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1 Fiscal stimulus (Certification exam, 2009)

Consider an economy populated by a large number of infinitely-lived agents with preferences over private consumption, $c_t$, and hours of work, $h_t$, of the form:

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t U(c_t - \gamma h_t^\theta)$$

The period utility function is assumed to be an increasing and concave function in $c_t$ and $-h_t$. Households face the wage rate $w_t$ and supply labor to competitive labor markets. They have access to complete asset markets and must pay lump-sum taxes. Furthermore, they receive profits from the ownership of firms, denoted $\Pi_t$, which households take as exogenously given. Assume that output is produced with labor as the only factor input, according to:

$$y_t = h_t^\alpha$$

where $\alpha \in (0, 1]$. Households are subject to a borrowing limit that prevents them from engaging in Ponzi schemes. The government finances government consumption, denoted $g_t$, by lump-sum taxes. Assume that the government follows a balanced-budget rule whereby the primary fiscal deficit is zero. For simplicity, further assume that initial government liabilities are zero.

Note the three key elements in this description of the economy:

- Preferences are of the GHH form, so that we know that there are no wealth and intertemporal substitution effects on labour supply. We saw this in problem set 3.
- There is no capital in this economy, so no way to shift resources over time.
- The government runs a balanced budget and has zero initial debt, so that each period, government outlays must be equal to government revenue:

$$T_t = g_t$$

Furthermore, taxation is lump-sum, therefore non-distortionary.

1.1 Question (a)

Suppose in period $t$ there is a purely temporary one-percent increase in government spending. Find the response of output, wages and consumption in period $t$. Discuss the size of the government spending multiplier and the extend to which public consumption crowds out private consumption.
The household’s problem is:

\[
\max \ E_0 \sum_{t=0}^{+\infty} \beta^t U(c_t - \gamma h_t^\theta) \\
\text{s.t. } c_t + \mathbb{E}_t r_{t,t+1} d_{t+1} + T_t \leq d_t + w_t h_t + \pi_t \ [\lambda_t] \\
\lim_{j \to +\infty} \mathbb{E}_t r_{t,t+j+1} d_{t+1} \geq 0 \\
d_0 \text{ given}
\]

The Lagrangian of the problem is:

\[
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t (U(c_t - \gamma h_t^\theta) + \lambda_t (d_t + w_t h_t + \pi_t - c_t - \mathbb{E}_t r_{t,t+1} d_{t+1} - T_t))
\]

The set of sufficient first order conditions characterizing the optimal choice of the household are:

\[
\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0 \text{ a.s., } \frac{\partial \mathcal{L}}{\partial c_t} = 0, \frac{\partial \mathcal{L}}{\partial h_t} = 0, \frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \text{ and } \lambda_t \geq 0
\]

They can be rewritten as:

\[
\begin{align*}
    r_{t,t+1} \lambda_t &= \beta \lambda_{t+1} \\
    u'(c_t - \gamma h_t^\theta) &= \lambda_t \\
    -\gamma \theta \theta_T^{-1} u'(c_t - \gamma h_t^\theta) &= -\lambda_t w_t \\
    \pi_t + w_t h_t + d_t &= \mathbb{E}_t r_{t,t+1} d_{t+1} + c_t + T_t, \quad \lambda_t \geq 0
\end{align*}
\]

The firm’s period maximization problem is:

\[
\max_{h_t} h_t^\alpha - w_t h_t
\]

The sufficient first order condition to this problem is:

\[
w_t = \alpha h_t^{\alpha-1}
\]

which leads to profits:

\[
\pi_t = (1 - \alpha) h_t^\alpha
\]
The government’s budget constraint is:

\[ T_t = g_t \]

Finally, the market clearing condition for financial assets is:

\[ d_{t+1} = 0 \text{ a.s.} \]

By Walras’ law, the budget constraint of the household (substituting in the expression for profits obtained above) along with the market clearing condition for the labour and financial market is equivalent to the goods’ market clearing conditions, or resource constraint:

\[ c_t + g_t = h_t^\alpha \]

