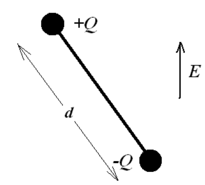
Physics GR6037 Professor Christ

due 12/10/2018

Problem Set #11

36. Let E_l be the lowest energy eigenvalue for all normalizable eigenstates with orbital angular momentum l, in a spherically symmetric potential $V(|\vec{r}|)$. Prove that if $l > l'$ then $E_l > E_{l'}$.

37. A rod of length d with point charges $+Q$ and $-Q$ at either end is constrained to rotate in the $x-y$ plane about its center. In addition there is a uniform electric field $\vec{E}$ parallel to the y-direction and the system has a moment of inertia I about its center.



   (a) What is the Hamiltonian for the system?
   (b) If the interaction with the electric field is treated as a perturbation what are the zeroth order energies?
   (c) The first order energies
   (d) The second order energies?

38. Consider the 2-dimension harmonic oscillator with Hamiltonian

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2).$$

   (a) Determine the energy eigenvalues and corresponding energy eigenstates.
   (b) If an extra perturbing term, λxy is added to the energy, use degenerate perturbation theory to find the first-order shift in the energies of the six lowest energy states.
   (c) Solve the problem exactly for this case.
   (d) If instead an extra perturbing term, $\lambda x^2 y^2$ is added to the energy, use degenerate perturbation theory to again find the first-order shift in the energies of the six lowest states.

39. Find the Clebsch-Gordon coefficients $(j, m|j_1, m_1; j_2, m_2)$ for

   (a) $j_1 = 1/2$, $j_2 = 1$
   (b) $j_1 = 1$, $j_2 = 1$
   (c) $j_1 = 1/2$, $j_2 =$ arbitrary.

40. Two spin-1 particles interact through the Hamiltonian

$$H = g\vec{S}_1 \cdot \vec{S}_2$$

   (a) What are the allowed energies and corresponding eigenstates for this Hamiltonian? Express the eigenstates in terms of the standard basis of states with definite values of $(S_1)_z$ and $(S_2)_z$.
   (b) If initially the system is in the state with $(S_1)_z = (S_2)_z = 0$, what is the probability of finding $(S_1)_z = \hbar$ at a later time t?