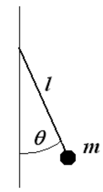
Physics GR6037 — Professor Christ

Final Examination (practice) — 12/15/2017

Answer each of the following **three (3)** questions. Please give a complete description of your method of solution since *partial credit* will be given.



1. A particle of mass m swings under the influence of gravity in a vertical plane at the end of a massless rod of length l attached to a fixed, frictionless pivot.

   (a) Write down the Hamiltonian for this system and determine its eigenvalues and describe its eigenstates in the approximation that the amplitude of oscillation is small. [13 points]

   (b) Calculate the first order corrections to the energy eigenvalues coming from this small amplitude approximation. [20 points]

2. Two particles, each with mass m and positions $\vec{r}_1$ and $\vec{r}_2$ move in a 3-dimensional harmonic oscillator potential $V(\vec{r}_1, \vec{r}_2) = \frac{1}{2}m\omega^2(\vec{r}_1^{\,2} + \vec{r}_2^{\,2})$.

   (a) Write down the allowed energies and describe the corresponding eigenstates of this system. [5 points]

   (b) If each particle carries a charge q and they interact through the Coulomb potential $q^2/|\vec{r}_1 - \vec{r}_2|$ find the shift in the ground state energy caused by this electrostatic repulsion to order q^2. (The formula $\int_{-\infty}^{\infty} dz\, e^{-z^2} = \pi$ may be useful when evaluating your result.) [16 points]

   (c) Improve on your result from part (b) by using the variational method with the trial wave function:
   $$Ne^{-\alpha \frac{m\omega}{2\hbar}(\vec{r}_1^{\,2} + \vec{r}_2^{\,2})}.$$
   Find the equations which determines the parameter α and solve it to first order in q^2. Compare the resulting first-order energy with that obtained in part (b). [12 points]

3. A particle of mass m moving in a square, 2-dimensional box of side L obeys periodic boundary conditions at the walls. A small perturbing potential energy $V(x, y) = \lambda \sin[2\pi(x - y)/L]$ also affects the particle.

   (a) Write down the energy eigenvalues and eigenstates for this problem to zeroth order in λ. [5 points]

   (b) Find the changes in the eigenvalues through first order in λ and their eigenstates to zeroth order in λ for the five states with the lowest unperturbed energies. (Be careful of degeneracies.) [15 points]

   (c) Find all eigenvalues through first order in λ and their corresonding eigenstates through zeroth order in λ. [14 points]