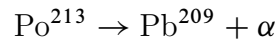


Problem Set #3

Be careful not to miss problems 10 and 11 on second sheet.

8. Using simple uncertainty principle arguments, determine:

- (a) The size of the hydrogen atom in terms of the electronic charge and mass.
- (b) The spread of α -particle energies observed in the decay:

if the Po^{213} nucleus has a lifetime of 4×10^{-6} seconds.

- (c) The maximum length of time a pencil can be balanced on its point. (Assume the pencil has a perfect point and that it is no longer balanced when it makes a noticeable angle, perhaps 10 - 15° , with the vertical.)

9. Consider a free particle of mass m moving in one dimension.

- (a) If at $t = 0$ the wave function $\psi(x)$ is the Gaussian

$$\psi(x) = \frac{e^{-\frac{x^2}{4d^2}}}{d^{\frac{1}{2}}(2\pi)^{\frac{1}{4}}} e^{ip_0x/\hbar}$$

show that at a later time t the wave function becomes:

$$\psi(x, t) = \frac{e^{-i\frac{p_0^2}{2m}t/\hbar}}{(1 + \frac{i\hbar t}{2md^2})^{\frac{1}{2}}(d)^{\frac{1}{2}}(2\pi)^{\frac{1}{4}}} e^{ip_0x/\hbar} \exp\left(-\frac{(x - \frac{p_0}{m}t)^2}{4d^2(1 + \frac{i\hbar t}{2md^2})}\right)$$

- (b) Define the Green's function for one-dimensional free particle motion by the expression

$$G(x - x', t) = \langle x | e^{-iHt/\hbar} | x' \rangle,$$

where $H = p^2/2m$. Show that $G(x, t)$ solves the equation:

$$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} G(x, t) = i\hbar \frac{\partial}{\partial t} G(x, t)$$

and obeys the initial condition $G(x, 0) = \delta(x)$.

- (c) Show that given $\psi(x, t = 0)$ we can use $G(x, t)$ to obtain $\psi(x, t)$ from

$$\psi(x, t) = \int_{-\infty}^{\infty} dx' G(x - x', t) \psi(x', t = 0)$$

- (d) Find the Green's function $G(x, t)$ as an explicit function of x and t . (Note: G can be obtained as an appropriately normalized, $d \rightarrow 0$ limit of the wave function studied in part (a)).

10. If $\Delta x \Delta p = \frac{\hbar}{2}$ for a state ψ , show that $\psi(x)$ must be a Gaussian wave function.
11. If the commutator $[A, B]$ itself commutes with both A and B , *i.e.* $[A, [A, B]] = [B, [A, B]] = 0$, show that

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A, B]}.$$