Physics G6037 Professor Christ

due 9/28/2018

Problem Set #3

7. Prove the Schwartz inequality:

$$|\langle A|B\rangle|^2 \leq \langle A|A\rangle\langle B|B\rangle.$$

[Hint: you might use a method similar to our proof of the uncertainty relation from the commutator $[x, p] = i\hbar$, examining the norm of the vector $|A\rangle + \alpha|B\rangle$ as a function of the variable α.]

8. Using simple uncertainty principle arguments, determine:

   (a) The size of the hydrogen atom in terms of the electronic charge and mass.

   (b) The spread of α-particle energies observed in the decay:

$$\mathrm{Po}^{213} \rightarrow \mathrm{Pb}^{209} + \alpha$$

   if the Po^{213} nucleus has a lifetime of 4×10^{-6} seconds.

   (c) The maximum length of time a pencil can be balanced on its point. (Assume the pencil has a perfect point and that it is no longer balanced when it makes a noticible angle, perhaps 10-15°, with the vertical.)

9. If $\Delta x\, \Delta p = \frac{\hbar}{2}$ for a state ψ, show that $\psi(x)$ must be a Gaussian wave function.

10. If the commutator $[A, B]$ itself commutes with both A and B, *i.e.* $[A, [A, B]] = [B, [A, B]] = 0$, show that

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]}.$$