Physics G6037 Professor Christ
due 9/29/2017

Problem Set #3

Be careful not to miss problems 10 and 11 on second sheet.

8. Using simple uncertainty principle arguments, determine:

   (a) The size of the hydrogen atom in terms of the electronic charge and mass.

   (b) The spread of α-particle energies observed in the decay:

   $$\mathrm{Po}^{213} \rightarrow \mathrm{Pb}^{209} + \alpha$$

   if the Po^{213} nucleus has a lifetime of 4×10^{-6} seconds.

   (c) The maximum length of time a pencil can be balanced on its point. (Assume the pencil has a perfect point and that it is no longer balanced when it makes a noticible angle, perhaps 10-15°, with the vertical.)

9. Consider a free particle of mass m moving in one dimension.

   (a) If at $t = 0$ the wave function $\psi(x)$ is the Gaussian

   $$\psi(x) = \frac{e^{-\frac{x^2}{4d^2}}}{d^{\frac{1}{2}}(2\pi)^{\frac{1}{4}}} e^{ip_0 x/\hbar}$$

   show that at a later time t the wave function becomes:

   $$\psi(x,t) = \frac{e^{-i\frac{p_0^2}{2m}t/\hbar}}{(1+\frac{i\hbar t}{2md^2})^{\frac{1}{2}}(d)^{\frac{1}{2}}(2\pi)^{\frac{1}{4}}} e^{ip_0 x/\hbar} \exp\Big(-\frac{(x-\frac{p_0}{m}t)^2}{4d^2(1+\frac{it\hbar}{2md^2})}\Big)$$

   (b) Define the Green's function for one-dimentional free particle motion by the expression

   $$G(x-x',t) = \langle x|e^{-iHt/\hbar}|x'\rangle,$$

   where $H = p^2/2m$. Show that $G(x,t)$ solves the equation:

   $$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}G(x,t) = i\hbar\frac{\partial}{\partial t}G(x,t)$$

   and obeys the initial condition $G(x,0) = \delta(x)$.

   (c) Show that given $\psi(x,t=0)$ we can use $G(x,t)$ to obtain $\psi(x,t)$ from

   $$\psi(x,t) = \int_{-\infty}^{\infty} dx' G(x-x',t)\psi(x',t=0)$$

   (d) Find the Green's function $G(x,t)$ as an explicit function of x and t. (Note: G can be obtained as an appropriately normalized, $d \rightarrow 0$ limit of the wave function studied in part (a)).

10. If $\Delta x \, \Delta p = \frac{\hbar}{2}$ for a state ψ, show that $\psi(x)$ must be a Gaussian wave function.

11. If the commutator $[A, B]$ itself commutes with both A and B, *i.e.* $[A, [A, B]] = [B, [A, B]] = 0$, show that

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]}.$$