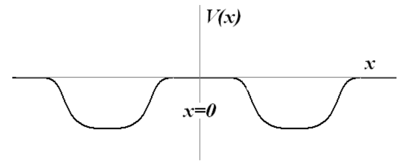
Physics G6037 Professor Christ

due 11/10/2017

Problem Set #8

26. Consider a one-dimensional potential $V(x)$ of the form shown, symmetrix about $x = 0$: $V(x) = V(-x)$.



(a) Prove that the eigenstates of the Hamiltonian

$$H = \frac{\partial^2}{\partial x^2}\frac{\hbar^2}{2m} + V(x)$$

can be chosen to obey $\psi(x) = \pm\psi(-x)$.

(b) If the two potential wells are far separated then the normalizable eigenstates are approximately the superpositions:

$$\psi_{n,\pm}(x) = \phi_n^L(x) \pm \phi_n^R(x)$$

where $\phi_n^L(x) = \phi_n^R(-x)$ is a normalizable eigenfunction for the potential

$$V'(x) = \begin{cases} V(x) & x < 0 \\ 0 & 0 < x. \end{cases}$$

Use the WKB approximation to find the difference in energy of $\psi_{n,+}$ and $\psi_{n,-}$ for the case of a large but finite separation between the wells.

27. A spin-1/2 particle with magnetic moment $\vec{\mu} = \gamma\vec{s}$ is placed in a magnetic field $\vec{B}_0 = B_0\hat{z}$ in the z-direction. Use eigenstates $|\pm\frac{1}{2}\rangle$ of s_z with eigenvalues $\pm\hbar/2$.

(a) For an initial state $|\psi(0)\rangle = a_+|+\frac{1}{2}\rangle + a_-|-\frac{1}{2}\rangle$, find the resulting state $|\psi(t)\rangle$ at a later time t.

(b) Next a time-dependent magnetic field

$$\vec{B}_1(t) = B_1\hat{x}\cos(\omega t) - B_1\hat{y}\sin(\omega t) \tag{1}$$

is added to the original field $\vec{B}_0$. Solve the resulting time-dependent Schrödinger equation for these combined magnetic fields by working in a "rotating" frame: deriving and solving the Schrödinger equation obeyed by the transformed state:

$$|\psi'(t)\rangle = e^{-i\omega t\hat{z}\cdot\vec{s}/\hbar}|\psi(t)\rangle. \tag{2}$$

[Note: while it offers insight to view this as a transformation to a rotating frame, Eq. (2) can be viewed as a simple mathematical transformation which makes the problem easy to solve without any need for physical intuition.]

(c) Describe the resulting motion of the particle's spin as ω is varied through the Larmor precession frequency $\omega_L = \gamma B_0$.