A rational expectations equilibrium of this economy is a set of 6 processes:

\[ \{c_t, h_t, r_{t,t+1}, \lambda_t, w_t, d_{t+1}\} \]

such that the following 7 conditions hold:

\[
\begin{align*}
    r_{t,t+1} \lambda_t & = \beta \lambda_{t+1} \quad \text{(Pricing kernel/ Euler equation)} \\
    u'(c_t - \gamma h_t^\alpha) & = \lambda_t \quad \text{(S.V. of income = MUC)} \\
    -\gamma \theta h_t^{\theta-1} u'(c_t - \gamma h_t^\gamma) & = -\lambda_t w_t \quad \text{(LS)} \\
    w_t & = \alpha h_t^{\alpha-1} \quad \text{(LD)} \\
    c_t + g_t & = h_t^\alpha \quad \text{(Resource constraint)} \\
    d_{t+1} & = 0 \quad \text{(Market clearing for asset markets)} \\
    \lim_{j \to +\infty} \mathbb{E}_t r_{t,t+j+1} d_{t+1} & = 0 \quad \text{(TVC)}
\end{align*}
\]

Note that this is a static economy. Indeed, the allocation is entirely determined by equilibrium on the labour market, and the resource constraint. To see this, note that by combining the FOC with respect to consumption and with respect to hours, we get that equilibrium in the labour market can be written as:

\[ \alpha h_t^{\alpha-1} = \gamma \theta h_t^{\theta-1} \]

So that:

\[ h_t = h = \left( \frac{\alpha}{\gamma \theta} \right)^{\frac{1}{\theta-\alpha}} \]
Using the resource constraint, this implies that:

\[ c_t + g_t = y_t = y = \left( \frac{a}{\gamma \theta} \right)^{\frac{\theta}{\gamma - \alpha}} \]

Therefore, the responses of output, wages and consumption to a change in current government spending are, respectively:

\[ \frac{\partial y_t}{\partial g_t} = 0 \]
\[ \frac{\partial w_t}{\partial g_t} = 0 \]
\[ \frac{\partial c_t}{\partial g_t} = -1 \]

In this economy, there is complete crowding out of private consumption by government expenditures, and the government multiplier is 0.

**How would your answer change if government spending remained unchanged but it was learned in period \( t \) that government spending will temporarily increase by one percent in period \( t + 1 \) ?**

Following the previous discussion, because this is a static economy, the announcement in period \( t \) of higher expenditures in period \( t + 1 \) has no effect in period \( t \). In period \( t + 1 \), the effects are identical to the ones describes in period \( t \).

**Provide intuition.**

In this simple economy, labour demand and labour supply are unaffected by spending decisions of the government, both because of the fact that the government runs a balanced budget (implying that any current change in expenditures must be financed with taxation in the same period) and because preferences are such that there are no wealth or intertemporal substitution effects on labour supply. Because labour is the only input into production, total output is unaffected by change in government spending. Therefore, there must be complete crowding out of private consumption for the resource constraint to hold.

If we relax the assumption of GHH preferences, then, since current expenditures have an effect on the current marginal utility of wealth, there may also be an effect of expenditures on hours, and therefore on output. However, the economy is still static - there is no way to shift resources over time. There would still be no response to news about future expenditures.
1.2 Question (b)

Now assume that the government no longer has access to lump-sum taxation and instead must finance its expenditures by means of a proportional sales tax paid by firms. In period $t$, tax revenue from the sales tax is given by:

$$ T_t = \tau_t y_t $$

Consider the effects of a fiscal stimulus packet taking the form of a purely temporary one-percent increase in government spending in period $t$ on output, hours, wages, consumption and the tax rate. Assume that $\alpha = 1$ and that $\theta(1 - s_g) < 1$, where $s_g$ denotes the share of government spending in output in the deterministic steady-state. Use a log-linear approximation to the equilibrium conditions to answer this question.

