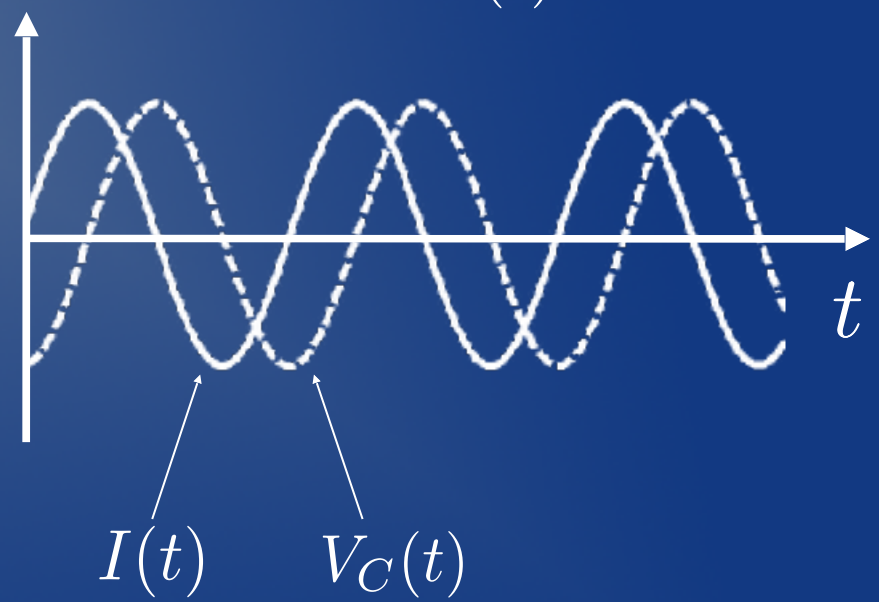


- The voltage is sinusoidal:

$$V_C(t) = -\frac{I_{\max}}{\omega C}\cos(\omega t)$$
$$= -V_{\max}\cos(\omega t)$$
$$= V_{\max}\sin\left(\omega t - \frac{\pi}{2}\right)$$

- The extra *π/2* in the expression is the **phase of the voltage**.
- Voltage across the capacitor lags behind the current by:

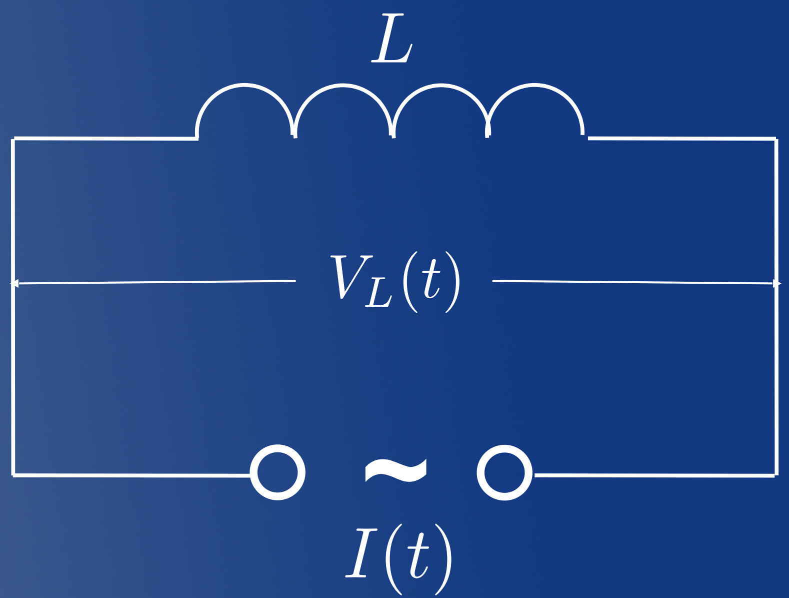
$$\phi = -\pi/2$$



AC circuits: inductors

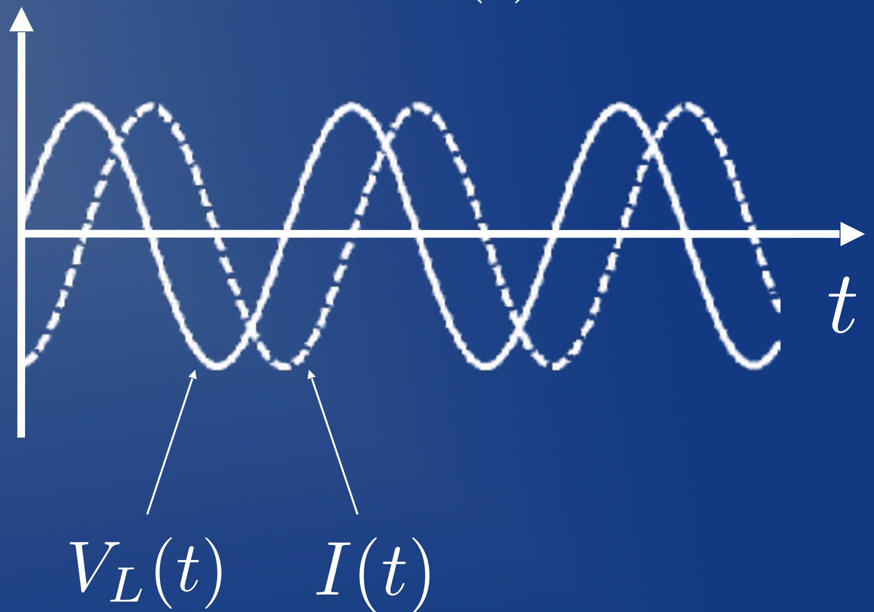
- The voltage is still sinusoidal:

$$\begin{aligned} V_L(t) &= L\frac{dI}{dt} \\ &= \omega L I_{\max}\cos(\omega t) \\ &= V_{\max}\sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned}$$



- Inductor voltage is also **phase shifted** w.r.t. current.
- Voltage across the inductor anticipates the current through it by:

$$\phi = \pi/2$$



Voltage maxima: a closer look

- Given our expression for V_R , the ***maximum value*** of the voltage across the resistor is just given by Ohm's Law:

$$V_R^{\max} = I_{\max} R$$

- The maximum voltage across the capacitor is a function of ω:

$$V_C^{\max} = \boxed{\frac{1}{\omega C}} I_{\max} \equiv I_{\max} X_C$$

Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

Given an oscillating input current the capacitor voltage is higher for small frequencies and lower for high frequencies

- But, the maximum voltage across the inductor is also a function of the driving frequency:

$$V_L^{\max} = \boxed{\omega L} I_{\max} \equiv I_{\max} X_L$$

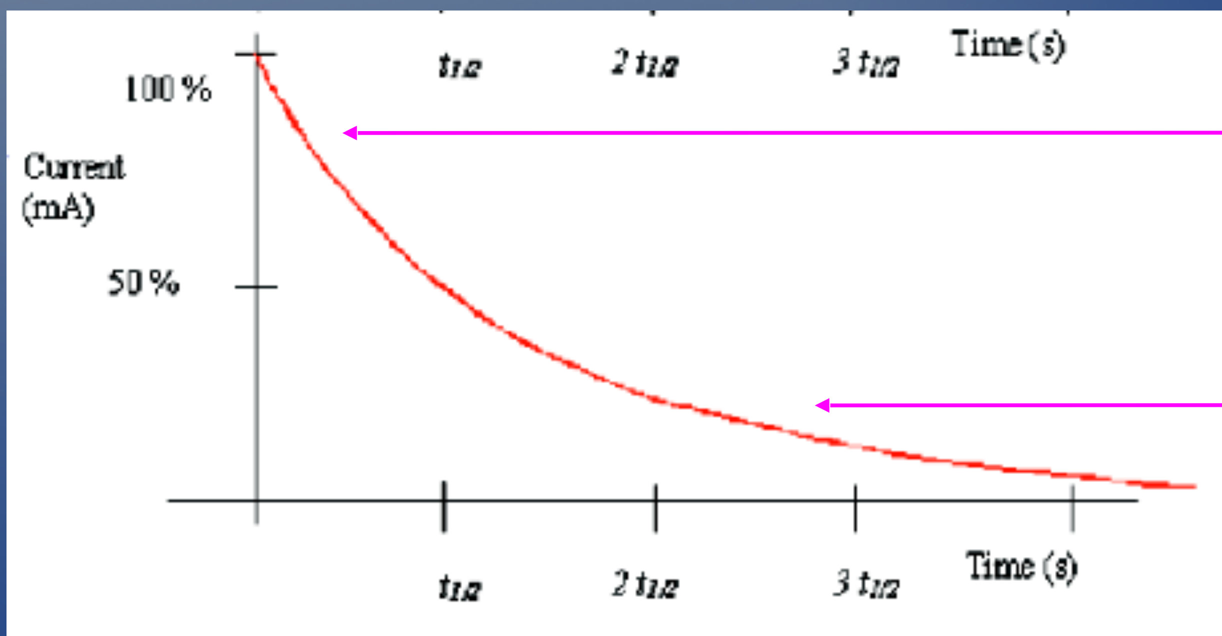
Inductive reactance

$$X_L = \omega L$$

The inductor voltage is instead higher for large frequencies and lower for small ones

