Experiment 9: AC circuits

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INTRO TO EXPERIMENTAL PHYS-LAB
1493/1494/2699
Introduction

- **Last week (RC circuit):**
  - **Constant** Voltage power source (constant over time)

- **This week:**
  - A new component: the **inductor**
  - **Alternating Current (AC) circuits**
    - Time dependent voltage source
  - Leads to:
    - Time dependent currents (alternating currents)
    - Phase shifts in voltage and currents in components with respect to one another
    - Resonance
Varying electromagnetic fields

- The 2\textsuperscript{nd} and 4\textsuperscript{th} Maxwell equations in vacuum and with no sources read:
  \[
  \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
  \]
  \[
  \nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}
  \]

- Varying magnetic field generates electric field and vice versa!

- We therefore expect from a changing $B$ field to give rise to a current or potential difference.

- This is summarized by Faraday’s law:
  \[
  \mathcal{E} = -\frac{d\Phi_B}{dt}
  \]

- Change in magnetic flux creates an e.m.f.
  - Magnetic flux can be varied in two ways:
    1. Change the $B$ field
    2. Change the surface

Magnetic $B$ field

Flux = $\Phi_B = B \, A$

Surface ($A$)
Introducing the inductor

- It stores energy in form of magnetic fields (analogous to the capacitor).
- From Faraday’s law one deduces the expression for the potential difference at the two ends of the inductor:
  \[ V_L = -L \frac{dI}{dt} \]
- The inductor is only sensitive to the change in current! No change = no voltage.
- Negative sign indicates that the inductor opposes any change in current (Lenz's Law).
Why AC circuits?

- Sensitive to input **frequency** (i.e. function generator frequency)
- Serve as **signal frequency filters**:
  - High-frequency filters
  - Low-frequency filters
  - Band-pass filters
- Transformers
  - **Induction effects** - Ability to raise or lower the voltage amplitude.
- Generators and Motors
AC circuits: resistors

- AC circuits have an enormous range of applications. Here we cover the most important aspects.
- For this lab:
  - Consider only sources that vary \textit{sinusoidally}:

\[ I(t) = I_{\text{max}} \sin(\omega t) \]

- Simple example:
  - Function generator + resistor
- **Ohm’s Law**: voltage across the resistor is just \( V_R = IR \)

\[ V_R(t) = I(t)R = I_{\text{max}}R \sin(\omega t) \equiv V_{\text{max}} \sin(\omega t) \]
AC circuits: capacitors

- More interesting case: connect a capacitor to the AC voltage source
- Last time we saw that the voltage across a capacitor is given by:

\[ Q = CV_C \]

\[ \downarrow \]

\[ \int I(t) \, dt = CV_C \]

- Therefore, when the current is sinusoidal the voltage is given by:

\[ V_C(t) = \frac{1}{C} \int I(t) \, dt = \frac{1}{C} \int I_{max} \sin(\omega t) \, dt = -\frac{I_{max}}{\omega C} \cos(\omega t) \]
AC circuits: capacitors

- The voltage is sinusoidal:

\[
V_C(t) = -\frac{I_{\max}}{\omega C} \cos(\omega t) = -V_{\max} \cos(\omega t) = V_{\max} \sin\left(\omega t - \frac{\pi}{2}\right)
\]

- The extra \( \pi/2 \) in the expression is the phase of the voltage.
- Voltage across the capacitor lags behind the current by:

\[
\phi = -\pi/2
\]
AC circuits: inductors

- The voltage is still sinusoidal:

\[
V_L(t) = L \frac{dI}{dt} = \omega LI_{\text{max}} \cos(\omega t) = V_{\text{max}} \sin \left(\omega t + \frac{\pi}{2}\right)
\]

- Inductor voltage is also \textbf{phase shifted} w.r.t. current.

- Voltage across the inductor anticipates the current through it by:

\[
\phi = \pi / 2
\]
Voltage maxima: a closer look

- Given our expression for $V_R$, the maximum value of the voltage across the resistor is just given by Ohm’s Law:
  \[ V_R^{\text{max}} = I_{\text{max}} R \]

- The maximum voltage across the capacitor is a function of $\omega$:
  \[ V_C^{\text{max}} = \frac{1}{\omega C} I_{\text{max}} \equiv I_{\text{max}} X_C \]

  Given an oscillating input current the capacitor voltage is higher for small frequencies and lower for high frequencies.