First, the equilibrium conditions are unchanged, except for (LD). The profit maximization problem of the firm is now:

$$ \max_{h_t} (1 - \tau_t)^{\alpha} h_t - w_t h_t $$

so that the labour demand curve is now:

$$ w_t = \alpha (1 - \tau_t)^{\alpha - 1} h_t $$

The log-linearized form of the labour demand curve is:

$$ \hat{w}_t = -\frac{\tau}{1 - \tau} \hat{h}_t + (\alpha - 1) \hat{h}_t $$

Second, the labour supply curve can be log-linearized as:

$$ \hat{w}_t = (\theta - 1) \hat{h}_t $$

Third, the government budget constraint is now:

$$ \tau_t y_t = g_t $$

In steady-state,

$$ \tau y_t = g = s_g y $$

so that:

$$ \tau = s_g $$

In log-linear form,

$$ \hat{\tau}_t = \hat{g}_t - \hat{y}_t $$
Fourth, the production function can be log-linearized as:

\[ \hat{y}_t = \alpha \hat{h}_t \]

Finally, the resource constraint can be log-linearized as:

\[ (1 - s_g)\hat{c}_t + s_g \hat{g}_t = \hat{y}_t \]

Equating the labour demand and labour supply schedules and using the fact that \( \alpha = 1 \), we get that:

\[ (\theta - 1)\hat{h}_t = -\frac{\tau}{1 - \tau} \hat{t}_t \]

Then, plugging in the fact that \( \hat{y}_t = \hat{h}_t \) and \( \hat{t}_t = \hat{g}_t - \hat{y}_t \), and re-arranging, we get that:

\[ \hat{y}_t = \frac{s_g}{1 - \theta(1 - s_g)} \hat{g}_t \]

The output multiplier is therefore positive and equal to:

\[ \frac{s_g}{1 - \theta(1 - s_g)} \]

Since \( y_t = h_t \), hours also increase as a response to the government spending shock. The wage response is:

\[ \hat{w}_t = (\theta - 1) \frac{s_g}{1 - \theta(1 - s_g)} \hat{g}_t \]

Again, the multiplier is positive, but not necessarily greater than 1.

Using the log-linearized resource constraint, we get:

\[ \hat{c}_t = \frac{\theta s_g}{1 - \theta(1 - s_g)} \hat{g}_t \]

The multiplier on consumption is positive, but not necessarily greater than 1.

Finally, the effect on the tax rate is given by:

\[ \hat{\tau}_t = \frac{(1 - s_g)(1 - \theta)}{1 - \theta(1 - s_g)} \]
For the labour curve to be downward sloping we need that $\theta > 1$, so that the equilibrium response of the tax rate is negative. This is consistent with the fact that the wage rate increases: the labour demand curve shifts up, in response to the decrease in the tax rate.

One way to understand the equilibrium response of the economy is to view the shock as an exogenous decrease in the tax rate. This decrease increases marginal revenue, and therefore marginal cost, for the profit-maximizing firms, so that labour demand shifts up. This implies an increase in hours worked. When $\theta(1 - s_g) = \theta(1 - \tau) < 1$, this increase in hours worked more than offsets the decrease in the wage rate, so that government expenditures actually increase (as a result of higher government revenue). This is a world of self-financing tax cuts.

**Discuss to what extent the behaviour of wages and consumption in response to a government spending shock is consistent with the existing empirical evidence from SVAR studies**

The positive response of both consumption and hours to a government spending shock is in line with the evidence discussed, for example, in Perotti (2007).

## 2 Countercyclical markups (midterm, 2009)

**In the data, it is observed that wages fail to move countercyclically even conditional on shocks other than productivity shocks**

### 2.1 Question (a)

**Explain why this empirical observatino poses a problem for competitive models of the business cycle that assume a concave, homogeneous of degree 1 production function in which capital and labor are the only factor inputs**

In a perfectly competitive model of the business cycle, the wage is set to be equal to the marginal product of labor:

$$w_t = A_t F_2(K_t, L_t) = A_t F_2(K_t, 1)$$

The labour supply schedule takes the form:

$$w_t = w(L_t, \lambda_t)$$

where $\lambda_t$ is the marginal utility of consumption. Consider a shock that does not affect productivity $A_t$. Since capital is fixed in the short-run, the labour demand curve does not shift. Therefore, any comovement in the
economy must occur along the labour demand curve. This implies necessarily that wages and hours, and therefore wages and output, move in opposite directions, contrary to what is observed in the data.

