Physics GR6037

Professor Christ
due 11/30/2018

Problem Set #10

32. Find the first and second order effects of the perturbing potential $V(x) = \lambda x$ on the spectrum of the one-dimensional harmonic oscillator with the unperturbed Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

using the standard formalism of perturbation theory. Show that your results agree with the exact solution to the perturbed problem obtained by using a shifted position variable.

33. Consider the Hamiltonian $H = H_0 + \lambda V$ and assume that the spectrum of H_0 is non-degenerate.

   (a) Use Rayleigh-Schrödinger perturbation theory to determine the eigenvalues of H to third order in λ in terms of $E_n^{(0)}$ and $V_{n',n}$ where

   $$H_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle, \quad \langle\phi_{n'}|V|\phi_n\rangle = V_{n',n}$$

   (b) Show that your result is consistent with what you would obtain using Wigner-Brillouin perturbation theory.

34. A particle of mass m moves on the surface of a sphere of radius R under the influence of a uniform gravitational field near the Earth's surface. Assume that the sphere is sufficiently small that $\lambda = m^2 g R^3/\hbar^2$ can be treated as a small expansion parameter.

   (a) What are the two lowest eigenvalues and the corresponding eigenfunctions to zeroth order in λ?

   (b) What are the corrections to each of these two eigenvalues to first order in λ?

   (c) What are the corrections to second order in λ? (Note: rules for decomposing the products of spherical harmonics limit the possible intermediate states.)

35. Consider a potential function $V(x)$ which is everywhere finite and which obeys:

$$V(x) \leq 0 \text{ for all } x \quad \text{and} \quad V(x) = 0 \text{ for } |x| > x_0.$$

Show using the variational method that:

   (a) The 1-dimensional potential $\lambda^2 V(x)$ for $-\infty < x < +\infty$ always possesses at least one bound state for arbitrarily small λ.

   (b) The 3-dimensional, spherically symmetric potential $\lambda^2 V(|\vec{r}|)$ possesses no bound state if λ is made sufficiently small.

Hint: A good strategy is to compare to the known solutions for the case that $V(x)$ or $V(|\vec{r}|)$ is an appropriately chosen square well potential.