Physics GR6037 Professor Christ

due 10/12/2018

Problem Set #5

16. A particle of mass m moves in one dimension under the influence of the potential $\frac{1}{2}m\omega^2 x_{op}^2$ where x_{op} is the particle's position operator.

    (a) Show that the operator $\exp\{i\Delta p\, x_{op}/\hbar\}$ increases the particle's momentum by the amount Δp.

    (b) The particle, initially in the ground state, undergoes a sudden collision with a second particle and its momentum is increased by Δp. Assuming that the effect of this collision can be described by an application of the operator $\exp\{i\Delta p\, x_{op}/\hbar\}$, what is the probability that the particle remains in the ground state?

    (c) What is the probability that it is raised to the n^{th} excited state?

17. A charged, spinless particle moves in a uniform magnetic field $\vec{B}$ pointing in the z-direction. Use the gauge choice $A_x = A_z = 0$, $A_y = |\vec{B}|x$.

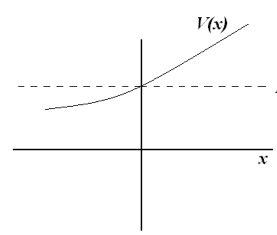
    (a) Find the allowed energies and corresponding eigenstates for this system. Note, these energy eigenstates can be chosen also be eigenstates of p_y.

    (b) Use your results to construct a time-dependent state which solves the Schrodinger equation and for which the expectation values of the position operators $x_{\rm op}$ and $y_{\rm op}$ approximate the usual, classical circular motion. One possible approach:

        i. Create a SHO coherent state with fixed p_y.

        ii. Take a Gaussian superposition of states with different p_y.

        iii. Find the location where the wave function is largest as a function of time using the stationary phase approximation.



18. A particle of mass m and energy E moves in one dimension under the influence of the potential $V(x)$. If $V(0) = E$ and the particle's wavefunction has the approximate form:

$$\psi(x) = \frac{C}{[E - V(x)]^{\frac{1}{4}}} \cos\left\{\frac{1}{\hbar}\int_x^0 \sqrt{2m(E - V(x'))}dx' + \frac{\pi}{4}\right\}$$

for x large and negative. Find $\psi(x)$ for x large and positive assuming the WKB approximation to be valid both for x sufficiently small to permit a first order Taylor aproximation to $V(x)$ about $x = 0$.

19. Consider a one-dimensional simple harmonic oscillator with lowering operator

$$a = \sqrt{\frac{m\omega}{2\hbar}}x + \frac{1}{\sqrt{2m\omega\hbar}}ip.$$

Define the hermitian phase operator ϕ_{op} by:

$$e^{-i\phi_{op}} = \frac{1}{\sqrt{1 + a^\dagger a}}a$$

(a) Show that $e^{-i\phi_{op}}|n\rangle = |n-1\rangle$ where $a^\dagger a|n\rangle = n|n\rangle$.

(b) Show that $e^{i\phi_{op}}|n\rangle = \left(e^{-i\phi_{op}}\right)^\dagger |n\rangle = |n+1\rangle$.

(c) Show that the phase operator ϕ_{op} and the number operator $N_{op} = a^\dagger a$ are conjugate in the sense that:

$$[N_{op}, e^{-i\phi_{op}}] = -e^{-i\phi_{op}},$$

so that N_{op} acts like $-i\frac{\partial}{\partial\phi}$.

(d) Construct the eigenstates $|\phi_0\rangle$ of the hermitian operator

$$\begin{aligned} \sin(\phi_{op}) &= \frac{1}{2i}\left(e^{+i\phi_{op}} - e^{-i\phi_{op}}\right), \\ \sin(\phi_{op})|\phi_0\rangle &= \sin(\phi_0)|\phi_0\rangle \end{aligned} \quad (1)$$

which satisfy the normalization condition

$$\langle\phi'|\phi_0\rangle = \delta(\phi' - \phi_0).$$

where $-\pi/2 < \phi_0 \leq \pi/2$ and the eigenstates $|\phi_0\rangle$ should be expressed in terms of the usual energy eigenstates $|n\rangle$:

$$|\phi_0\rangle = \sum_{n=0}^{\infty} c_n(\phi_0)|n\rangle.$$

(e) Find the eigenstates and eigenvalues of the operator

$$\cos(\phi_{op}) = \frac{1}{2}\left(e^{+i\phi_{op}} + e^{-i\phi_{op}}\right).$$