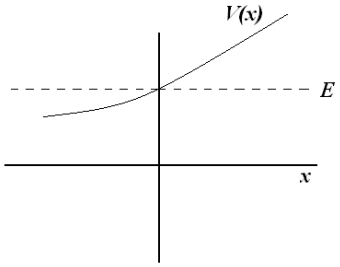
Physics G6037 Professor Christ
due 10/20/2017

Problem Set #6

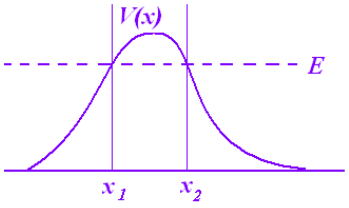
19. A particle of mass m and energy E moves in one dimension under the influence of the potential $V(x)$. If $V(0) = E$ and the particle's wavefunction has the approximate form:



$$\psi(x) = \frac{C}{[E - V(x)]^{\frac{1}{4}}} \cos\left\{\frac{1}{\hbar}\int_x^0 \sqrt{2m(E - V(x'))}dx' + \frac{\pi}{4}\right\}$$

for x large and negative. Find $\psi(x)$ for x large and positive assuming the WKB approximation to be valid both for x sufficiently small to permit a first order Taylor approximation to $V(x)$ about $x = 0$.

20. A particle with energy E and mass m is incident from the left on the potential barrier $V(x)$.



(a) What conditions as $x \to \pm\infty$ should be imposed on the energy eigenfunction $\psi_E(x)$ appropriate to this problem?

(b) Use the WKB approximation to obtain an expression for the probability of finding the particle to the right of the barrier at a much later time t for the case that E is less than the maximum value of V. Be careful to use only the information that is available from the known asymptotic form of the solution, avoiding the addition of elements of the solution which are not known without a more complete solution.

21. Find the three matrices $(J_i)_{mm'} = \langle j, m|J_i|j, m'\rangle$, $1 \le i \le 3$, explicitly for the case $j = 1$. Verify the commutation relation: $[J_1, J_2] = i\hbar J_3$.