Suppose the baseline competitive business cycle is modified as follows. Assume that goods are supplied by monopolistically competitive firms, each facing a demand function of the type:

\[ a_{i,t} = p_{i,t}^{-\eta} a_t \]

where \( \eta > 1 \) denotes the price elasticity of demand, \( p_{i,t} \) the relative price charge by firm \( i \), and \( a_t \) denotes the level of aggregate demand. The production function is given by \( F(k_{i,t}, h_{i,t}) \), where \( F \) is increasing and homogeneous of degree 1, \( k_{i,t} \) denotes capital services rented by firm \( i \) in period \( t \) and \( h_{i,t} \) denotes the number of hours of labor hired by firm \( i \) in period \( t \). Assume that factor markets are perfectly competitive.

2.2 Question (b)

Find the labor demand schedule of firm \( i \)

Firm \( i \) solves the following problem:

\[
\max_{p_{i,t}, h_{i,t}, k_{i,t}} p_{i,t} a(p_{i,t}) - w_t h_{i,t} - u_t k_{i,t} \\
\text{s.t.} \quad F(k_{i,t}, h_{i,t}) \geq a_{i,t} \quad [mc_{i,t}]
\]

The Lagrangian of the problem is:

\[
L = p_{i,t} a(p_{i,t}) - w_t h_{i,t} - u_t k_{i,t} + mc_{i,t}(F(k_{i,t}, h_{i,t}) - a_{i,t})
\]

The sufficient first order conditions to the problem are:

\[
\frac{\partial L}{\partial p_{i,t}} = 0, \quad \frac{\partial L}{\partial h_{i,t}} = 0, \quad \frac{\partial L}{\partial k_{i,t}} = 0, \quad mc_{i,t} \frac{\partial L}{\partial mc_{i,t}} = 0, \quad mc_{i,t} \geq 0
\]

The first order condition with respect to \( p_{i,t} \) yields the pricing condition:

\[
p_{i,t} = \frac{\eta}{\eta - 1} mc_{i,t}
\]

As usual, under monopolistic pricing, price is a markup over marginal cost.
The first order conditions with respect to hours and capital yield:

\[ w_t^D = mc_{i,t} F_2(k_{i,t}, h_{i,t}) \]
\[ u_t^D = mc_{i,t} F_1(k_{i,t}, h_{i,t}) \]

Homogeneity of degree one of the production function then implies that the capital to labour ratio is identical across firms:

\[ \frac{k_{i,t}}{h_{i,t}} = \frac{k_t}{h_t} \]

In turn, this marginal cost, and therefore price are identical across firms:

\[ p_{i,t} = p_t, \quad mc_{i,t} = mc_t \]

The labour demand schedule of the firm is identical across \( i \) and is given by:

\[ w_t = mc_t A_t F_2 \left( \frac{k_t}{h_t}, 1 \right) \]

### 2.3 Question (c)

*Consider an equilibrium in which all firms charge the same price. Find the equilibrium markup of prices over the marginal cost.*

From the derivations above, we see that the markup of price over marginal cost is

\[ \frac{\eta}{\eta - 1} \]

### 2.4 Question (d)

*Does this model allow for the possibility that wages move procyclically in the absence of technology shocks?*

No, because the markup is time-invariant, so it cannot be countercyclical. Wage setting of the absence of a countercyclical mark-up will look like in question (a).
2.5 Question (e)

Now assume that the price elasticity of demand $\eta_t$ is time-varying, in particular, procyclical. The individual firm takes $\eta_t$ as exogenously given. Find the cyclicality of the equilibrium markup. Discuss whether this model affords the possibility that wages move procyclically even in the absence of productivity shocks.

The labour demand schedule becomes:

$$w_t = \frac{\eta_t - 1}{\eta_t} F_2(k_t, h_t, 1)$$

The markup, $\mu_t = \frac{m}{y-1}$, is now countercyclical.

In this case, suppose we have additively separable preferences in leisure and consumption. Then, $(LS)$ is given by:

$$w_t = f(h_t)$$

where $f$ is some increasing function of $h$.

If the markup fluctuates countercyclically, then in times of expansion, the markup decreases and the labour demand schedule shifts out. The labour demand schedule does not change. In this class of model, we may therefore see comovement in wages, hours and output, as the economy shifts along the LS curve.