Physical explanation: capacitors

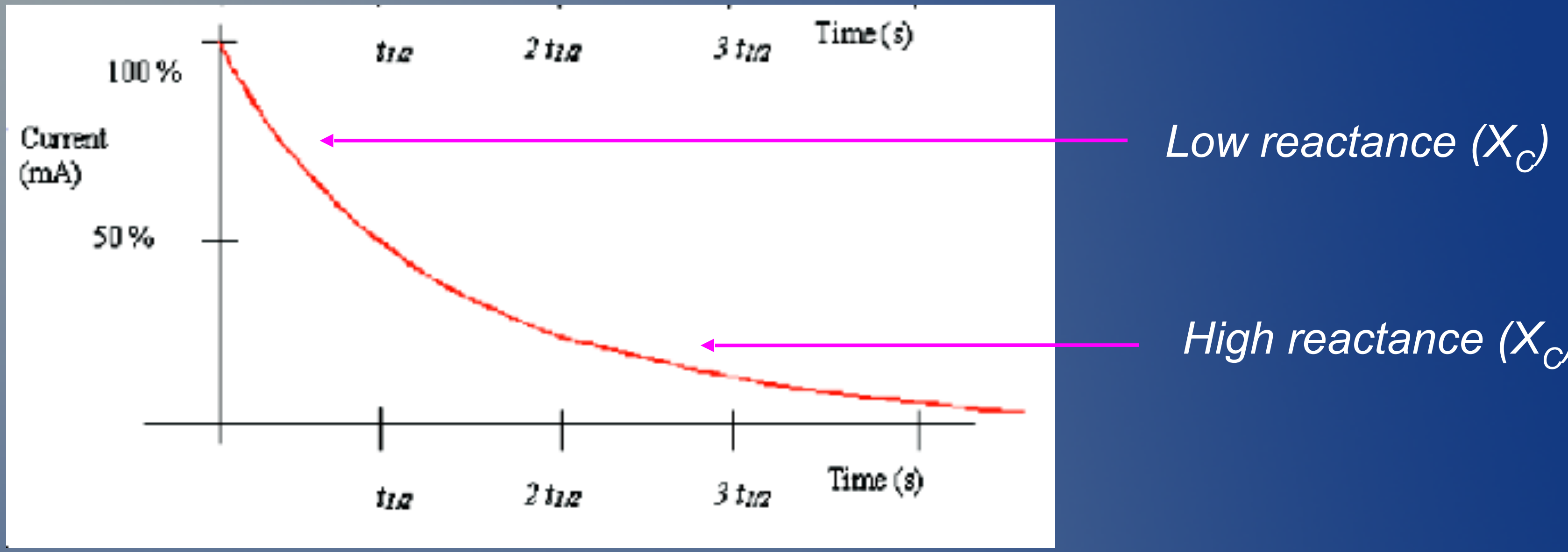
- **Question:** Why does the capacitor *resist low-frequency signals more than high-frequency ones*?
- Last time: when charging/discharging the capacitor, the current – *the rate at which you can charge it* – decreases exponentially. ***It becomes harder and harder to push in more charge as the capacitor fills up.***



Easy to charge = low reactance (X_C)

Hard to charge = high reactance (X_C)

Physical explanation: capacitors



- ***Rapidly varying signals*** (high frequency) quickly charge/discharge capacitor before it fills with charge → low impedance.
- ***Slowly varying signals*** (low frequency) charge the capacitor to its limit, slowing down the rate: that is, decreasing the current!
- Now that we have introduced the language of reactances, <u>you can think about the capacitor somehow as a resistor with ω-dependent resistance</u>

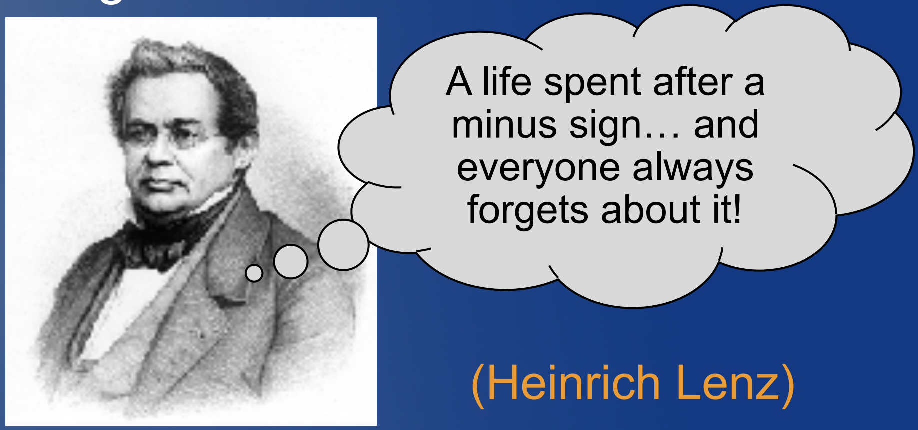
Physical explanation: inductors

- **Question:** Why does the *inductor resist high-frequency signals more than low-frequency ones*?

- Think about the nature of an inductor: it is a ***coil of wire***. If the current in the wire changes, then the magnetic flux through the coil changes → **induction**!