- But, the maximum voltage across the inductor is also a function of the driving frequency:
  \[ V_L^{\text{max}} = \omega L I_{\text{max}} \equiv I_{\text{max}} X_L \]

  The inductor voltage is instead higher for large frequencies and lower for small ones.
Physical explanation: capacitors

**Question:** Why does the capacitor resist low-frequency signals more than high-frequency ones?

**Last time:** when charging/discharging the capacitor, the current – *the rate at which you can charge it* – decreases exponentially. *It becomes harder and harder to push in more charge as the capacitor fills up.*

![Graph showing current over time](image)

- Easy to charge = low reactance ($X_C$)
- Hard to charge = high reactance ($X_C$)
Physical explanation: capacitors

- **Rapidly varying signals** (high frequency) quickly charge/discharge capacitor before it fills with charge → low impedance.

- **Slowly varying signals** (low frequency) charge the capacitor to its limit, slowing down the rate: that is, decreasing the current!

- Now that we have introduced the language of reactances, you can think about the capacitor somehow as a resistor with $\omega$-dependent resistance.
Physical explanation: inductors

**Question:** Why does the inductor resist high-frequency signals more than low-frequency ones?

Think about the nature of an inductor: it is a coil of wire. If the current in the wire changes, then the magnetic flux through the coil changes → induction!

**Lenz’s Law:** a coil will oppose changes in magnetic flux. Self-induced EMF is:

\[
\mathcal{E} = N \frac{d\Phi_B}{dt} = L \frac{dI}{dt}
\]

- Rapidly varying signals strongly change the flux, so the inductor “pushes back” harder against the flow of current!
  - Voltage is maximum (and opposing) when \( I \) changes most rapidly (high frequency)
  - Voltage = 0 when \( I \) is constant (low frequency)

A life spent after a minus sign… and everyone always forgets about it!

(Heinrich Lenz)
RLC circuits

- Let’s see what happens when we combine all these three components in a series:

- From Kirchhoff’s first law (loops):

\[ V(t) = V_R + V_L + V_C = IR + L \frac{dI}{dt} + \frac{Q}{C} \]

\[ = I_{\text{max}} \left[ R \sin \omega t + X_L \sin \left( \omega t + \frac{\pi}{2} \right) + X_C \sin \left( \omega t - \frac{\pi}{2} \right) \right] \]

\[ = I_{\text{max}} \left[ R \sin \omega t + (X_L - X_C) \cos \omega t \right] \]

\[ \sin(\omega t \pm \pi/2) = \pm \cos(\omega t) \]

Use what we learned about inductors and capacitors a few slides ago.

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**RLC circuits: phase shift**

- After passing through the three components the voltage will have some phase shift.

- Let’s then impose to $V(t)$ to look like:

$$V(t) = V_{\text{max}} \sin(\omega t + \phi)$$

$$= V_{\text{max}} [\sin(\omega t) \cos \phi + \cos(\omega t) \sin \phi]$$

- Comparing with the equation from the previous slide it must necessarily be:

$$\begin{cases} 
V_{\text{max}} \cos \phi = I_{\text{max}}R \\
V_{\text{max}} \sin \phi = I_{\text{max}}(X_L - X_C)
\end{cases}$$

- And hence the **phase shift** is:

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$$

- The phase shift will depend both on the characteristics of the circuit ($R, C, L$) and on the **frequency** of the input signal!
**RLC circuits: phase shift**

- What about the maximum amplitude for the voltage?
- Let take again:
  \[
  \begin{align*}
  V_{\text{max}} \cos \phi &= I_{\text{max}} R \\
  V_{\text{max}} \sin \phi &= I_{\text{max}} (X_L - X_C)
  \end{align*}
  \]
- Let’s now square both equations and add them together:
  \[
  V_{\text{max}} = I_{\text{max}} \sqrt{R^2 + (X_L - X_C)^2} = I_{\text{max}} Z
  \]
- The quantity $Z$ is called the **impedance** of the RLC circuit.
- **NOTE:** the previous equation resembles very closely Ohm’s law for resistors!
- This procedure can actually be generalized introducing the so-called **phasor formalism**
Resonant frequency

