

Problem Set #3

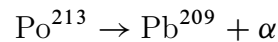
7. Prove the Schwartz inequality:

$$|\langle A|B \rangle|^2 \leq \langle A|A \rangle \langle B|B \rangle.$$

[Hint: you might use a method similar to our proof of the uncertainty relation from the commutator $[x, p] = i\hbar$, examining the norm of the vector $|A\rangle + \alpha|B\rangle$ as a function of the variable α .]

8. Using simple uncertainty principle arguments, determine:

- (a) The size of the hydrogen atom in terms of the electronic charge and mass.
- (b) The spread of α -particle energies observed in the decay:



if the Po^{213} nucleus has a lifetime of 4×10^{-6} seconds.

- (c) The maximum length of time a pencil can be balanced on its point. (Assume the pencil has a perfect point and that it is no longer balanced when it makes a noticeable angle, perhaps 10 - 15° , with the vertical.)
9. If $\Delta x \Delta p = \frac{\hbar}{2}$ for a state ψ , show that $\psi(x)$ must be a Gaussian wave function.
10. If the commutator $[A, B]$ itself commutes with both A and B , *i.e.* $[A, [A, B]] = [B, [A, B]] = 0$, show that

$$e^A e^B = e^{A+B} e^{\frac{1}{2}[A, B]}.$$