- ***Lenz's Law:*** a coil will ***oppose changes*** in magnetic flux. Self-induced EMF is:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

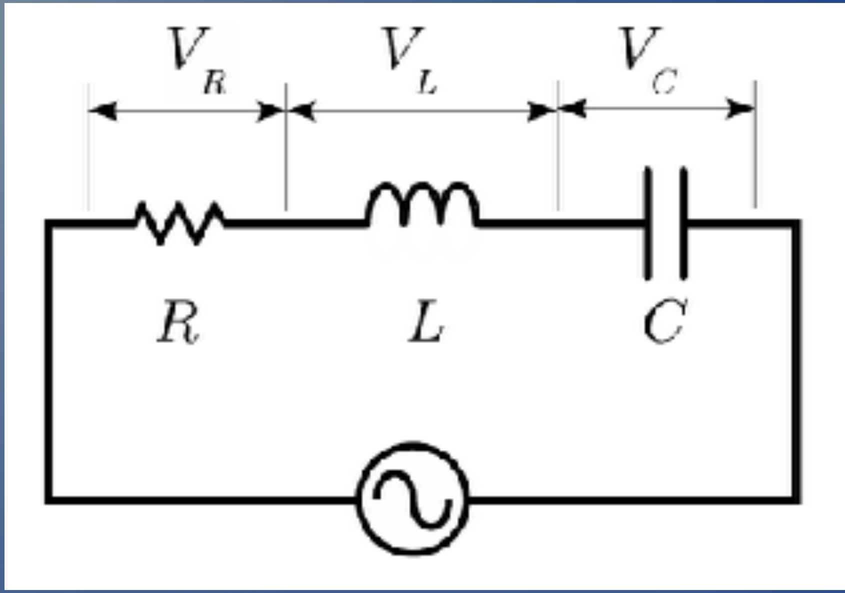


(Heinrich Lenz)

- *Rapidly varying* signals strongly change the flux, so the inductor "pushes back" harder against the flow of current!
  - Voltage is maximum (and opposing) when I changes *most rapidly* (high frequency)
  - Voltage = 0 when I is *constant* (low frequency)

RLC circuits

- Let's see what happens when we combine all these three components in a series:



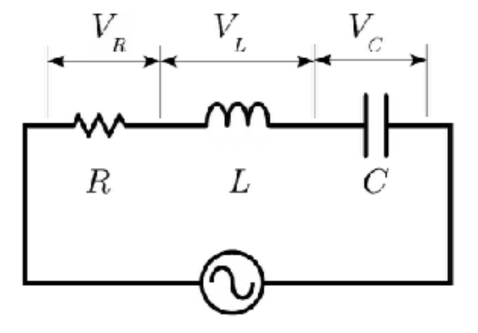
- From Kirchhoff's first law (loops):

$$
\begin{aligned}
V(t) =& V_R + V_L + V_C = IR + L\frac{dI}{dt} + \frac{Q}{C} \\
=& I_{\max}\left[R\sin\omega t + X_L\sin\left(\omega t + \frac{\pi}{2}\right) + X_C\sin\left(\omega t - \frac{\pi}{2}\right)\right] \\
=& I_{\max}\left[R\sin\omega t + (X_L - X_C)\cos\omega t\right]
\end{aligned}
$$

Use what we learned about inductors and capacitors a few slides ago

sin(ωt ± π/2) = ± cos(ωt)

RLC circuits: phase shift



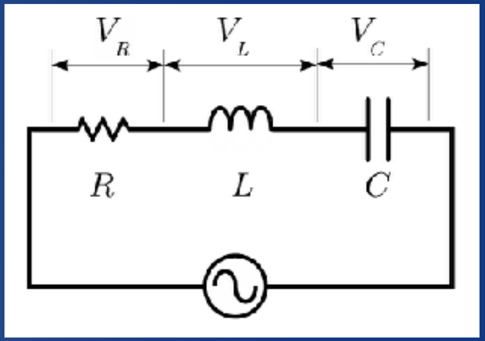
- After passing through the three components the voltage will have some phase shift
- Let's then impose to *V(t)* to look like:

$$\begin{aligned} V(t) &= V_{\max} \sin(\omega t + \phi) \\ &= V_{\max} \left[\sin(\omega t) \cos\phi + \cos(\omega t) \sin\phi\right] \end{aligned}$$

- Comparing with the equation from the previous slide it must necessarily be: $\begin{cases} V_{\max} \cos\phi = I_{\max} R \\ V_{\max} \sin\phi = I_{\max}(X_L - X_C) \end{cases}$
- And hence the **phase shift** is: $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{\omega L - 1/\omega C}{R}$
- The phase shift will depend both on the characteristics of the circuit (*R, C, L*) and on the **frequency** of the input signal!

RLC circuits: phase shift

- What about the maximum amplitude for the voltage?



- Let take again: $\begin{cases} V_{\max}\cos\phi = I_{\max}R \\ V_{\max}\sin\phi = I_{\max}(X_L - X_C) \end{cases}$
- Let's now square both equations and add them together:

$$\boxed{\begin{aligned} V_{\max} &= I_{\max}\sqrt{R^2 + (X_L - X_C)^2} \\ &= I_{\max} Z \end{aligned}}$$

- The quantity *Z* is called the **impedance** of the *RLC* circuit
- NOTE: the previous equation resembles very closely Ohm's law for resistors!
- This procedure can actually be generalized introducing the so-called phasor formalism

Resonant frequency

- So the whole RLC system has this peculiar frequency dependent "effective resistance". In particular:
  - <u>High-frequencies</u>: killed by the *inductor*
  - <u>Low-frequencies</u>: killed by the *capacitor*
- We therefore expect to have a particular frequency (ω_0) in the middle range that goes through the system almost untouched