- So the whole RLC system has this peculiar frequency dependent “effective resistance”. In particular:
  - **High-frequencies**: killed by the *inductor*
  - **Low-frequencies**: killed by the *capacitor*
- We therefore expect to have a particular frequency \( \omega_0 \) in the middle range that goes through the system almost untouched

\[
I_{\text{max}} = \frac{V_{\text{max}}}{Z} = \frac{V_{\text{max}}}{\sqrt{R^2 + (X_L - X_C)^2}}
\]

- For a given input voltage, the current in the circuit is maximum when \( Z \) is **minimum** i.e. when \( X_L = X_C \).
- The **resonant frequency** is given by:

\[
\omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}
\]
Resonance

Terminology

FWHM: **Full Width at Half Maximum**

Is the full width of the resonance peak at the point where its height is halfway between zero and the maximum.

Recall that the resistor voltage $V_R$ is directly proportional to the magnitude of current.
The Experiment
Main goals

- **Resonance** of RLC circuit:
  - Measure the resonant frequencies and FWHM for three known circuits
  - Compute the unknown inductance of a copper coil by finding the resonant frequency of the whole system

- Observe the **phase shift**, $\varphi$, between the driving signal and the three components (R, L and C) of the circuit
- Compare with expected value
Experimental setup

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Experimental setup

• Recommendations:
  • Set the function generator peak-to-peak voltage to 20 V, the maximum allowed.
    - There is a 0-2V / 0-20V selector button in addition to the voltage knob.
  • Make sure the oscilloscope is set to trigger on channel 1, the function generator signal. You can do this by pressing the TRIGGER button and checking in the window menu that CH 1 is selected.
  • Use the MEASURE tools to observe peak-peak amplitudes, signal periods, and signal frequencies. Let the scope do the work for you!
  • Make sure that both peaks are in the viewable range of the scope!
Resonance measurements

- Set the oscilloscope to look at the potential between the two ends of the resistor
- First localize $\omega_0$ by looking at when the amplitude of the signal gets amplified
- Then, for about 20 frequencies above and below $\omega_0$, record peak-peak voltage across resistor
- Normalize your values such that the maximum is $V_{pp} = 1$
- Repeat for three values of the resistance ($10 \, \Omega$, $50 \, \Omega$, $500 \, \Omega$)
- Plot $V_{pp}$ vs. $\omega$ as shown in the figure
Resonance measurements

- Use the plots to determine the value of the *resonant frequencies (with errors!)*
- Also use the plots to measure the *FWHM* of the three curves.
- Compare the results from the three measurements
- When you’re finished with this part, replace the inductor with the large copper coil
- Repeat the previous measures and from the value of the average resonant frequency compute the *inductance, $L$, of the wire*
Phase shift measurement

- Replace the copper ring with the known capacitor again and set $R = 30 \, \Omega$
- Locate the resonant frequency and for 5 values at, above and below it measure the phase shift across the resistor
- Now increase the frequency well above the resonant one. This makes the inductor much more important than the capacitor. Measure $\phi$
- Now make the capacitor more important by going way below the resonant frequency. Measure $\phi$ again

\[ \Delta t = t_R - t_d \]

\[ \phi = 2\pi \frac{t_R - t_d}{T_d} \]

Compare the results obtained with the expected ones
Tips

- Don’t get confused! The frequency reported by the oscilloscope (in the MEASURE mode) is $f$. It is related to what we called frequency so far by $\omega = 2\pi f$.

- Once you manage to locate the maximum of the $V_{pp}$ curve and you took 20 points above and below it, try to take more points right around you maximum. This will reduce your uncertainty on $\omega_0$.

- When plotting $V_{pp}$ vs $\omega$ remember to take enough data to measure the FWHM. When $R$ is large, the peak is very broad. Keep taking data until you pass half height.

- The resistor is old but if you look carefully on each knob there is a label telling you how many $\Omega$ correspond to that knob.