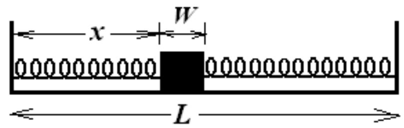
Physics G6037 | Professor Christ
Midterm Examination (practice) | 10/27/2017

Answer **two (2)** of the following three questions. Please give a complete description of your method of solution since *partial credit* will be given.

1. A quantum mechanical block of mass M, width W and located by the variable x slides without friction on a horizonal surface. It is attached by two identical massless springs of spring constant k and equilibrium length l to a left and right wall separated by a distance L. The block is constrained to move in one dimension parallel to the springs and oscillates about its equilibrium position at the center.



   (a) Write down the quantum Hamiltonian for this system as well as the corresponding energy eigenvalues. What is the wave function $\psi_0(x)$ for the ground state? [15 points]

   (b) At $t = 0$ the right hand the spring is suddenly broken. What is the new Hamiltonian for the system, the new energy eigenvalues and the wave function $\psi_0'(x)$ for the new ground state? [15 points]

   (c) Assume that for $t \leq 0$ the system is in its ground state and that at $t = 0$ the right hand spring is broken so quickly that the quantum state of the system remains unchanged after the spring is broken. Find the probability that block will be found in the ground state of the new Hamiltonian. [20 points]

2. A particle with spin one and magnetic moment $\vec{\mu} = \gamma\vec{S}$ is initially polarized in the $+z$ direction.

   (a) Express this initial state as a linear combination of the three eigenstates $|s = 1, m_s\rangle$ of $\vec{S}^2$ and S_z with eigenvalues $\hbar^2 s(s+1)$ and $\hbar m_s$. [2 points]

   (b) A magnetic field $\vec{B}$ is imposed pointing in the $+x$ direction. Write down the Hamiltonian for the system. [5 points]

   (c) Find the resulting state after a time t has elapsed. (Hint, representing the spin 1 state using a basis of unit vectors $\hat{e}_x$, $\hat{e}_y$, and $\hat{e}_z$ may allow you to more easily perform the rotation.) [23 points]

   (d) What are the probabililies for measuring S_z at the time t and getting each of the values $+\hbar$, 0 and $-\hbar$? [20 points]

3. An electrically neutral, spin-1/2 particle of mass m, position $\vec{r}$, spin angular momentum $\vec{s}$ and magnetic moment $\vec{\mu} = \gamma\vec{s}$ moves in a three-dimensional simple harmonic potential $V(\vec{r}) = \frac{1}{2}m\omega^2\vec{r}^2$.

   (a) Describe the energy eigenstates and write down the energy eigenvalues for this system. [10 points]

   (b) Determine the energy eigenvalues if a uniform magnetic field $\vec{B} = B\hat{z}$ is introduced, where $\hat{z}$ is a unit vector in the z direction? [10 points]

   (c) If instead the field is inhomogenous, with a strength which depends linearly on x, $\vec{B} = \kappa x\hat{z}$, determine the new energy eigenstates and eigenvalues. [10 points]

   (d) For this case of an inhomogenous magnetic field, the ground state is doubly degenerate. Find the particular ground state with $\langle\vec{s}_z\rangle = \langle\vec{s}_y\rangle = 0$. [5 points]

   (e) Calculate $\langle\vec{s}_x\rangle$ for this state. [15 points]