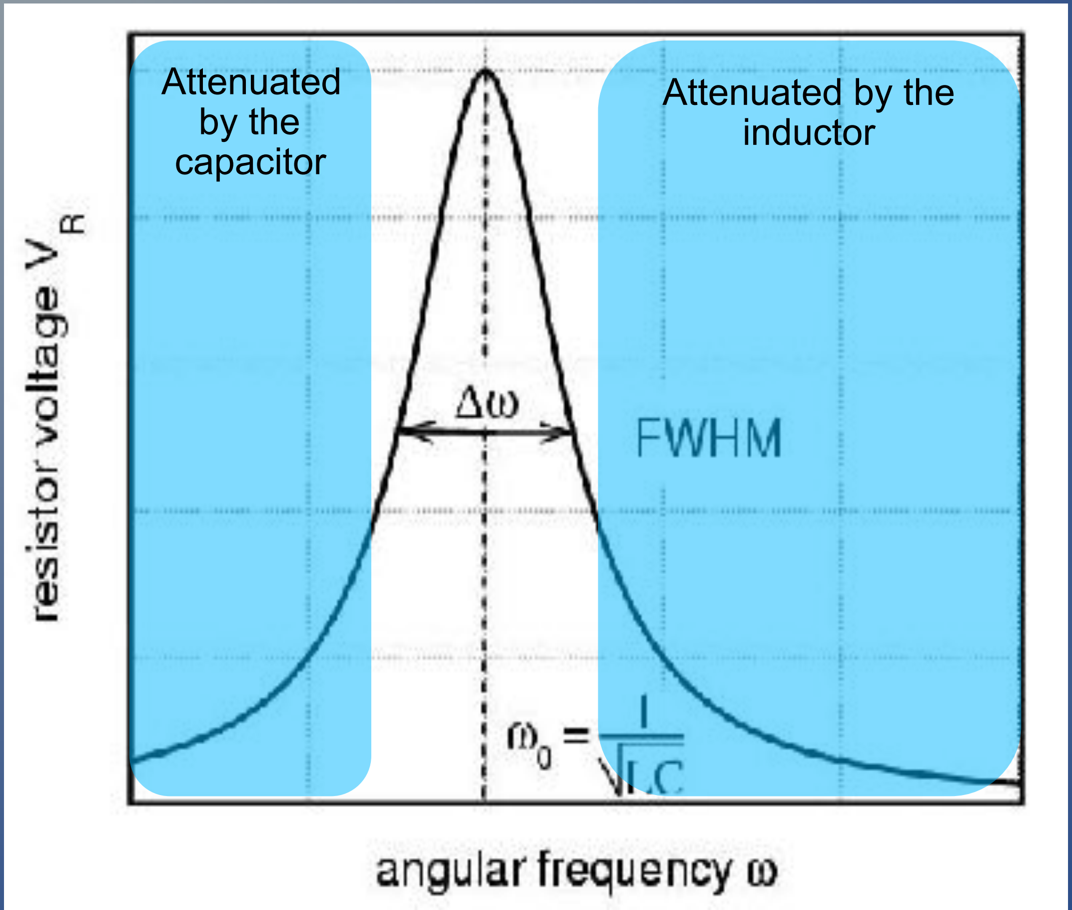
High Resonant Low

$$I_{\max} = \frac{V_{\max}}{Z} = \frac{V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

- For a given input voltage, the current in the circuit is maximum when Z is **minimum** *i.e.* when $X_L = X_C$.
- The ***resonant frequency*** is given by:

$$\omega_0 L = \frac{1}{\omega_0 C} \implies \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

Resonance



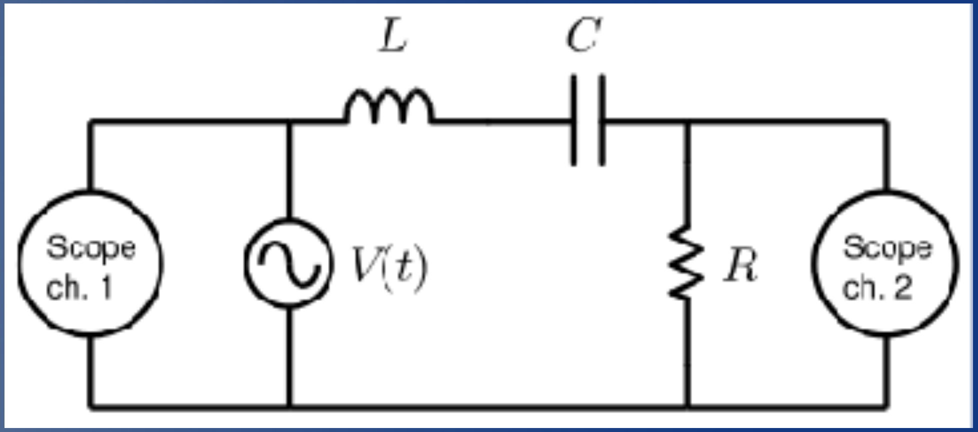
Terminology

FWHM: **<u>F</u>ull <u>W</u>idth** at **<u>H</u>alf <u>M</u>aximum**
Is the full width of the resonance peak at the point where its height is halfway between zero and the maximum.

Recall that the resistor voltage V_R is directly proportional to the magnitude of current.

The Experiment

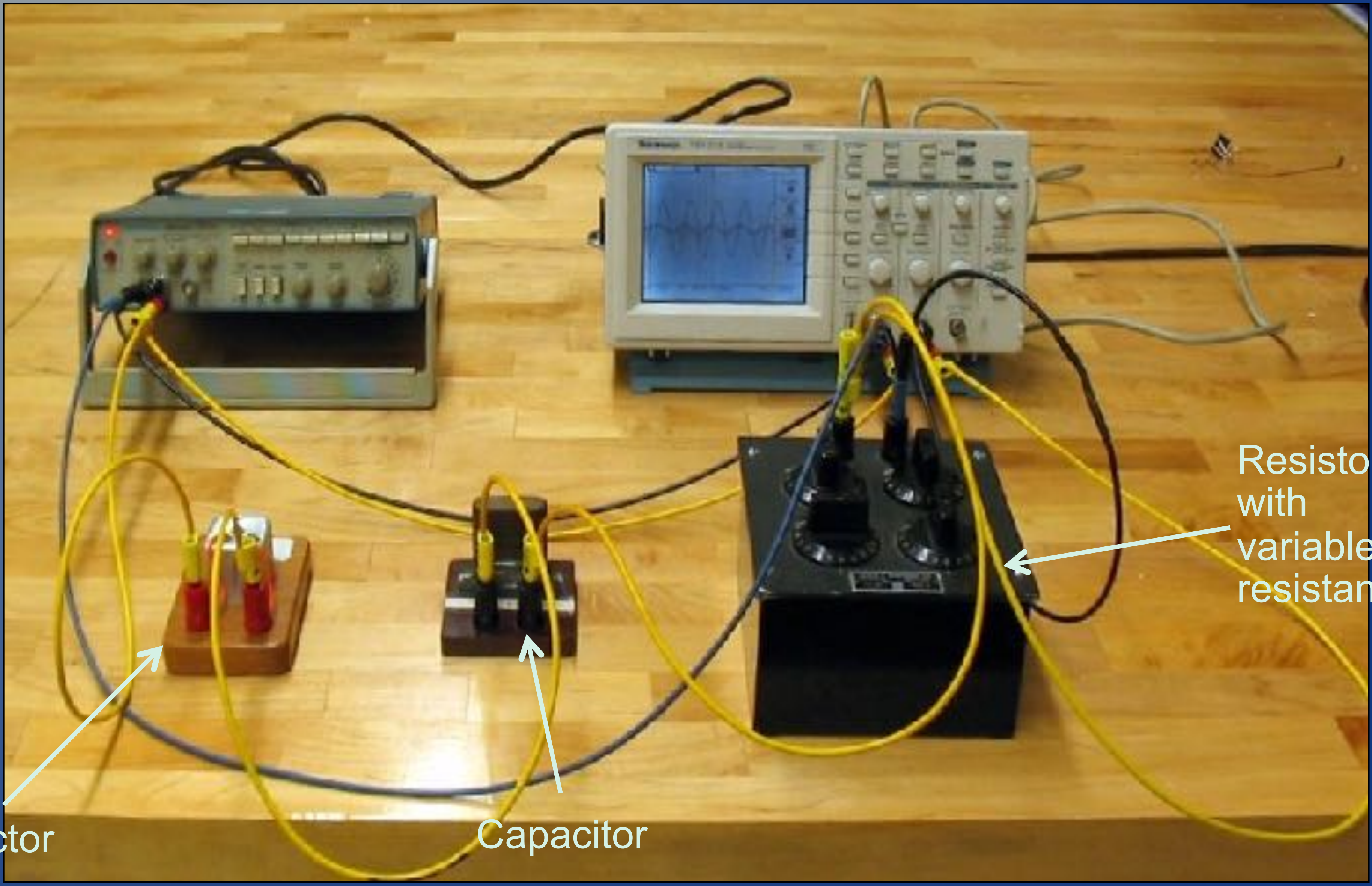
Main goals



- Resonance of RLC circuit:
  - Measure the resonant frequencies and FWHM for three known circuits
  - Compute the unknown inductance of a copper coil by finding the resonant frequency of the whole system

- Observe the phase shift, φ, between the driving signal and the three components (R, L and C) of the circuit
- Compare with expected value

Experimental setup

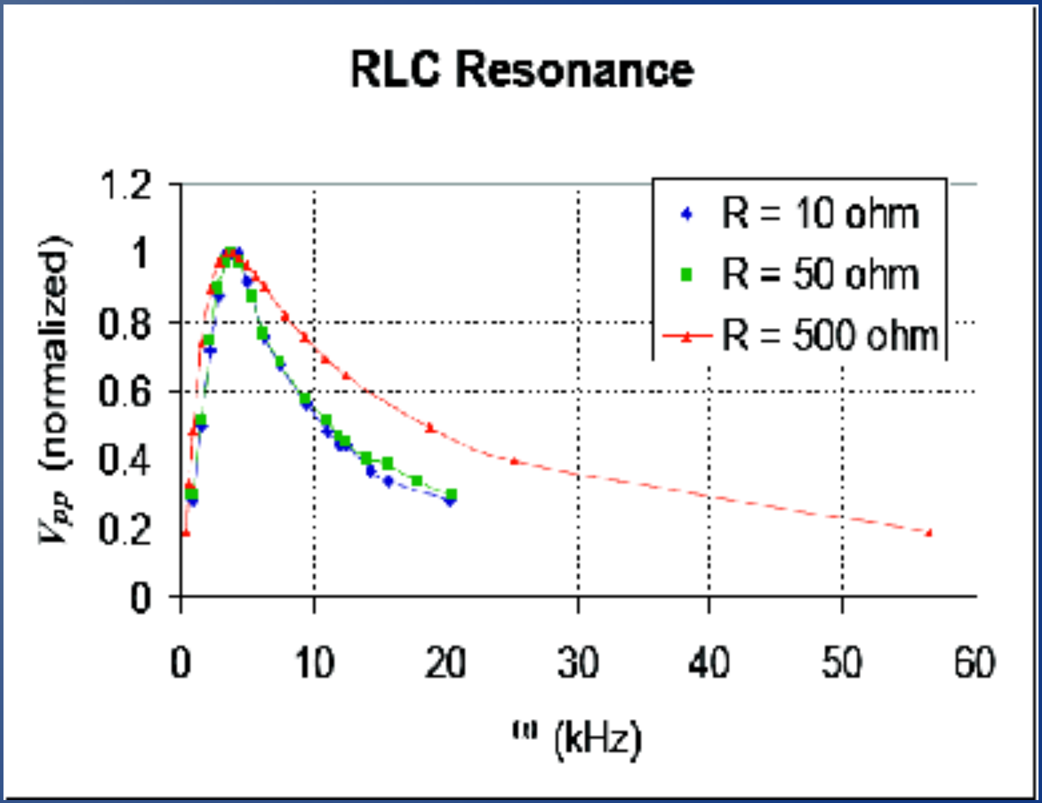


Experimental setup

- Recommendations:
  - Set the function generator peak-to-peak voltage to 20 V, the maximum allowed.
    - *There is a 0-2V / 0-20V selector button in addition to the voltage knob.*
  - Make sure the oscilloscope is set to trigger on channel 1, the function generator signal. You can do this by pressing the **TRIGGER** button and checking in the window menu that CH 1 is selected.
  - Use the **MEASURE** tools to observe peak-peak amplitudes, signal periods, and signal frequencies. Let the scope do the work for you!
  - Make sure that both peaks are in the viewable range of the scope!

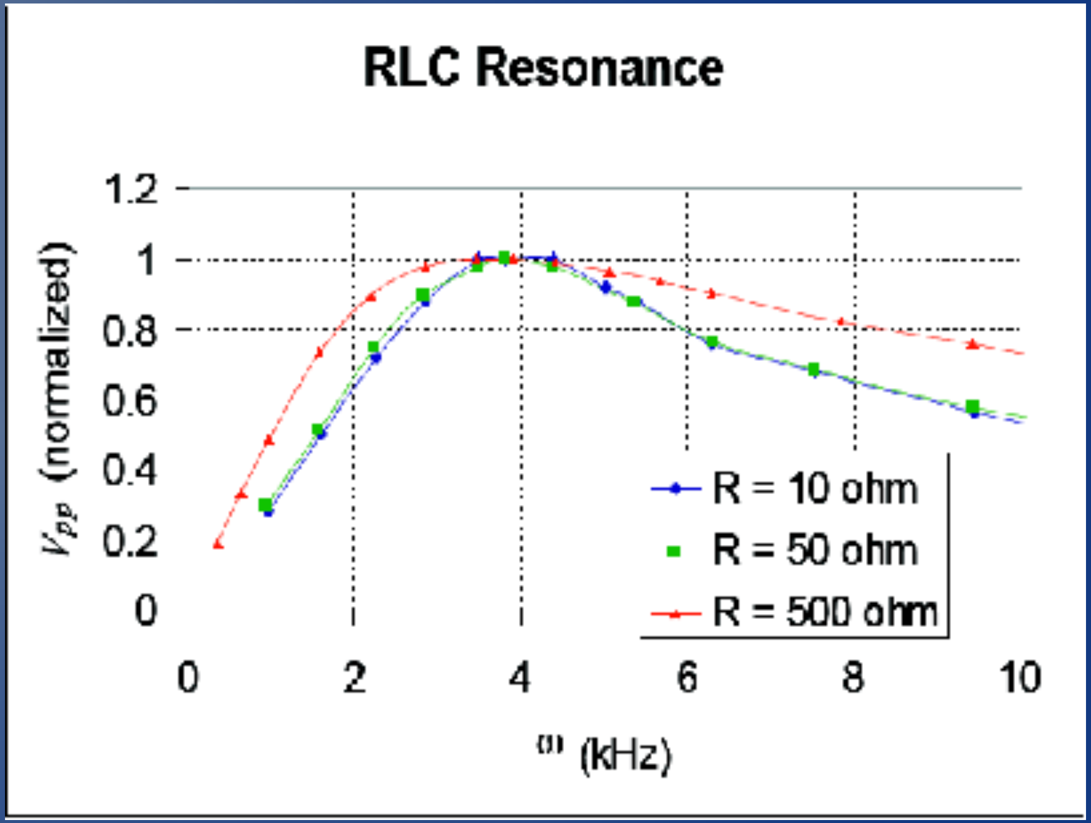
Resonance measurements

- Set the oscilloscope to look at the potential between the two ends of the resistor
- First localize ω_0 by looking at when the amplitude of the signal gets amplified
- Then, for about ***20 frequencies*** above and below ω_0, record peak-peak voltage across resistor
- Normalize your values such that the maximum is $V_{pp} = 1$
- Repeat for three values of the resistance *(10 Ω, 50 Ω, 500 Ω)*
- Plot V_{pp} *vs.* ω as shown in the figure



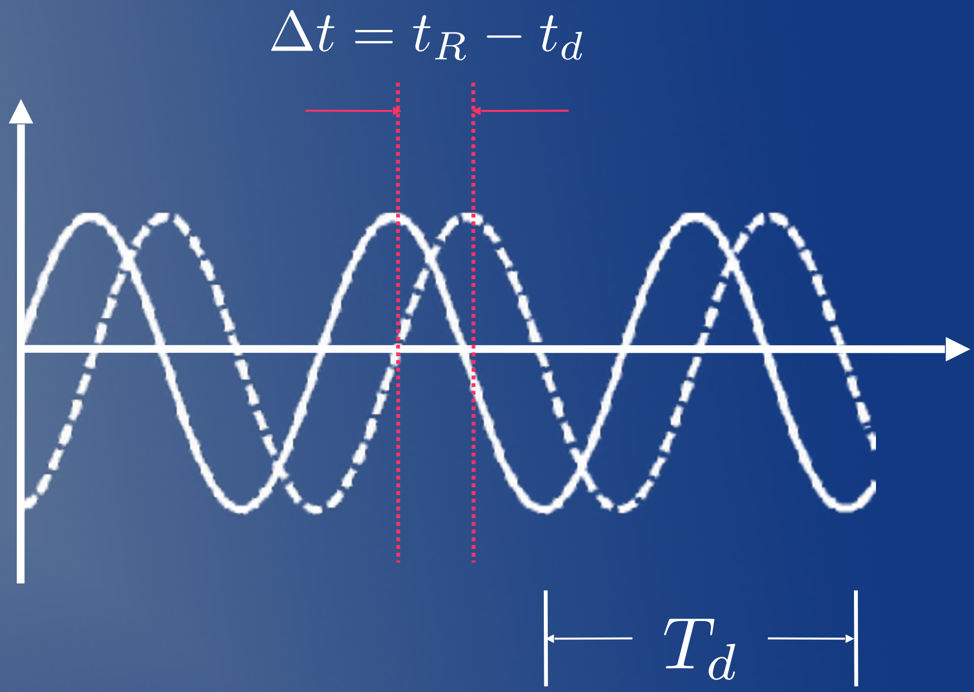
Resonance measurements

- Use the plots to determine the value of the ***resonant frequencies (with errors***!)
- Also use the plots to measure the ***FWHM*** of the three curves.
- <u>Compare the results from the three measurements</u>
- When you're finished with this part, <u>replace the inductor with the large copper coil</u>
- Repeat the previous measures and from the value of the average resonant frequency compute the inductance, *L*, of the wire



Phase shift measurement

- Replace the copper ring with the known capacitor again and set R = *30 Ω*
- Locate the resonant frequency and for 5 values at, above and below it measure the phase shift across the resistor
- Now increase the frequency well above the resonant one. This makes the inductor much more important than the capacitor. Measure φ
- Now make the capacitor more important by going way below the resonant frequency. Measure φ again



$$\phi = 2\pi \frac{t_R - t_d}{T_d}$$

<u>Compare the results obtained with the expected ones</u>

Tips

- Don't get confused! The frequency reported by the oscilloscope (in the MEASURE mode) is *f*. It is related to what we called frequency so far by $\omega = 2\pi f$.
- Once you manage to locate the maximum of the V_{pp} curve and you took 20 points above and below it, try to take more points right around you maximum. This will reduce your uncertainty on ω_0
- When plotting V_{pp} vs ω remember to take enough data to measure the FWHM. When R is large, the peak is very broad. Keep taking data until you pass half height
- The resistor is old but if you look carefully on each knob there is a label telling you how many Ω correspond to that knob.